ECCD-induced Tearing Mode Stabilization via Active Control in Coupled NIMROD/GENRAY HPC Simulations

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Center for Simulation of RF Wave Interactions with Magnetohydrodynamics

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Problem: modeling the mitigation and control of neoclassical tearing modes by ECCD

- NTMs generate magnetic islands in tokamaks
  - -local flattening in plasma pressure profile
  - -altered plasma bootstrap current profile
  - -helical, self-reinforcing perturbations
- Island structures replace nested flux surfaces at rational surface

- Islands grow to macroscopic scales before nonlinearly saturating, causing degraded confinement and the possibility of disruption

DIII-D shot 122898

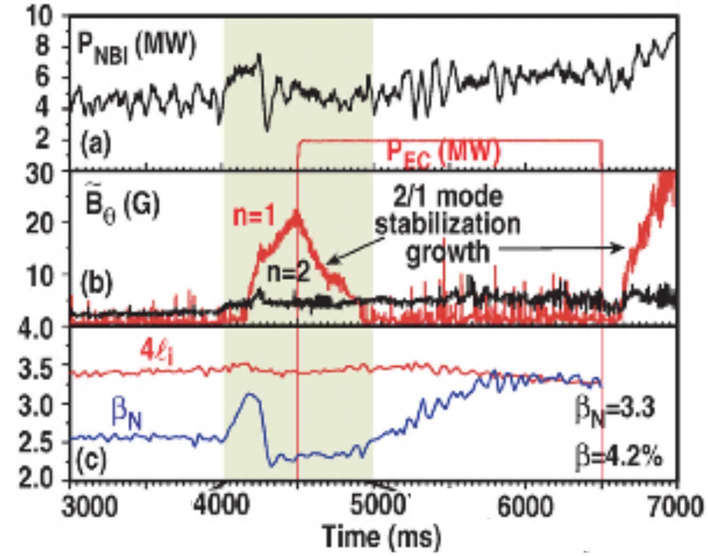
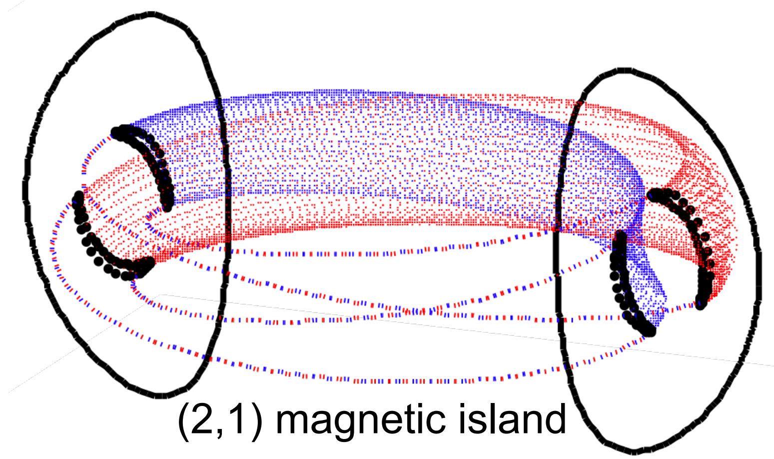


Figure from Prater *et al*., Nucl. Fusion **47**, 371 (2007).

- NTM control in ITER will be critical



- RF waves resonant with electron cyclotron motion can drive currents that alter or suppress island structures.

- For quantitative numerical prediction, need:
  - -self-consistent theoretical approach
  - -implementation of physics components (fluid, RF, control system codes)
  - -computational infrastructure

Separation of fluid and RF spatiotemporal scales underlies the theoretical framework

•Hegna and Callen [*Phys. Plasmas* **16**, 112501 (2009)] outline general formalism; Ramos [*Phys. Plasmas* **17**, 082502 (2010); **18**, 102506 (2011)] gives more rigorous detail.

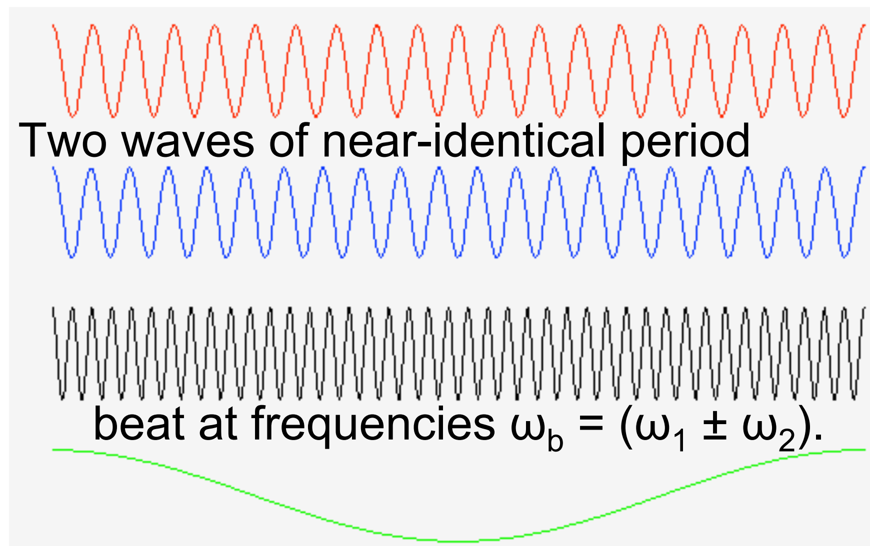
•On fluid timescales: average $\langle ... \rangle$ over RF timescale

$$f_\alpha = \langle f_\alpha \rangle + f_\alpha^{RF} \quad ; \quad \vec{E} = \langle \vec{E} \rangle + \vec{E}^{RF} \quad ; \quad B = \langle \vec{B} \rangle + \vec{B}^{RF}$$

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha + \frac{q_\alpha}{m_\alpha} \left[\left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \frac{\partial f_\alpha}{\partial \vec{v}} \right] = \sum_\beta C(f_\alpha, f_\beta)$$

•On RF timescales, fluid is static

•Kinetic equation has quadratic RF terms
-beating, quasilinear velocity-space diffusion on fluid timescales



Fluid equations are modified by quasilinear RF terms (though Maxwell equations are not)

•Building upon Hegna-Callen formalism:

$$f_\alpha = f_{M\alpha}(\vec{x},\vec{v},t) + \delta F_\alpha(\vec{x},\vec{v},t) + \varepsilon \,\mathrm{Re}\left[f_\alpha^{RF}(\vec{x},\vec{v},t) e^{i\psi(\vec{x},t)} \right]$$

local Maxwellian + kinetic distortion + RF-induced perturbation

$$\vec{E} = \vec{E}_0(\vec{x},t) + \varepsilon \,\mathrm{Re}\left[\vec{E}^{RF}(\vec{x},t) e^{i\psi(\vec{x},t)} \right]$$

$$\vec{B} = \vec{B}_0(\vec{x},t) + \varepsilon \,\mathrm{Re}\left[\vec{B}^{RF}(\vec{x},t) e^{i\psi(\vec{x},t)} \right]$$

background field + RF fields

-only the phase term $\psi(\vec{x},t)$ varies on RF spatiotemporal scales

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla}\cdot\left(n_\alpha \vec{V}_\alpha\right) = 0$$

RF does not create or destroy density

$$m_\alpha n_\alpha \left(\frac{\partial \vec{V}_\alpha}{\partial t} + \left(\vec{V}_\alpha \cdot \vec{\nabla}\right)\vec{V}_\alpha \right) = -\vec{\nabla}\left(n_\alpha T_\alpha\right) - \vec{\nabla}\cdot\overleftrightarrow{\Pi}_\alpha + q_\alpha n_\alpha\left(\vec{E}_0 + \vec{V}_\alpha \times \vec{B}_0\right) + \vec{R}_\alpha$$

$$+ \left\langle e^{-2\,\mathrm{Im}(\psi)} \right\rangle \frac{\varepsilon^2 q_\alpha}{2} \mathrm{Re}\left[\int f_\alpha^{*RF} \left(\vec{E}^{RF} + \vec{v}\times\vec{B}^{RF} \right) d^3\vec{v} \right]$$

RF contributes momentum (also, current)

$$\frac{3}{2} n_\alpha \left(\frac{\partial T_\alpha}{\partial t} + \left(\vec{V}_\alpha \cdot \vec{\nabla}\right) T_\alpha \right) + n_\alpha T_\alpha \vec{\nabla}\cdot\vec{V}_\alpha = -\vec{\nabla}\cdot\vec{q}_\alpha - \overleftrightarrow{\Pi}_\alpha : \vec{\nabla}\vec{V}_\alpha + Q_\alpha$$

$$+ \left\langle e^{-2\,\mathrm{Im}(\psi)} \right\rangle \frac{\varepsilon^2 q_\alpha}{2} \mathrm{Re}\left[\vec{E}^{RF} \cdot \int f_\alpha^{*RF} \vec{v}\, d^3\vec{v} \right]$$

RF contributes energy

•Can now make extended MHD approximations (quasineutrality, etc.)

•Need to solve closure problem (what are $\vec{q}_\alpha$ and $\overleftrightarrow{\Pi}_\alpha$?), calculate the RF propagation, and evaluate quasilinear terms

Compatible orderings/closures in fusion-relevant regime are rigorously addressed by Ramos

•Ramos [*Phys. Plasmas* **15**, 082106 (2008); **17**, 082502 (2010); **18**, 102506 (2011)] has developed a rigorous, self-consistent closure scheme for low-collisionality, NTM-relevant regimes using moments of a drift-kinetic equation.

$$\overleftrightarrow{\Pi}_\alpha = m_\alpha \int F_\alpha \left(\vec{v} - \vec{V}_\alpha\right)\left(\vec{v} - \vec{V}_\alpha\right) d^3\vec{v}$$

$$\vec{q}_\alpha = \frac{m_\alpha}{2} \int F_\alpha \left(\vec{v} - \vec{V}_\alpha\right)\left[\left(\vec{v} - \vec{V}_\alpha\right)\cdot\left(\vec{v} - \vec{V}_\alpha\right)\right] d^3\vec{v}$$

•The ensuing scheme is compatible with the addition of an RF source
-additional RF terms arise in closure calculation

•Extended MHD code can be used to model mode growth in the presence of RF – we use NIMROD.

Fokker-Planck physics effects are captured by the closure terms

- RF interaction increases electron $v_\perp$
- Lower collisionality $\sim v^{-3}$
- Net momentum transfer between ions and electrons; current

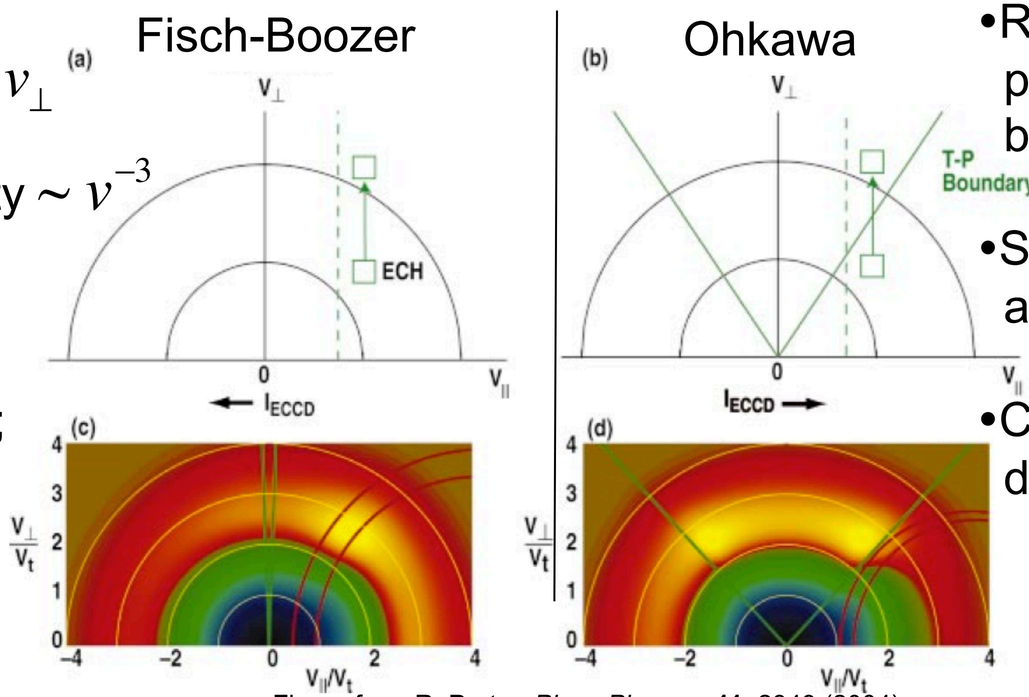


Figure from R. Prater, *Phys. Plasmas* **11**, 2349 (2004).

- RF interaction moves particles across T-P boundary
- Symmetric detrapping; asymmetric trapping
- Current (opposite direction)

Linear wave propagation through inhomogeneous fluid profiles captures the salient RF physics

$\psi\left(\vec{x},t\right)=\vec{k}\cdot\vec{x}-\omega t$ is the phase of the RF wave (varies rapidly in time and space)

•Dominant RF terms (ray optics approximation) describe linear wave propagation:

$$\vec{k}\cdot\vec{E}^{RF}=-\sum_{\alpha}\frac{iq_\alpha}{\varepsilon_0}\int f_\alpha^{RF}d^3\vec{v}$$

$$\vec{k}\times\vec{E}^{RF}=\omega\vec{B}^{RF}$$

$$c^2\vec{k}\times\vec{B}^{RF}=-\omega\vec{E}^{RF}-\sum_{\alpha}\frac{iq_\alpha}{\varepsilon_0}\int f_\alpha^{RF}\vec{v}\,d^3\vec{v}$$

$\vec{k}$ is complex (imaginary part dissipates RF momentum and energy into plasma)

•Distribution function is [generalizing Kennel/Engelmann, *Phys. Fluids* **9**, 2377 (1966)]:

$$f_\alpha^{RF}=e^{iz\sin\phi}\sum_{n=-\infty}^{\infty}\frac{ie^{-in\phi}q_\alpha f_{M\alpha}}{\omega T_\alpha}\left[-J_n(z)V_{\parallel\alpha}\hat{b}+\underbrace{\frac{\left(\omega-k_\parallel V_{\parallel\alpha}\right)}{\left(\omega-k_\parallel v_\parallel-n\Omega_\alpha\right)}}_{\text{cyclotron resonance, Doppler shift, etc.}}\left(\frac{nv_\perp J_n(z)}{z}\hat{k}_\perp+iv_\perp J_n'(z)(\hat{b}\times\hat{k}_\perp)+v_\parallel J_n(z)\hat{b}\right)\right]\cdot\vec{E}^{RF}$$

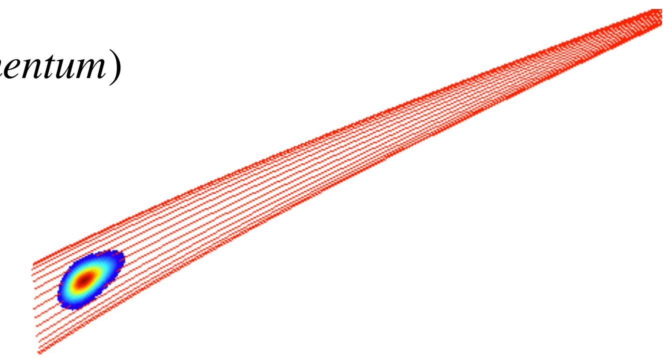
from which a dispersion relation can be constructed.

•Characteristic solutions along trajectories of constant ψ can be determined from ray tracing codes (e.g. GENRAY).

Knowing the RF solutions allows us to construct the quasilinear terms analytically

$$\left\langle e^{-2\,\mathrm{Im}(\psi)}\right\rangle \frac{\varepsilon^2 q_\alpha}{2} \mathrm{Re}\left[\int f_\alpha^{*RF}\left(\vec{E}^{RF}+\vec{v}\times\vec{B}^{RF}\right)d^3\vec{v}\right]=\vec{k}_r H_\alpha \qquad (momentum)$$

$$\left\langle e^{-2\,\mathrm{Im}(\psi)}\right\rangle \frac{\varepsilon^2 q_\alpha}{2} \mathrm{Re}\left[\vec{E}^{RF}\cdot\int f_\alpha^{*RF}\vec{v}\,d^3\vec{v}\right]=\omega H_\alpha \qquad (energy)$$



[details in Jenkins/Kruger, *Phys. Plasmas* **19**, 122508 (2012)]

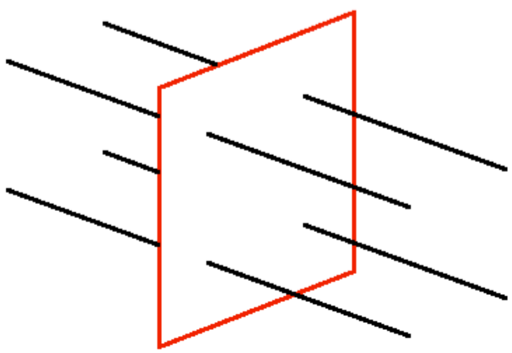
$$H_\alpha=\left\langle e^{-2\vec{k}_i\cdot\vec{x}}\right\rangle \sum_{n=-\infty}^{\infty}\frac{q_\alpha^2 n_\alpha\sqrt{\pi}\left(\omega-k_{\parallel r}V_{\parallel\alpha}\right)}{4\sqrt{2}\,\omega^2 k_{\parallel r}\sqrt{m_\alpha T_\alpha}}\exp\left(-\frac{k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\right)\exp\left(-\frac{m_\alpha\left(\omega-k_{\parallel r}V_{\parallel\alpha}-n\Omega_\alpha\right)^2}{2k_{\parallel r}^2 T_\alpha}\right)$$

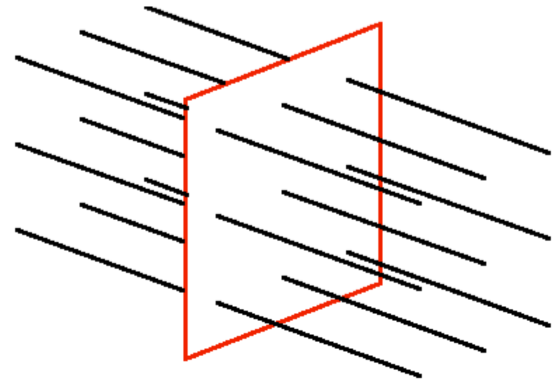
$$\left\{\left[I_n\left(\frac{k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\right)-I_{n+1}\left(\frac{k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\right)\right]\left(\frac{2k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\left|E_y^{RF}\right|^2+n\left|E_x^{RF}-iE_y^{RF}+\frac{k_{\perp r}\left(\omega-n\Omega_\alpha\right)E_z^{RF}}{n\Omega_\alpha k_{\parallel r}}\right|^2\right)\right.$$

$$\left.+\left[I_n\left(\frac{k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\right)-I_{n-1}\left(\frac{k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\right)\right]\left(\frac{2k_{\perp r}^2 T_\alpha}{m_\alpha\Omega_\alpha^2}\left|E_y^{RF}\right|^2-n\left|E_x^{RF}+iE_y^{RF}+\frac{k_{\perp r}\left(\omega-n\Omega_\alpha\right)E_z^{RF}}{n\Omega_\alpha k_{\parallel r}}\right|^2\right)\right\}$$

H_α is only known along ray trajectories – but we need the global solution for RF fields

How do solutions along discrete ray trajectories relate to the global RF solution?





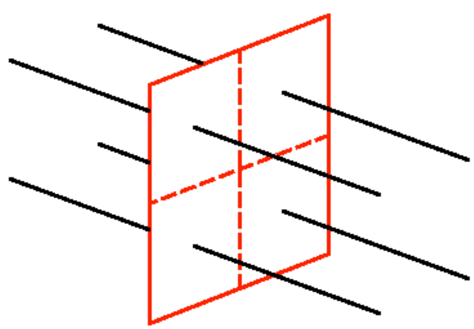
•Increasing the number of rays shouldn't change the global physics

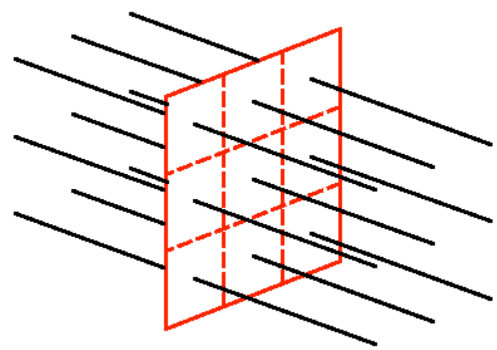
-RF ray bundle must carry same total power P_0

-Each ray must then carry a smaller fraction of P_0 if N is increased

•Power flux through the plane should be constant regardless of N, if converged

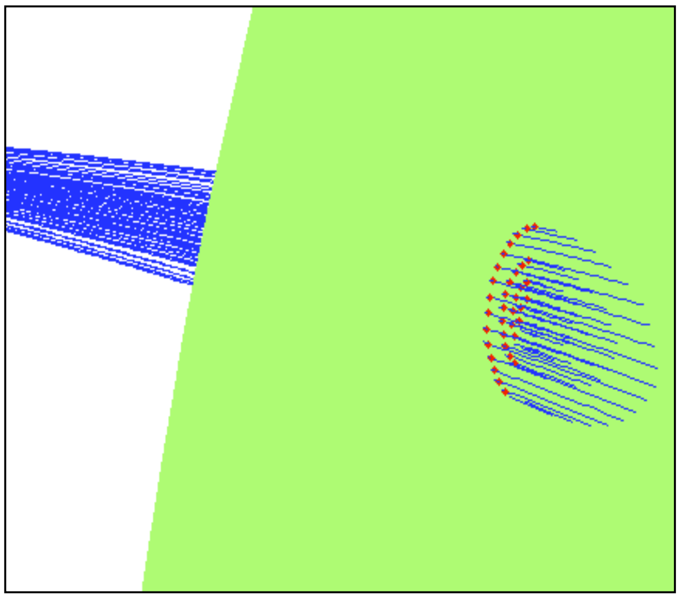
-Effective area associated with each ray is smaller



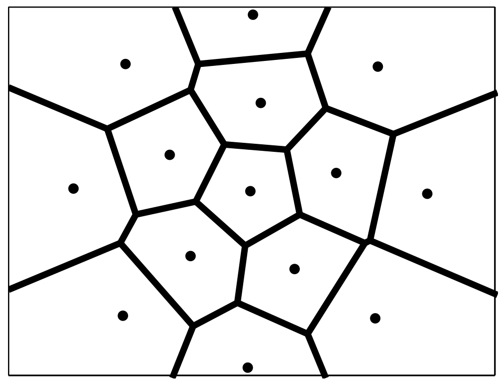


Local field values must conserve the total power as the rays diverge

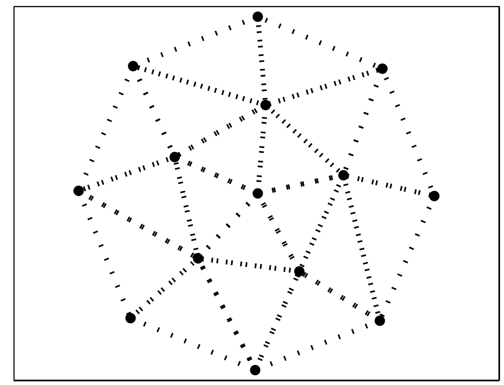
- To calculate divergence, need local area to calculate Poynting flux
- Area elements relate discrete values (along trajectories) to global quantities



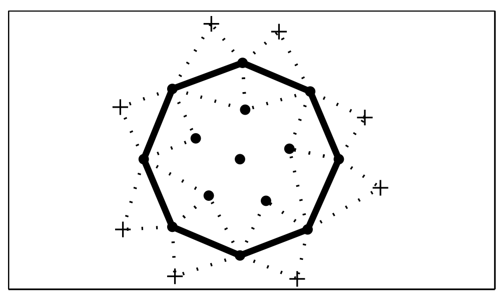
- Rays must be closely packed enough that inter-ray spacing distance << characteristic xMHD scale lengths



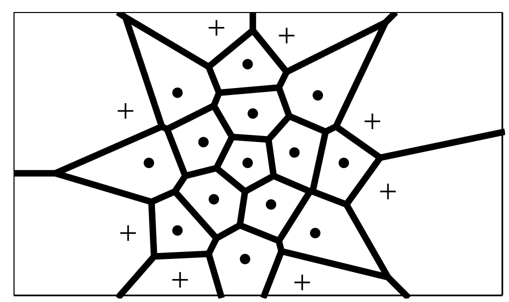
Voronoi tessellation (some infinite areas?)



Delaunay triangulation



Reflection over convex hull



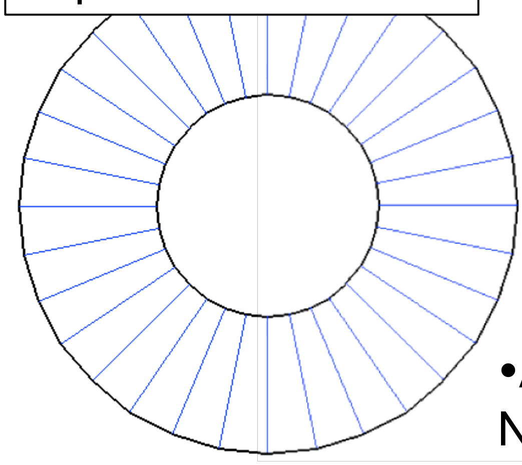
Bounded area elements

- Now, have exact RF solution, but still only at trajectory points…

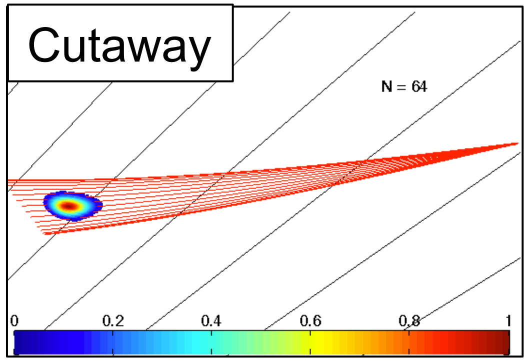
Exact RF solutions need to be interpolated onto NIMROD basis functions

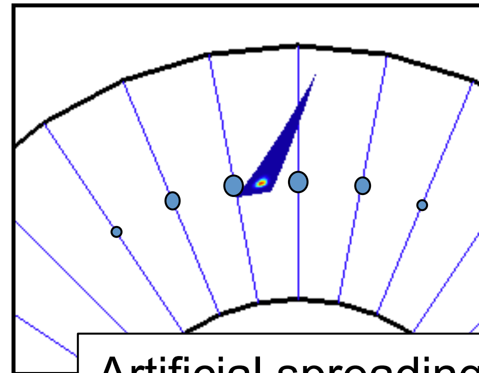
•NIMROD uses a Fourier representation in the toroidal direction – more Fourier modes = more collocation planes around the torus

Top view of tokamak



Cutaway



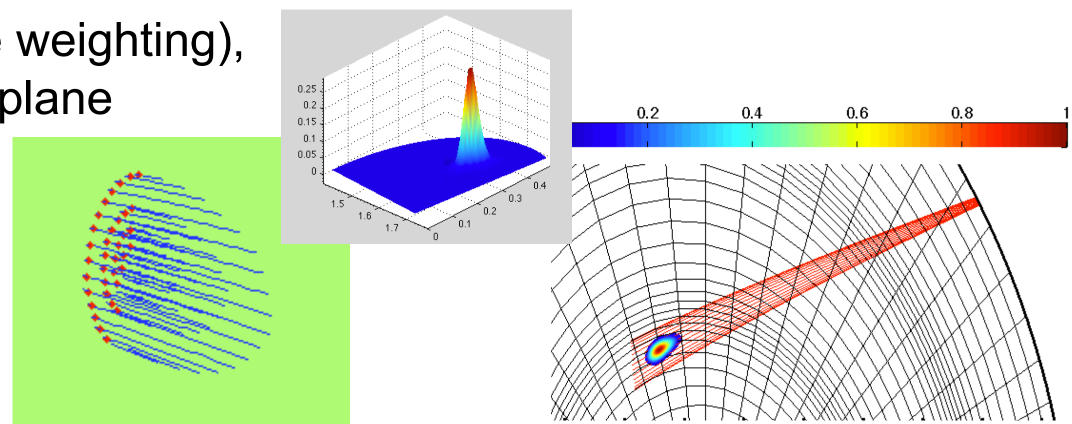


Artificial spreading?

•Alternatively, just use high Fourier resolution (good scaling at NERSC/OLCF with >33k processors and 512 Fourier modes)

•In NIMROD poloidal planes, a finite element representation is used

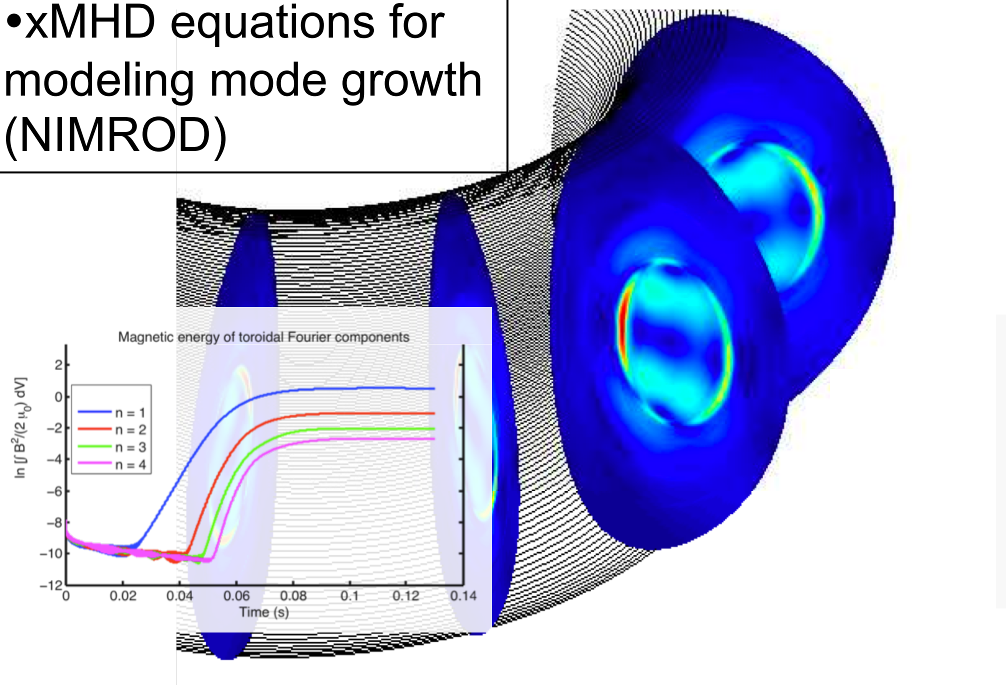
•Shepard algorithm (inverse distance weighting), applied to crossing points in poloidal plane
  -yields a smooth function
  -project this function onto FE basis
  -increased resolution generally not needed



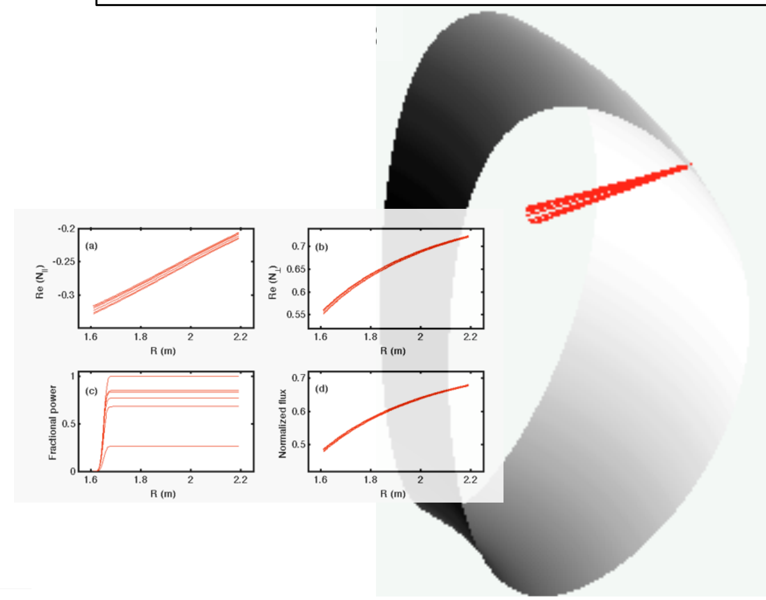
We need to determine how the physics components in the simulation will interact

All the physics components are in place:

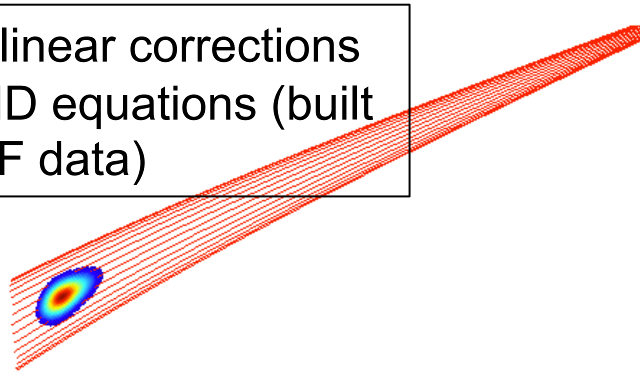
•xMHD equations for modeling mode growth (NIMROD)



•Ray tracing equations for linear RF propagation (GENRAY)



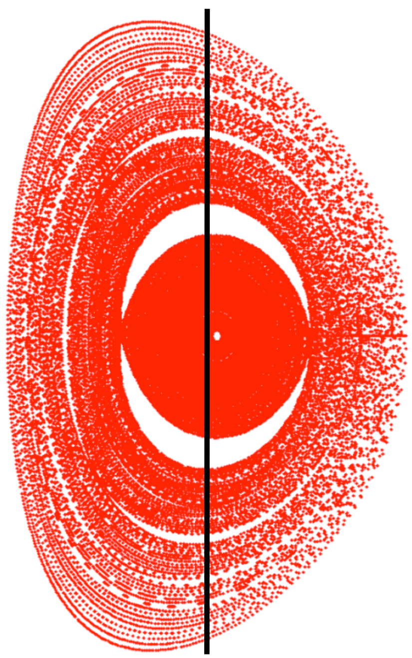
•Quasilinear corrections to xMHD equations (built from RF data)



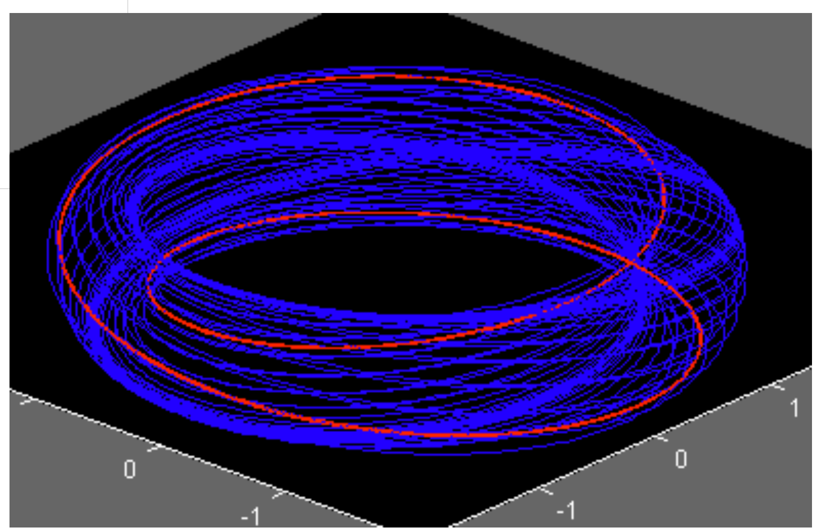
•Interpolation methods to relate RF and xMHD representations

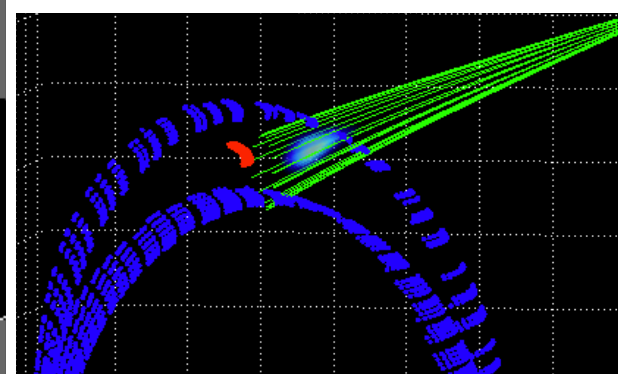
•Where should we put the RF? How do we control it? To what does it respond?

Depositing RF at island O-point: a 3D targeting problem whose solution may vary in time



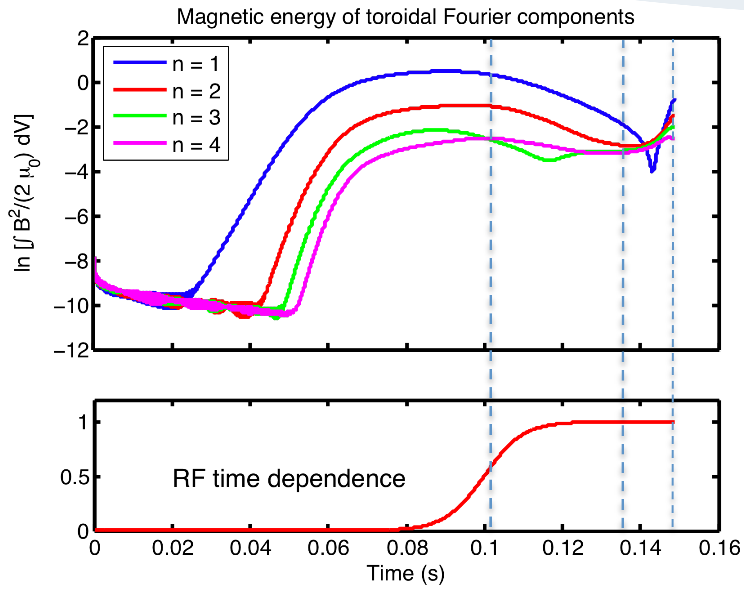
- Target island O-point for optimum mode suppression
  - -Hegna & Callen, *Phys. Plasmas* **4**, 2940 (1997)
  - -Pletzer & Perkins, *Phys. Plasmas* **6**, 1589 (1999)
- Constraints: cyclotron frequency primarily varies with toroidal field
  - -RF frequency determines resonance location
  - -toroidal launcher position constrained by machine geometry
- Experimental approaches:
  - -Alter toroidal field or plasma position (computationally complicated, not relevant to ITER)
  - -steerable mirrors to alter RF path (our approach)



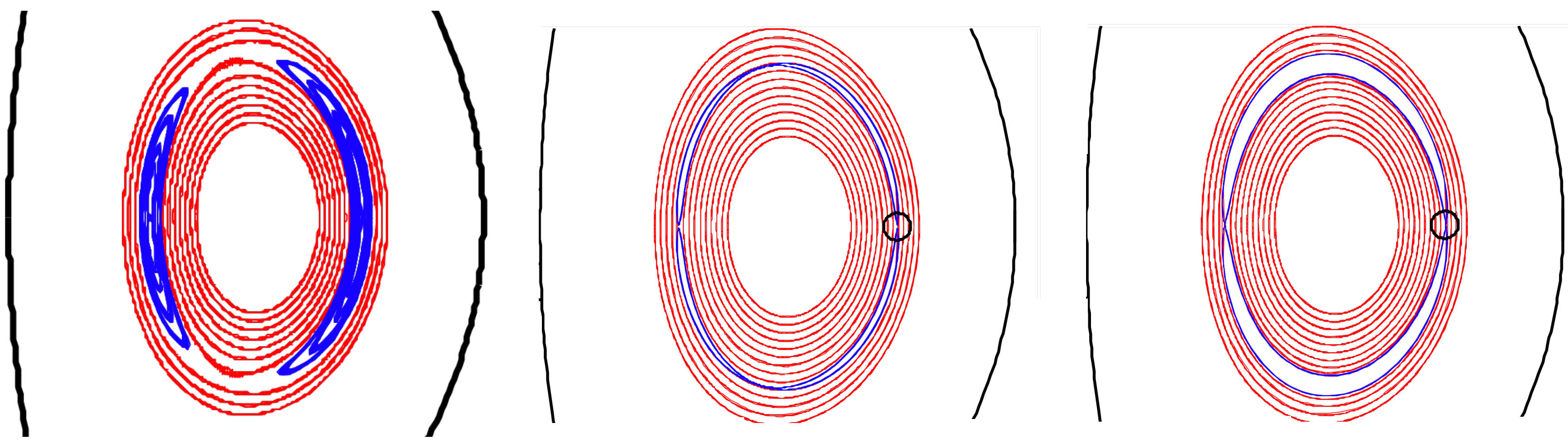


- Toroidal rotation = O-point rotation in a fixed poloidal plane; cannot always hit island O-point

In the absence of toroidal rotation, fixed RF stabilizes the resistive TM only temporarily

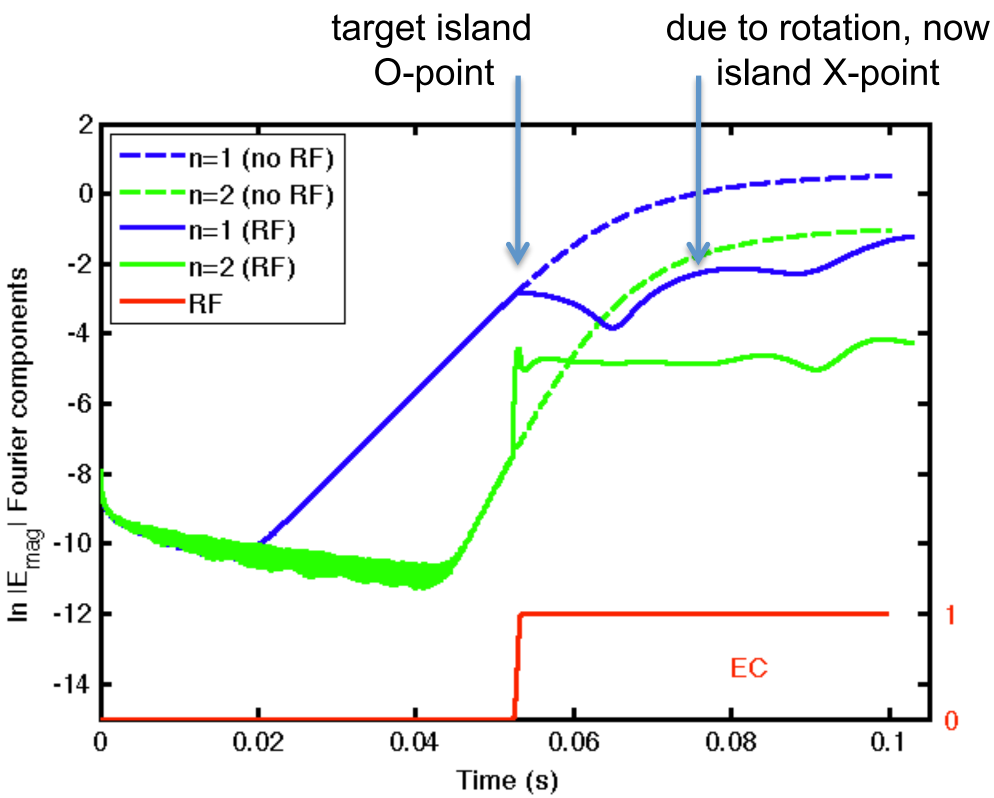


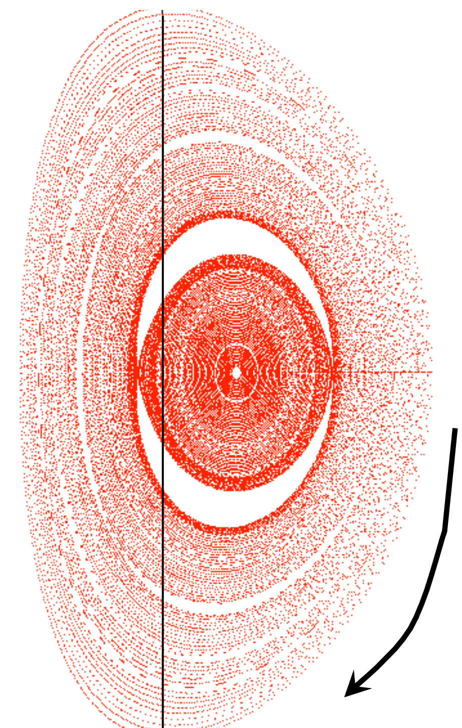
- Inject RF at O-point of saturated (2,1) island
- (4,2) island forms, mode energy decreases (stabilization?)
- (2,1) island with different O-point grows up again



- Here, island size and RF hotspot size are initially comparable.

With toroidal rotation, holding RF fixed in space only partially impedes resistive TM growth





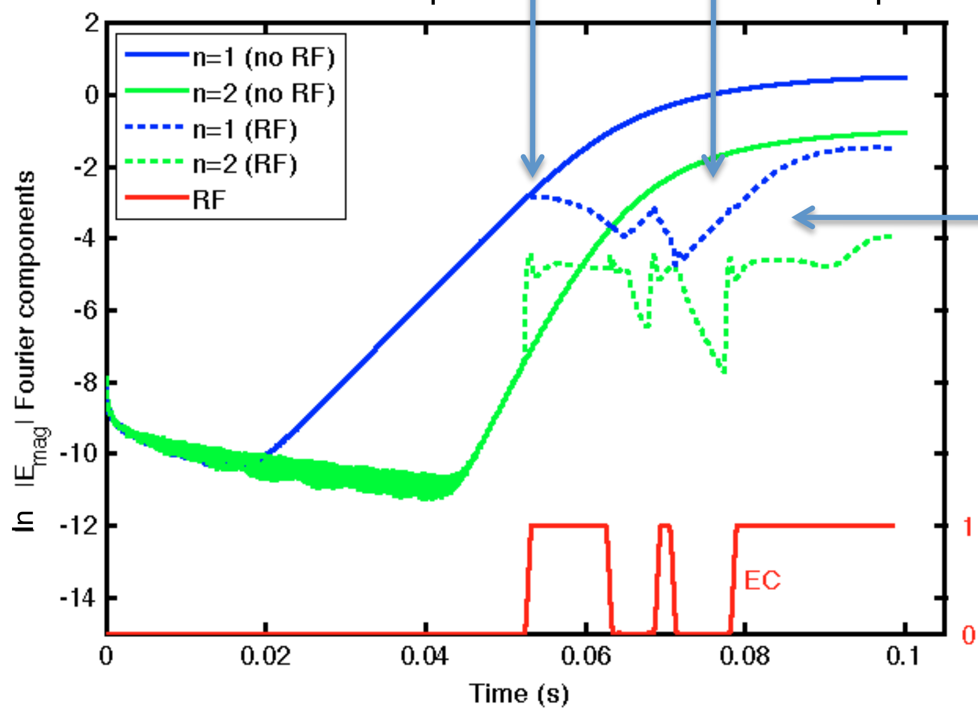
Actual islands are much smaller (not visible at suppression point)

- Growth is initially reversed, but then resumes at a slower rate
- Different RF positioning can reduce or enhance growth rate

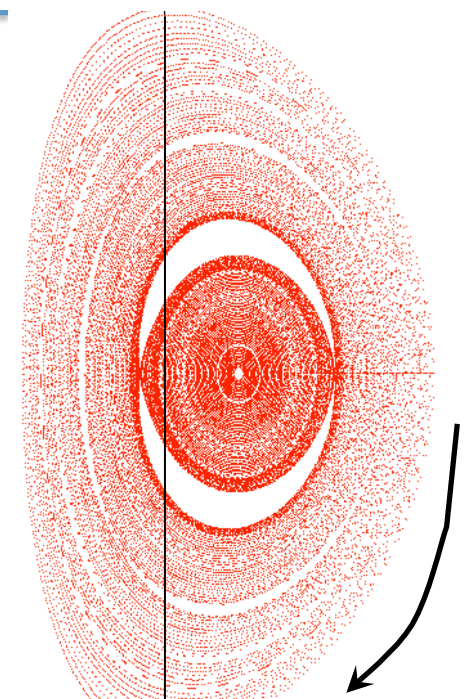
RF can be switched on and off strategically to promote stabilization

target island O-point

due to rotation, now island X-point



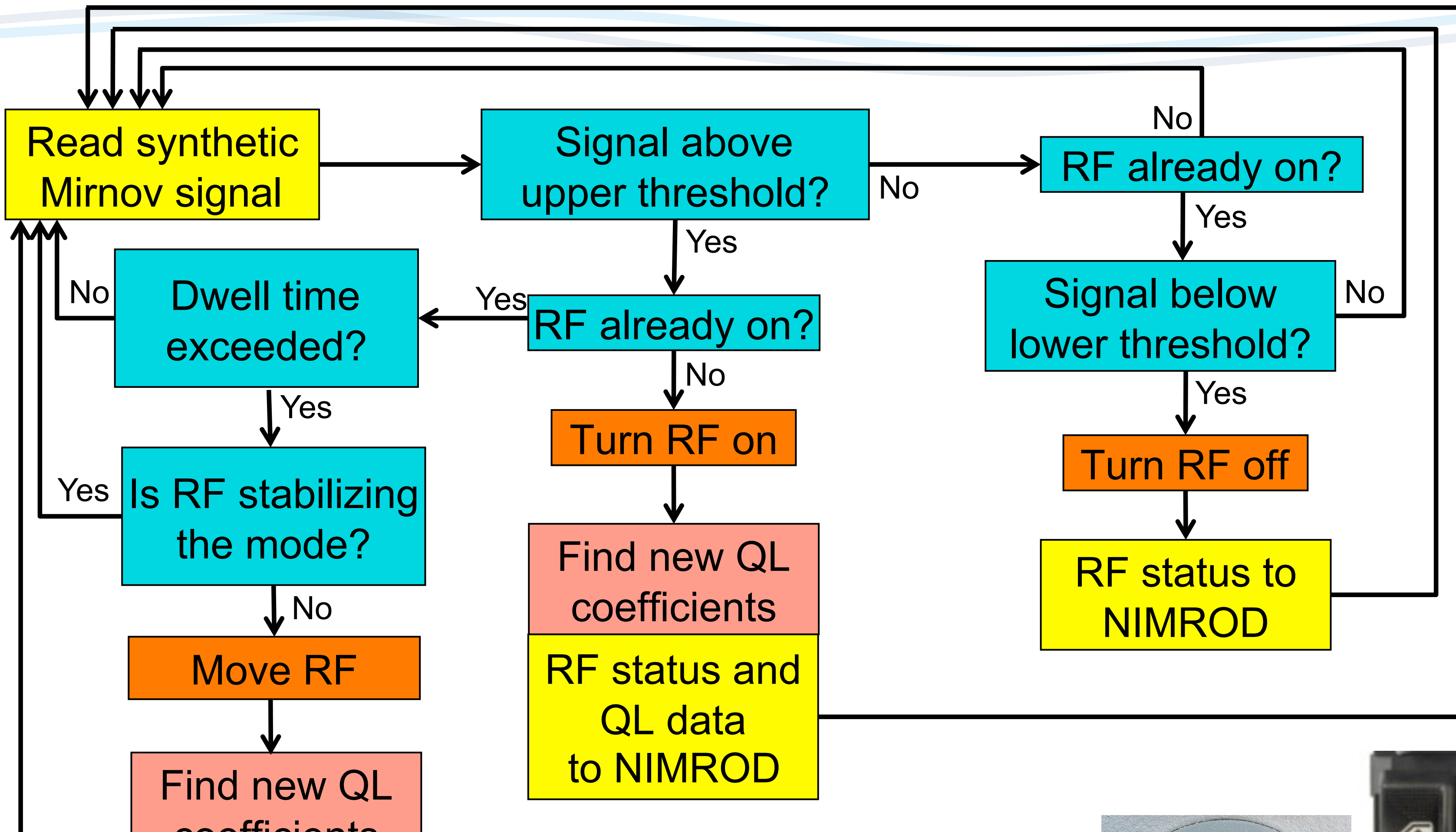
n=1 linear growth rate the same, suggesting no Δ' modification by RF

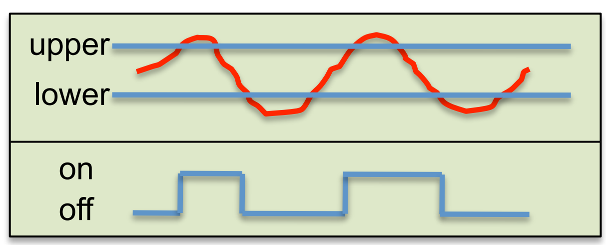


- Saturation level is decreased, so some success here…

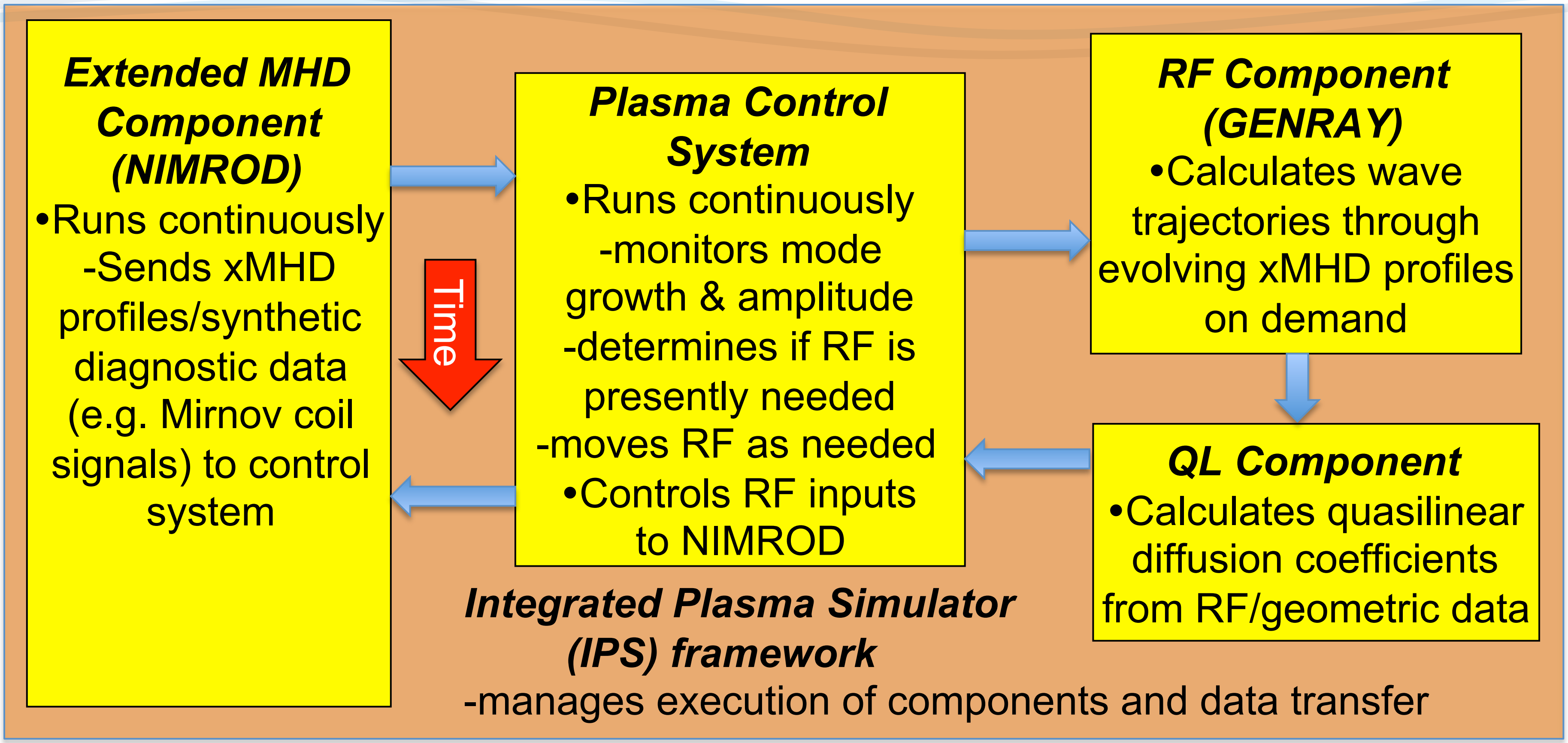
Actual islands are much smaller (not visible at suppression point)

Basic control system: an RF thermostat



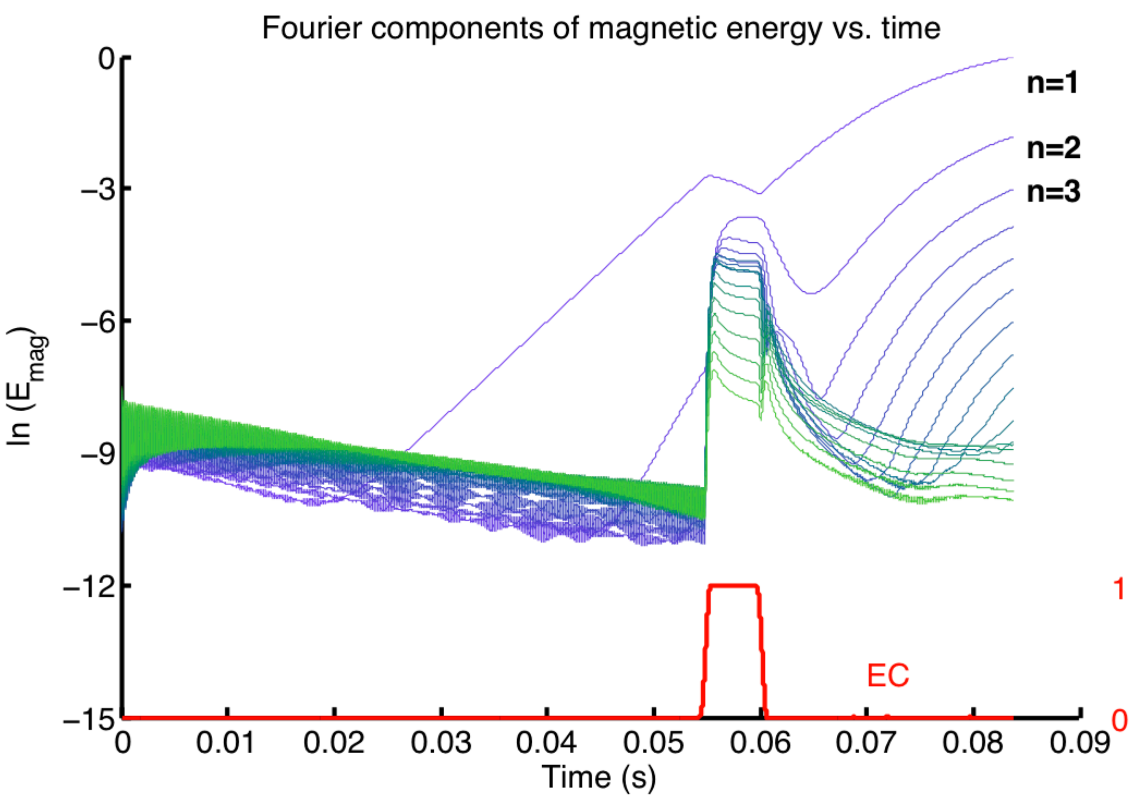


The control system is an additional physics component in the coupled simulation



- All physics components run in a larger simulation framework (IPS)
- Explicit coupling exploits the timescale separation between RF and xMHD

Initial results of coupled simulations are promising



- Control system aligns RF, halts mode growth, shrinks island.

- Growth resumes when RF is shut off.

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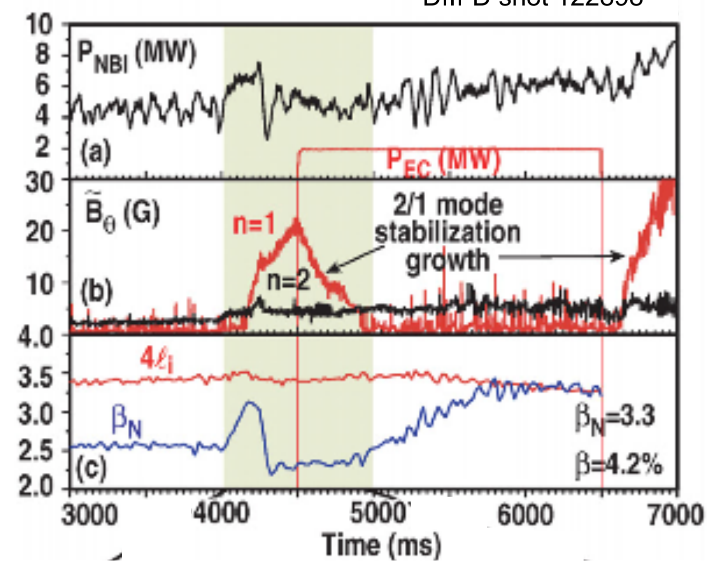
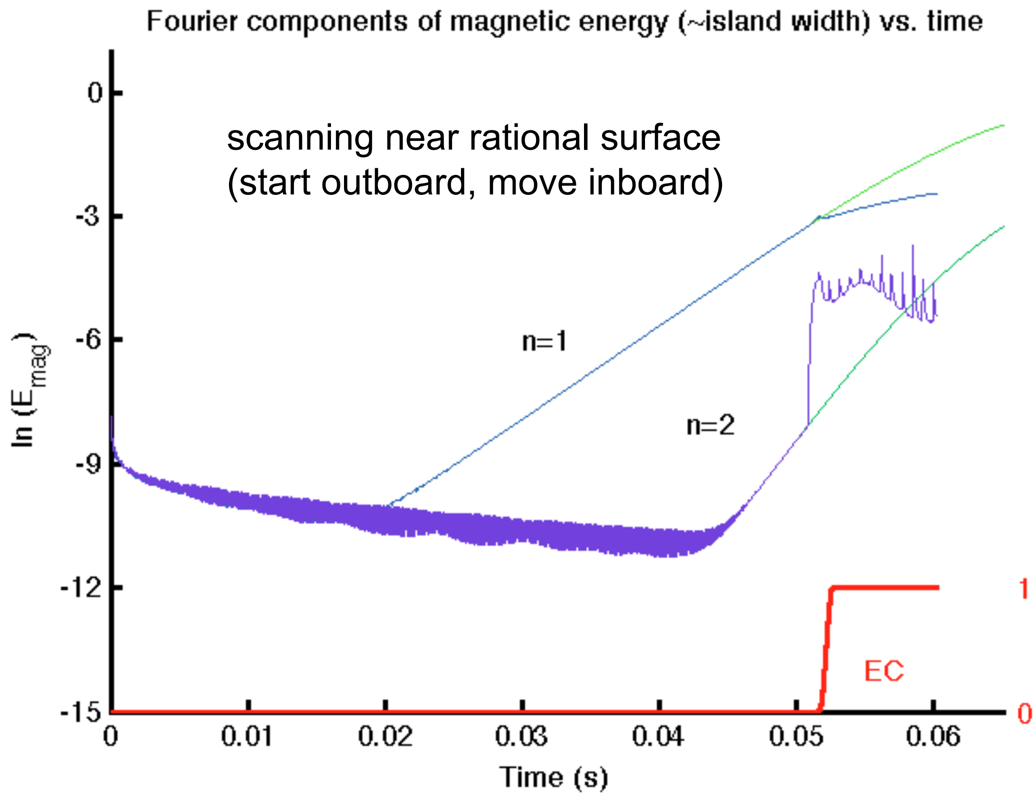
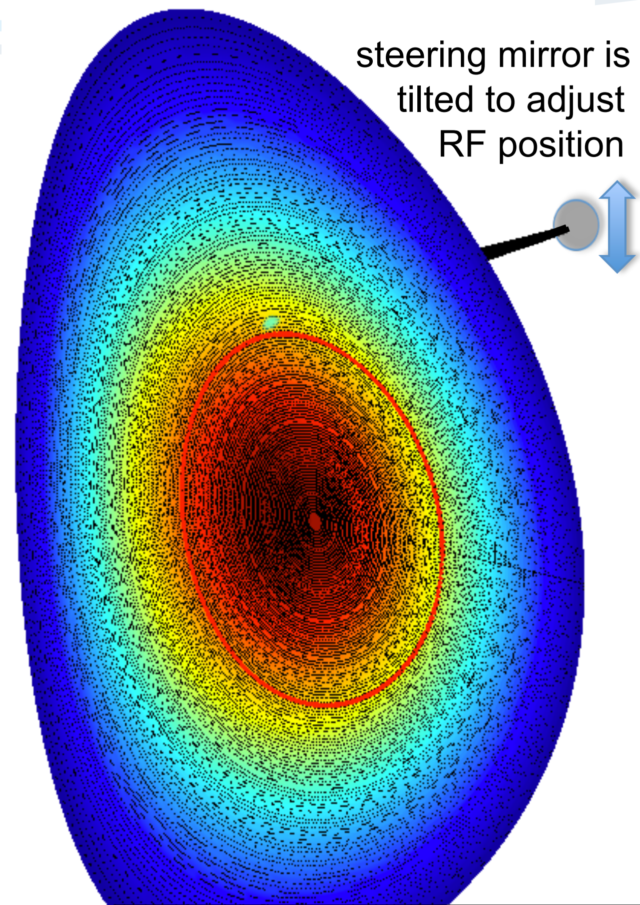


Optimal control system parameters are still under investigation





Equilibrium toroidal current, original rational surface and Poincaré map, ray trajectories, and RF hotspot

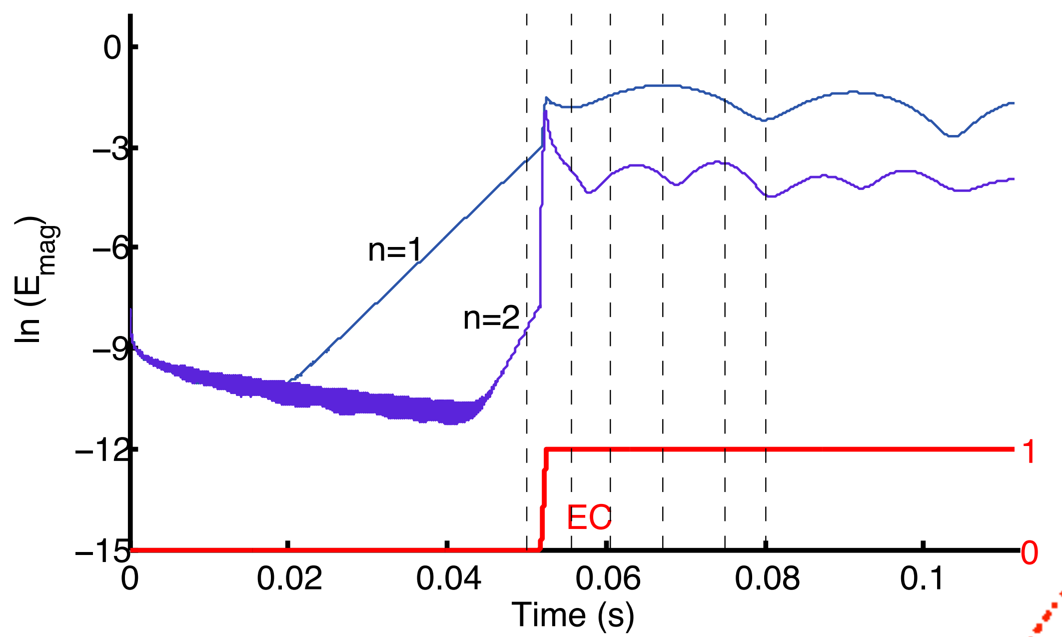
- -Dwell time (how long does RF stay in one place?),
- -Step size (how much does RF move when it moves?)
- -Directional logic (which way should it move?)
- -Power content, targeting strategies, etc.

- Exploring the physics of static RF in rotating plasmas provides some insight

Misaligned RF can stop island growth, though it doesn't necessarily shrink the islands

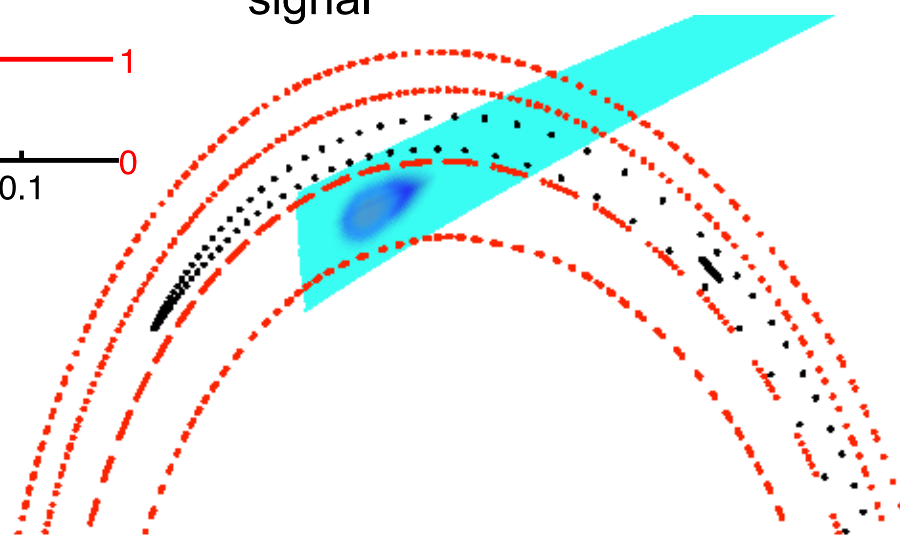
•Experimentally, initial misalignment is unavoidable…

Fourier components of magnetic energy (~island width) vs. time



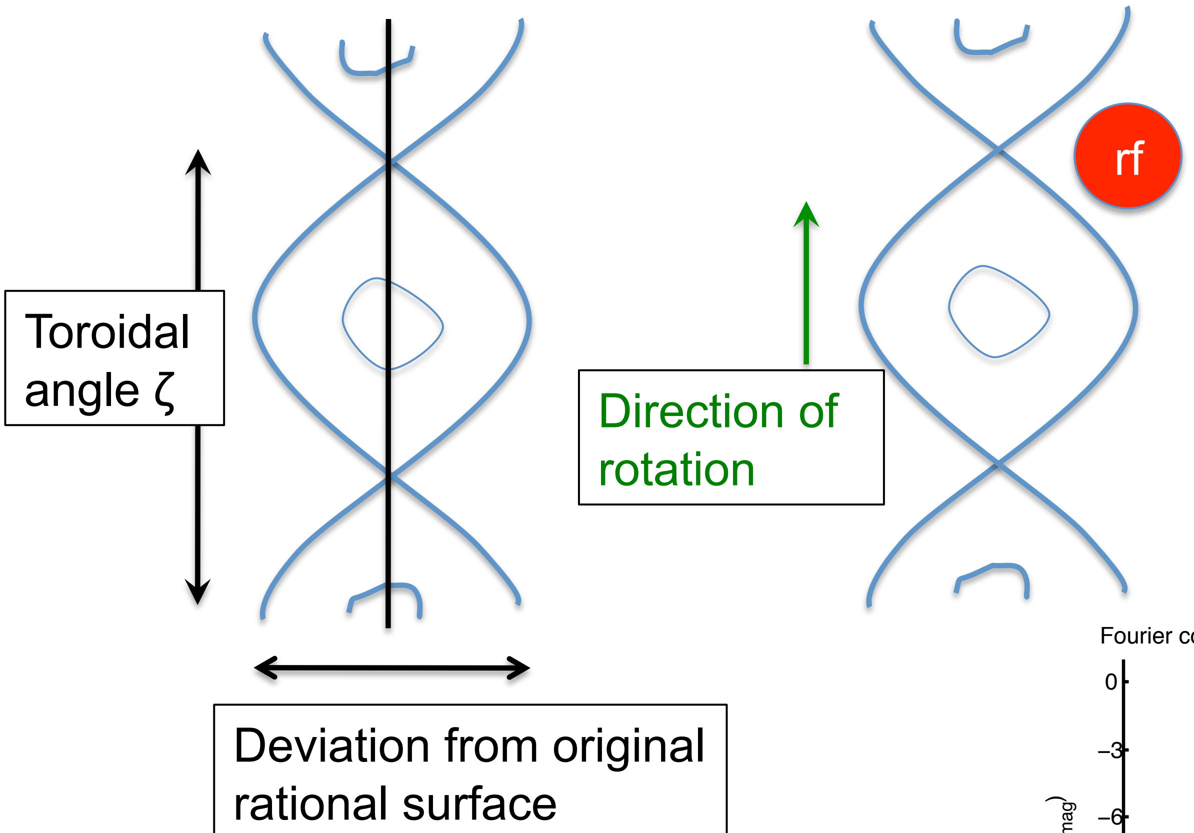
•Here, RF is 4.1 cm vertically misaligned (inwardly) from rational surface and remains fixed in space (counter-ECCD).

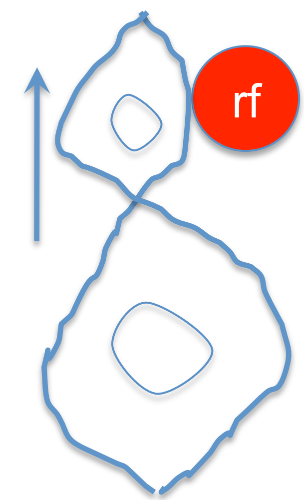
•Mode growth is halted, island size remains largely fixed; oscillations in magnetic energy signal

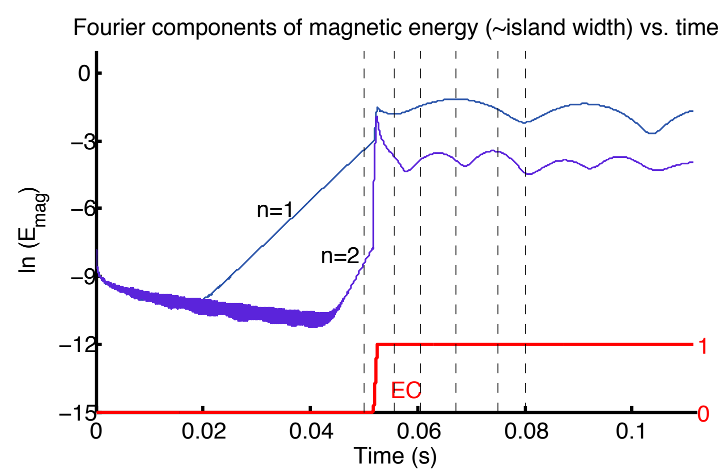


•Period of signal oscillation is the rotation period – this represents (2,1) and (4,2) components of the island rotating past the fixed RF

Conceptual picture - RF acts like a speed bump





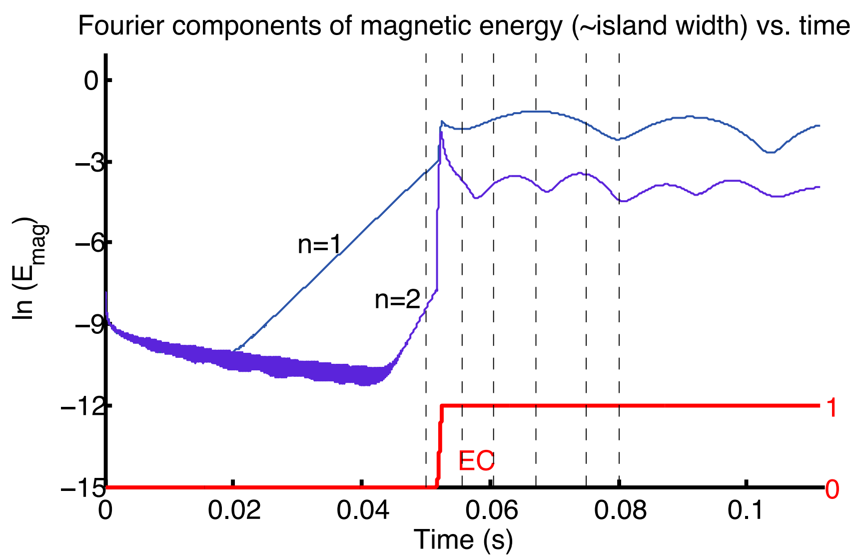


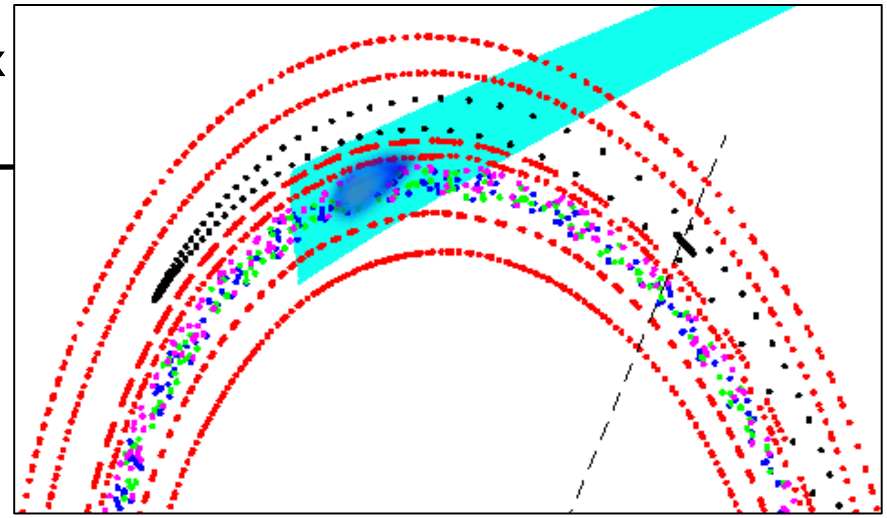
•Rational surfaces are not stationary [Jenkins et al., *Phys. Plasmas* **17**, 012502 (2010)], so the picture is actually more complicated…

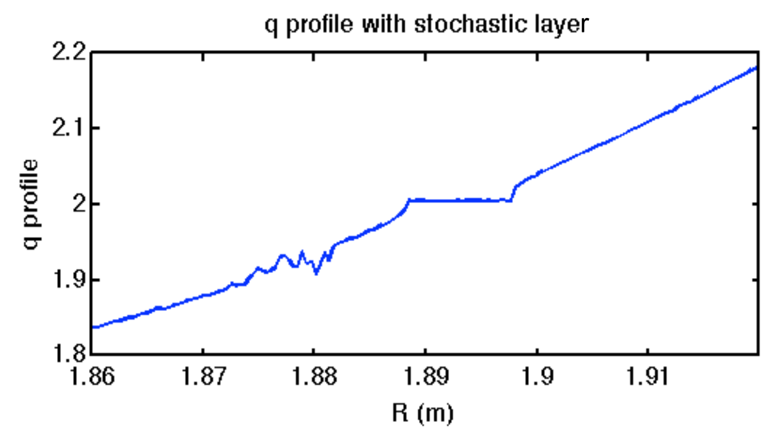
Misaligned RF may create a region where fieldlines are stochastic

- Higher RF powers tend to create a layer of stochastic field lines, which replace flux surfaces where RF was deposited

t = 0.0557 s

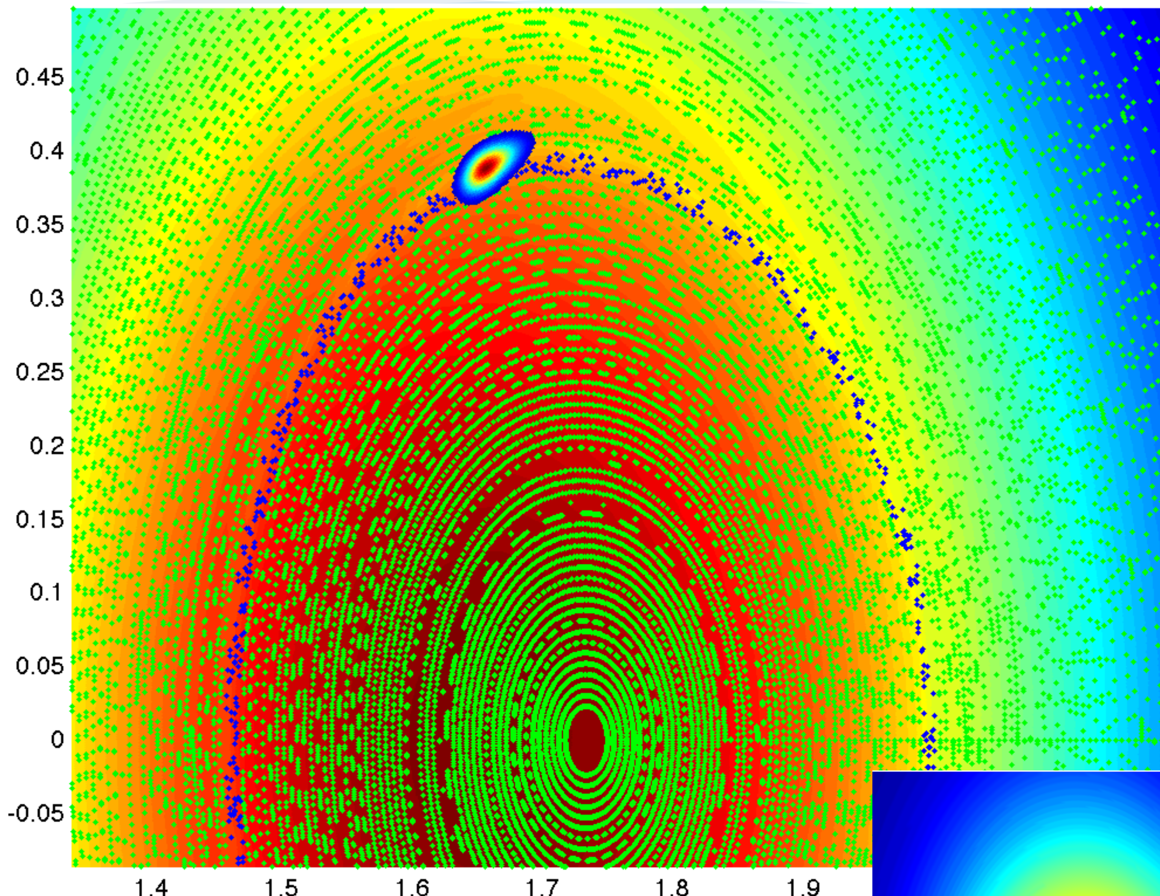




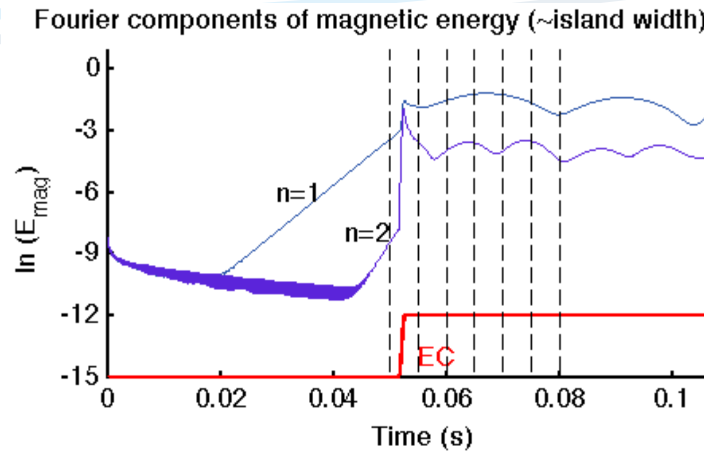


- q in range [1.9 -1.95] in stochastic region – overlap of (19,10), (21,11), (23,12), islands?

The toroidal current profile is modified at the deposition layer

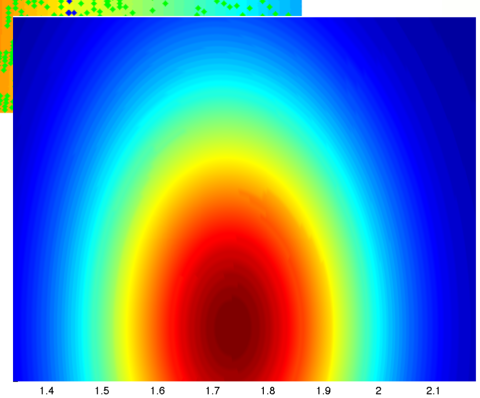


Total toroidal current with Poincaré map and quasilinear diffusion (counterclockwise island rotation)

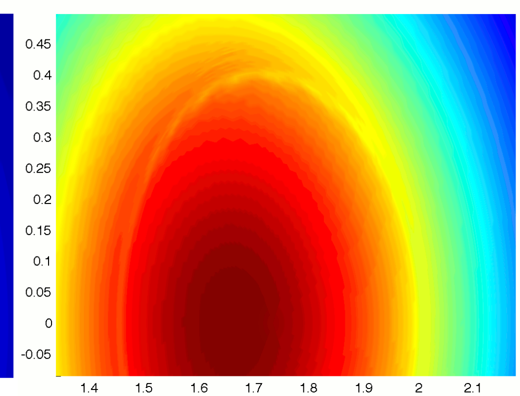


- Toroidal current is not a flux function, but RF-induced perturbations to it are confined to flux surfaces

- Stochastic layers are associated with relatively large toroidal current perturbations (= “higher” RF powers)

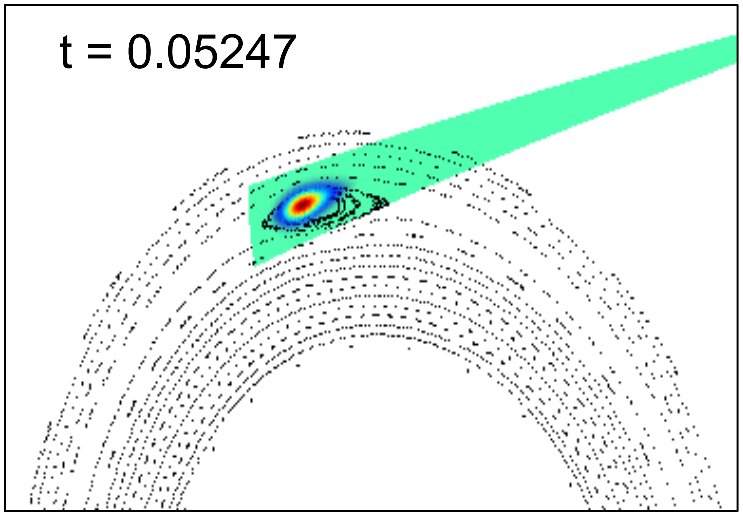


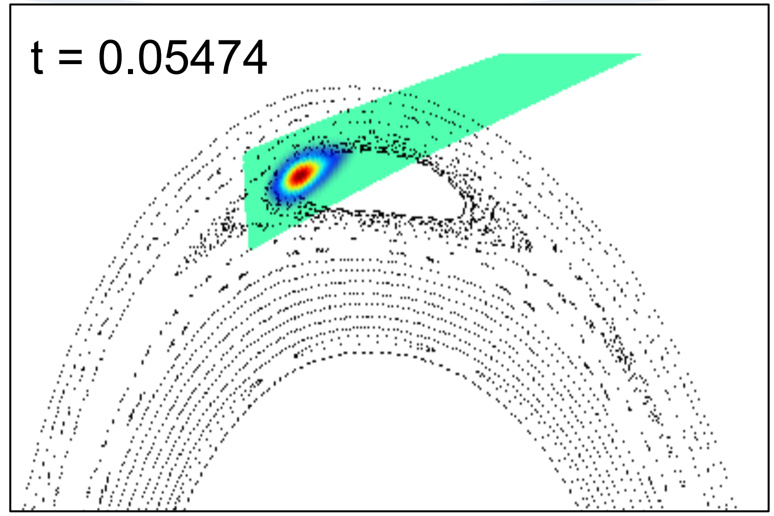
Total pressure

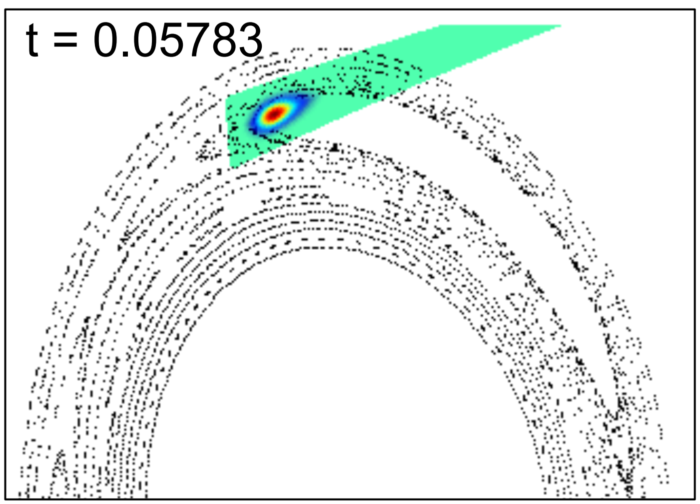


Total toroidal current

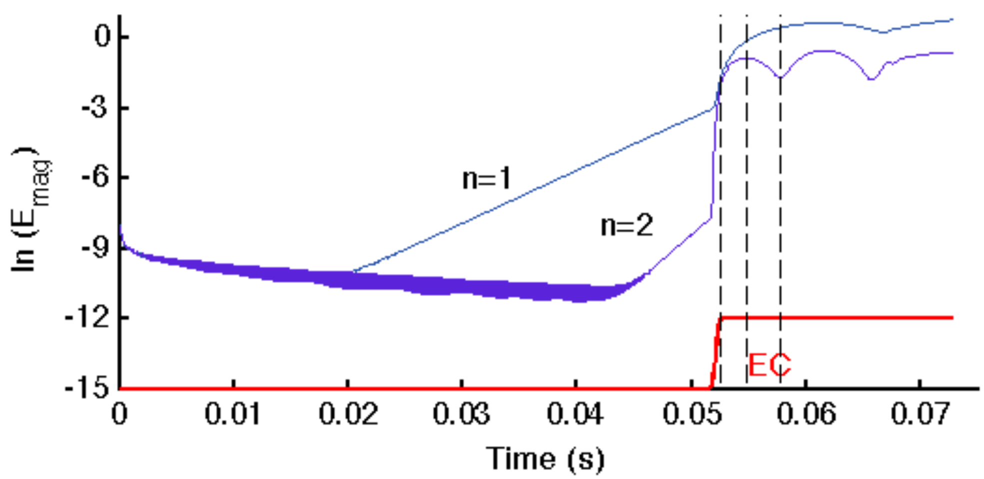
RF aligned with the rational surface can tear open islands at high input power levels







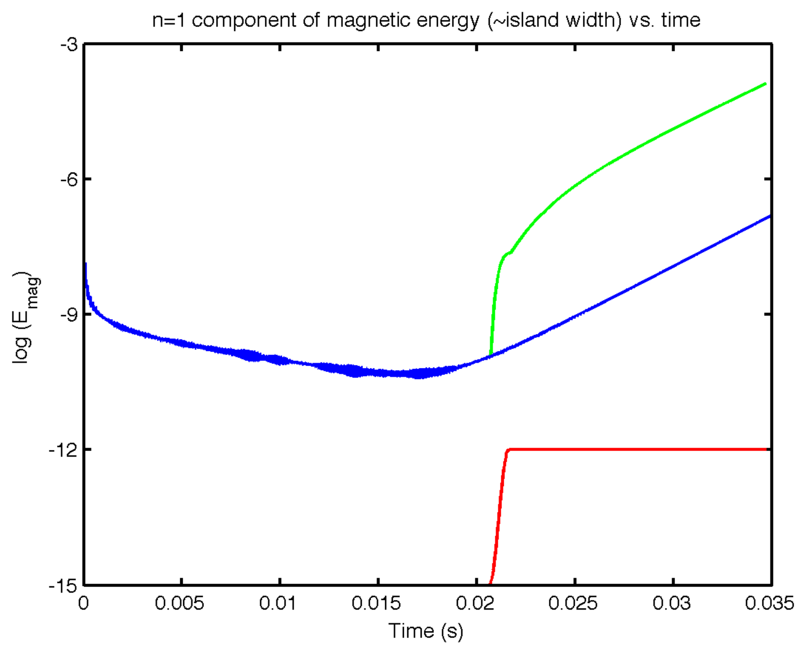
Fourier components of magnetic energy (~island width) vs. time

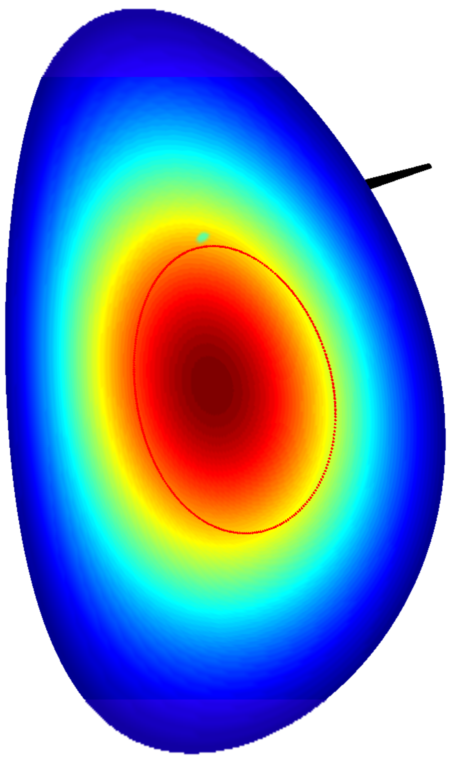


• Comparatively large currents driven (relative to mode currents) for this case

Low-power RF can trigger the mode without substantially affecting its growth rate

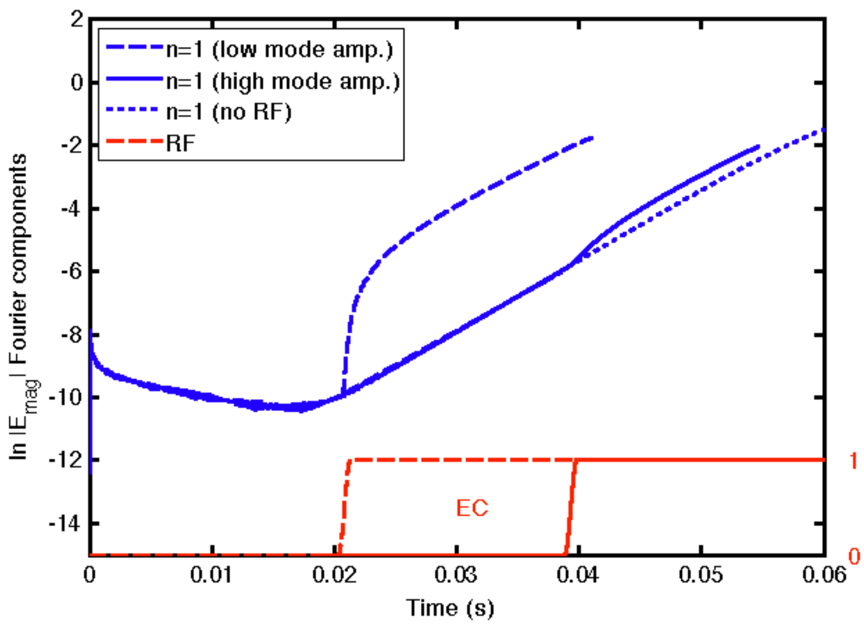
- RF powers injected near the rational surface can trigger linear growth of unstable modes; growth rates are not substantially affected

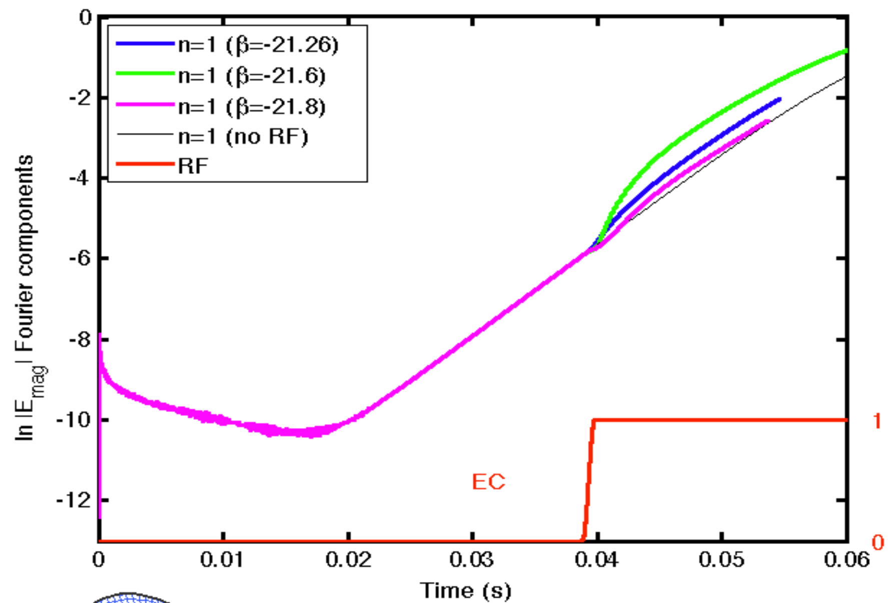




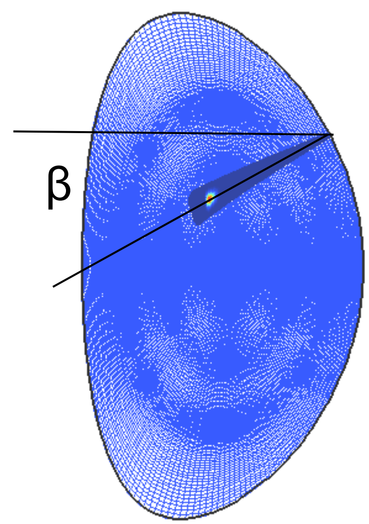
- Effects are most pronounced near the rational surface (co-ECCD)

Response to injected RF depends on relative size of mode and RF amplitudes



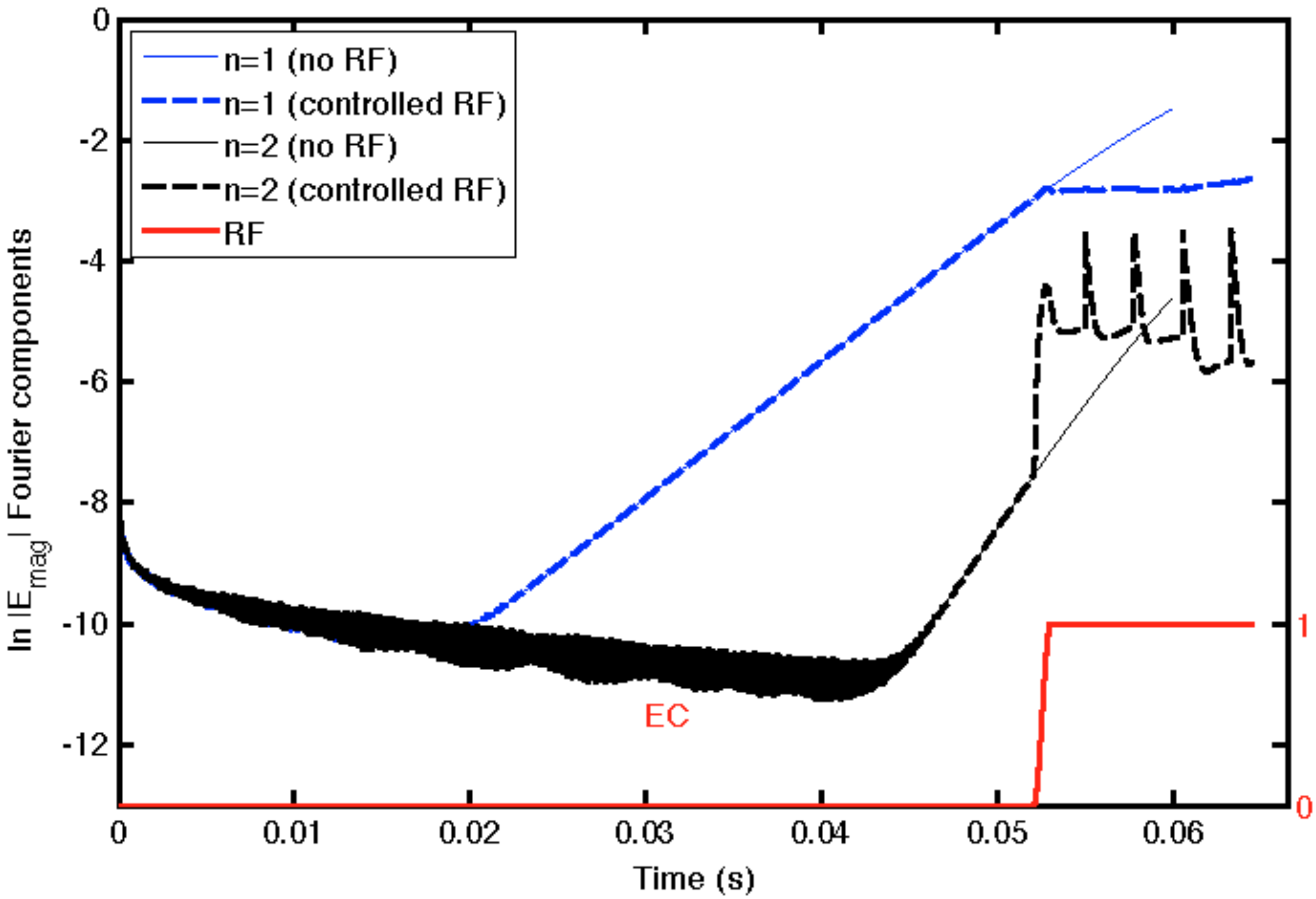


- Larger-amplitude modes will require higher RF power input for suppression



- Control system response is less pronounced when RF-induced physics is comparable to physics of the mode.

Control system enables the suppression of modes in the linear growth phase



- Control system adjustments to RF position as simulation runs are able to keep mode amplitude low.
- Control algorithms are being refined and improved.

- Rutherford regime of greater experimental relevance, but control system development easier in linear regime (physics is well understood)

Present status/future plans

- Developments to the control system algorithms, and exploration of physics effects imparted by RF, are ongoing

- Improvements to neoclassical closure physics in NIMROD will allow quantitative assessments of Ohkawa and Fisch-Boozer currents, experimental validation
  - -Under active development by Eric Held

- Better equilibria closer to stability boundary also needed, for NTM studies

- Paper recently published in Phys. Plasmas, covering the details of RF/MHD interaction [T. G. Jenkins and S. E. Kruger, *Phys. Plasmas* **19**, 122508 (2012)]

- Computational methods paper nearly completed (J. Comp. Phys.) - how to analytically/numerically relate the various physics objects in this problem
  - -continuous xMHD solutions
  - -discrete RF solutions along ray trajectories
  - -collective properties of the RF ray bundle

Summary

- We have developed and are refining a tool capable of modeling the active control of tearing instabilities by RF

- It is built on a well-developed theoretical foundation, and allows us to explore:
  - -Where do we want to aim the RF?
  - -How much power should we inject?
  - -What is the optimal control algorithm?
  - -What are the physics effects imparted by the RF?
  - -How do these effects interact with the physics of the tearing mode?

- Our model is already capturing physics not seen in analytic theories
  - -Generation of stochastic layers
  - -Stabilization effects when RF is not aligned with island O-points

- It will become even more useful as increasingly accurate neoclassical closures become available within NIMROD, enabling quantitative experimental comparisons

Extra slides

This presentation, together with other SWIM-related work, will be downloadable from my website

http://nucleus.txcorp.com/~tgjenkins/

within a few days.

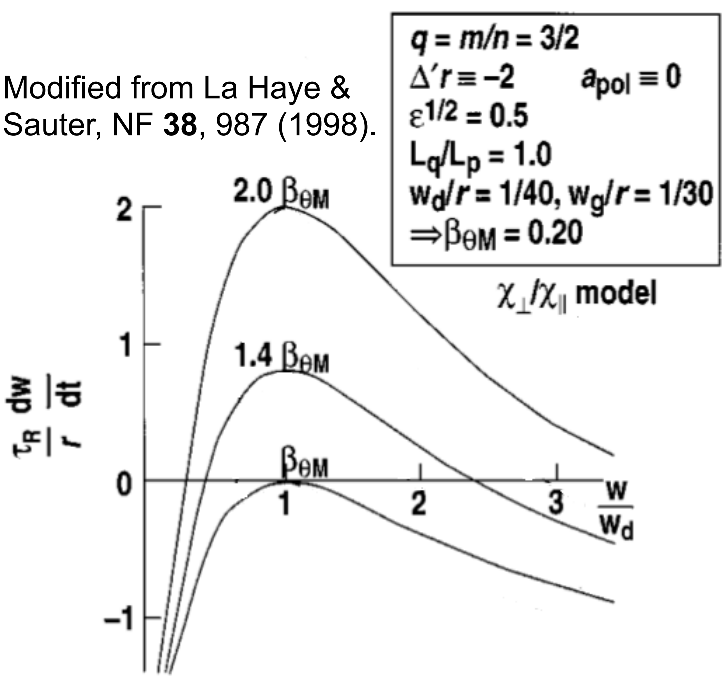
Active control of neoclassical tearing modes will likely be easier due to the excitation threshold

•For resistive tearing modes, small islands are not self-stabilizing.

•For NTMs, small islands are self-stabilizing below some threshold width.

$$\frac{\tau_R}{r}\frac{dw}{dt} = \Delta' r + \frac{\sqrt{\varepsilon}\beta_\theta r w (L_q / L_p)}{w^2 + w_d^2} - \alpha w + \ldots$$

Modified from La Haye & Sauter, NF **38**, 987 (1998).



•ECCD-induced changes to Δ' drag curves down (stabilizing) – other changes to curves also ensue as missing bootstrap current is replaced.

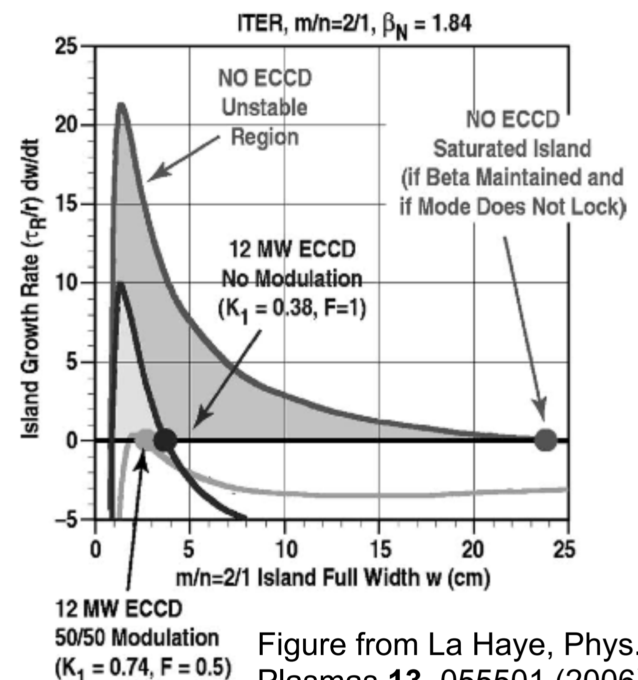


Figure from La Haye, Phys. Plasmas **13**, 055501 (2006).