PROGRESS ON A NEW ELECTRON DRIFT-KINETIC EQUATION SOLVER FOR COUPLED NEOCLASSICAL-MAGNETOHYDRONAMIC SIMULATIONS

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Introduction

How can one model neoclassical tearing modes?

Neoclassical tearing mode modeling

- NTM stability place a severe limit on maximum β
- Most common cause of disruptions on JET[1]
- High-fidelity simulations required for prediction, control, avoidance, and understanding of NTMs
  - Especially important for ITER operation, in which very few disruptions can be tolerated[2]
- NTMs incorporate a lot of physics
  - Cause: Neoclassical kinetic theory (bootstrap current)
  - Effect: MHD destabilization (island growth)
  - Requires a hybrid model

[1] P.C. de Vries, et al., Nucl. Fusion **51**, 053018 (2011)
[2] T.C. Hender, et al., Nucl. Fusion **47**, S128-S202 (2007)

Framework for hybrid solver

- Use existing MHD time-evolution code (e.g., M3D-C^1, NIMROD)
- Desirable traits for neoclassical drift–kinetic equation (DKE) solver
  - Three-dimensional toroidal geometry
    - Study nonaxisymmetric geometries with magnetic islands
  - Full Fokker-Planck-Landau collision operator
    - Use of model collision operators can lead to errors of 5%-10%[3]
  - Continuum model
    - Good convergence properties, especially for long times
    - Straight-forward coupling to MHD solvers
    - Potentially more computationally efficient than PIC

[3] E.A. Belli and J. Candy, Plasma Phys. Control. Fusion **54**, 015015 (2012)

Ramos Form of DKE

- J.J. Ramos (Phys. Plasmas 2010 & 2011) provides analytic framework for a neoclassical solver appropriate for core plasma instability simulations
- DKE evolves f_{NMs}, difference between full distribution function and shifting Maxwellian (similar to delta-f)
- Small parameters for high-temperature fusion plasmas

$$\boxed{\delta \sim \rho_i/L \ll 1} \qquad \boxed{\nu_* \sim L/\lambda_{\mathrm{mfp}} \sim \delta}$$

- Important properties:
  - Maintained to collisional inverse timescale of $O\left(\delta^3 v_{the}/L\right)$
    - Conventional neoclassical banana regime for electrons
  - Velocity referenced to each species' macroscopic flow
  - Perturbed distribution carries no density, parallel momentum, or kinetic energy

Analytic & Numerical Formulation

Axisymmetric case

Overview of next step

- NIES code (previously presented at 6/12 & 10/12 CEMM meetings) successfully solved axisymmetric Ramos DKEs to zeroth order in collisionality
- We'll retain axisymmetric geometry for now
- Want to solve the full Ramos DKE without further expansions in collisionality
  - Extends result to first-order in collisionality
  - Allows solution to vary poloidally
  - Solves for trapped <u>and</u> passing particles' distribution functions
- Will couple directly to reduced MHD equations

MHD equations

- Besides Maxwell's Eqs., we have:
  - Ohm's Law

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \frac{1}{en}\mathbf{F}_e^{coll} + \frac{1}{en}\left\{\mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \cdot \left[\left(p_{e\|} - p_{e\perp}\right)\left(\mathbf{bb} - \overleftrightarrow{\mathbf{I}}/3\right)\right]\right\}$$

  - Momentum evolution

$$nm_i\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) + \nabla p - \mathbf{J} \times \mathbf{B} + \nabla \cdot \overleftrightarrow{\mathbf{\Pi}}_{GV} + \nabla \cdot \left[\left(p_{\|} - p_{\perp}\right)\left(\mathbf{bb} - \overleftrightarrow{\mathbf{I}}/3\right)\right] = 0$$

  - Pressure evolution

$$\frac{3}{2}\left[\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}_e)\right] + p_e \nabla \cdot \mathbf{u}_e + \nabla \cdot \left(q_{e\|}\mathbf{b} + \frac{5nT_e}{2eB}\mathbf{b} \times \nabla T_e\right) - G_e^{coll} = 0$$

- Use the 2-field representation to start (no pressure eq.)

$$\mathbf{B} = \psi_0 \nabla \tilde{\psi} \times \nabla \zeta + I(\tilde{\psi}) \nabla \zeta \qquad \mu_0 \mathbf{J} = -\psi_0 \Delta^* \tilde{\psi} \nabla \zeta \qquad \mathbf{u} = R^2 \nabla U \times \nabla \zeta$$

Required Moments for Closure

- Pressure Anisotropy

$$p_{s\|} - p_{s\perp} = \frac{1}{2} m_s \int d^3v \, \left(3v_\|^2 - v^2\right) f_{NMs}$$

- Parallel Heat Flux

$$q_{s\|} = \frac{1}{2} m_s \int d^3v \; v^2 v_\| f_{NMs}$$

- Collisional Friction Force

$$\mathbf{F}_e^{coll} = m_e \int d^3v \; \mathbf{v} C_{ei} \left[f_{Me} + f_{NMe}, f_{Mi} \right]$$

- Collisional Heat Source

$$G_e^{coll} = \frac{1}{2} m_e \int d^3v \; v^2 C_{ei} \left[f_{Me} + f_{NMe}, f_{Mi} \right]$$

- All of these moments are given by the solution to appropriate DKEs
  - We'll only consider the electron DKE here

Reduced Electron DKE

- Assume flat, stationary temperature & density profiles with equal ion & electron temperatures
- Work in axisymmetric 4D phase space
  - $\tilde{\psi}$ denotes a flux surface, θ is the poloidal angle
  - v is the total velocity, $y = \cos\chi$ is cosine of the pitch angle
- Electron DKE simplifies to

$$\frac{\partial f_{NMe}}{\partial t} + vy\mathbf{b}\cdot\nabla f_{NMe} - \frac{1}{2}v\left(1-y^2\right)\mathbf{b}\cdot\nabla\ln B\frac{\partial f_{NMe}}{\partial y} = \left\{\frac{vy}{nT_e}\mathbf{b}\cdot\left[\frac{2}{3}\nabla\left(p_{e\|}-p_{e\perp}\right)-\left(p_{e\|}-p_{e\perp}\right)\nabla\ln B-\mathbf{F}_e^{coll}\right]\right.$$
$$\left.+P_2(y)\frac{v^2}{3v_{the}^2}\left(\nabla\cdot\mathbf{u}_e-3\mathbf{b}\cdot\left[\mathbf{b}\cdot\nabla\mathbf{u}_e\right]\right)+\frac{1}{3nT_e}\left(\frac{v^2}{v_{the}^2}-3\right)\nabla\cdot\left(q_{e\|}\mathbf{b}\right)\right\}f_{Me}+\left\langle C_{ee}+C_{ei}\right\rangle$$

where

$$\left\langle C_{ee}+C_{ei}\right\rangle = \nu_{De}(v)\mathcal{L}[f_{NMe}] + \frac{\nu_e v_{the}^3}{v^2}\frac{\partial}{\partial v}\left\{\xi_e\left[v\frac{\partial f_{NMe}}{\partial v}+\frac{v^2}{v_{the}^2}f_{NMe}\right]+\xi_i\left[v\frac{\partial f_{NMe}}{\partial v}+\frac{m_e v^2}{m_i v_{thi}^2}f_{NMe}\right]\right.$$
$$+\frac{\nu_e v_{the}}{n}f_{Me}\left(4\pi v_{the}^2 f_{NMe}-\Phi_e[f_{NMe}]+\frac{v^2}{v_{the}^2}\frac{\partial^2\Psi_e[f_{NMe}]}{\partial v^2}\right)+\nu_e f_{Me}\frac{v_{the}}{v_{thi}^2}\frac{\mathbf{b}\cdot\mathbf{J}}{en}\xi_i y$$

Time evolution of Electron DKE

$$\frac{f_{NMe}^{n+1}}{\Delta t} - vy\frac{\psi_0}{\mathcal{J}B}\frac{\partial f_{NMe}^{n+1}}{\partial\theta} + \frac{1}{2}v\left(1-y^2\right)\frac{\psi_0}{\mathcal{J}B^2}\frac{\partial B}{\partial\theta}\frac{\partial f_{NMe}^{n+1}}{\partial y} - \left[\langle C_{ee}+C_{ei}\rangle - \nu_e f_{Me}\frac{v_{the}}{v_{thi}^2}\frac{J_\parallel}{en}\xi_i y\right]^{n+1}$$

$$= \frac{f_{NMe}^{n}}{\Delta t} - \frac{1}{3nT_e}\left(\frac{v^2}{v_{the}^2}-3\right)f_{Me}\frac{\psi_0}{\mathcal{J}B}\frac{\partial}{\partial\theta}\left(\frac{q_{e\parallel}^n}{B}\right)$$

$$-\frac{vy}{nT_e}f_{Me}\left\{\frac{2}{3}\frac{\psi_0}{\mathcal{J}B}\frac{\partial}{\partial\theta}\left(p_{e\parallel}-p_{e\perp}\right)^n - \frac{\psi_0}{\mathcal{J}B^2}\frac{\partial B}{\partial\theta}\left(p_{e\parallel}-p_{e\perp}\right)^n + \left[F_{e\parallel}^{coll} - \frac{2m_e\nu_e}{3\sqrt{2\pi}e}J_\parallel\right]^n\right\}$$

$$+P_2(y)\frac{v^2}{3v_{the}^2}\left(\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot\left[\mathbf{b}\cdot\nabla\mathbf{u}_e\right]\right) + \nu_e f_{Me}\frac{v_{the}}{v_{thi}^2}\frac{J_\parallel}{en}\xi_i y - \frac{2}{3\sqrt{2\pi}}\nu_e f_{Me}\frac{v}{v_{the}^2}\frac{J_\parallel}{en}y$$

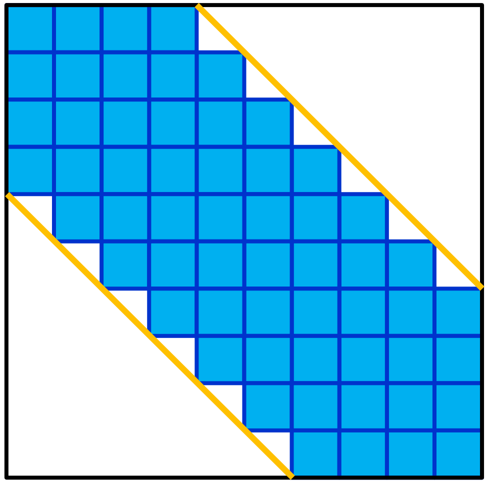
- First line consists of convective flow and homogeneous collision operator and is treated implicitly
- Second and third lines consist of moments of the solution and are treated explicitly
  - No stability constraints expected since these are integrals over the solution.
- Last line consists of the inhomogeneous drive terms

Expansions in DKE

- Velocity
  - Cubic B-spline finite elements for v
- Pitch angle
  - Legendre polynomials in $y = \cos\chi$
  - May try finite elements as well
- Configuration Space
  - Fourier modes in θ
  - ψ is just a parameter (each flux surface treated locally)
  - May try finite elements in θ or in (R, Z)

DKE Solution Method

- Poisson equations for Rosenbluth potentials solved simultaneously with DKE at each time step
- Galerkin method with cubic B-spline finite elements creates a block septadiagonal matrix in v



- Each block contains information on y and θ derivatives
- Solve as a sparse banded matrix using ScaLAPACK
  - May transition to SuperLU at some point to take advantage of sparsity within blocks

Timescales

Machine	n (m^{-3})	T (keV)	B (T)	a (m)	R (m)
LTX	3.15×10^{19}	0.2	0.34	0.26	0.4
NSTX	9.04×10^{19}	1	0.45	0.65	0.85
DIII-D	1.13×10^{20}	5	2.1	0.65	1.67
ITER	1.19×10^{20}	20	5.3	2.0	6.2

Machine	τ_{Alfven} (s)	$\tau_{e,conv}$ (s)	$\tau_{i,conv}$ (s)	$\tau_{e,coll}$ (s)	$\tau_{i,coll}$ (s)	$\tau_{resistive}$ (s)
LTX	3.0×10^{-7}	6.7×10^{-8}	2.9×10^{-6}	5.8×10^{-7}	2.5×10^{-5}	3.3×10^{-1}
NSTX	8.2×10^{-7}	6.4×10^{-8}	2.7×10^{-6}	2.0×10^{-6}	8.6×10^{-5}	2.0×10^{1}
DIII-D	3.9×10^{-7}	5.6×10^{-8}	2.4×10^{-6}	1.6×10^{-5}	6.7×10^{-4}	2.0×10^{2}
ITER	5.9×10^{-7}	1.0×10^{-7}	4.5×10^{-6}	1.1×10^{-4}	4.6×10^{-3}	1.3×10^{4}

- Difficult to consider DKE time dependently
  - In DKE, collision time 10-10^3 longer than convective time
  - MHD resistive time 10^6-10^8 longer than collision time
- Reasonable to expect the distribution function to evolve to steady state within an MHD time step

Proposed solution iteration

Solve DKE(s) to steady state to get distribution function for given equilibrium

Take moments to get necessary closures for MHD equations

(e.g., friction force)

Evolve MHD equations to get new equilibrium using (modified) M3D-C^1

17

Current Progress

Status of code

- All terms discussed have been implemented
- Can reproduce known result with good agreement
- Currently debugging some computational issues
  - Convergence with number of velocity finite elements
  - Spurious density, parallel momentum, and kinetic energy formation

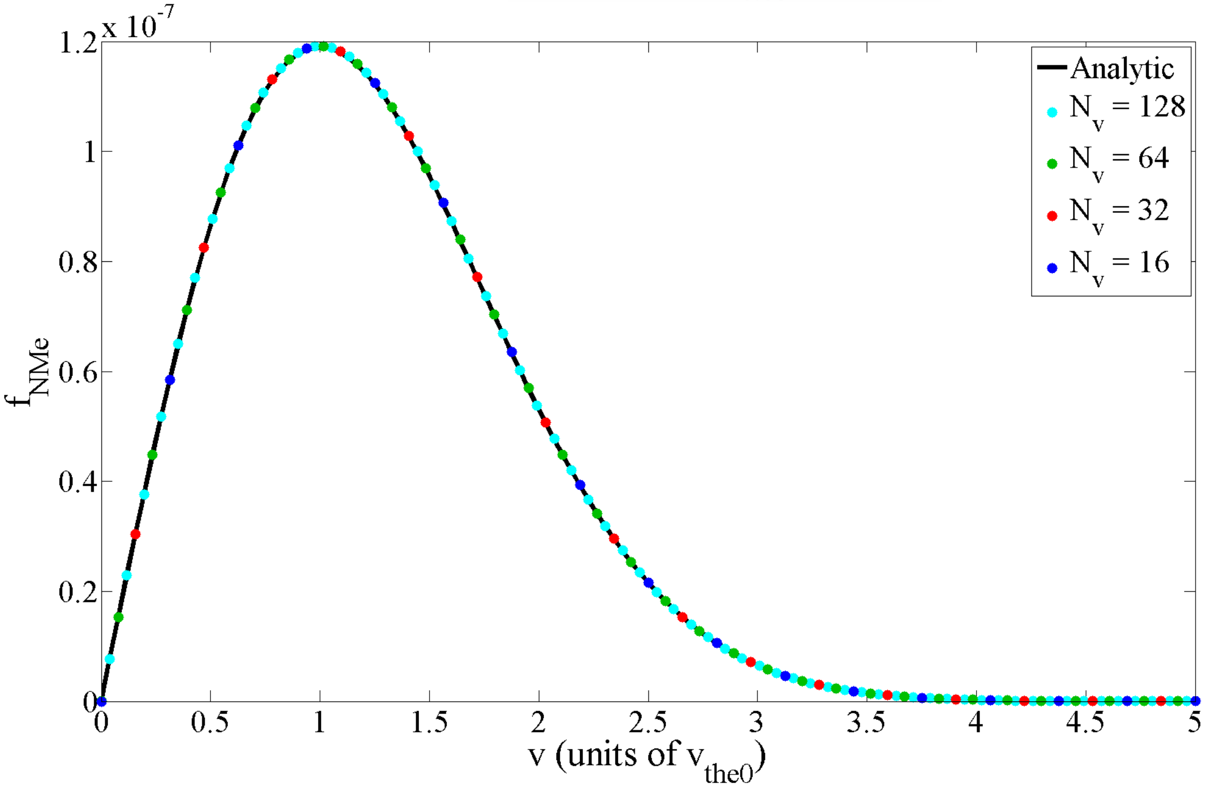
Adiabatic Solution Test

- A reduced version of the steady-state electron DKE:

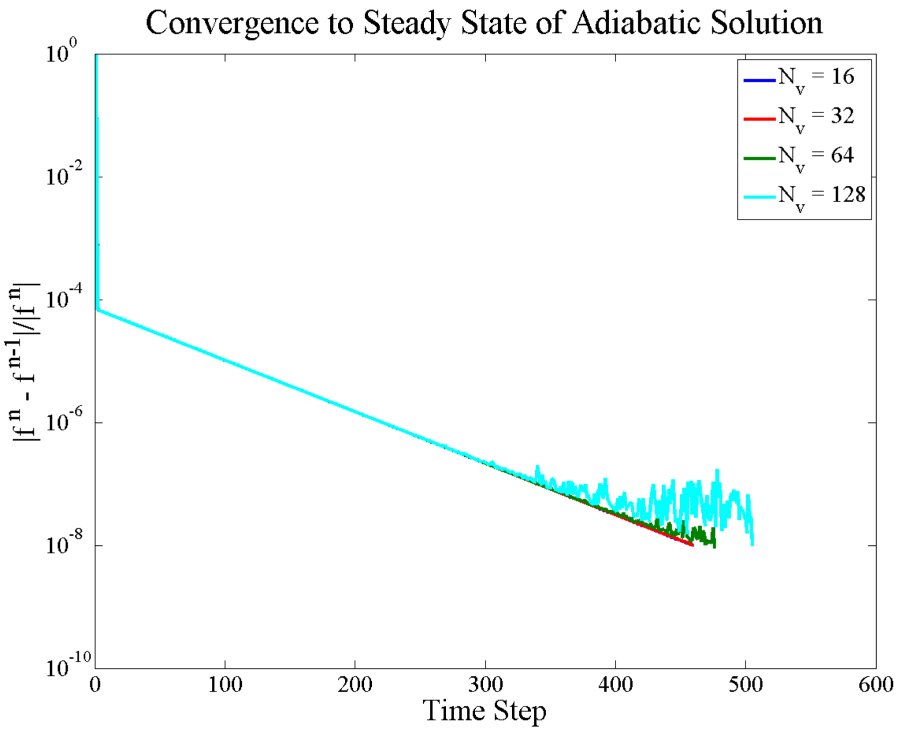
$$vy\mathbf{b}\cdot\nabla f_{NMe} - \frac{1}{2}v\left(1-y^2\right)\mathbf{b}\cdot\nabla\ln B\frac{\partial f_{NMe}}{\partial y} = P_2(y)\frac{v^2}{3v_{the}^2}\left(\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot[\mathbf{b}\cdot\nabla\mathbf{u}_e]\right)f_{Me}$$

has a known particular solution: $f_{NMe} = -\frac{\mathbf{b}\cdot\mathbf{u}_e}{v_{the}^2}vyf_{Me}$

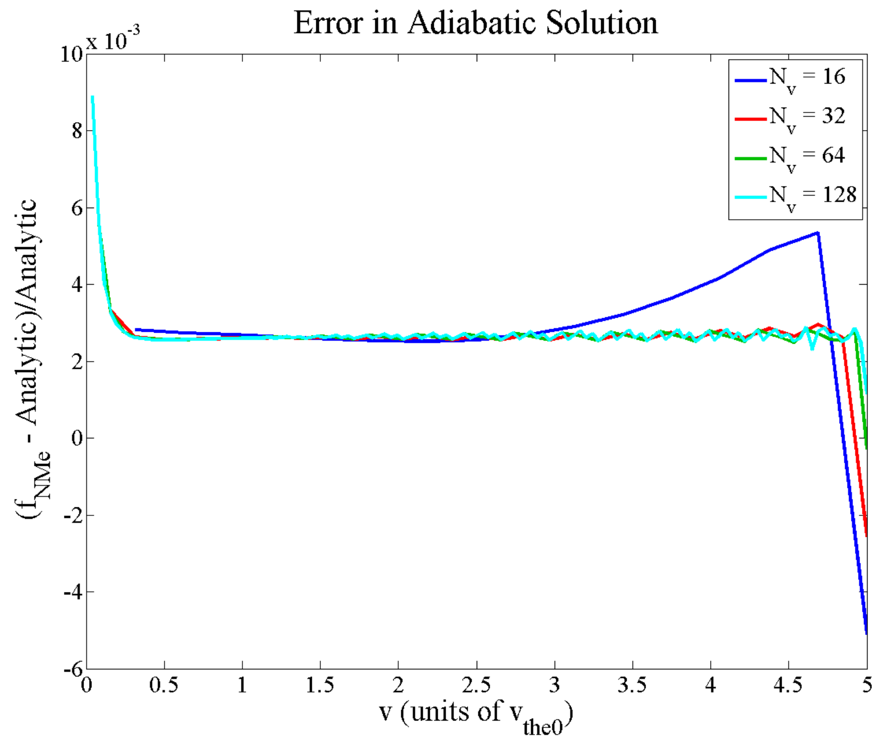
- Our computed steady-state solution to this equation



Convergence to Adiabatic Solution?



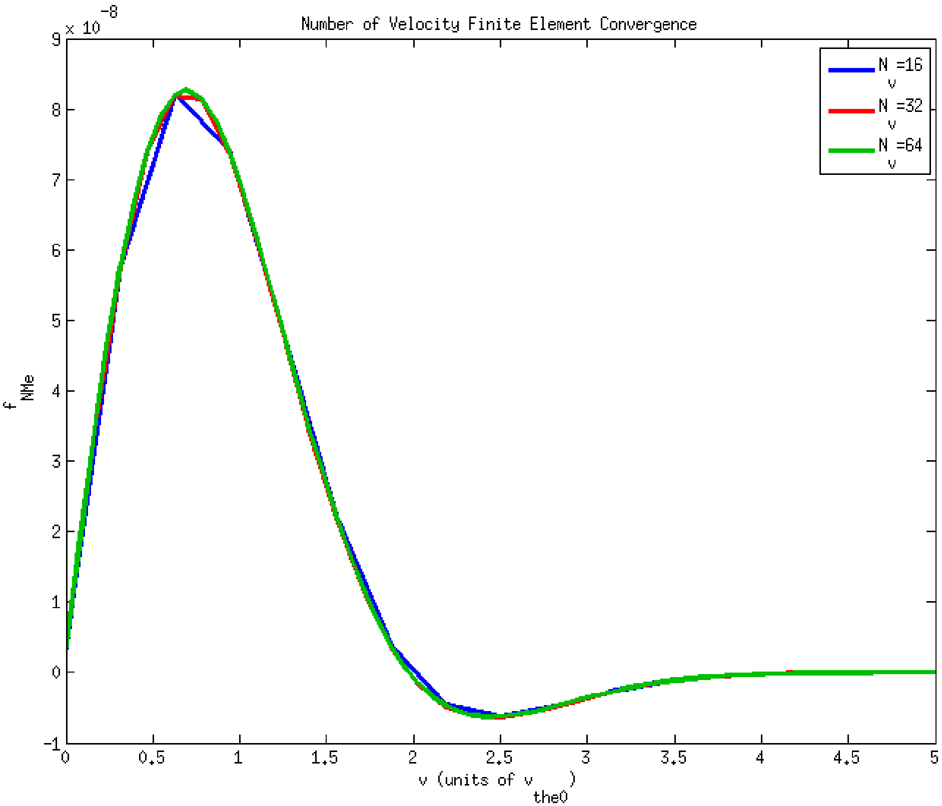
- Level of random error can be reduced with smaller time step or larger grid spacing
- Possible stability issue?

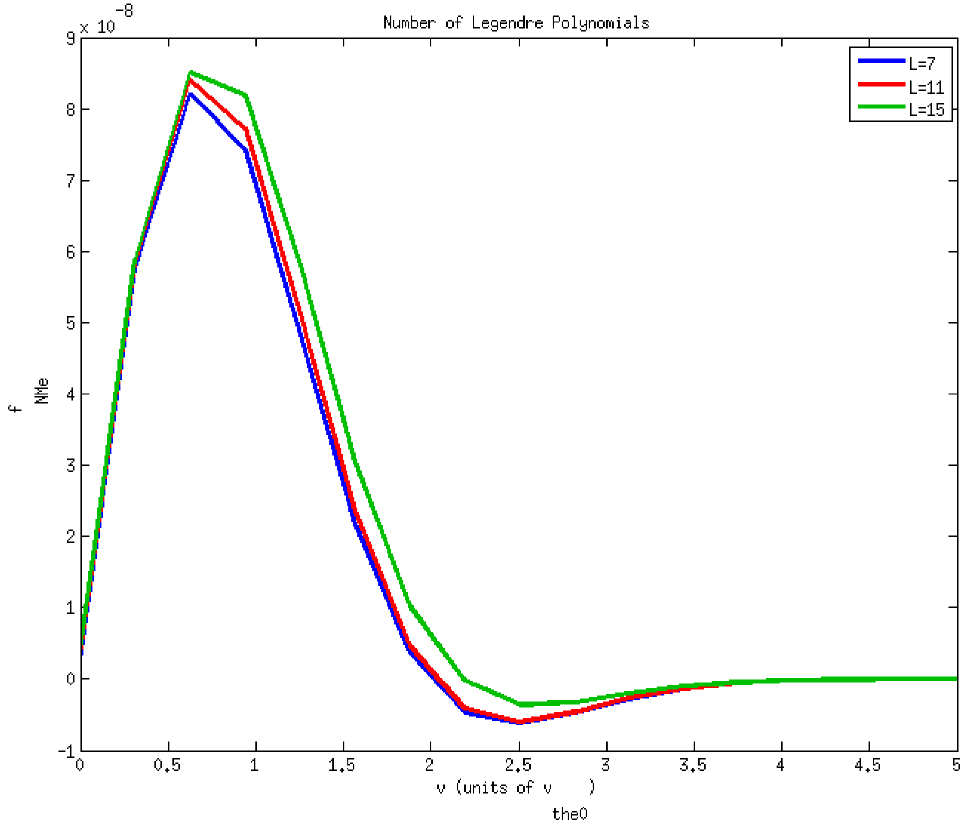


- Possible boundary condition problems
  - Eq. ill-defined at origin?
- Cause of oscillations?

Full DKE Solutions

- Don't observe same issue in steady-state convergence (except at small magnitude ~10^{-11}); many time steps for convergence though
- Currently working on numerical convergence





Conservation Laws

- Define: $$\delta n_e = \int d^3v\, f_{NMe} \qquad \delta u_{e,\|} = \frac{1}{n}\int d^3v\, v_{\|} f_{NMe}$$

$$\delta K_e = \frac{m_e}{2n}\int d^3v\, v^2 f_{NMe}$$

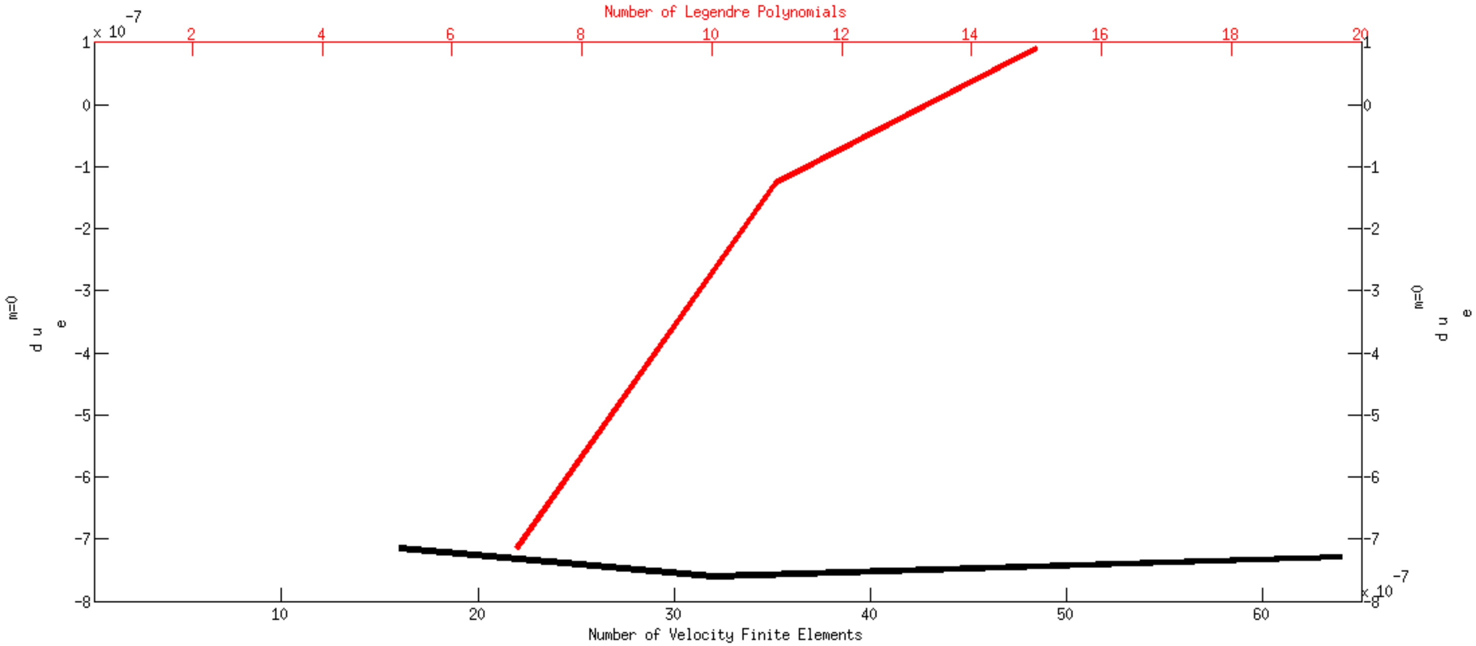
- One can show that the analytic electron DKE should enforce several conservation laws

$$\frac{\partial \delta n_e}{\partial t} + \mathbf{B}\cdot\nabla\left(\frac{\delta u_{e,\|}}{B}\right) = 0 \qquad \frac{\partial \delta u_{e,\|}}{\partial t} + \frac{1}{3}\mathbf{b}\cdot\nabla\left(\delta K_e\right) = 0 \qquad \frac{\partial \delta K_e}{\partial t} = 0$$

- Expect numerical equations to deviate from these, but spurious values should converge as solution converges

Using Conservation Laws to Debug

- Producing spurious momentum of ~1% of mean flow



- Derivation of these laws show which terms balance, e.g.,
  - Convective terms balance parallel heat flux to produce no δK_e
  - No other terms should contribute to δK_e
  - Preliminary tests show that this balance is not converging, though spurious kinetic energy is small (~10^{-6} of electron temperature)

Calculating Neoclassical Conductivity

- Despite these problems, it would be useful to calculate the neoclassical conductivity given by our computed solution
- Parallel Ohm's Law gives

$$\langle \mathbf{B} \cdot \mathbf{F}_e^{coll} \rangle \; + \; \langle (p_{e\|} - p_{e\perp}) \, \mathbf{b} \cdot \nabla B \rangle \; = \; - \, en \, V_0 \, I \, \langle R^{-2} \rangle$$

- Thus, the neoclassical conductivity is

$$\sigma_{neo} = \frac{\langle \mathbf{J} \cdot \mathbf{B} \rangle}{\langle \mathbf{E} \cdot \mathbf{B} \rangle} = - \frac{e^2 n^2 \, \langle \mathbf{u}_e \cdot \mathbf{B} \rangle}{\langle \mathbf{B} \cdot \mathbf{F}_e^{coll} \rangle + \left\langle \left(p_{e\|} - p_{e\perp} \right) \mathbf{b} \cdot \nabla B \right\rangle}$$

- Should be done soon (perhaps by my poster Monday)

Future work & Conclusion

Test problem

- Diffusion of current into a toroidal plasma due to a loop voltage at its edge
- Current evolves self-consistently with equilibrium
- Should observe neoclassical conductivity reduction
  - Trapped particles carry no net current
  - Can benchmark to theoretical and numerical results

Extensions to axisymmetric code

- When current code is working, we will
  - Allow separate ion and electron temperatures
  - Relax constraints on density and temperature profiles
- Will have to solve separate, but similar, ion DKE
- Will allow for simulations of the inductive formation of the bootstrap current
- Use full six-field MHD model with M3D-C^1 to self-consistently evolve pressure as well

Summary

- The operation of ITER and other future MCF experiments requires predictive capabilities for core plasma instabilities (e.g., Sawtooths, NTMs)
- To date, no neoclassical code exists that is well-suited for such simulations (work by E. Held excepted)
- We are creating such a code based on the Ramos drift-kinetic formulation
  - Axisymmetric hybrid code currently being debugged
  - Hope to start work on nonaxisymmetric code in late 2013
- My poster: Monday, Session II, #24