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EXACT CONSERVATION LAWS IN THE CLOSED FLUID-KINETIC MODELING OF LOW-COLLISIONALITY, SLOW DYNAMICS*

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INTRODUCTION

- A CLOSED THEORETICAL MODEL APPLICABLE TO SLOW, MACROSCOPIC INSTABILITIES AT LOW COLLISIONALITY (SUCH AS THE NTM) IS BEING PROPOSED, BASED ON THE ORDERINGS:

$$\delta \sim \rho_\imath/L \sim \nu_s L/v_{ths} \sim (m_e/m_\imath)^{1/2} \sim u_\imath/v_{th\imath} \ll 1$$

AND KEEPING ACCURACY TO THE INVERSE TIME SCALE:

$$\omega \sim \delta\ \omega_{*s} \sim \delta^2 v_{th\imath}/L \sim \delta^3 \Omega_{c\imath} \sim \delta^3 v_{the}/L.$$

- HYBRID FLUID-KINETIC MODEL: FLUID CONSERVATION EQUATIONS FOR PARTICLE NUMBER, MOMENTUM AND ENERGY, AND DRIFT-KINETIC GYROTROPIC (ALSO CALLED PARALLEL) CLOSURES.

- **ELECTRON SIDE OF THE THEORY IN Phys. Plasmas 17, 082502 (2010) [1]. ION SIDE IN POSTER 1P34 AT THIS SHERWOOD CONFERENCE [2]. IN THOSE WORKS THE CONSERVATION OF MOMENTUM AND ENERGY ARE FULFILLED WITHIN THE ACCURACY OF THE PERTURBATION ORDERS KEPT CONSISTENTLY:**

$$\frac{\partial}{\partial t}\Big[n(m_e\mathbf{u}_e + m_\iota\mathbf{u}_\iota)\Big] \ + \ \nabla\cdot(...) \ = \ O(\delta^3 n m_\iota v_{th\iota}^2/L) \ ,$$

$$\frac{\partial}{\partial t}\left[\frac{n}{2}(m_e u_e^2 + m_\iota u_\iota^2) + \frac{3n}{2}(T_e + T_\iota) + \frac{1}{2}B^2\right] \ + \ \nabla\cdot(...) \ = \ O(\delta^3 n m_\iota v_{th\iota}^3/L) \ .$$

- **HERE, A SUMMARY OF THE COMPLETE MODEL WILL BE PRESENTED, WITH A PARTIAL SET OF HIGHER-ORDER TERMS ADDED, WHICH ENSURES EXACT MOMENTUM AND ENERGY CONSERVATION LAWS.**

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CHAPMAN-ENSKOG-LIKE, FLUID-KINETIC FRAMEWORK

- **QUASINEUTRAL MAXWELL AND CONTINUITY EQUATIONS WITH SINGLE ION SPECIES OF UNIT CHARGE [$s = (\iota, e),\ e_\iota = -e_e = e$]:**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}\ , \qquad \mathbf{j} = en(\mathbf{u}_\iota - \mathbf{u}_e) = \nabla \times \mathbf{B}\ , \qquad \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_s) = 0\ .$$

- **KINETIC EQUATIONS:**

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \sum_{s'} C_{ss'}[f_s, f_{s'}]\ ,$$

where $C_{ss'}$ are Fokker-Planck-Landau operators for Coulomb collisions.

- **NEAR MAXWELLIAN DISTRIBUTION FUNCTIONS WITH SMALL PARALLEL TEMPERATURE GRADIENTS:**

$$f_s = f_{Ms} + f_{NMs} = \frac{n}{(2\pi)^{3/2}\ v_{ths}^3} \exp\left(-\frac{|\mathbf{v} - \mathbf{u}_s|^2}{2\ v_{ths}^2}\right) + f_{NMs} \quad \textbf{with} \quad v_{ths}^2 \equiv T_s/m_s\ ,$$

$$f_{NM\iota} \sim \delta\ f_{M\iota}\ , \qquad f_{NMe} \sim \delta^2\ f_{Me}\ , \qquad \mathbf{b} \cdot \nabla T_\iota \sim \delta\ T_\iota/L\ , \qquad \mathbf{b} \cdot \nabla T_e \sim \delta^2\ T_e/L\ .$$

- MAXWELLIAN AND NON-MAXWELLIAN PARTS OF THE DISTRIBUTION FUNCTIONS, $f_{Ms} + f_{NMs}$, EVALUATED IN THE REFERENCE FRAMES OF THE MEAN FLOWS, $\mathbf{u}_s$, WHICH ARE DERIVED FROM THE FLUID MOMENTUM EQUATIONS.

- DENSITY AND TEMPERATURES CARRIED BY THE MAXWELLIANS AND ALSO DERIVED FROM THE FLUID CONTINUITY AND ENERGY EQUATIONS.

- 1, $\mathbf{v} - \mathbf{u}_s$ AND $|\mathbf{v} - \mathbf{u}_s|^2$ VELOCITY-SPACE MOMENTS OF f_{NMs} EQUAL TO ZERO.

- DRIFT-KINETIC EQUATIONS FOR f_{NMs}, TO PROVIDE FLUID CLOSURE TERMS. SINCE f_{NMs} ARE OBTAINED IN THE REFERENCE FRAMES OF THE MACROSCOPIC FLOWS, THE EVALUATION OF THE STRESS AND HEAT FLUX TENSORS IS DIRECT WITHOUT THE NEED OF SUBTRACTING THE MEAN FLOWS.

FLUID MOMENTUM AND TEMPERATURE EQUATIONS

$$m_e n \left[\frac{\partial \mathbf{u}_e}{\partial t} + (\mathbf{u}_e \cdot \nabla)\mathbf{u}_e \right] + \nabla(nT_e) + \nabla \cdot \left[(p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathsf{I}/3) \right] + en(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \mathbf{F}_e^{coll} = 0$$

$$m_\iota n \left[\frac{\partial \mathbf{u}_\iota}{\partial t} + (\mathbf{u}_\iota \cdot \nabla)\mathbf{u}_\iota \right] + \nabla(nT_\iota) + \nabla \cdot \left[(p_{\iota\|} - p_{\iota\perp})(\mathbf{bb} - \mathsf{I}/3) + \mathsf{P}_\iota^{GV} \right] - en(\mathbf{E} + \mathbf{u}_\iota \times \mathbf{B}) - \mathbf{F}_\iota^{coll} = 0$$

$$\frac{3n}{2} \left(\frac{\partial T_e}{\partial t} + \mathbf{u}_e \cdot \nabla T_e \right) + nT_e \nabla \cdot \mathbf{u}_e + \left[(p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathsf{I}/3) \right] : (\nabla \mathbf{u}_e) + \nabla \cdot (q_{e\|}\mathbf{b} + \mathbf{q}_{e\perp}) - G_e^{coll} = 0$$

$$\frac{3n}{2} \left(\frac{\partial T_\iota}{\partial t} + \mathbf{u}_\iota \cdot \nabla T_\iota \right) + nT_\iota \nabla \cdot \mathbf{u}_\iota + \left[(p_{\iota\|} - p_{\iota\perp})(\mathbf{bb} - \mathsf{I}/3) + \mathsf{P}_\iota^{GV} \right] : (\nabla \mathbf{u}_\iota) + \nabla \cdot (q_{\iota\|}\mathbf{b} + \mathbf{q}_{\iota\perp}) - G_\iota^{coll} = 0$$

CONSERVATION LAWS IN THE FLUID SYSTEM

With the momentum and energy conservation properties of the collisional moments,

$$\mathbf{F}_e^{coll} \;=\; -\,\mathbf{F}_\iota^{coll} \qquad\qquad \textbf{and} \qquad\qquad G_e^{coll} \;=\; -\,G_\iota^{coll} \;+\; \frac{\mathbf{j}\cdot\mathbf{F}_e^{coll}}{en}\;,$$

the above fluid system has the exact particle, momentum and energy conservation laws:

$$\frac{\partial n}{\partial t} \;+\; \nabla\cdot(n\mathbf{u}_s) \;=\; 0\;,$$

$$\frac{\partial}{\partial t}\Big[n(m_e\mathbf{u}_e + m_\iota\mathbf{u}_\iota)\Big] \;+\; \nabla\cdot\left[n(m_e\mathbf{u}_e\mathbf{u}_e + m_\iota\mathbf{u}_\iota\mathbf{u}_\iota) + \mathsf{P}_e^{CGL} + \mathsf{P}_\iota^{CGL} + \mathsf{P}_\iota^{GV} + \frac{1}{2}B^2\mathsf{I} - \mathbf{BB}\right] \;=\; 0\;,$$

$$\frac{\partial}{\partial t}\left[\frac{n}{2}(m_e u_e^2 + m_\iota u_\iota^2) + \frac{3n}{2}(T_e + T_\iota) + \frac{1}{2}B^2\right] \;+$$

$$+\;\nabla\cdot\left[\frac{n}{2}(m_e u_e^2\mathbf{u}_e + m_\iota u_\iota^2\mathbf{u}_\iota) + \frac{3n}{2}(T_e\mathbf{u}_e + T_\iota\mathbf{u}_\iota) + \mathsf{P}_e^{CGL}\cdot\mathbf{u}_e + \left(\mathsf{P}_\iota^{CGL} + \mathsf{P}_\iota^{GV}\right)\cdot\mathbf{u}_\iota + \mathbf{q}_e + \mathbf{q}_\iota + \mathbf{E}\times\mathbf{B}\right] = 0.$$

MAXWELLIAN AND NON-GYROTROPIC (PERPENDICULAR) CLOSURES

$$\mathbf{F}_{e\perp}^{coll} \;=\; -\;\mathbf{F}_{\iota\perp}^{coll} \;=\; \frac{2\nu_e m_e}{3(2\pi)^{1/2}e}\;\mathbf{j}_\perp \;-\; \frac{\nu_e m_e n}{(2\pi)^{1/2}eB}\;\mathbf{b}\times\nabla T_e$$

$$G_e^{coll} \;=\; \frac{2\nu_e n m_e}{(2\pi)^{1/2}m_\iota}\;(T_\iota - T_e) \;+\; \frac{\mathbf{j}\cdot\mathbf{F}_e^{coll}}{en}$$

$$G_\iota^{coll} \;=\; \frac{2\nu_e n m_e}{(2\pi)^{1/2}m_\iota}(T_e - T_\iota)$$

$$\mathsf{P}_{\iota,jk}^{GV} \;=\; \frac{1}{4}\;\epsilon_{jlm}b_l\;\mathsf{K}_{\iota,mn}\;(\delta_{nk}+3b_nb_k) \;+\; (j\leftrightarrow k)$$

$$\mathsf{K}_{\iota,jk} \;=\; \frac{m_\iota}{eB}\left[\;nT_\iota\,\frac{\partial u_{\iota,k}}{\partial x_j} \;+\; \frac{\partial(q_{\iota T\|}b_k)}{\partial x_j} \;+\; (2q_{\iota B\|}-3q_{\iota T\|})\;b_j\kappa_k \;+\; \frac{\partial}{\partial x_j}\left(\frac{nT_\iota}{eB}\;\epsilon_{klm}\;b_l\;\frac{\partial T_\iota}{\partial x_m}\right)\;\right] \;+\; (j\leftrightarrow k)$$

$$\mathbf{q}_{e\perp} \;=\; -\;\frac{5nT_e}{2eB}\;\mathbf{b}\times\nabla T_e$$

$$\mathbf{q}_{\iota\perp} = \frac{\mathbf{b}}{eB}\times\left\{\frac{5}{2}nT_\iota\nabla T_\iota + \frac{5}{6}T_\iota\nabla(p_{\iota\|}-p_{\iota\perp}) + T_\iota(p_{\iota\|}-p_{\iota\perp})\left[\frac{1}{3}\nabla\ln(nT_\iota) - \frac{5}{2}\boldsymbol{\kappa}\right] + \nabla\hat{r}_{\iota\perp} + (\hat{r}_{\iota\|}-\hat{r}_{\iota\perp})\boldsymbol{\kappa}\right\}$$

GYROTROPIC (PARALLEL) CLOSURES TO BE DETERMINED KINETICALLY

$$F_{e\|}^{coll} \;=\; -\;F_{\iota\|}^{coll} \;=\; \frac{2\nu_e m_e}{3(2\pi)^{1/2}e}\;j_\| \;-\; \nu_e m_e v_{the}^3 \int d^3\mathbf{v}'\;\frac{v'_\|}{v'^3}\;\bar{f}_{NMe}\;,$$

$$(p_{s\|}-p_{s\perp}) \;=\; \frac{m_s}{2}\int d^3\mathbf{v}'\;(2v_\|'^2 - v_\perp'^2)\;\bar{f}_{NMs}\;,$$

$$q_{s\|} = \frac{m_s}{2}\int d^3\mathbf{v}'\;v'_\|\;v'^2\;\bar{f}_{NMs}\;,$$

$$q_{\iota B\|} = \frac{m_\iota}{2}\int d^3\mathbf{v}'\;v_\|'^3\;\bar{f}_{NM\iota}\;, \qquad q_{\iota T\|} = q_{\iota\|} - q_{\iota B\|}\;,$$

$$\hat{r}_{\iota\|} = \frac{m_\iota^2}{2}\int d^3\mathbf{v}'\;v_\|'^2\;v'^2\;\bar{f}_{NM\iota}\;, \qquad \hat{r}_{\iota\perp} = \frac{m_\iota^2}{4}\int d^3\mathbf{v}'\;v_\perp'^2\;v'^2\;\bar{f}_{NM\iota}\;,$$

where

$$\mathbf{v}' \;=\; \mathbf{v} \;-\; \mathbf{u}_s(\mathbf{x},t) \;=\; v'_\|\;\mathbf{b}(\mathbf{x},t) \;+\; v'_\perp\;[\cos\alpha\;\mathbf{e}_1(\mathbf{x},t)+\sin\alpha\;\mathbf{e}_2(\mathbf{x},t)]\;,$$

$$\bar{f}_{NMs}(v'_\|,v'_\perp,\mathbf{x},t) \;=\; \langle f_{NMs}\rangle_\alpha(v'_\|,v'_\perp,\mathbf{x},t) \;=\; (2\pi)^{-1}\oint d\alpha\;f_{NMs}(v'_\|,v'_\perp,\alpha,\mathbf{x},t)\;.$$

DRIFT-KINETIC EQUATIONS FOR THE GYROPHASE-AVERAGED, NON-MAXWELLIAN PARTS OF THE DISTRIBUTION FUNCTIONS

In terms of the velocity-space coordinates $(v'_\parallel, v'_\perp)$ in the reference frames of the macroscopic flows, the gyrophase-averaged, non-Maxwellian parts of the distribution functions, $\bar{f}_{NMs}(v'_\parallel, v'_\perp, \mathbf{x}, t)$ satisfy drift-kinetic equations of the form:

$$\frac{d_s \bar{f}_{NMs}}{dt} \equiv \frac{\partial \bar{f}_{NMs}}{\partial t} + \dot{\mathbf{x}}_s \cdot \frac{\partial \bar{f}_{NMs}}{\partial \mathbf{x}} + \dot{v}'_{\parallel s} \frac{\partial \bar{f}_{NMs}}{\partial v'_\parallel} + \dot{v}'_{\perp s} \frac{\partial \bar{f}_{NMs}}{\partial v'_\perp} = D_s \; f_{Ms} + \mathcal{Q}_s^{coll}$$

where, consistent with the desired accuracy of $\bar{f}_{NMe} = O(\delta^2 f_{Me}) + O(\delta^3 f_{Me})$ **and** $\bar{f}_{NM\iota} = O(\delta f_{M\iota}) + O(\delta^2 f_{M\iota})$ **:**

$$d_\iota/dt = O(v_{th\iota}/L) + O(\delta v_{th\iota}/L) \; , \qquad d_e/dt = O(v_{the}/L) \; ,$$

$$D_\iota = O(\delta v_{th\iota}/L) + O(\delta^2 v_{th\iota}/L) \; , \qquad D_e = O(\delta^2 v_{the}/L) \; ,$$

$$\mathcal{Q}_\iota^{coll} = O(\delta^2 v_{th\iota}/L \;\; f_{M\iota}) \; , \qquad \mathcal{Q}_e^{coll} = O(\delta^3 v_{the}/L \;\; f_{Me}) \; .$$

KEY POINTS IN THE DERIVATION OF THE DRIFT-KINETIC EQUATIONS

- **SYSTEMATIC USE OF GALILEAN TRANSFORMATIONS TO THE MOVING FRAMES OF THE MACROSCOPIC FLOWS.**
- **FARADAY'S LAW SUBSTITUTED FOR** $\partial \mathrm{b}/\partial t$**.**
- **FLUID MOMENTUM CONSERVATION EQUATIONS USED TO ELIMINATE THE ELECTRIC FIELD ALGEBRAICALLY, WITHOUT ADDITIONAL APPROXIMATIONS. INERTIAL PIECE OF THE ELECTRIC FIELD CANCELLED BY INERTIAL FORCE FROM TRANSFORMATION TO THE MOVING REFERENCE FRAME.**
- **FLUID CONTINUITY AND TEMPERATURE EQUATIONS SUBSTITUTED FOR THE TIME DERIVATIVE OF THE MAXWELLIANS.**
- **NO UNBALANCED** $O(v_{th\imath}/L)$ **OR** $O(v_{the}/L)$ **TERMS IN** $D_\imath$ **OR** D_e **WITH CONDITIONS** $\mathrm{b} \cdot \nabla T_\imath \sim \delta \; T_\imath/L$ **AND** $\mathrm{b} \cdot \nabla T_e \sim \delta^2 \; T_e/L$**, WHILE** $\mathrm{b} \cdot \nabla n$ **REMAINS ARBITRARY.**

COLLISIONLESS STREAMING OPERATORS

The coefficient functions in the operators

$$\frac{d_s}{dt} \equiv \frac{\partial}{\partial t} + \dot{\mathbf{x}}_s \cdot \frac{\partial}{\partial \mathbf{x}} + \dot{v}'_{\parallel s} \frac{\partial}{\partial v'_\parallel} + \dot{v}'_{\perp s} \frac{\partial}{\partial v'_\perp}$$

are

$$\dot{\mathbf{x}}_e = v'_\parallel \mathbf{b} \ ,$$

$$\dot{v}'_{\parallel e} = \frac{T_e}{m_e} \mathbf{b} \cdot \nabla \ln n - \frac{v'^2_\perp}{2} \mathbf{b} \cdot \nabla \ln B \ ,$$

$$\dot{v}'_{\perp e} = \frac{v'_\perp v'_\parallel}{2} \mathbf{b} \cdot \nabla \ln B \ ,$$

$$\dot{\mathbf{x}}_\iota \;=\; v'_\parallel \mathbf{b} \;+\; \mathbf{u}_\iota - \mathbf{u}_{D\iota} \;+\; \frac{v'^2_\perp}{2}\nabla\times\left(\frac{\mathbf{b}}{\Omega_{c\iota}}\right) \;+\; \left(v'^2_\parallel - \frac{v'^2_\perp}{2}\right)\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}} \qquad \textbf{with} \qquad \mathbf{u}_{D\iota} \;=\; \frac{\mathbf{b}\times\nabla(nT_\iota)}{m_\iota n\Omega_{c\iota}} \;,$$

$$\dot{v}'_{\parallel\iota} \;=\; \frac{\mathbf{b}\cdot(\nabla\cdot\mathsf{P}_\iota^{CGL})}{m_\iota n} \;-\; \frac{v'^2_\perp}{2}\mathbf{b}\cdot\nabla\ln B \;-\; v'_\parallel \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;+\; \frac{v'_\parallel v'^2_\perp}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right) \;,$$

and

$$\dot{v}'_{\perp\iota} = \frac{v'_\perp}{2}\;\left\{v'_\parallel \mathbf{b}\cdot\nabla\ln B \;+\; \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;-\; \nabla\cdot(\mathbf{u}_\iota - \mathbf{u}_{D\iota}) \;-\; v'^2_\parallel \nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)\right\} \;.$$

$$\dot{\mathbf{x}}_\iota \;=\; v'_\parallel \mathbf{b} \;+\; \mathbf{u}_\iota - \mathbf{u}_{D\iota} \;+\; \frac{v'^2_\perp}{2}\nabla\times\left(\frac{\mathbf{b}}{\Omega_{c\iota}}\right) \;+\; \left(v'^2_\parallel - \frac{v'^2_\perp}{2}\right)\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}} \qquad \textbf{with} \qquad \mathbf{u}_{D\iota} \;=\; \frac{\mathbf{b}\times\nabla(nT_\iota)}{m_\iota n \Omega_{c\iota}} \;,$$

$$\dot{v}'_{\parallel\iota} \;=\; \frac{\mathbf{b}\cdot(\nabla\cdot\mathbf{P}^{CGL}_\iota)}{m_\iota n} \;-\; \frac{v'^2_\perp}{2}\mathbf{b}\cdot\nabla\ln B \;-\; v'_\parallel\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;+\; \frac{v'_\parallel v'^2_\perp}{2}\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right) \;,$$

and

$$\dot{v}'_{\perp\iota} = \frac{v'_\perp}{2}\left\{v'_\parallel\mathbf{b}\cdot\nabla\ln B \;+\; \mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota - \mathbf{u}_{D\iota})] \;-\; \nabla\cdot(\mathbf{u}_\iota - \mathbf{u}_{D\iota}) \;-\; v'^2_\parallel\nabla\cdot\left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)\right\} \;.$$

THESE FULFILL THE PHASE-SPACE VOLUME CONSERVATION CONDITIONS

$$\frac{\partial}{\partial\mathbf{x}}\cdot\dot{\mathbf{x}}_s \;+\; \frac{\partial\dot{v}'_{\parallel s}}{\partial v'_\parallel} \;+\; \frac{1}{v'_\perp}\frac{\partial(v'_\perp\dot{v}'_{\perp s})}{\partial v'_\perp} \;=\; 0 \;,$$

so that

$$\frac{d_s\bar{f}_{NMs}}{dt} \;=\; \frac{\partial\bar{f}_{NMs}}{\partial t} \;+\; \frac{\partial}{\partial\mathbf{x}}\cdot(\bar{f}_{NMs}\dot{\mathbf{x}}_s) \;+\; \frac{\partial}{\partial v'_\parallel}\left(\bar{f}_{NMs}\dot{v}'_{\parallel s}\right) \;+\; \frac{1}{v'_\perp}\,\frac{\partial}{\partial v'_\perp}\left(v'_\perp\bar{f}_{NMs}\dot{v}'_{\perp s}\right) \;.$$

COLLISIONAL TERMS

The collisional terms in the drift-kinetic equations are

$$\mathcal{Q}_e^{coll} = \langle C_{ee}[f_{NMe}, f_{Me}] + C_{ee}[f_{Me}, f_{NMe}] + C_{e\iota}[f_{NMe}, f_{M\iota}] + C_{e\iota}[f_{Me}, f_{M\iota}]\rangle_\alpha \; -$$

$$- \;\frac{v'_\parallel}{nT_e}\; F_{e\parallel}^{coll} \;-\; \frac{1}{3nT_e}\left(\frac{m_e v'^2}{T_e} - 3\right) G_e^{coll}$$

and

$$\mathcal{Q}_\iota^{coll} \;=\; \langle C_{\iota\iota}[f_{NM\iota}, f_{M\iota}] + C_{\iota\iota}[f_{M\iota}, f_{NM\iota}]\rangle_\alpha \; .$$

COLLISIONAL TERMS

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$$- \; \frac{v'_\parallel}{nT_e} \; F_{e\parallel}^{coll} \; - \; \frac{1}{3nT_e} \left(\frac{m_e v'^2}{T_e} - 3 \right) G_e^{coll}$$

and

$$\mathcal{Q}_\iota^{coll} \;=\; \langle C_{\iota\iota}[f_{NM\iota}, f_{M\iota}] + C_{\iota\iota}[f_{M\iota}, f_{NM\iota}] \rangle_\alpha \; .$$

THESE HAVE PARTICLE, MOMENTUM AND ENERGY CONSERVATION PROPERTIES:

$$\int d^3\mathbf{v}' \; (1, v'_\parallel, v'^2) \; \mathcal{Q}_s^{coll} \;=\; 0 \; .$$

MOMENTS OF THE COLLISION-INDEPENDENT DRIVING TERMS

The complete expressions of the $D_s f_{Ms}$ terms are in Refs.[1,2]. Their first three moments are:

$$\int d^3\mathbf{v}'\ D_e\ f_{Me}\ =\ 0\ ,$$

$$m_e \int d^3\mathbf{v}'\ v'_{\|}\ D_e\ f_{Me}\ =\ \mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\|}-p_{e\perp}) - \left(p_{e\|}-p_{e\perp}\right)\nabla \ln B\right]\ ,$$

$$\frac{m_e}{2}\int d^3\mathbf{v}'\ v'^2\ D_e\ f_{Me}\ =\ \nabla\cdot(q_{e\|}\mathbf{b})\ ,$$

$$\int d^3\mathbf{v}'\ D_\iota\ f_{M\iota}\ =\ -\nabla\cdot\left\{\frac{\mathbf{b}}{m_\iota\Omega_{c\iota}}\times\left[\frac{1}{3}\nabla(p_{\iota\|}-p_{\iota\perp}) - (p_{\iota\|}-p_{\iota\perp})\boldsymbol{\kappa}\right]\right\}\ ,$$

$$m_\iota \int d^3\mathbf{v}'\ v'_{\|}\ D_\iota\ f_{M\iota}\ =\ \mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{\iota\|}-p_{\iota\perp}) - \left(p_{\iota\|}-p_{\iota\perp}\right)\nabla \ln B\right]\ +$$

$$+\ \nabla\cdot\left\{\frac{\mathbf{b}}{\Omega_{c\iota}}\times\left[\nabla q_{\iota T\|} + 2(q_{\iota B\|}-q_{\iota T\|})\boldsymbol{\kappa}\right]\right\}\ +\ \left(\frac{\mathbf{b}\times\boldsymbol{\kappa}}{\Omega_{c\iota}}\right)\cdot\nabla q_{\iota T\|}\ ,$$

$$\frac{m_\iota}{2}\int d^3\mathbf{v}'\ v'^2\ D_\iota\ f_{M\iota}\ =\ \nabla\cdot(q_{\iota\|}\mathbf{b})\ +\ \nabla\cdot\left\{\frac{\mathbf{b}}{m_\iota\Omega_{c\iota}}\times\left[\nabla\hat{r}_{\iota\perp} + (\hat{r}_{\iota\|}-\hat{r}_{\iota\perp})\boldsymbol{\kappa}\right]\right\}\ +$$

$$+\ (p_{\iota\|}-p_{\iota\perp})\left\{\mathbf{b}\cdot\left[(\mathbf{b}\cdot\nabla)(\mathbf{u}_\iota-\mathbf{u}_{D\iota})\right] - \frac{1}{3}\nabla\cdot(\mathbf{u}_\iota-\mathbf{u}_{D\iota})\right\}\ .$$

From the definition of v' as relative to the mean flows and the condition that the fluid density and temperatures be those of the Maxwellians,

$$\int d^3\mathbf{v}' \ (1, v'_\parallel, v'^2) \ \bar{f}_{NMs} \ = \ 0 \ .$$

Using these conditions along with the expressions for $\dot{\mathrm{x}}_s, \ \dot{v}'_{\parallel s}, \ \dot{v}'_{\perp s},$ it is then verified that

$$\int d^3\mathbf{v}' \ (1, v'_\parallel, v'^2) \ \frac{d_s \bar{f}_{NMs}}{dt} \ = \ \int d^3\mathbf{v}' \ (1, v'_\parallel, v'^2) \ D_s \ f_{Ms} \ .$$

THEREFORE, CONSISTENT WITH THE CONDITIONS $\int d^3\mathbf{v}' \ (1, v'_\parallel, v'^2) \ \bar{f}_{NMs} = 0$, THE $(1, v'_\parallel, v'^2)$ MOMENTS OF THE (FIRST-ORDER IN ρ_e/L) ELECTRON AND (SECOND-ORDER IN ρ_ι/L) ION DRIFT-KINETIC EQUATIONS ARE SATISFIED IDENTICALLY.

SUMMARY

A CLOSED FLUID AND DRIFT-KINETIC MODEL FOR THE MACROSCOPIC SIMULATION OF MAGNETIZED PLASMAS HAS BEEN DEVELOPED, BASED ON SLOW DYNAMICS AND LOW COLLISIONALITY ORDERINGS. IT HAS THE EXACT CONSERVATION LAWS:

$$\frac{\partial n}{\partial t} + \nabla \cdot (...) = 0$$

$$\frac{\partial}{\partial t}\Big[n(m_e \mathbf{u}_e + m_\iota \mathbf{u}_\iota)\Big] + \nabla \cdot (...) = 0$$

$$\frac{\partial}{\partial t}\left[\frac{n}{2}(m_e u_e^2 + m_\iota u_\iota^2) + \frac{3n}{2}(T_e + T_\iota) + \frac{1}{2}B^2\right] + \nabla \cdot (...) = 0$$

$$\frac{d_s \bar{f}_{NMs}}{dt} = \frac{\partial \bar{f}_{NMs}}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (...) + \frac{\partial}{\partial v'_\parallel}(...) + \frac{1}{v'_\perp}\frac{\partial}{\partial v'_\perp}(v'_\perp \; ...)$$

$$\int d^3\mathbf{v}' \; (1, v'_\parallel, v'^2) \; \frac{d_s \bar{f}_{NMs}}{dt} = \int d^3\mathbf{v}' \; (1, v'_\parallel, v'^2) \; (D_s f_{Ms} + \mathcal{Q}_s^{coll}) \qquad \textbf{with} \qquad \int d^3\mathbf{v}' \; (1, v'_\parallel, v'^2) \; \bar{f}_{NMs} = 0$$