Comparison of Kinetic and Extended MHD Models for the Ion Temperature Gradient Instability in Slab Geometry

D. D. Schnack
University of Wisconsin – Madison
Jianhua Cheng, S. E. Parker
University of Colorado – Boulder
D. C. Barnes
TriAlpha Energy, Inc.

Un-indicted Co-Conspirators

- Ping Zhu
- Chris Hegna
- Eric Held
- Jake King
- Scott Kruger
- Carl Sovinec

Goals

- Verify the NIMROD code for the ITG instability
  - Are the extended MHD equations being solved correctly?
- Validate the extended MHD model for the ITG
  - When can extended MHD be used as a physical model for the ITG?
  - Quantify the differences between extended MHD and fully kinetic model

Ion Temperature Gradient Instability

- *Parallel sound wave* destabilized by interaction with a *perpendicular drift wave* in the presence of an *ion temperature gradient*
  - L is gradient scale length
  - Perturbed perp. drift motions convect heat via $V_x \, dT_{i0}/dx$
  - Can amplify temperature perturbation in sound wave if phase and frequency are right
- Requires FLR/two-fluid effects for instability
  - Stable in ideal and resistive MHD
  - Threshold in ϱ_i/L or $k_y\varrho_i$ for instability
  - Differs from g-mode, which is MHD unstable and is stabilized by FLR effects
- Good test for extended MHD model
  - How far can the model be pushed into the kinetic regime?

Approach

- Solve *local* kinetic and fluid dispersion relations for complex eigenvalue
- Solve extended MHD model with NIMROD code for complex eigenvalue and *global* eigenfunction
- Solve Vlasov + field equations with hybrid kinetic δf code (Cheng, et. al.) for complex eigenvalue and *global* eigenfunction
- Compare all results for a range of $k_y \varrho_i$ and ϱ_i/L

Equilibrium

- Slab (x, y, z) geometry
  - Quasi-neutral $n_{i0} = n_{e0} = n_0$
  - $T_{i0}(\mathrm{x})$, $\mathbf{B}(x) = B_0(x)\,\mathbf{e}_z$, n_0 = const, T_{e0} = const.
  - z is parallel, y is perpendicular, no shear
- Species force balance: $\mathbf{E}_0 + \mathbf{V}_{s0} \times \mathbf{B}_0 + \dfrac{1}{n_0 q_s} \nabla P_s = 0$
  - Specify P, determine B from MHD force balance
- E_{0x} determines frame of reference
  - $E_{0x} = 0$ for all calculations here
  - Ion drift velocity explicitly included in equilibrium

Local Kinetic Dispersion Relation

- No external forces or field line curvature; electrostatic
- Perturbations: $f = \tilde{f} e^{i\left(k_y y + k_z z - \omega t\right)}$
  - Ignore *x*-dependence: *local approximation*
- Low frequency: $|\omega| << |\Omega_{e,i}|$

$$1 + \sum_s \frac{1}{\left(k\lambda_{Ds}\right)^2}\left\{1 + \frac{\omega - \omega'_{ds}}{\omega}\left[W\left(\frac{\omega}{|k_z| V_{ths}}\right) - 1\right] I_0\left(\xi_s\right) e^{-\xi_s}\right\} = 0$$

$$\xi_s = \left(k_y \rho_i\right)^2 \qquad \omega'_{ds} = \frac{T_s k_y}{q_s B}\frac{dT_s}{dx}\frac{\partial}{\partial T_s} \qquad W(\zeta) = \left(\zeta / \sqrt{2}\right) Z\left(\zeta / \sqrt{2}\right) + 1$$

- Fluid limit: $\left(\omega^2 - \omega_s^2\right)\omega + \omega_{se}^2 \omega_{Ti}^* = 0$

$$\omega_{se}^2 = k_z^2 T_e / M \qquad \omega_s^2 = \omega_{se}^2 + k_z^2 (5/3) T_i / M \qquad \omega_{Ti}^* = \frac{k_y}{eB}\frac{dT_i}{dx}$$

Local Extended MHD Dispersion Relation

- XMHD *mathematically equivalent* to "two-fluid" model – has same dispersion relation
- FLR effects captured through Braginskii closures ($k_y \varrho_i$ << 1):

$$\Pi_{\mathrm{i}}^{\mathrm{gv}} = \frac{\eta_3}{2}\left[\hat{\mathbf{b}} \times \mathrm{W} \cdot \left(\mathrm{I} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) + \text{transpose}\right], \quad \eta_3 = \frac{P_i}{2B}$$

$$\mathrm{W}_{i,j} = \frac{\partial V_j}{\partial x_i} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\delta_{i,j}\nabla \cdot \mathbf{V} \qquad \mathbf{q}_i^{gv} = \kappa_i^{gv}\hat{\mathbf{b}} \times \nabla T_i \quad , \quad \kappa_i^{gv} = \frac{5}{2}\frac{P_i}{B}$$

- Assume complete GV cancellations+electrostatic, k_z/k_y << 1:

$$\left(\omega^2 - \omega_s^2\right)\omega + \omega_{se}^2\omega_{Ti}^* = 0$$

- Same as fluid limit of kinetic equation!
  - Similar equation if GV cancellations are "incomplete"

Cubic Dispersion Relation

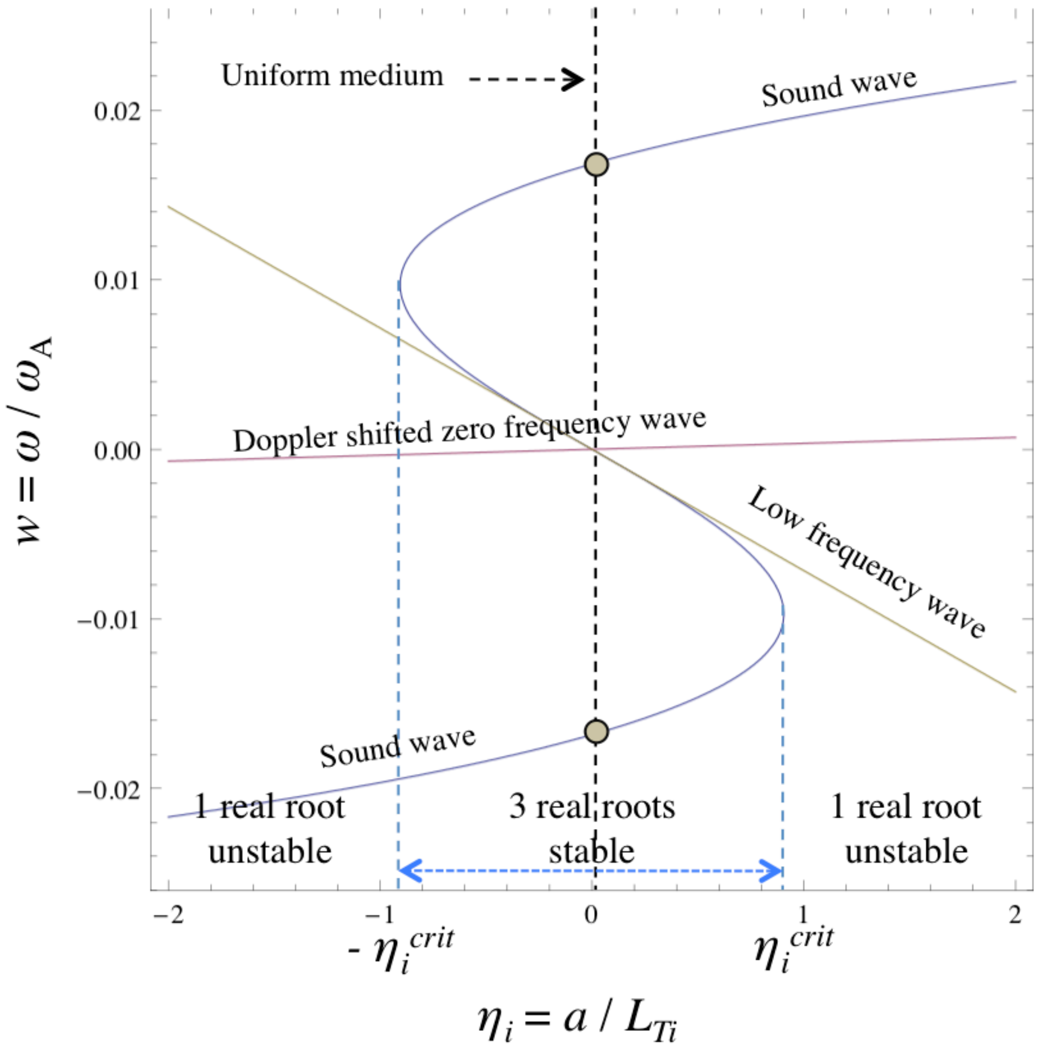
$$\underbrace{\omega^3}_{\text{Cubic}} - \underbrace{\omega_s^2 \omega}_{\text{Linear}} + \underbrace{\omega_{se}^2 \omega_{Ti}^*}_{\text{Constant}} = 0$$

- High frequency, or dT_i/dx small:
  - Cubic ~ Linear => Parallel sound waves: $\omega^2 = \omega_s^2$
- Low frequency, small dT_i/dx :
  - Linear ~ Constant => Drift wave: $\omega = -\dfrac{\omega_{se}^2 \omega_{Ti}^*}{\omega_s^2} \sim \dfrac{dT_i}{dx}$
- High frequency, Large dT_i/dx :
  - Cubic ~ Constant => Instability:

$$\omega = \left(\omega_{se}^2 \omega_{Ti}^*\right)^{1/3} e^{2\pi i l/3} \ , \quad l = 0,1,2 \quad \gamma \sim L^{-1/3}$$

- Interaction between sound and drift waves lead to instability
- Electromagnetic dispersion relation is quintic – 2 new shear Alfven waves, same low frequency behavior

Behavior of Low Frequency Roots in Fluid Limit



Fluid Solution Depends on Single Non-dimensional Parameter

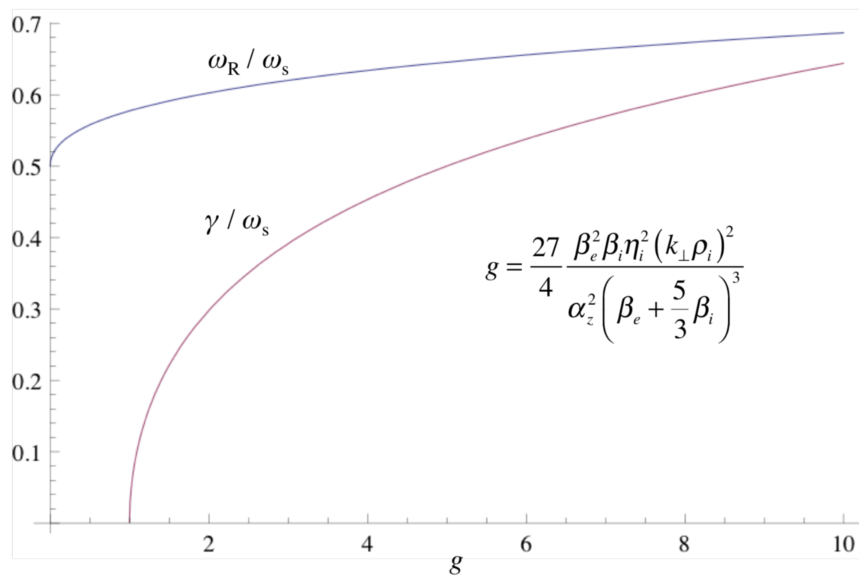
$$g = \frac{27}{4}\frac{\beta_e^2\beta_i}{\left(\beta_e + \frac{5}{3}\beta_i\right)^3}\left(\frac{k_\perp}{k_\parallel}\right)^2\left(\frac{\rho_i}{L}\right)^2 \quad , \qquad g > 1 \text{ for instability}$$

Growth Rate:

$$\frac{\gamma}{\omega_s} = \frac{1}{2}\left[\left(\sqrt{g}+\sqrt{g-1}\right)^{1/3} - \frac{1}{\left(\sqrt{g}+\sqrt{g-1}\right)^{1/3}}\right]$$

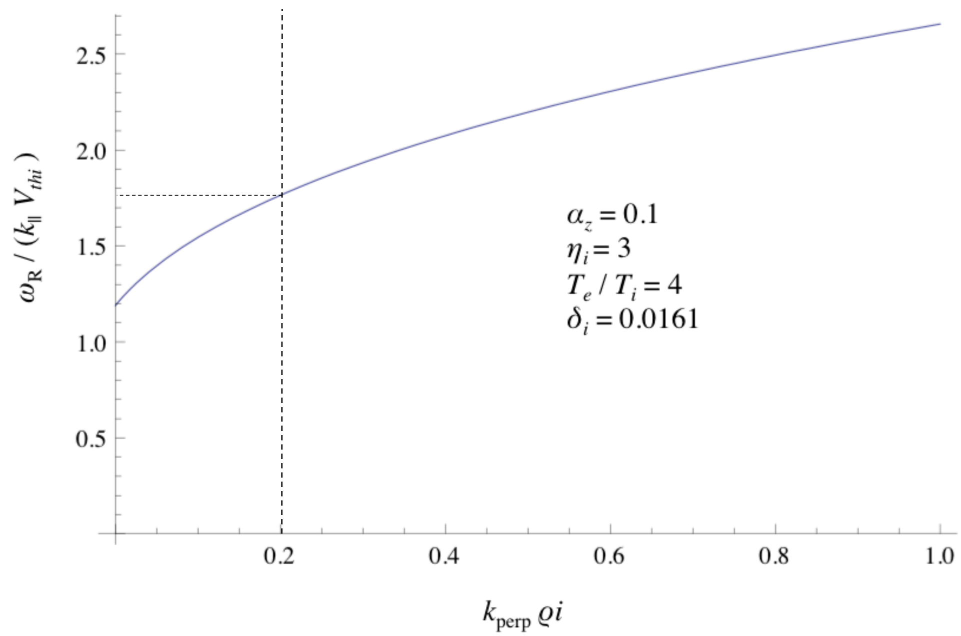
Real Frequency:

$$\frac{\omega_r}{\omega_s} = \frac{1}{2\sqrt{3}}\left[\left(\sqrt{g}+\sqrt{g-1}\right)^{1/3} + \frac{1}{\left(\sqrt{g}+\sqrt{g-1}\right)^{1/3}}\right]$$



Wave-Particle Interaction Effects

- Kinetic model includes wave-particle interaction effects (e.g., Landau damping)
- Not captured by extended MHD model
- Effects minimized when $\omega_r/(k_z\ V_{thi}) >> 1$ (few particles resonant with wave)
  - Also need $k_y \varrho_i << 1$



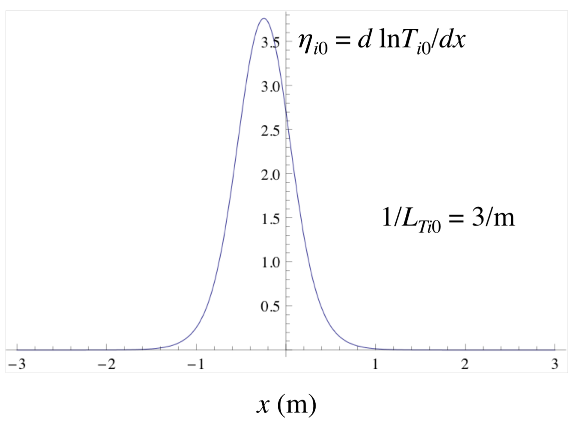
For $k_\perp \rho_i \sim 0.2,$ resonant fraction $\sim e^{-\left(\omega_r/(k_\parallel V_{thi})\right)^2} \sim e^{-(1.7)^2} = 0.055$

Equilibrium for Global Calculations

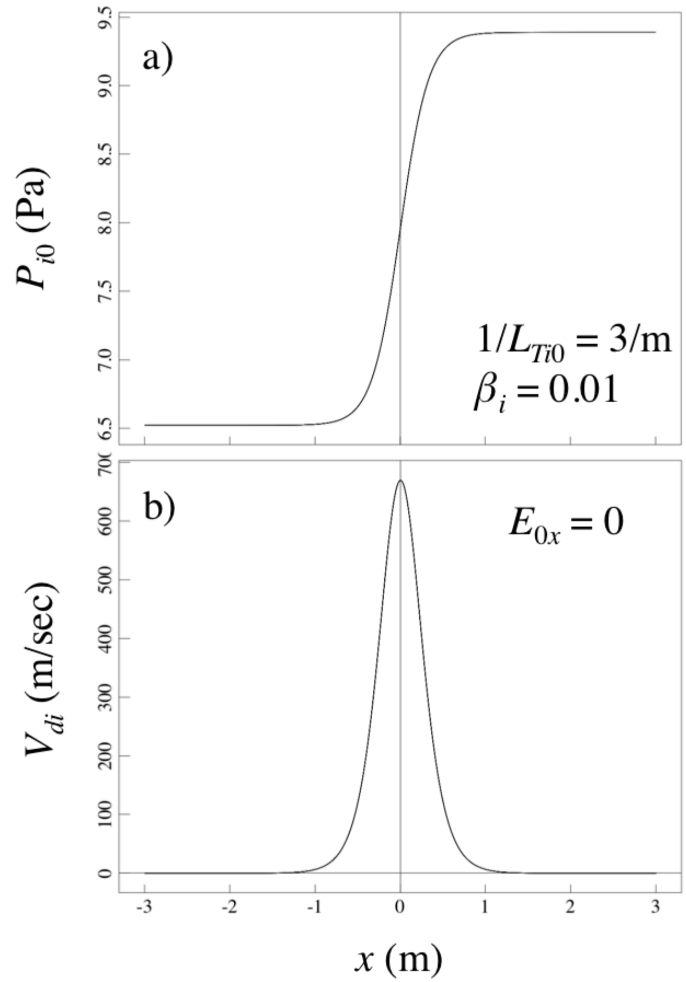
$$T_i(x) = T_{i0}\left[1 + 0.9\tanh\left(\frac{x}{L}\right)\right]$$

Walls are "far away"

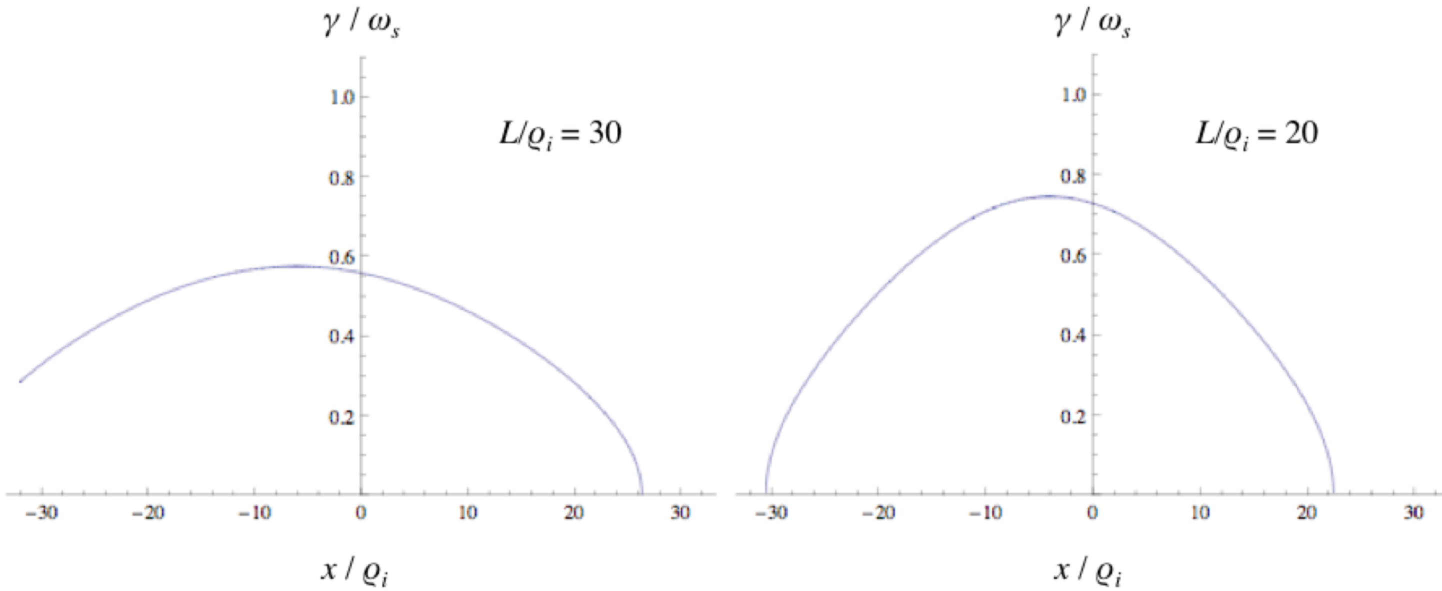
Used for both XMHD and kinetic calculations



η_i peaks at $x < 0$

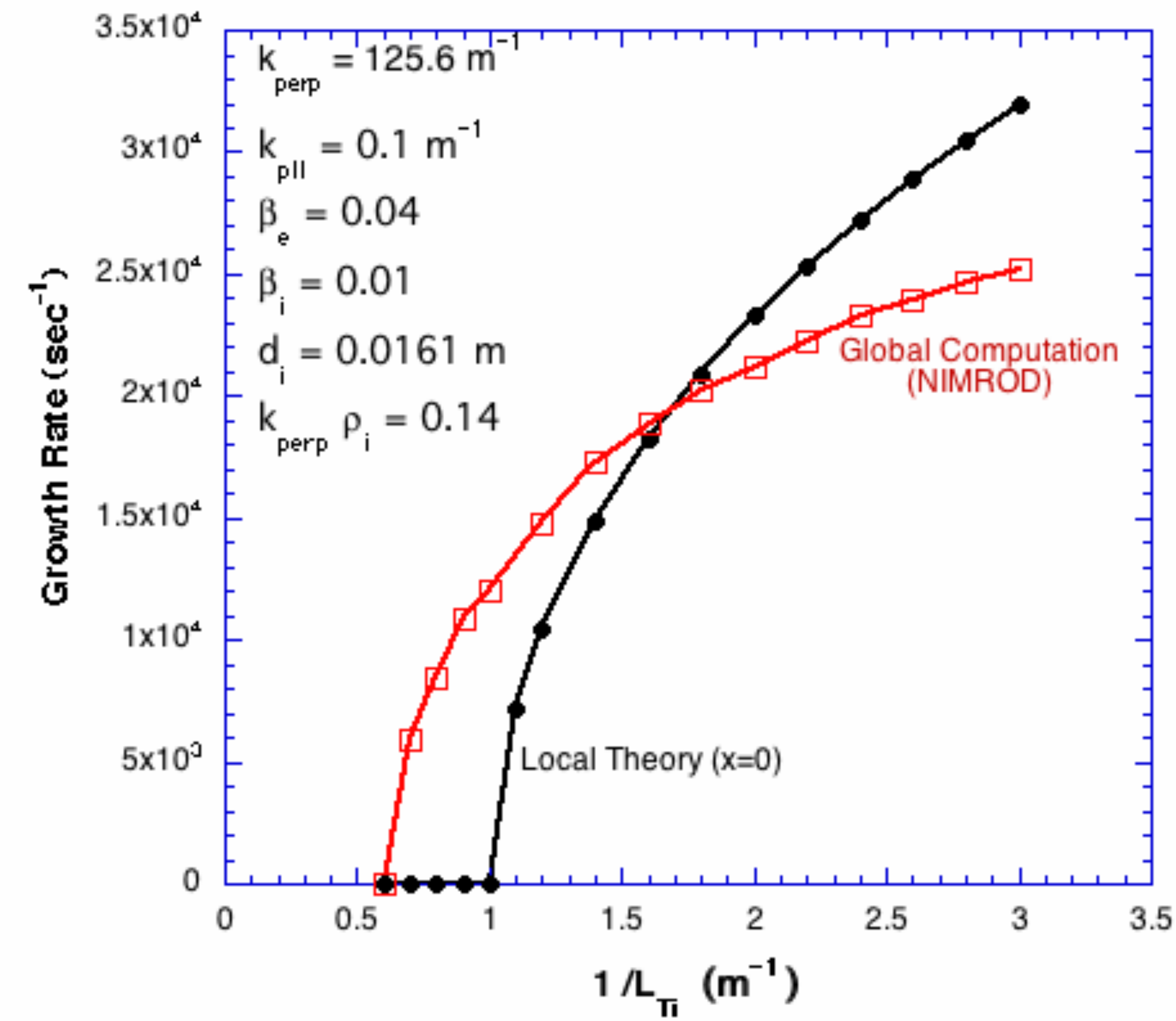


Local Fluid Growth Rate vs. x

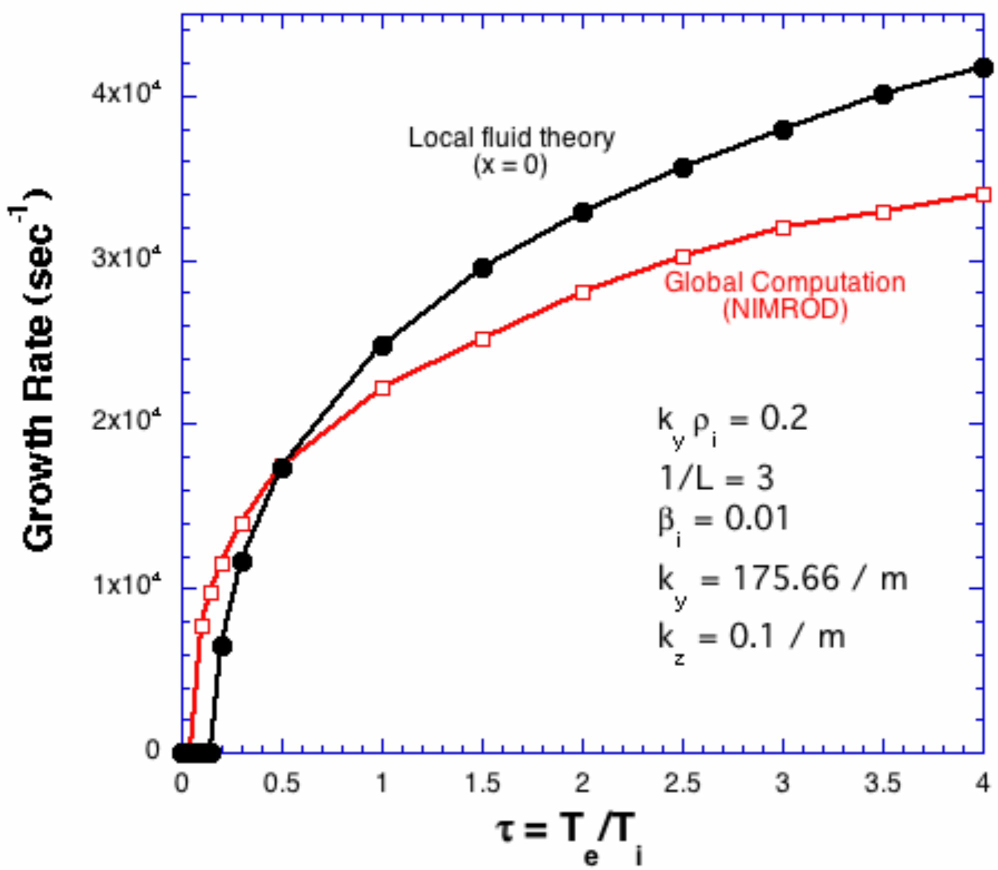


Maximum local growth rate biased toward $x < 0$

Comparison of NIMROD and Local Fluid Growth Rates



Growth Rate is a Function of T_e/T_i



When $T_e = 0$ the drift wave does not propagate

Comparison of Local Kinetic and Fluid Growth Rates with NIMROD Results

$$\rho_i / L = 4 \times 10^{-4}$$

$$1 / L = 3 / \mathrm{m}$$

$$k_{\|} = 0.1\ \mathrm{m}$$

$$\Omega_i = 1.9 \times 10^8\ / \sec$$

$$\beta_0 = \beta_e + \beta_i = 0.05$$

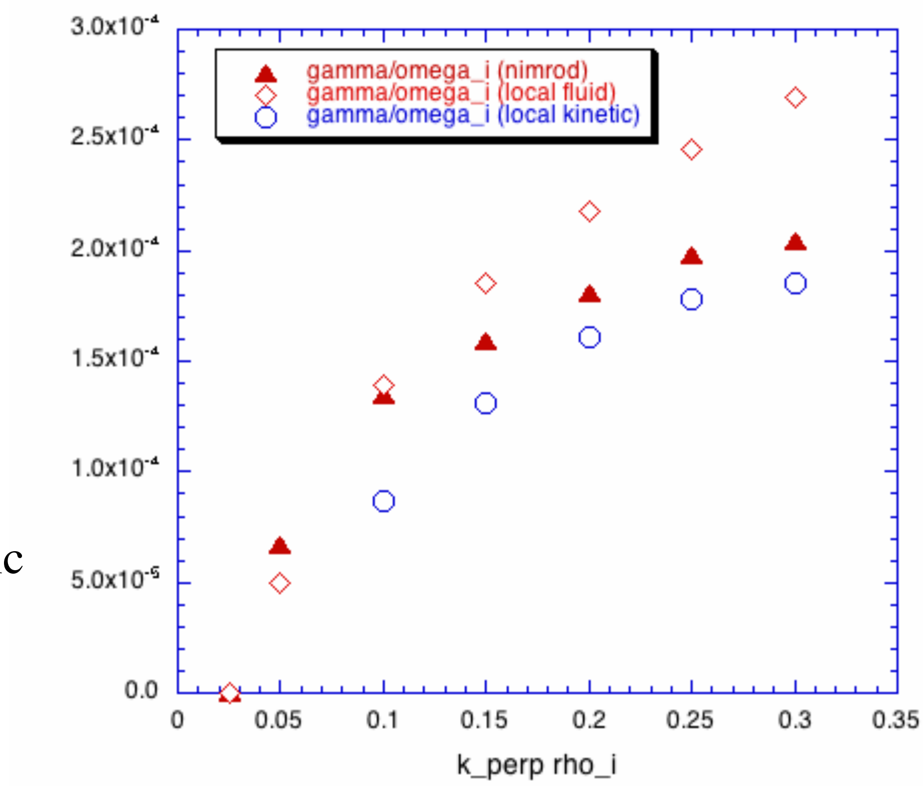
$$T_e / T_i = 4$$

NIMROD and local fluid in fair agreement for $k_y\varrho_i < 0.2$

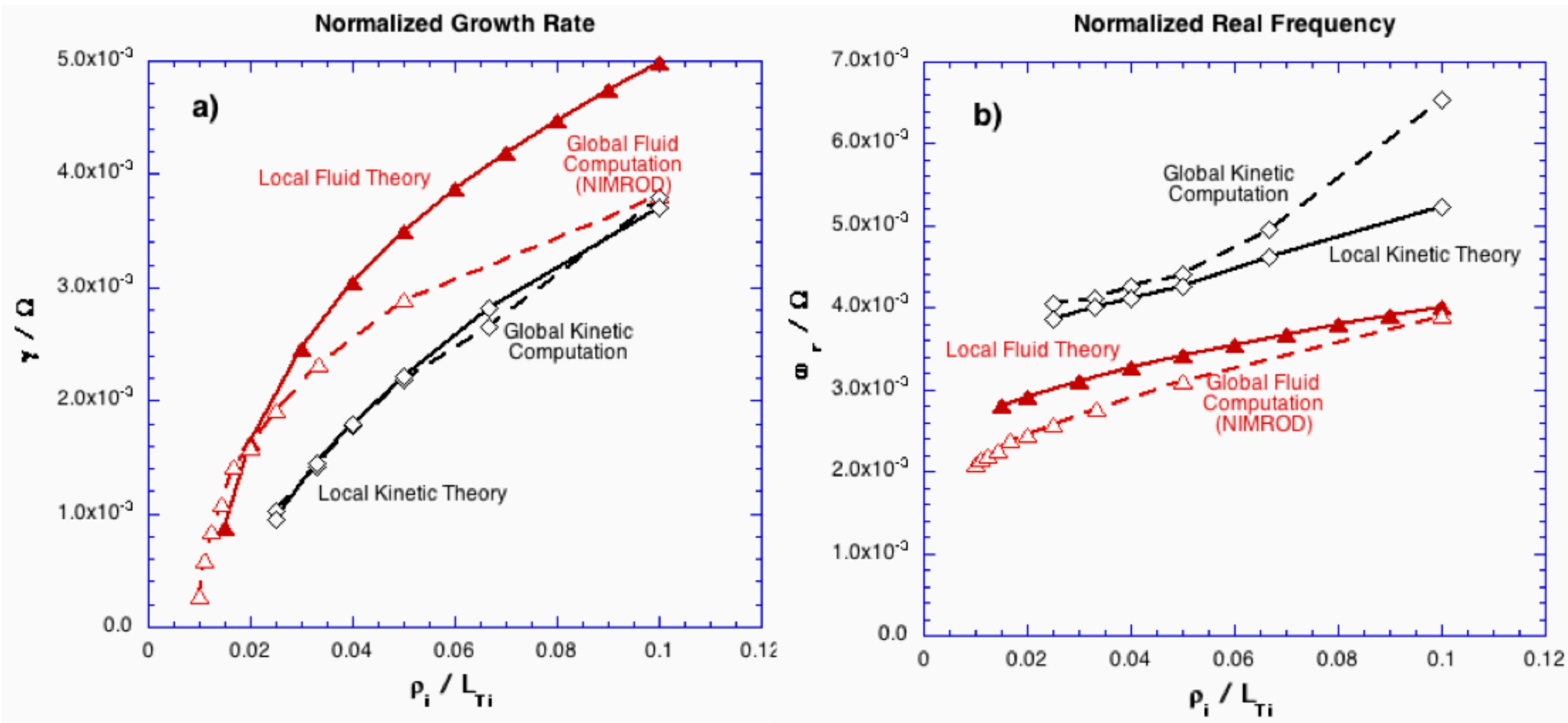
NIMROD, local fluid, and local kinetic agree on marginal point

Local kinetic stabilizes at $k_y\varrho_i \sim 1$

Global hybrid kinetic calculation impractical for this value of ϱ_i/L



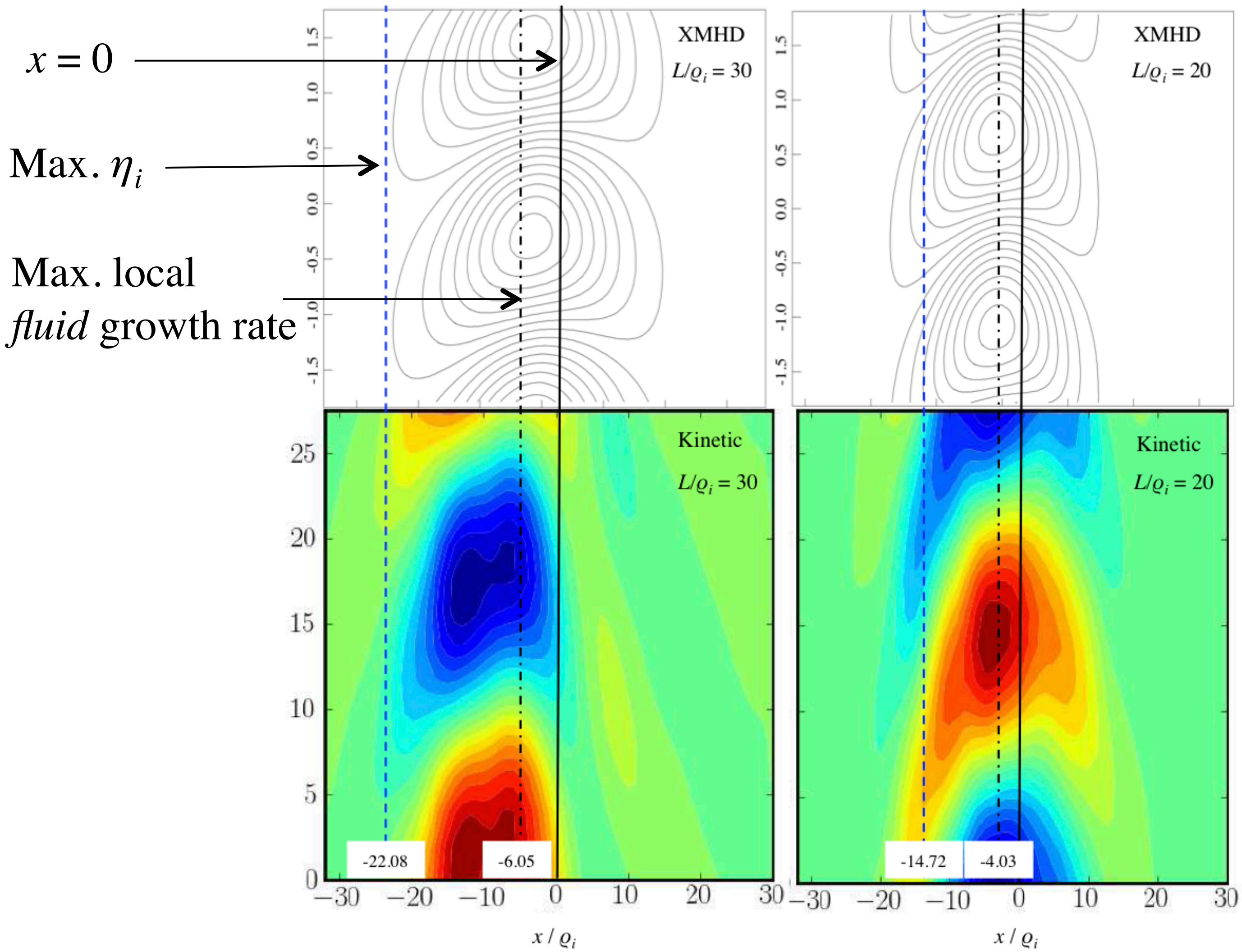
Comparison of Local and Global Kinetic and Fluid Results



Larger values of ϱ_i/L allow global kinetic calculation

$k_y\varrho_i = 0.2$ for all results

Comparison of Kinetic and Fluid Eigenfunctions



Verification of NIMROD

- When $\varrho_i/L << 1$, NIMROD growth rate in good agreement with local fluid theory as a function of $1/L$ for fixed $k_y\varrho_i = 0.14$
  - Difference at marginal point
- For fixed $\varrho_i/L = 4 \times 10^{-4}$, NIMROD growth rate in good agreement with local fluid theory as a function of $k_y\varrho_i$
  - Agreement on marginal point, $k_y\varrho_i = 0.025$
  - Excellent agreement for $k_y\varrho_i < 0.1$
  - Good agreement for $k_y\varrho_i < 0.2$
  - Divergence due to spatial dependence of equilibrium
- Accurate and correct solutions of extended MHD equations for this parameter range

NIMROD is verified for the ITG

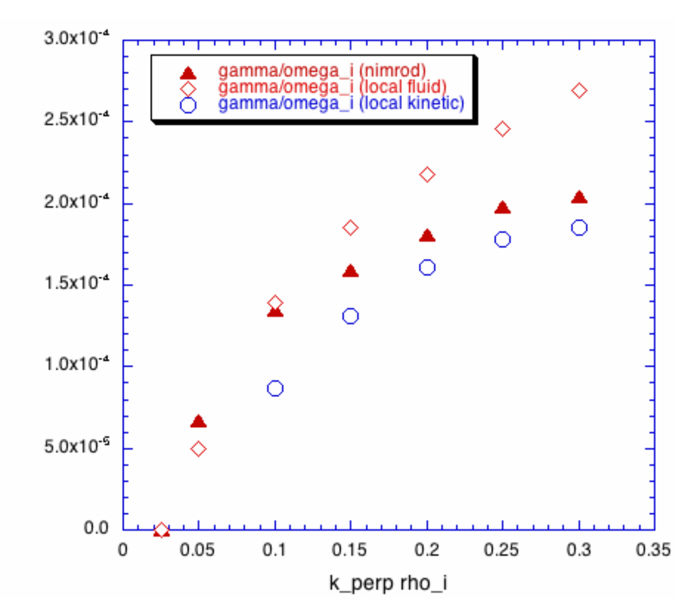
Hybrid Kinetic Model also Verified

Validation of Extended MHD Model in NIMROD

- Direct comparison with more physically accurate kinetic models (both local and global)
  - For $\varrho_i/L < 10^{-3}$, extended MHD has same marginal point in $k_y\varrho_i$ as local kinetic solution
  - Good agreement for $k_y\varrho_i < 0.05$
  - Begin significant divergence for $k_y\varrho_i > 0.2$
    - Wave particle interactions
  - For $k_y\varrho_i = 0.2$, agreement on marginal point in ϱ_i/L (= 0.013), but significant disagreement for larger ϱ_i/L
    - Wave particle interactions
  - Global extended MHD and hybrid kinetic eigenfunctions have similar character for $L/\varrho_i = 30$ and 20

Extended MHD is reliable physical model for $\varrho_i/L < 10^{-3}$ and $k_y\varrho_i < 0.2$, and is validated in this parameter range

Implications for Nonlinear Extended MHD Computations



Integrated model of MHD-scale dynamics in presence of ITG turbulence?

- ITG growth rate increases as $(k_y \varrho_i)^{1/3}$
  - g-mode (interchange) stabilized by large $k_y \varrho_i$
- Increasing resolution for nonlinear computations introduces modes with larger growth rates
  - Impossible to converge nonlinear spectrum?
- Kinetic model stabilizes for $k_y \varrho_\mathrm{i} \sim 1$
  - Suggests adding "hyper-dissipation" ~ $(k_y \varrho_i)^4$
  - Control unphysical large $k_y \varrho_i$ modes with little effect for $k_y \varrho_i < 0.2$

Future Directions

- Can we improve the closures in extended MHD?
  - Particle ions as part of bulk species?
  - Eric Held's kinetic closures (on grid in phase space)?
  - Can we go further into kinetic regime?
- Nonlinear ITG
  - Slab geometry
  - Hyper-dissipation
    - Thermal conductivity?
- ITG turbulence?
  - Effective transport?
  - Annulus calculations?
- Global toroidal simulations
  - Sawtooth + core ITG turbulence?
  - Can all this be captured in a single `"integrated" fluid calculation?