Coupled IPS/NIMROD/GENRAY simulations of ECCD-induced tearing mode stabilization

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Equations of the coupled RF/MHD system

- RF effects enter the kinetic equation as a quasilinear operator:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)$$

- Velocity moments ➔ two-fluid model ➔ MHD equations; RF terms contribute momentum, energy, and current. Calculate using data from GENRAY (RF code).

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \mathbf{J}\times\mathbf{B} - \nabla\cdot\mathbf{\Pi} + \sum_\alpha \mathbf{F}^{rf}_{\alpha 0}$$

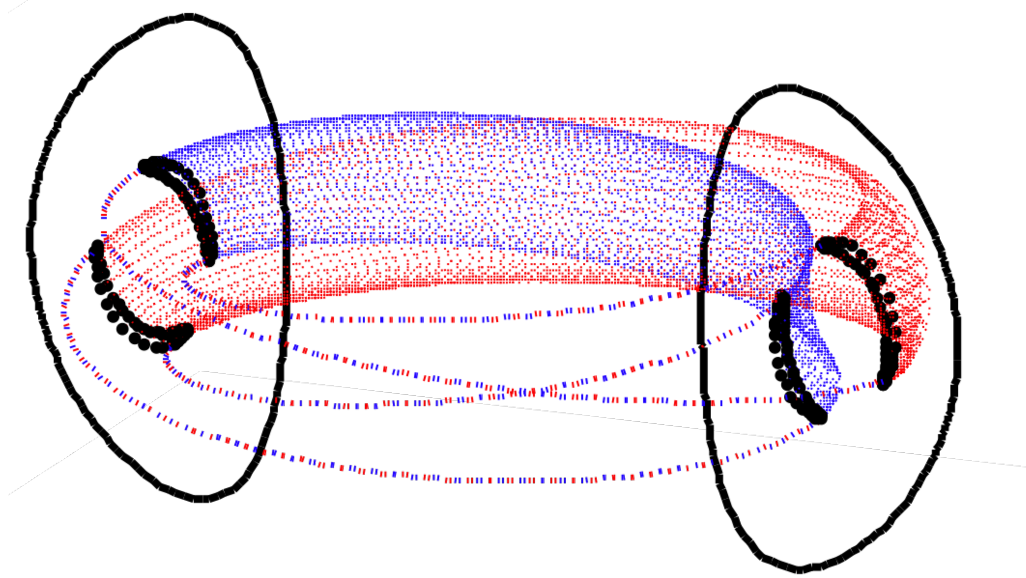
$$\mathbf{F}^{rf}_{\alpha 0} \equiv \int m_\alpha \mathbf{v} Q(f_\alpha) d\mathbf{v}$$

$$\frac{3}{2}n\left(\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T\right) + p\nabla\cdot\mathbf{u} = -\nabla\cdot\mathbf{q} - \mathbf{\Pi}:\nabla\mathbf{u} + Q + \sum_\alpha S^{rf}_{\alpha 0}$$

$$S^{rf}_{\alpha 0} \equiv \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha) d\mathbf{v}$$

$$\mathbf{E} + \mathbf{u}\times\mathbf{B} = \eta\mathbf{J} + \frac{\mathbf{F}^{rf}_{e0}}{n|q_e|}$$

- RF terms are highly localized. We want to drive current at the island O-point (helical symmetry) via electron cyclotron resonance (cylindrical symmetry).



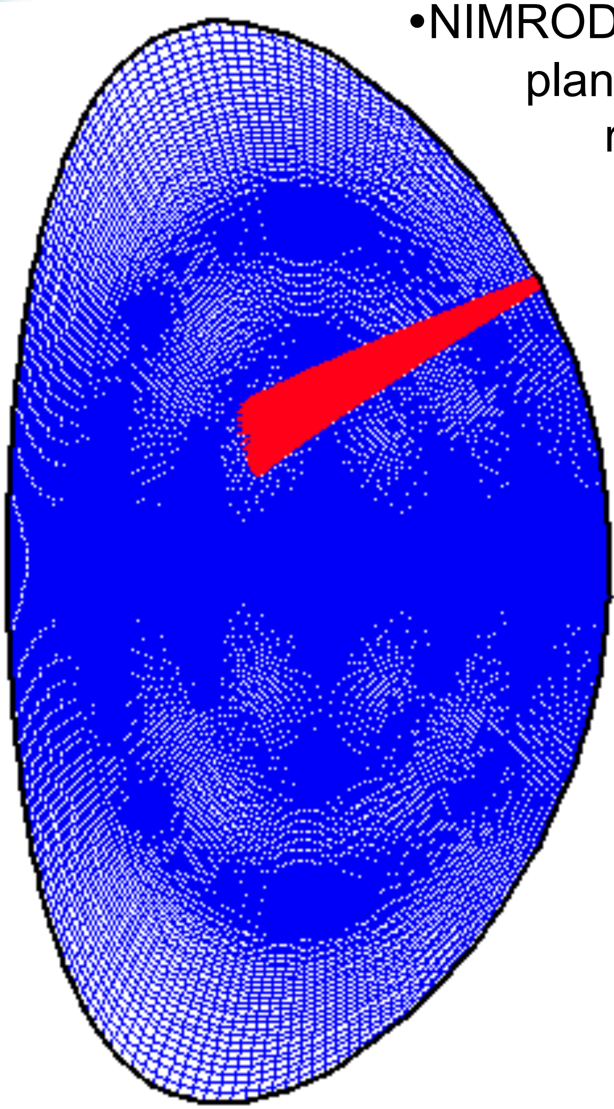
Scale lengths of RF deposition – GENRAY and NIMROD

- NIMROD uses finite elements in the R-Z plane, and a discrete Fourier representation in the toroidal direction.

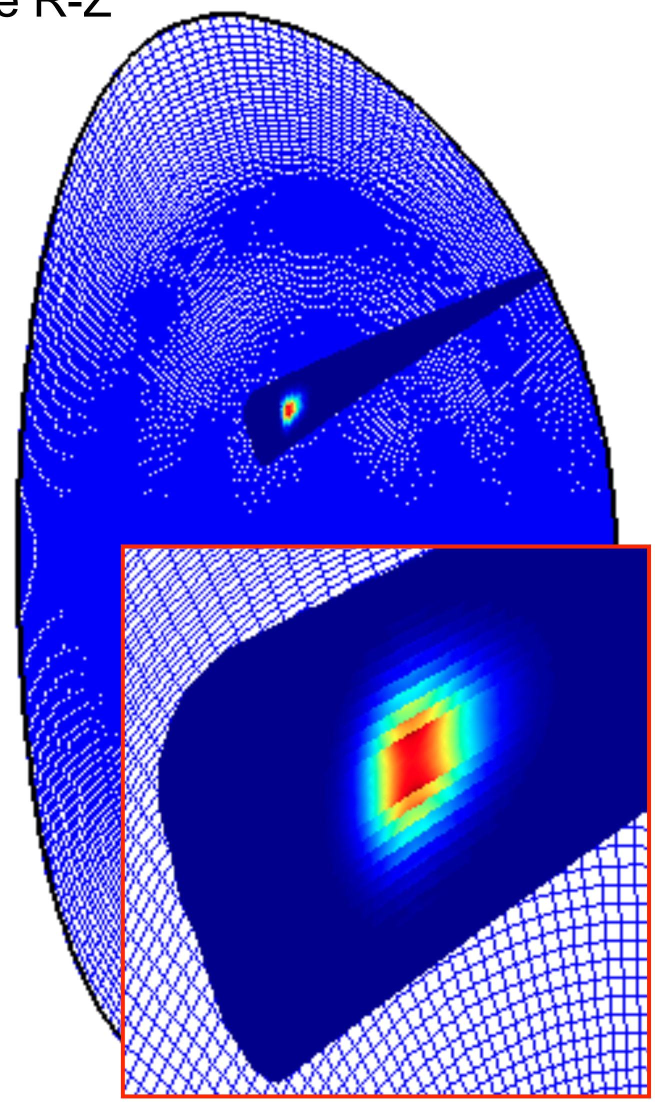
- Ray bundles are highly localized; quasilinear diffusion coefficients are only large near the electron cyclotron resonance, so we get even more localization…

$$\mathbf{F}^{\mathrm{rf}}_{\alpha 0} \sim \exp\left[-\left(\frac{\omega - n\Omega_e}{2k_{\parallel} v_{te}}\right)^2\right]$$

- Only a few points on the NIMROD R-Z grid are affected by the RF source.

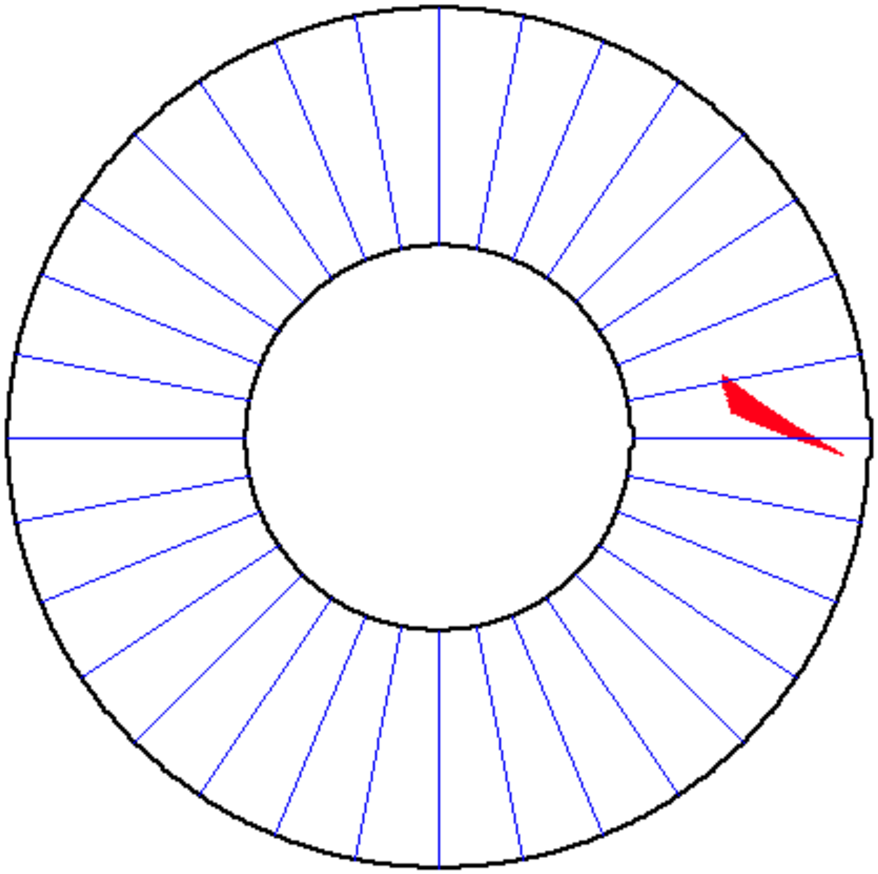


GENRAY ray bundle (420 rays) and NIMROD grid, R-Z projection

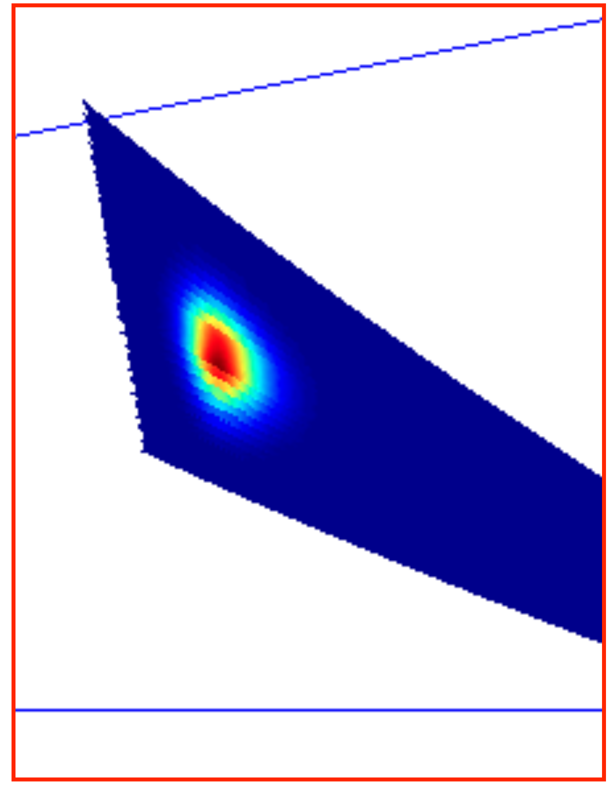


Quasilinear diffusion coefficient amplitude, R-Z projection

Scale lengths of RF deposition – GENRAY and NIMROD

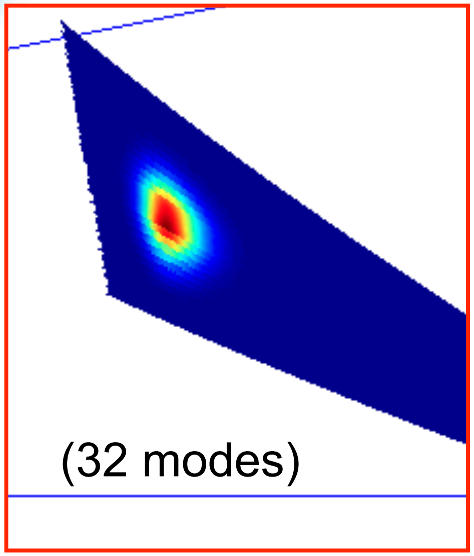


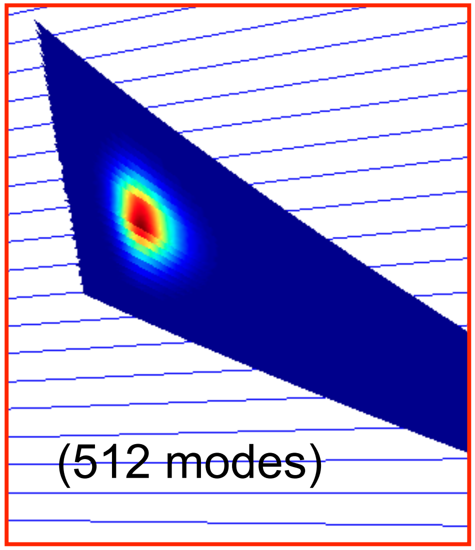
GENRAY ray bundle (420 rays) and NIMROD poloidal planes (corresponding to the 32 modes of the discrete Fourier representation), viewing tokamak from above.



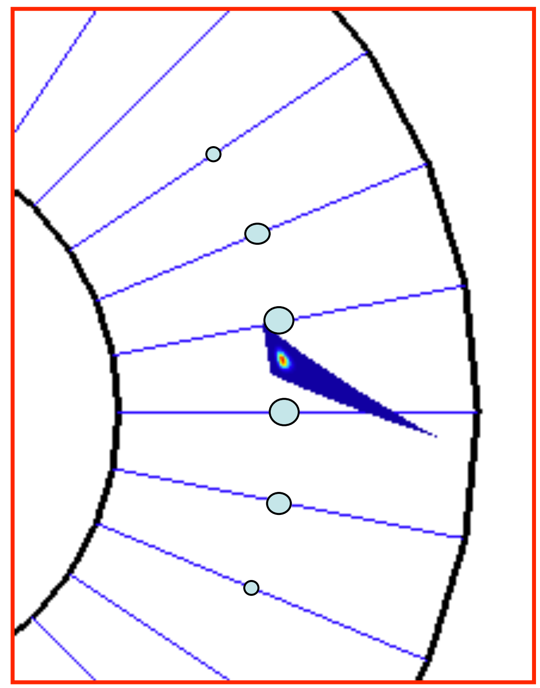
•At this toroidal resolution, the quasilinear diffusion will not be captured by NIMROD's toroidal representation at all!

Toroidally resolving the RF deposition is prohibitive





•Wasteful to sample the entire domain at fine resolution, if we only want to capture physics on a few grid points. (In addition, the number of toroidal modes required exceeds NIMROD's historical performance peak by a factor of eight).

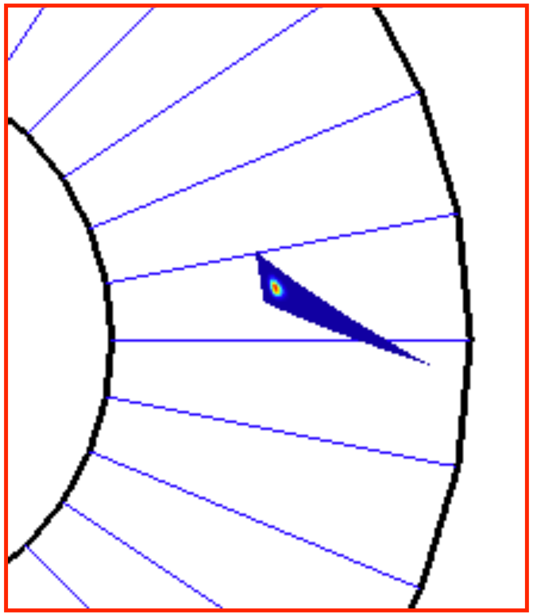


•Resonance region is not a function of toroidal angle; variation of island along toroidal coordinate is slow for low-helicity modes (which are of greatest concern). Toroidal approximation… average, and then spread the deposition toroidally;

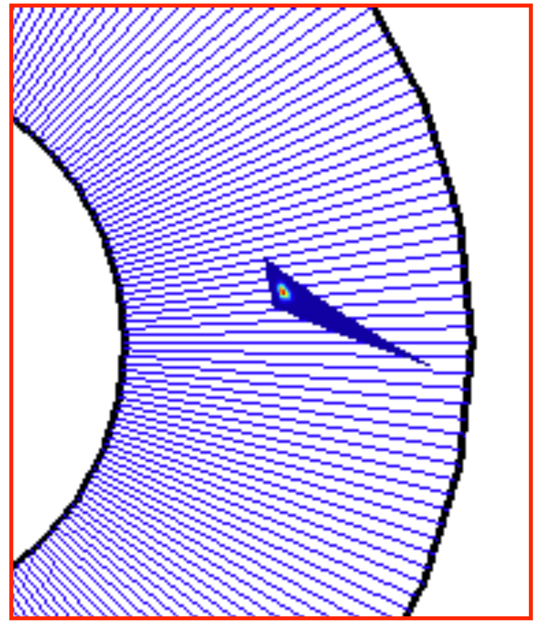
$$\mathbf{F}^{\mathrm{rf}}_{\alpha 0}(R,Z,\phi) \approx \left[\int_0^{2\pi} \frac{\mathbf{F}^{\mathrm{rf}}_{\alpha 0}(R,Z,\phi')}{2\pi}\, d\phi'\right] g(\phi)$$

for some normalized $g(\phi)$.

Accurate toroidal averaging is needed for energy conservation between NIMROD and GENRAY



•NIMROD's toroidal resolution doesn't have to be a limiting factor, if we are going to toroidally average…



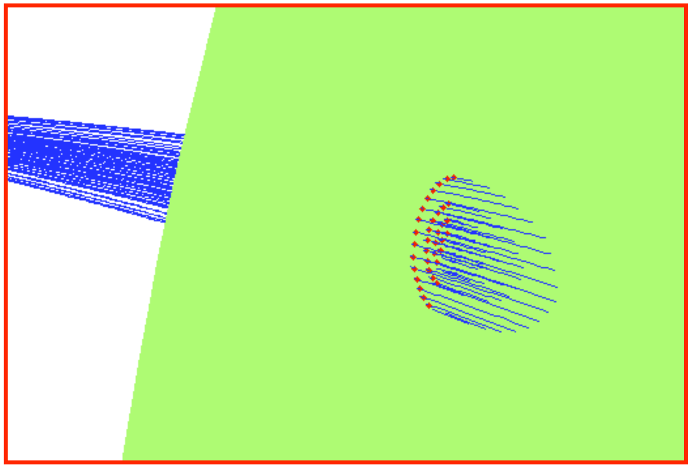
…we can just interpolate the data onto poloidal planes that are independent of regular NIMROD variables.

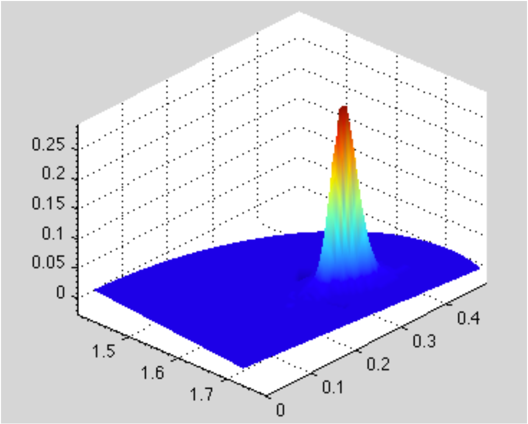
•Allows greater resolution in the toroidal integration. Ray data is very fine-grained relative to NIMROD scale lengths, so we need to do this very accurately to conserve energy.

$$\int_0^{2\pi} \frac{\mathbf{F}^{\mathrm{rf}}_{\alpha 0}(R, Z, \phi')}{2\pi}\, d\phi' \rightarrow \sum_{n=-N}^{N-1} \frac{\mathbf{F}^{\mathrm{rf}}_{\alpha 0 n}(R, Z)}{2N}$$

Toroidal averaging – numerical algorithms

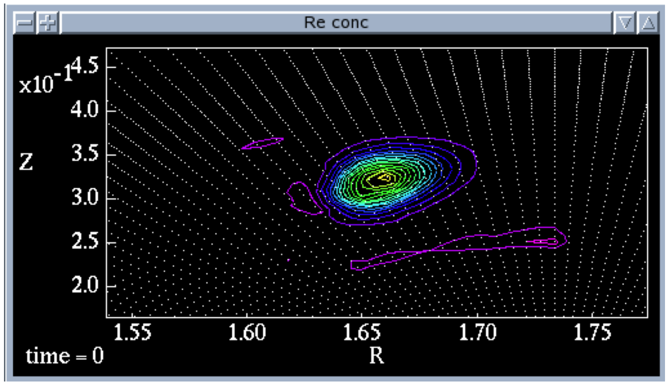
•Routines developed which (a) calculate crossing points for datasets that cross arbitrary poloidal planes (e.g. GENRAY rays, or field line traces), (b) use a weighted spline method (Shepard's algorithm) to represent the data at arbitrary points in the poloidal planes, and (c) evaluate the data at NIMROD gridpoints.





then numerically construct a function, using data values at the crossing points, that can be evaluated at arbitrary points...

•Example – quasilinear diffusion coefficients are calculated along ray paths. First, trace the rays to find the crossing points…



including NIMROD gridpoints.

Two basis sets for Shepard's algorithm were tested – CSHEP (cosine basis) and TSHEP (cubic polynomial basis). CSHEP appears to give slightly smoother results for this problem. Pitfalls…

Choosing the toroidal spreading function *g(ϕ)*

- NIMROD's discrete Fourier transform

$$B_m = \sum_{n=-N}^{N-1} \tilde{B}_n e^{imn\pi/N} \; ; \; m \in [-N \ldots N-1] \qquad \tilde{B}_n = \sum_{m=-N}^{N-1} \frac{B_m}{2N} e^{-imn\pi/N} \; ; \; n \in [-N \ldots N-1]$$

is an approximation to the exact Fourier series

$$B(\phi) = \sum_{n=-\infty}^{\infty} \tilde{B}_n e^{in\phi} \qquad \tilde{B}_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} B(\phi) e^{-in\phi} \, d\phi$$

- Choose an analytic function that's well-behaved and normalized;

$$B(\phi) = \frac{2\pi}{\phi_c} \cos\left(\frac{\pi(\phi - \phi_0)}{2\phi_c}\right)^2 \; ; \; |\phi - \phi_0| \le \phi_c \quad \text{(zero otherwise)}$$

- Get the coefficients:

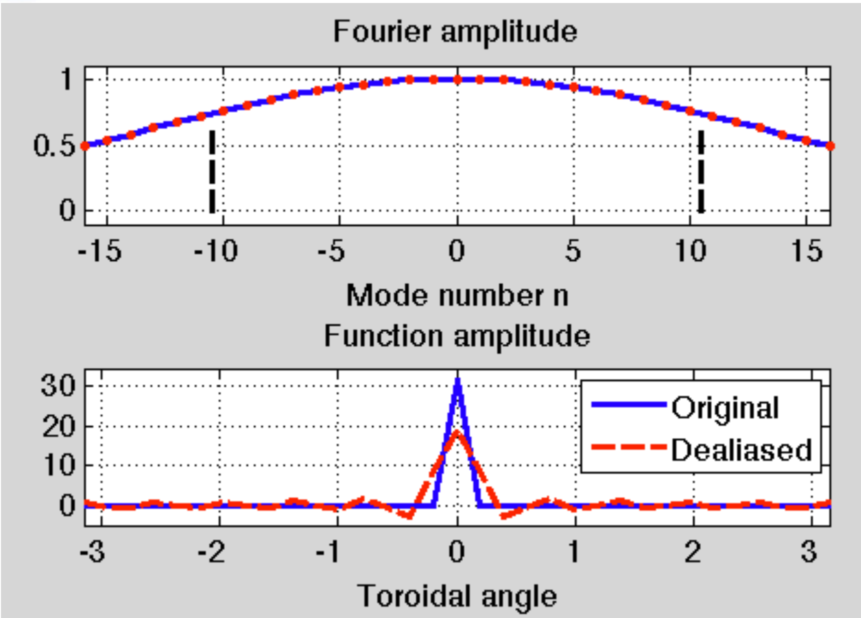
$$\tilde{B}_n = \frac{e^{-in\phi_0} \, \mathrm{sinc}(n\phi_c)\pi^2}{\pi^2 - n^2\phi_c^2}$$

ϕ_0 = location of peak deposition; get this from the RF data

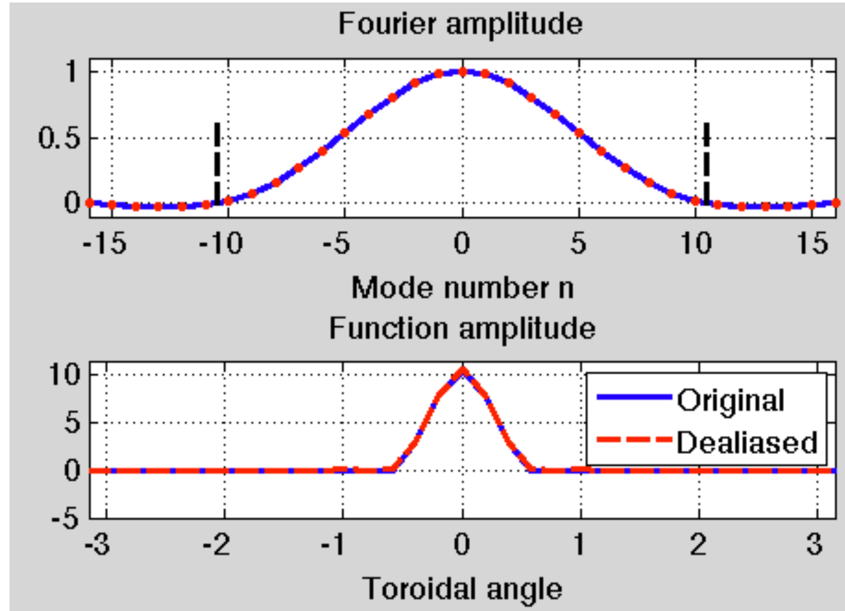
- Need to worry about dealiasing, since we throw away modes with

$$|n| > \mathrm{floor}\left(\frac{2N}{3}\right)$$

Toroidally broadening the RF deposition



ϕ_c = 0.2; 32 modes



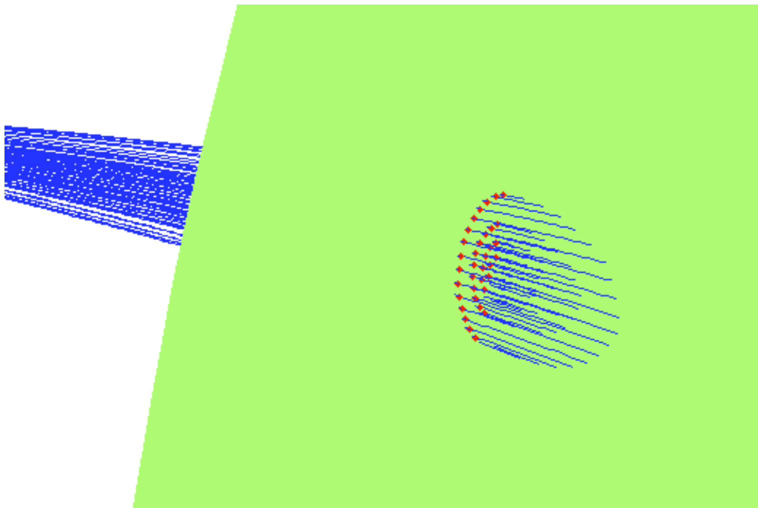
ϕ_c = 0.6; 32 modes

- For a given number of toroidal modes, if the RF data is too restricted toroidally, we cut off Fourier components that matter, and quasilinear diffusion occurs in places we don't want it (including near island X-points).

- In general, if the RF data crosses five or six toroidal planes, things are reasonably well-behaved.

- Relationship between RF broadening and plasma rotation – timescale arguments?

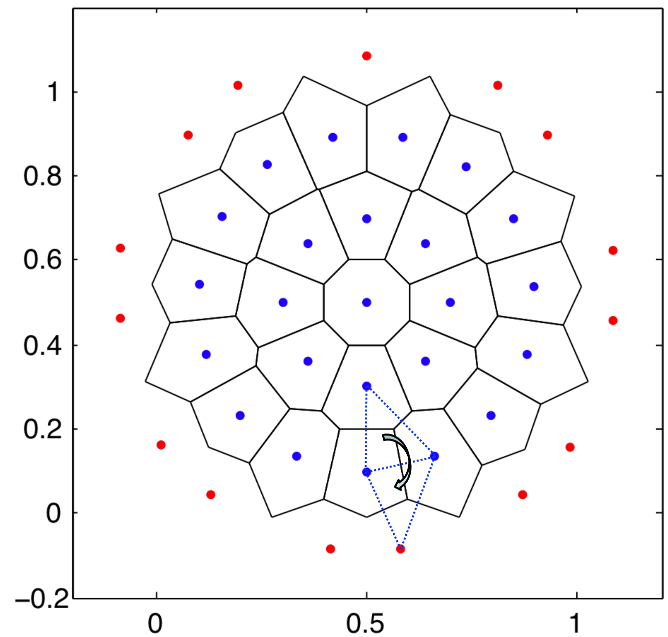
Computational geometry routines (QHULL) are also needed to calculate the quasilinear operator



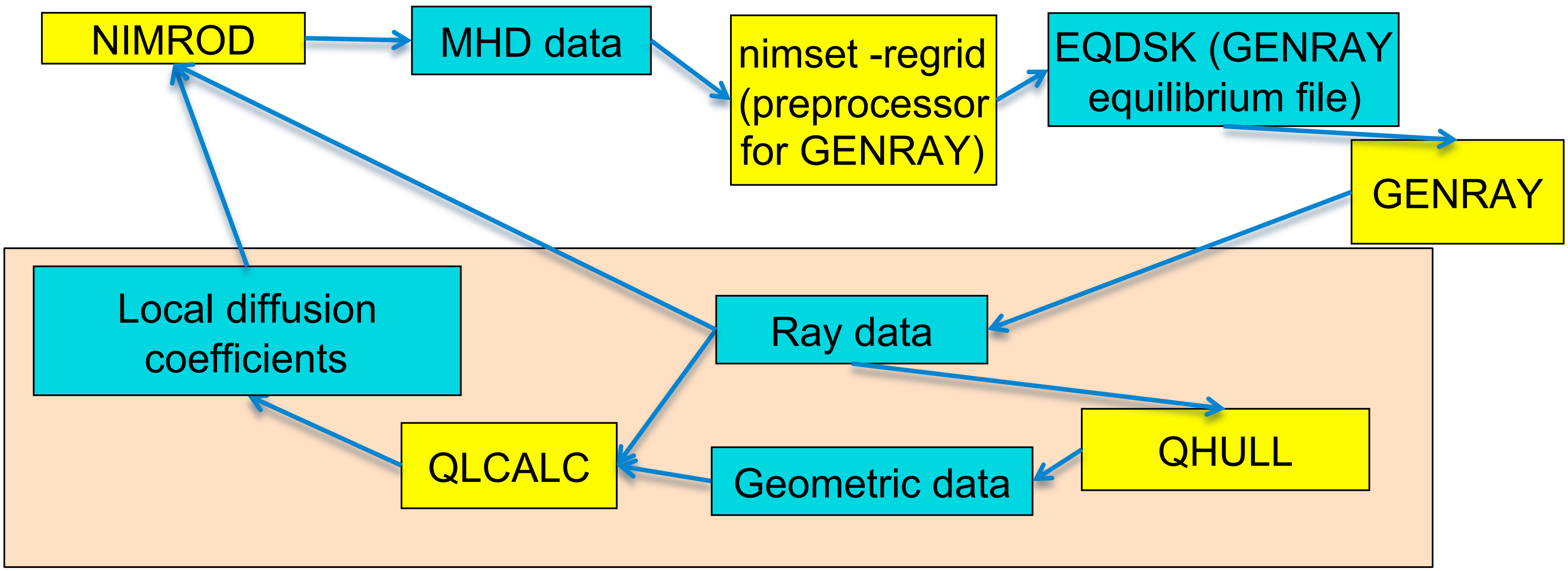
•Increasing the density of rays in the ray bundle (for constant RF power) lowers the power content of each individual ray, and decreases the surface area corresponding to that ray as the ray bundle passes through a given plane. The ratio of these quantities appears in the quasilinear terms (good for numerical convergence) but this area needs to be calculated (more difficult).

•An interface code (QLCALC) has been written which reads GENRAY data, interpolates it into planes along the path of the ray bundle, and (in tandem with the QHULL computational geometry package, available at www.qhull.org) calculates these areas.

•Delaunay triangulation is used on the initial dataset; additional ghost points are added by reflecting triangles over the set's outer boundary. A dual (Voronoi) mesh can then be constructed from the expanded dataset, bounding the points in the plane nearest to any one point in the set.



Other recent developments... altered workflow

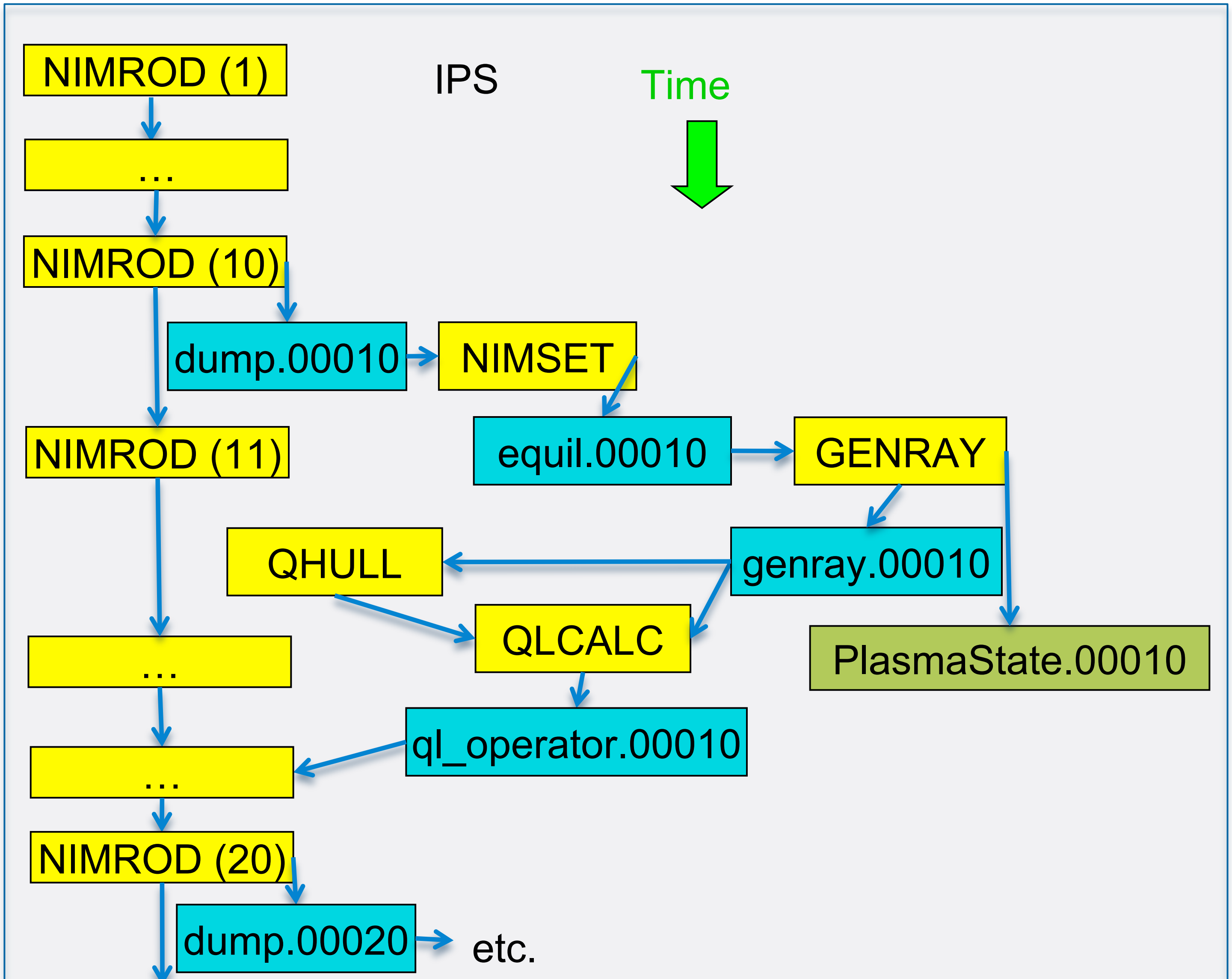


•Everything in the orange box used to be a (very unwieldy) NIMROD subroutine. Now, more compartmentalized and easy to follow.

•Python scripts written to interface various components to SWIM IPS framework.

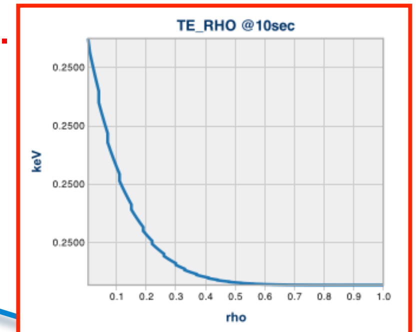
New workflow, under IPS (code management software)

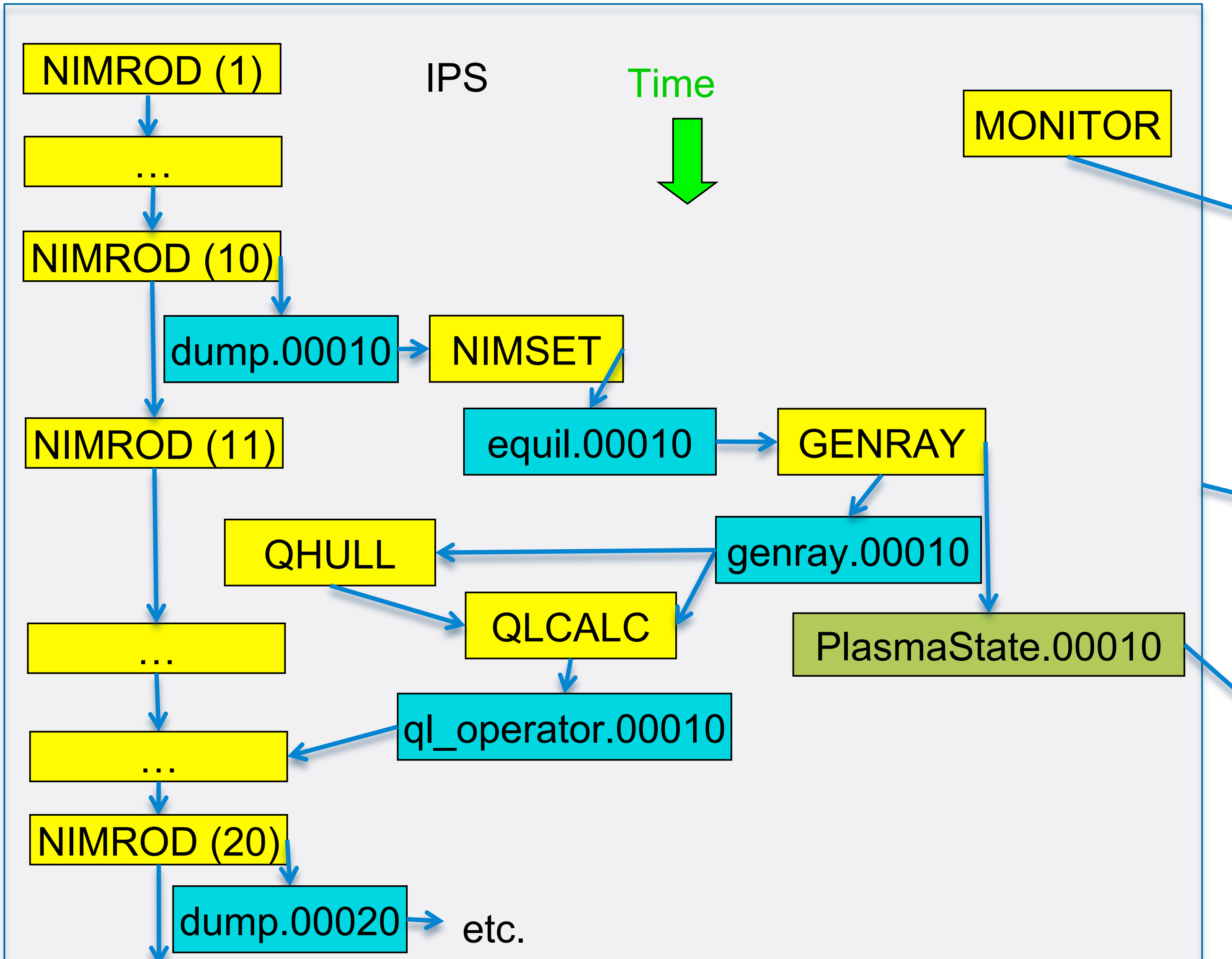
•NIMROD runs in server mode (always on); when dumpfile is generated, triggers structured launch of other codes that calculate the quasilinear operator.

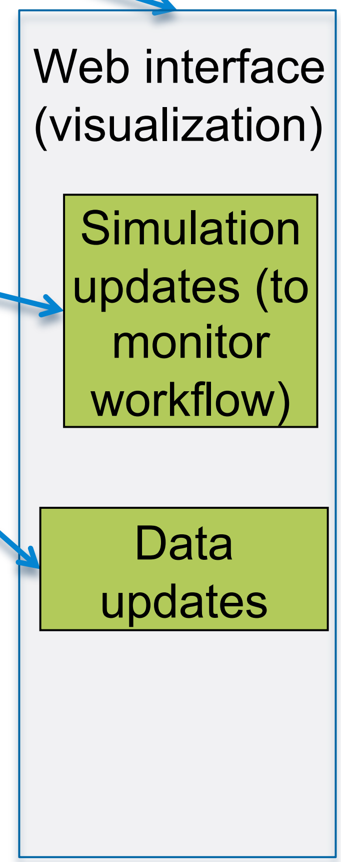


Realtime web monitoring capabilities are improving

- Monitoring component exports data and workflow information to an external web portal. Now with downloadable pdf's, web-based comments, etc...

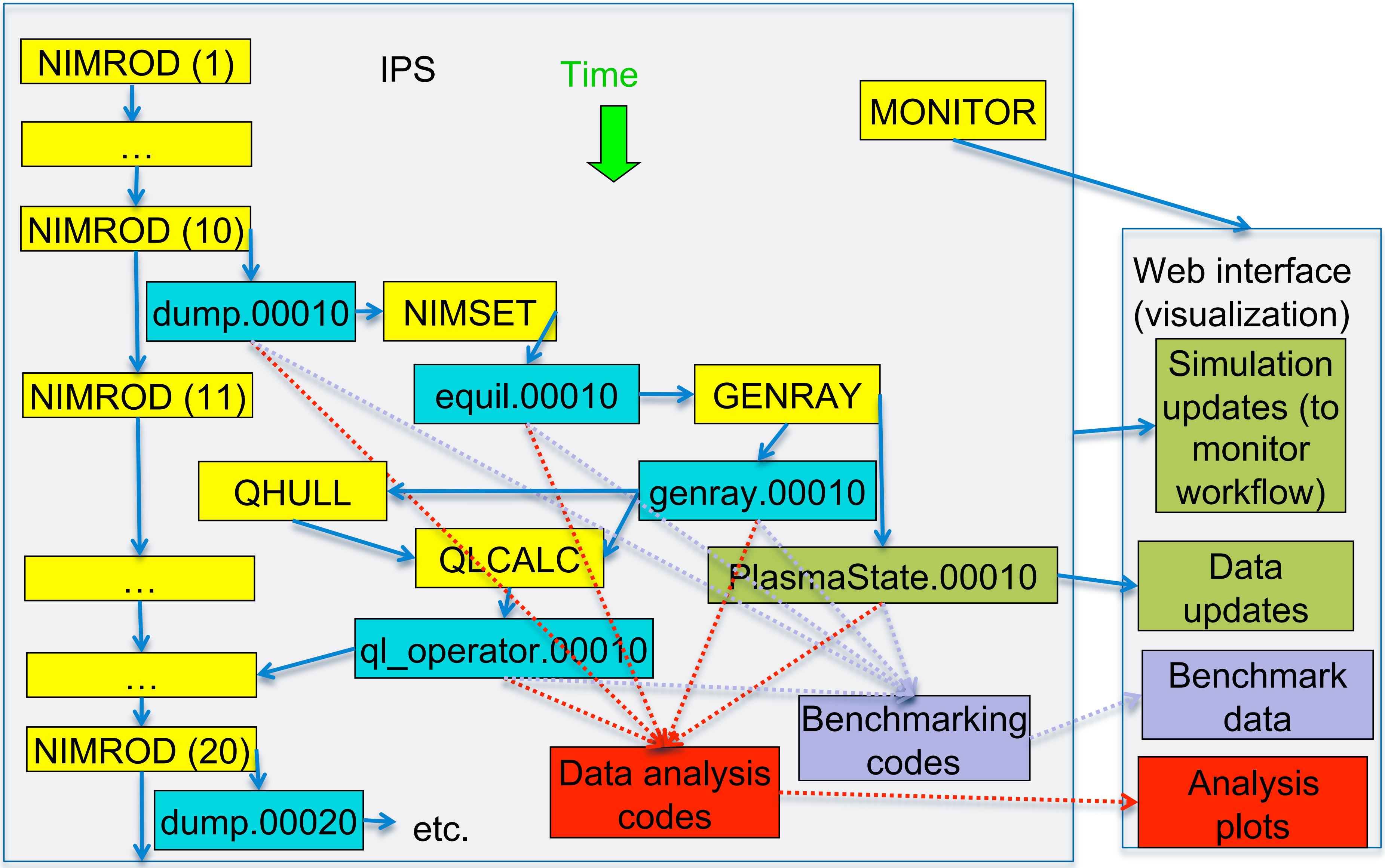






Realtime benchmarking and data analysis is possible

•Additionally, can add data analysis programs (nimfl, nimdraw, benchmarking, etc.) so that analysis can be done as the simulation proceeds. CQL3D?



Present status and plans

- RF-induced quasilinear diffusion coefficients can now be constructed from NIMROD and GENRAY data. The issue of toroidal resolution is being investigated; with toroidal spreading, the physics of the RF/MHD interaction still appears to be qualitatively correct. Next step – detailed numerical benchmark with CQL3D.

- A number of issues generally applicable to the interfacing of codes with considerably different spatial scales and datastructures have been addressed (unstructured – structured meshes).

- For NTM studies we will need better (higher-β) equilibria which are near the stability boundary for the NTM.

- Other plans – synthetic diagnostics, numerical plasma control system for mode stabilization, more accurate closures, etc.

- See my poster (Monday afternoon) for more…