# Altering NIMROD's Convergence on Interchange (+++)

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# Motivation

The objective of this work is to make NIMROD more robust and easier to use in application to nonlinear macroscopic extended-MHD dynamics of magnetically confined plasma.

# Outline

- Introduction
- Eigenvalue tests
- Time-dependent results
- Discussion and conclusions

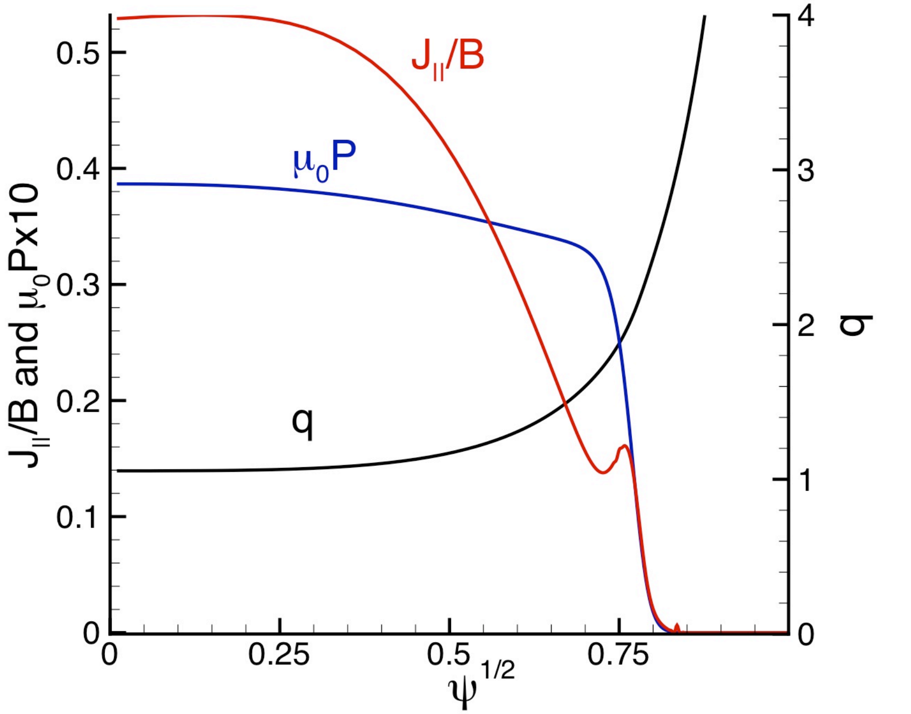
# Introduction

• In resistive-MHD with small or no viscous damping, growth rates of local interchange/ballooning instabilities tend to increase with perpendicular wavenumber.

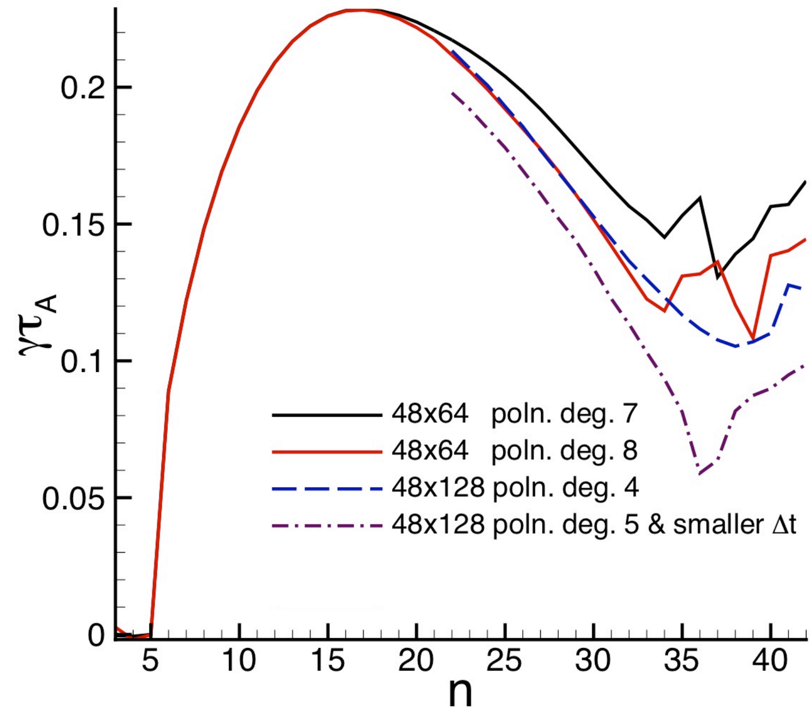
• In physical cases where these modes exist, drift and/or kinetic effects limit growth at small spatial scales.

• Many nonlinear computations of interest are impractical if nonlinear convergence on interchange is from the unstable side, even when including effects that stabilize high wavenumber modes physically.

# Numerical tests for edge-localized modes (ELMs) provide a demonstration of convergence from the unstable side.




Profiles of the dens8 circular-cross-section equilibrium by P. Snyder. Number density drops by 100, temperature drops by 10.




Growth rates computed with NIMROD including 2-fluid Ohm's law, gyro-viscous stress, and ion diamagnetic drift. Here, $d_i/a \cong 0.08$ and $\omega_{*i}\tau_A \cong 0.5$ for $n$=20.

- NIMROD is known to converge from the unstable side for MHD.
- The above results show that 2-fluid modeling does not change this property.
- Other physical parameters: S=$10^6$ (using central $T$), Pm=1, $\kappa_{||}/\kappa_{\perp} = 10^5$.

## **Eigenvalue tests:** a flexible computational tool helps us investigate many basis-function and formulation possibilities.

- The CYL_SPEC code generates matrices for generalized eigenmode problems (**Ax**-$\lambda$**Bx**=**0**) that are solved with LAPACK routines.
- 1D cylindrical geometry with $f \rightarrow f(r)e^{im\theta+ikz}$ is a compromise between non-trivial geometry and rapid development.
- Run-time parameters are used to select basis functions.
- Changing formulations requires minimal coding.
- First-order systems for **V**, **B**, *p*, and a scalar for the divergence constraint test possibilities for NIMROD's expansions.

The cylindrical profile used for testing internal kink in [Gruber and Rappaz] is also convenient for local interchange tests.

$$B_z(r) = B_0 \qquad B_\theta(r) = B_0 \frac{c_1 r}{1 + c_2^2 r^2} \qquad q(r) = \frac{1 + c_2^2 r^2}{R c_1}$$

$$P(r) = \frac{B_0^2}{2\mu_0}\left(\frac{c_1^2}{c_2^2}\right)\left[\frac{1}{\left(1 - c_2^2 r^2\right)^2} - \frac{1}{\left(1 - c_2^2\right)^2}\right]$$

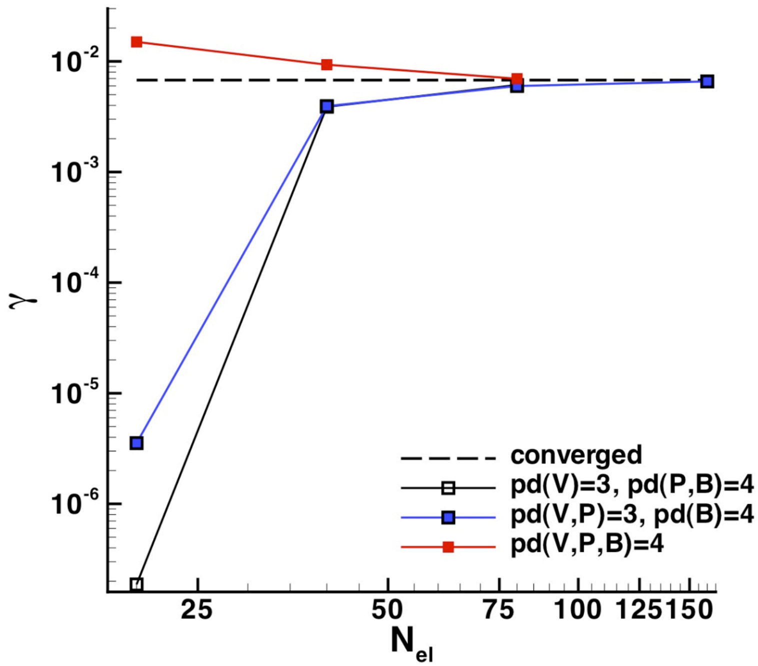
$$D_s(r) = -\frac{\mu_0}{r B_z^2}\left(\frac{q}{dq/dr}\right)^2\left(\frac{dP}{dr}\right) = \frac{c_1^2}{\left(1 + c_2^2 r^2\right) c_2^4 r^2}$$

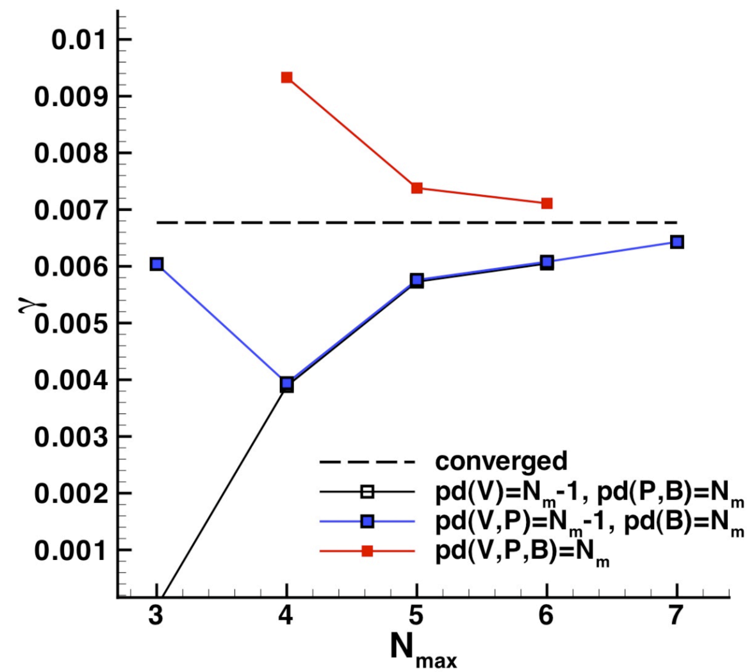
- The $q$ profile is parabolic and $D_s$ decreases monotonically in radius.
- Most cylindrical results reported here are computed with $c_1$=4/7 and $c_2$=10/7.

# A tractable modification of NIMROD's representation is to use different continuous basis functions for different fields.

• Expansions with **B** having larger polynomial degree than other fields is a generalization of the XTOR approach.

• CYL_SPEC tests of two possibilities on the unstable m=4, k=-1.78 cylinder mode at $r_s$=0.371 and $D_s(r_s)$=0.443 show convergence from the stable size.

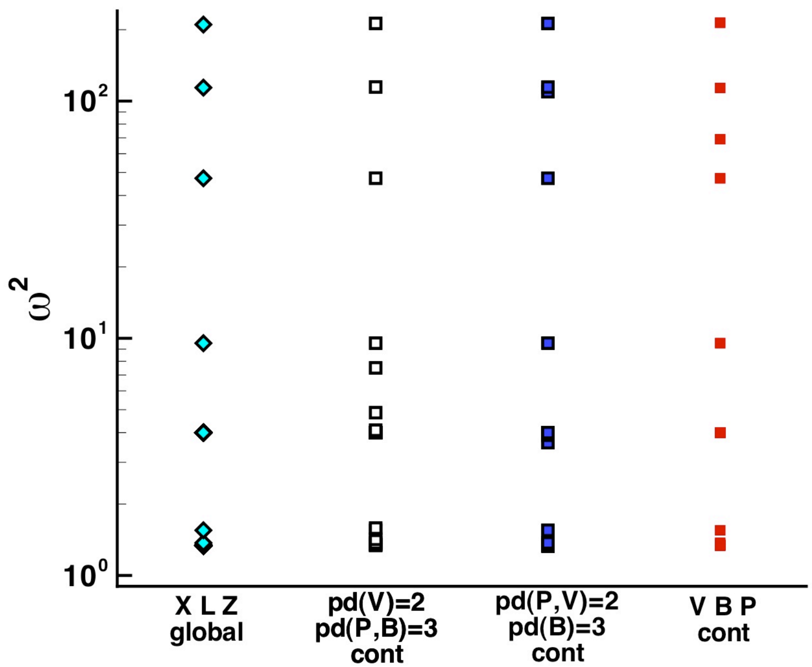


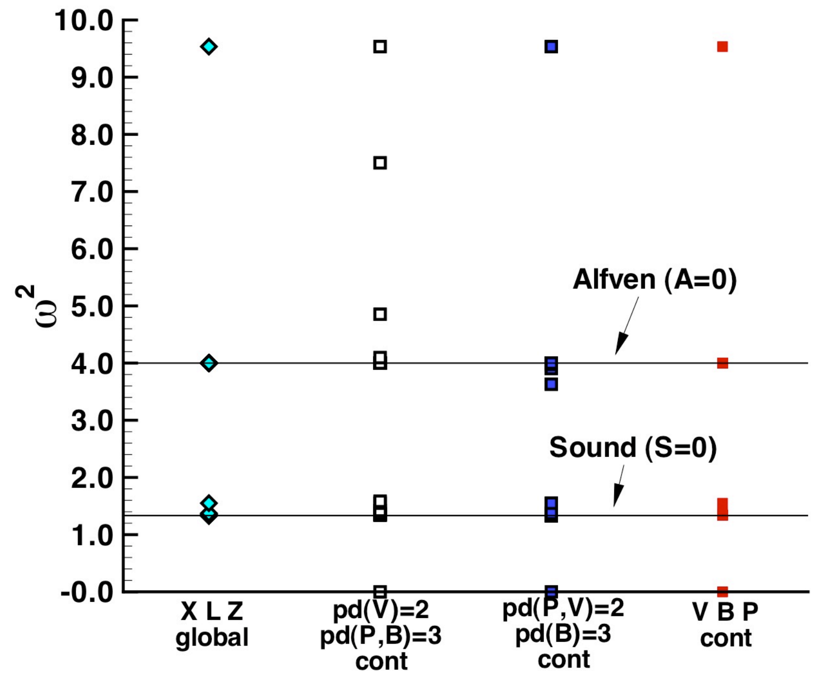




• The standard case (red) has all fields of the same polynomial degree.

• One 'modified' representation has only **V** of one degree lower (open), and the other has both **V** and *P* lower (blue).

## The expansion with both *P* and **V** at lower polynomial degree than that for **B** is more accurate for stable *m*=1 oscillations in uniform axial field.

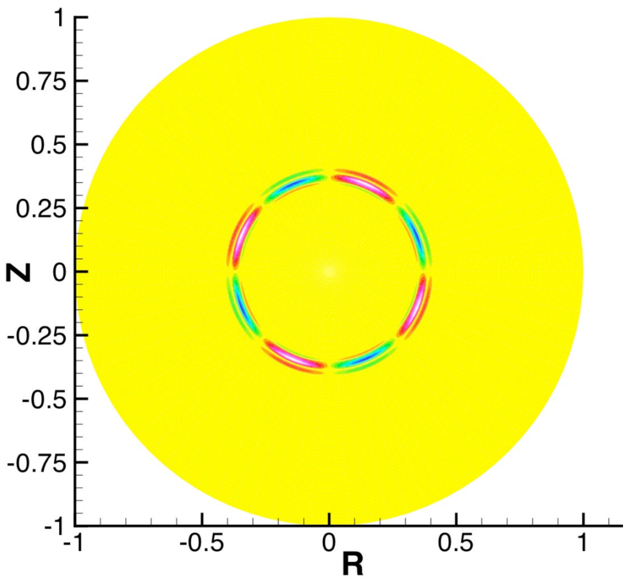


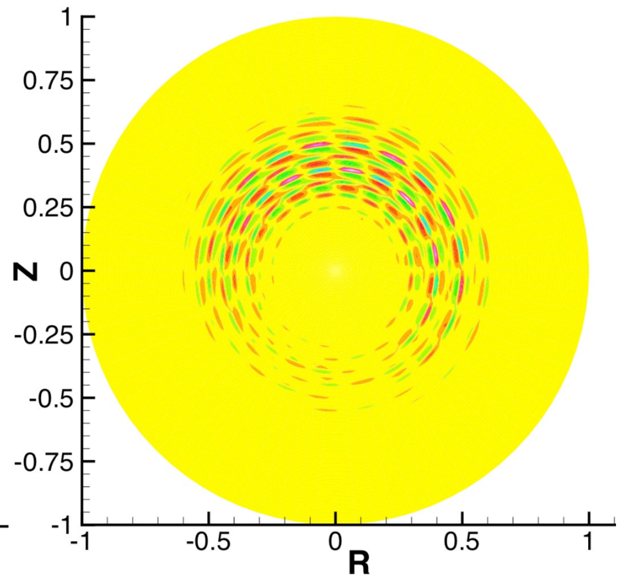

- Computations with the first-order systems use four elements.
- All of the continuous expansions have at least one zero-frequency mode, however.

**Time-dependent results:** a recent test branch of NIMROD allows independent specification of the polynomial degree of **B**, **V**, and *n*/*T*.

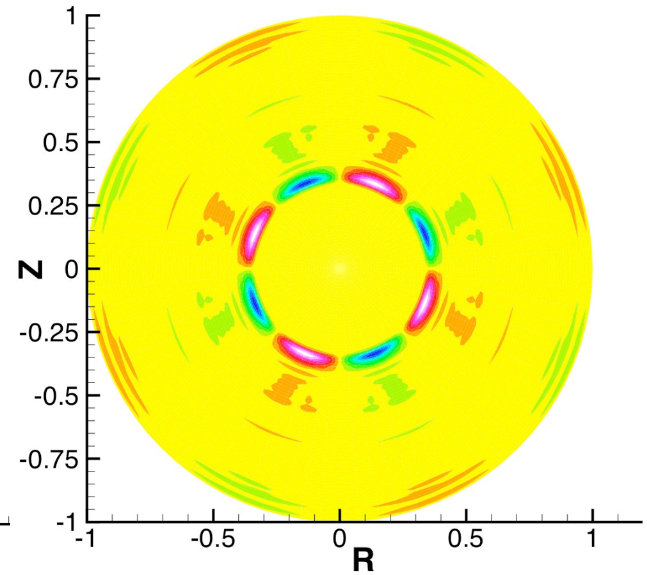
- This test is the same cylindrical profile with $c_1$=4/7 and $c_2$=10/7.
- The converged $m$=4, $k$=-1.78 ideal-MHD mode grows at $\gamma=6.77\times10^{-3}$.




**All fields have polynomial degree of 4, $\gamma=9\times10^{-4}$.**




**V, *n*, and *T* have reduced degree of 3, $\gamma=4\times10^{-3}$.**



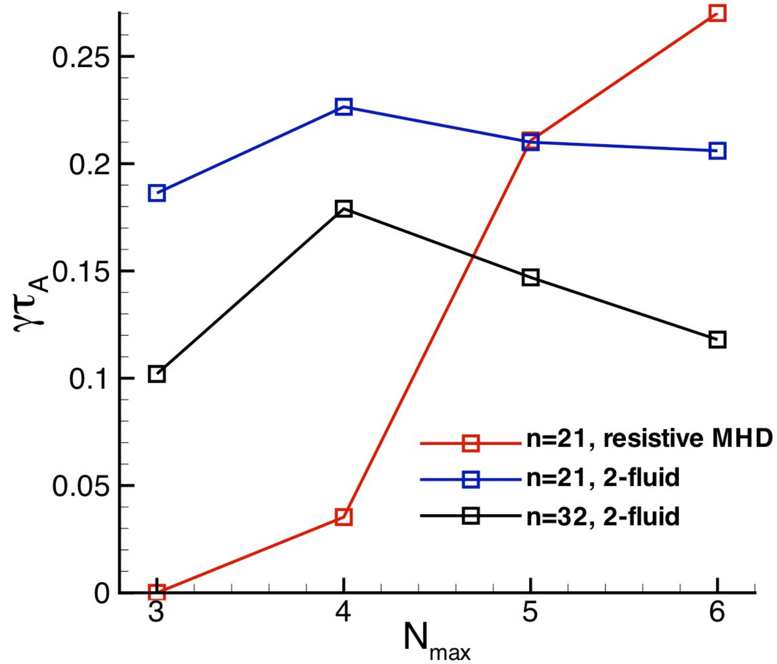

**Only V has reduced degree of 3, $\gamma=2\times10^{-5}$.**

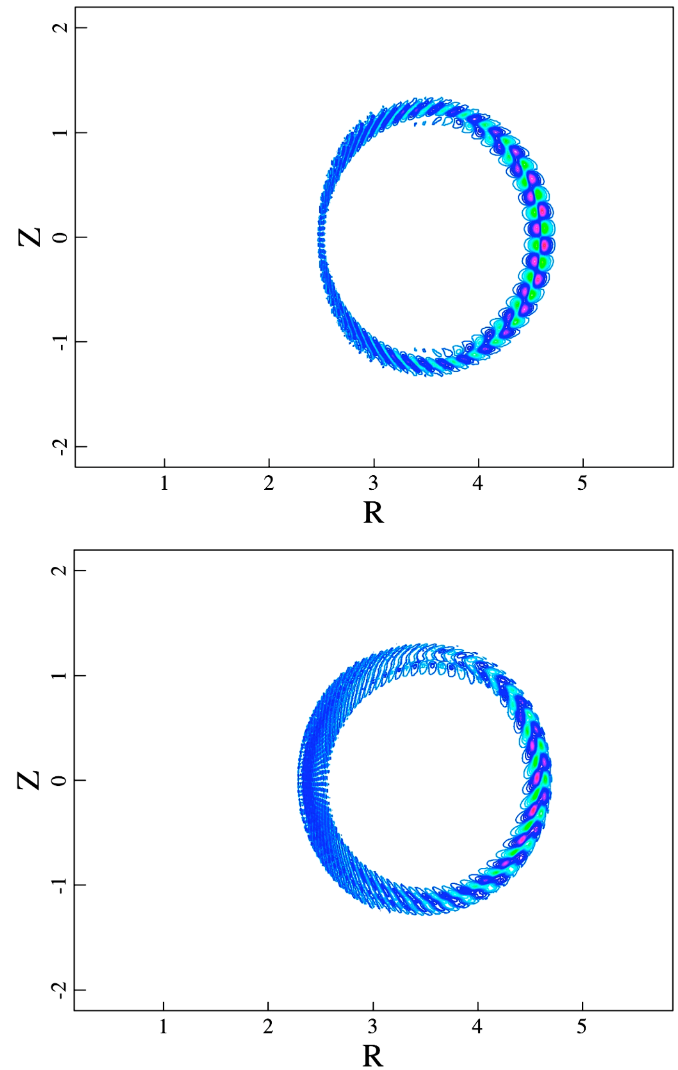
- These computations have no viscosity and $\chi_{iso}=\eta/\mu_0=10^{-8}$.
- With a uniform 20×20 mesh, the reduced-**V** & *n/T* case is overstable.

# The reduced-**V** expansion is the only representation of the three to avoid a numerical mode in computations for physically stable parameters.

- The equilibrium has $c_1$=1/2 and $c_2$=3/2, and $D_s$>1/4 for $r$<0.385.
- With $k$=-1.1 for $n$=1, $q(0)$=2.2 and m=2 is not resonant. The $m$=3 mode is resonant in the stable region.
- With a uniform 24×24 mesh, NIMROD computations with polynomial degree reduced for **V** show no exponential growth.
- The other two continuous representations show growing noise at $\gamma \cong 7\times10^{-3}$ for **V** expanded in polynomials of degree 4.

# Resistive-MHD computations using the reduced-**V** expansion on the dens8 ELM test show convergence from the stable side in toroidal geometry.




- The MHD computation has a 48×64 mesh, and the 2-fluid computations have a 42×96 mesh.

- The 2-fluid results may indicate competing numerical effects as resolution is increased.

**$V_\phi$ component of the n=21 mode for the MHD (top) and 2-fluid (bottom) models.**

# Discussion and Conclusions

- Expansions with continuous representations of flow-velocity and polynomial basis of one degree lower than those of other fields converges reliably from the stable side for MHD.
    - Ideal MHD spectra with 1D elements shows this property but points to unphysical stable modes between the Alfvén frequency and the first fast mode.
    - Tests with a modified version of NIMROD confirm convergence from the stable side with 2D spectral elements.
    - Tests with reduced number density and temperature representations allow growth of mesh-scale noise.
- More testing is needed for 2-fluid computations with the reduced-**V** expansion.
- The continuous expansions using lower polynomial degree for **V** may benefit from finite-element ‘stabilization’ or ‘filtering.’

# The Ion Temperature Gradient (ITG) Mode in NIMROD

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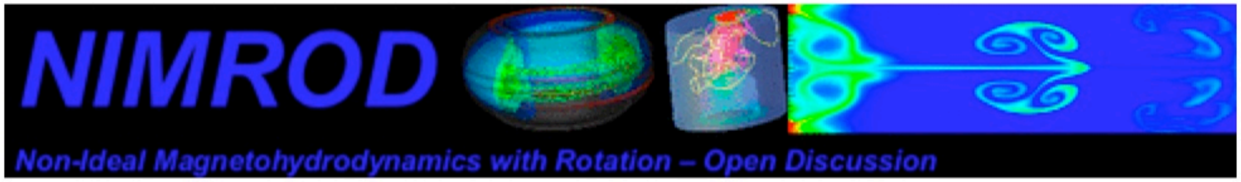


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# Equilibrium

- Profiles:

$$n_0(x) = \bar{n}_0 \exp\left(x / L_n\right)$$

$$P_{s0}(x) = n_0(x) T_{s0}(x) = \bar{n}_0 \exp\left(x / L_n\right) \bar{T}_{s0} \exp\left(x / L_{Ts}\right)$$

$$\equiv \bar{P}_{s0} \exp\left[\left(\eta_s + 1\right) x / L_n\right], \quad \eta_s \equiv L_n / L_{Ts}, \quad s = e, i$$

$$\frac{d}{dx}\left(P_{i0}(x) + P_{e0}(x) + \frac{B_0^2(x)}{2\mu_0}\right) = -M n_0(x) g$$

$B_0$ found by integration

- Drifts:

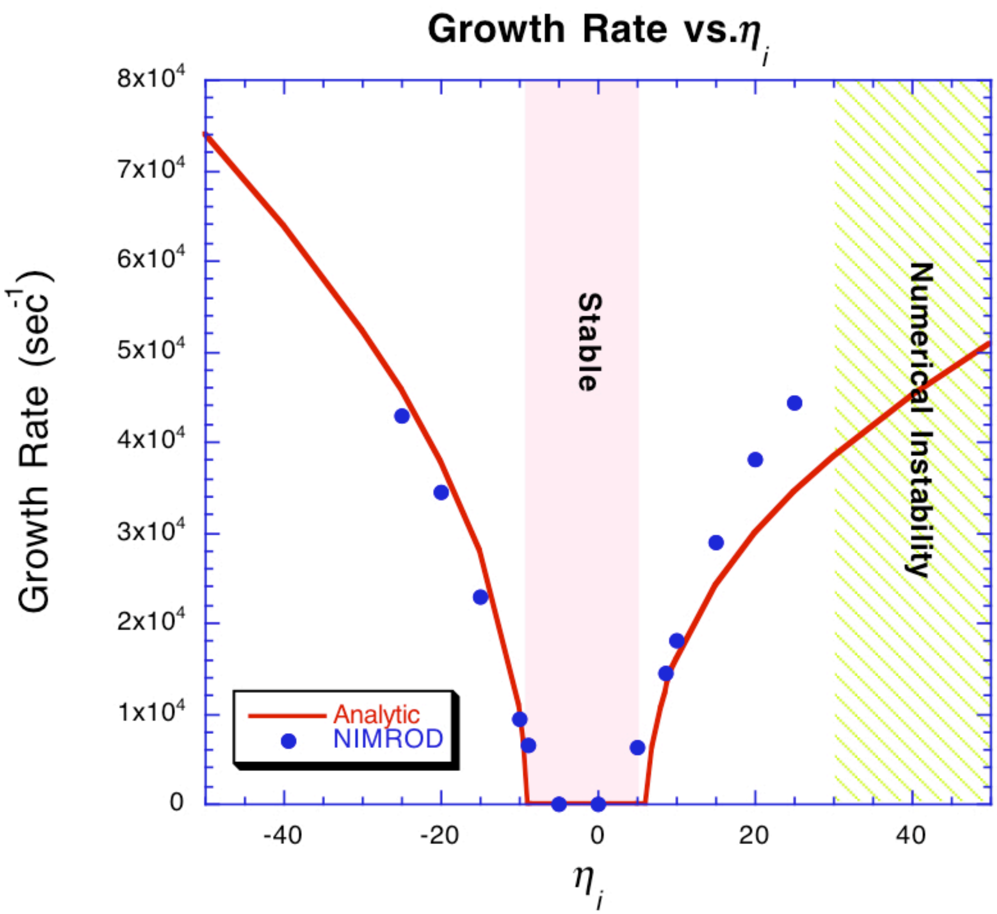
$$V_{yi0}(x) = \frac{E_{x0}}{B_0} + \frac{1}{n_0 e B_0} \frac{dP_{i0}}{dx} - \frac{g}{\Omega}$$

$$V_{ie0}(x) = \frac{E_{x0}}{B_0} - \frac{1}{n_0 e B_0} \frac{dP_{e0}}{dx}$$

$E_{x0}$ = constant
Coordinate transformation

Stable in ideal MHD with $g$ = 0.

# Preliminary NIMROD Results



# Convergence Properties of Spectral Elements on Thermal Transport in Chaotic Magnetic Fields

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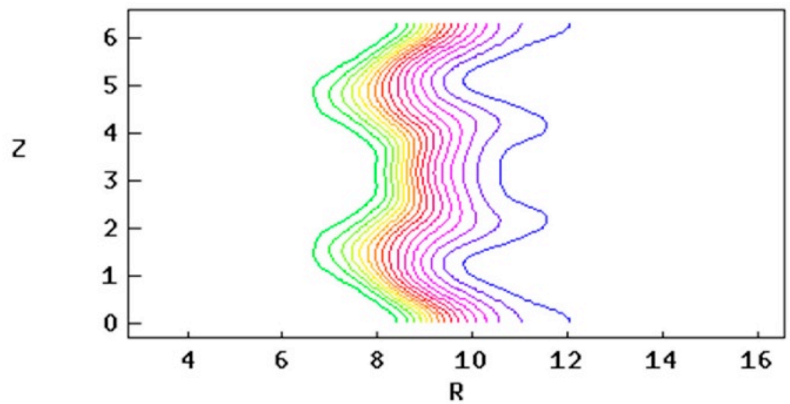
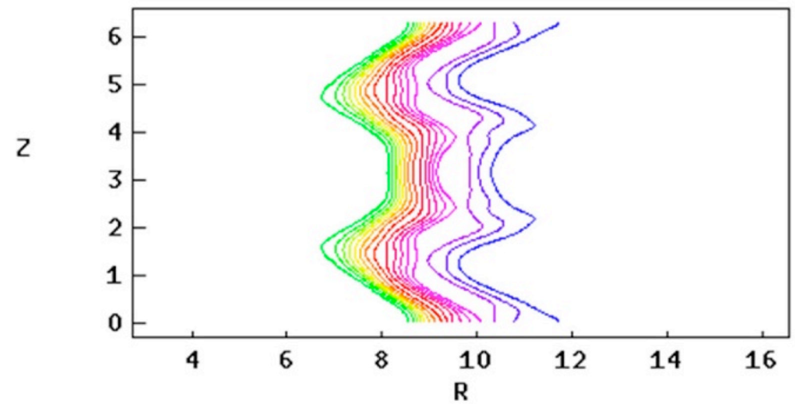
Tuesday, 3p2

As the anisotropy increases ($\kappa_{par}/\kappa_{perp}=10^8$), a higher poly. degree allows more features to be resolved.

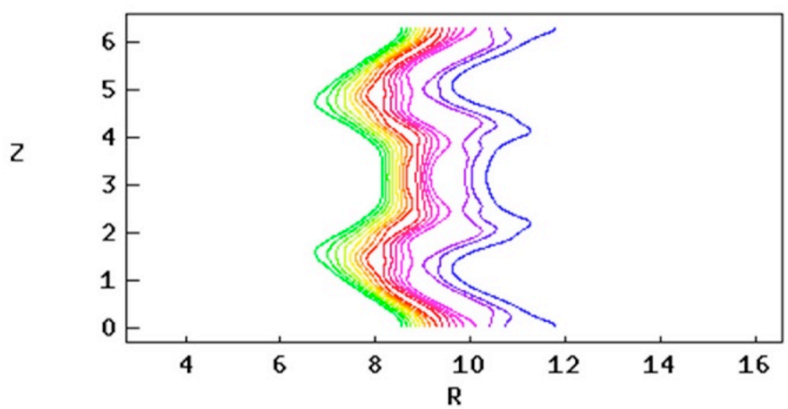
50x50 El.; P.D. 2; 22 F.C.



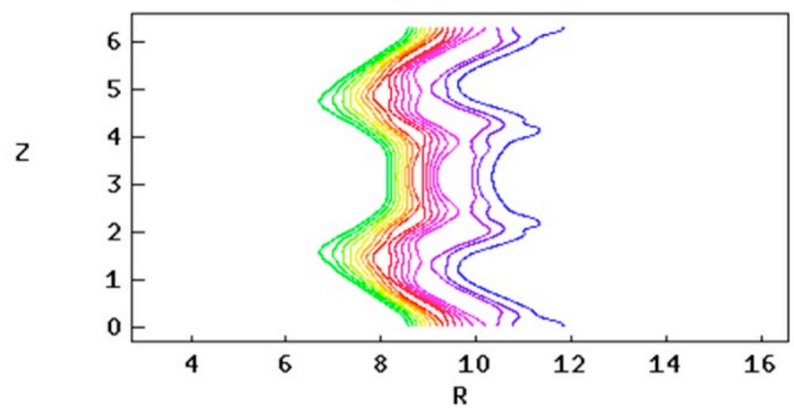

50x50 El.; P.D. 4; 22 F.C.




50x50 El.; P.D. 6; 22 F.C.

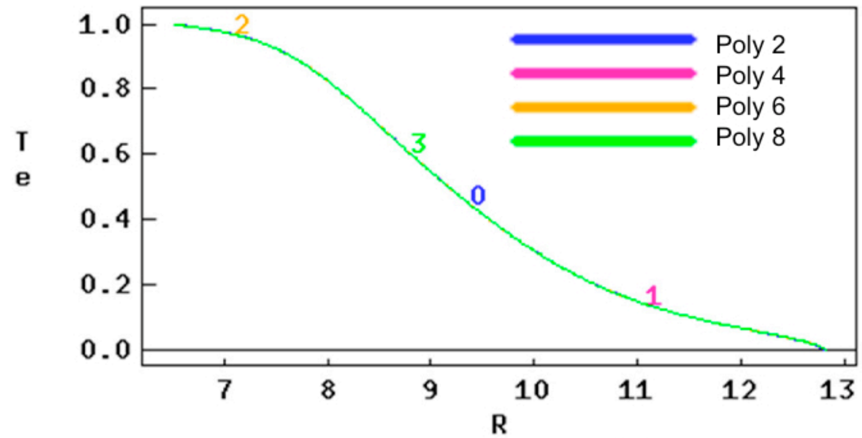



50x50 El.; P.D. 8; 86 F.C.

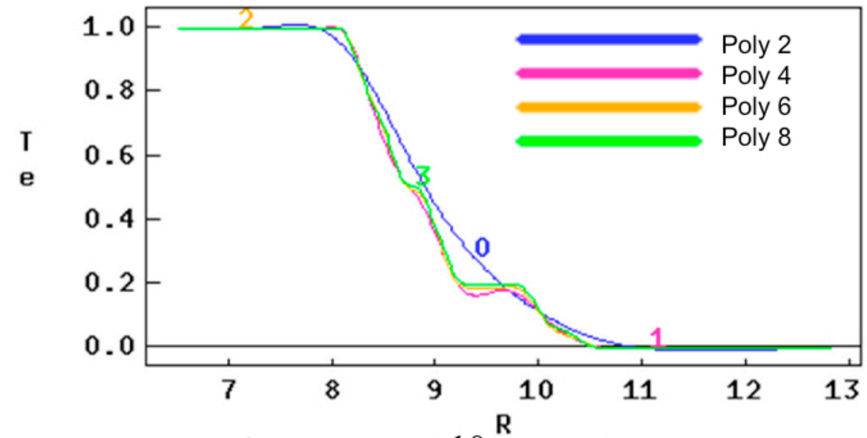


# For high anisotropy the temperature profile at the midplane begins to approximate a Devil's Staircase.

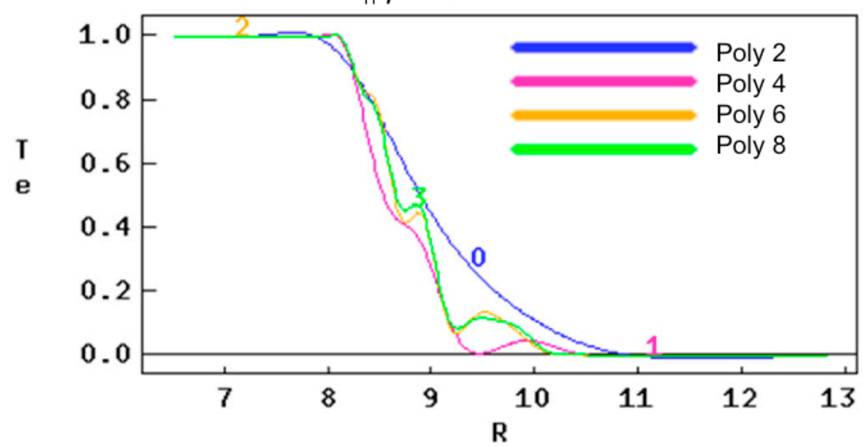
$\kappa_{\parallel}/\kappa_{\perp} = 10^5$



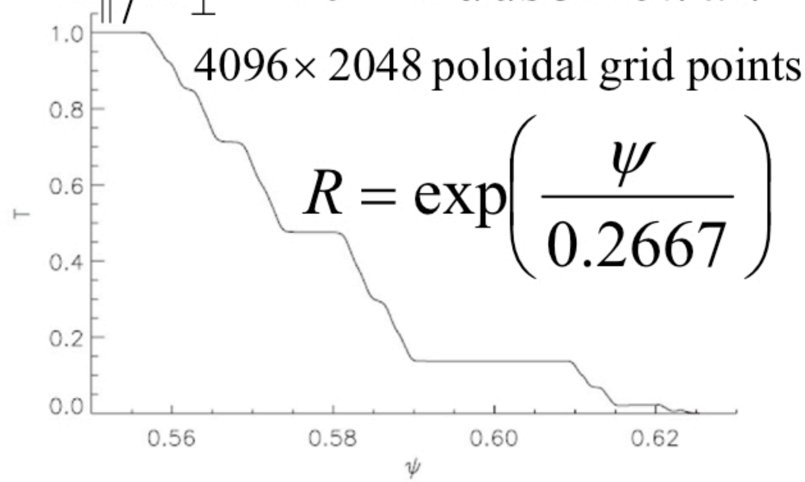

$\kappa_{\parallel}/\kappa_{\perp} = 10^8$




$\kappa_{\parallel}/\kappa_{\perp} = 10^{10}$




$\kappa_{\parallel}/\kappa_{\perp} = 10^{10}$ Hudson et. al.



# Helicity Injection in NSTX
# Status of Simulations

E. B. Hooper
in collaboration with Carl Sovinec

Nimrod Team meeting
April 28, 2011

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Security, LLC, Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344

LLNL-PRES-481328

# Plasma heating

**Plasma temperature — determined primarily by:**

- **ohmic heating**
- **thermal losses along open field lines to the wall**

$$3n\frac{dT}{dt} \approx \nabla_{\|}\left(\kappa_{\|}\cdot\nabla_{\|}T\right)+\eta_{\|}j_{\|}^{2}$$

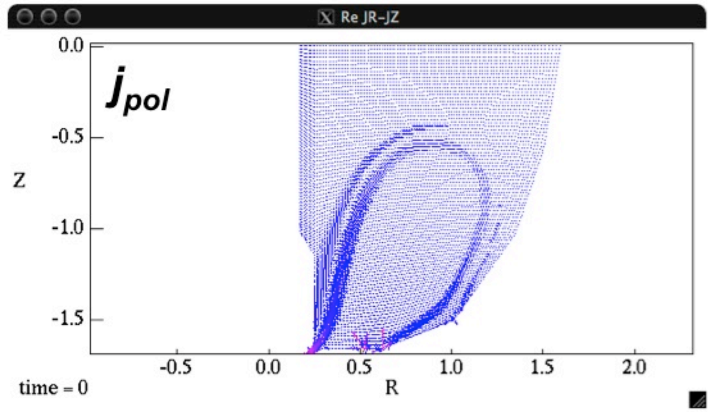
$$\kappa_{\|} \sim T^{5/2}\big/Z_{eff}$$

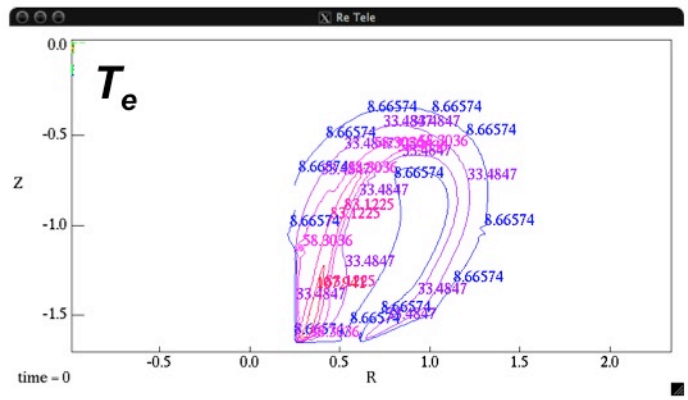
$$\eta_{\|} \sim Z_{eff}\big/T^{3/2}$$

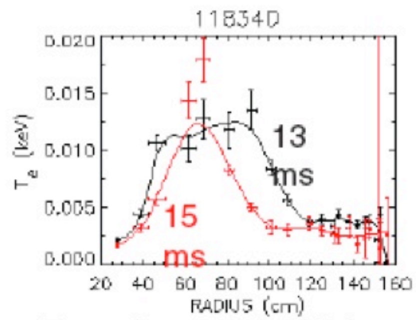
**so**

$$T \sim \left(Z_{eff}\, j_{\|}\ell\right)^{2/5}$$

**with $\ell$ an effective scale length**




Experiment at midplane

Simulation at 0.52 ms: $Z_{eff}$=1.

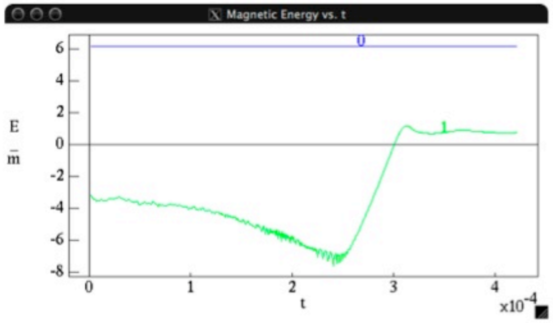
$T_e$ is highest (126 eV) near the lower left corner (small $R$) where poloidal flux tube areas ($2\pi Rw$) are small and $j_{\|}$ is large

Simulation temperatures

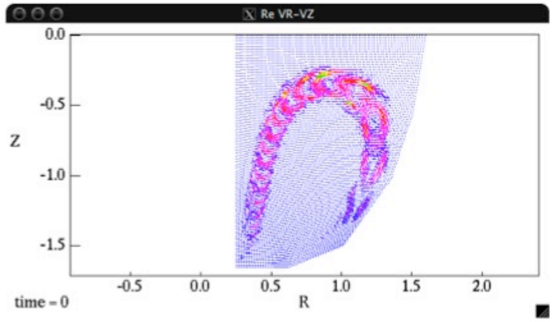
$T_e$ is consistent with experiment

# n=1 mode — PRELIMINARY RESULT

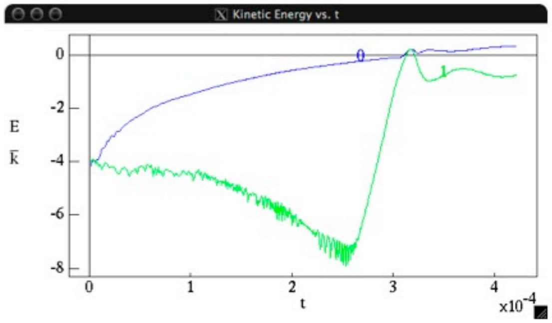
- Initial n=1 calculations show a mode in the discharge-current channel
- Fluid vortices and local current flow are generated in a region with large n=0 velocity and current shear



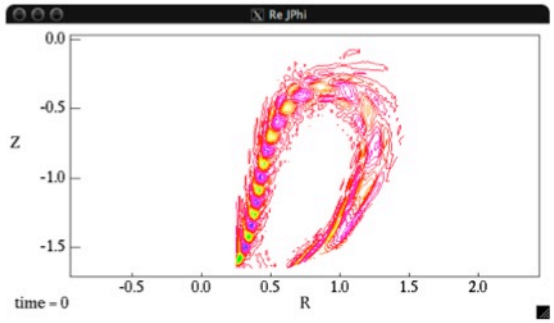

Magnetic energy




Poloidal velocity (vectors)




Kinetic energy




Azimuthal current (contours)

NSTX17/n=1A
t=0.41 ms