Sawteeth and 1/1 modes

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Topics

- Compressibility and finite aspect ratio corrections drastically change the *m*=1, *n*=1 MHD internal kink mode in a torus – better resembles experiments
  - Large aspect ratio and full MHD very different from RMHD
  - Compressible changes start in the linear 1/1 mode
  - Nonlinear MHD: X-layer, not Sweet-Parker reconnection!
    Fast crash phase with fast onset; rate nearly independent of η.
  - Large aspect ratio expansion breaks down nonlinearly at small $r_1/R \simeq 1/10$
- Compressibility ↔ evolving *dn/dt*
- New type of nonlinear 1/1 ``snake'' mode with a finite size density perturbation at *q*=1 resembles the early stage of heavy-impurity ion snakes in Alcator C-Mod

Compressible 1/1 internal kink in a torus

- Compressibility changes the *m*=1, *n*=1 MHD internal kink in a torus, linearly and nonlinearly
- Original toroidal large aspect ratio solution for linear 1/1 ideal MHD internal kink mode was incompressible (Bussac, 1975)
- Compressible large aspect ratio analytical linear mode solution exists (Wahlberg, J. Pl. Phys. 1999, done with symbolic algebra program), but hard to interpret.
- Nonlinear instability has been analyzed with RMHD, assuming linear mode 1/1 eigenfunction form and dropping higher order aspect ratio terms
  - Hazeltine, et al., PF 1986 neglected current in q=1 layer: exponential mode growth at $\gamma \lesssim \gamma_L$
  - Waelbrock, PF B 1989 used 1/1 magnetic island and helical magnetic flux conservation: modified Sweet-Parker reconnection layer and island width $W \sim \eta t^2$
  - Biskamp, PF B 1991 used 1/1 island and linear eigenfunction: Result similar to Waelbrock. Showed corresponding poloidal stream function growth $U \sim \eta t$ matched the numerical RMHD solution.

- Problem: $W \sim \eta t^2$ growth is too slow to explain observed the speed of sawtooth crashes in later plasmas at smaller resistivities

- One solution: outside MHD, nonlinear electron effects, parallel electron compressibility or electron inertia or other kinetic effects. can greatly speed up the instability (eg, Ayedmir PoP 1991, Wang PRL 1992).
  - These widen and shorten the narrow, poloidally elongated Sweet-Parker reconnection layer to an "X" shape.

- But non-MHD effects are NOT needed for fast crash!
  - RMHD model strongly constrains the perturbations to $m_o=1$, $n_o=1$ and m=n. This leads to formation of a narrow, poloidally elongated ($\theta \lesssim 0.8\pi/2$) reconnection and current layer, which constricts the plasma flow through the layer and reduces the attainable reconnection rate.
  - *Compressible MHD never lets a Sweet-Parker type layer develop*, due to the presence of m=2,0 n=1 harmonics from the linear mode.

Compressible large aspect ratio MHD model

$$\mathbf{v} = \epsilon R \nabla_\perp U \times \hat{\boldsymbol{\phi}} + \nabla_\perp \chi + V \hat{\boldsymbol{\phi}} \tag{1}$$

$$\mathbf{B} = \nabla_\perp \psi \times \nabla \phi + (1/R)\nabla_\perp F + (I/R)\hat{\boldsymbol{\phi}} \tag{2}$$

$$d\psi/dt = \partial U/\partial \phi + \eta \nabla_\perp^2 \psi \tag{3}$$

$$dw/dt = -(1/\rho)\mathbf{B}\cdot\nabla J_\phi - Yw + \mu\nabla^2 w \tag{4}$$

$$dp/dt = -\Gamma Y p + \nabla \cdot n\boldsymbol{\kappa}_T \cdot \nabla(p/n) \tag{5}$$

$$dI/dt = \mathbf{B}_\perp \cdot \nabla_\perp V - YI + \eta \nabla_\perp^2 I \tag{6}$$

$$dV/dt = (1/\rho)\mathbf{B}_\perp \cdot \nabla_\perp I - (1/\rho)\partial p/\partial \phi + \mu \nabla^2 V \tag{7}$$

$$w \equiv -\hat{\boldsymbol{\phi}} \cdot \nabla \times \mathbf{v} = \nabla_\perp^2 U + (1/R)\partial U/\partial R \tag{8}$$

$$Y \equiv \nabla \cdot \mathbf{v}_\perp = y + z \tag{9}$$

$$y = \nabla_\perp^2 \chi + (1/R)\partial \chi/\partial R \tag{10}$$

$$z = -2\epsilon(\partial U/\partial Z) \tag{11}$$

$$RJ_\phi = -\nabla_\perp^2 \psi + (1/R)\partial \psi/\partial R - (1/R)\partial F/\partial Z \tag{12}$$

$$d/dt = (\partial/\partial t) + \mathbf{v}\cdot\nabla \tag{13}$$

Compressible large aspect ratio MHD -2-

An equation for χ can be written from $\nabla r \cdot \partial \mathbf{v}_\perp / \partial t$ as

$$\begin{aligned}
\partial(\chi' + U^{\text{`}})/\partial t &= -(v_\perp^2/2)' + (1/\rho)\left[-(I^2/2)' - p' + J_\phi \psi'/R\right] \\
&\quad + w(U' - \chi^{\text{`}}) + \mu \nabla^2 (\chi' + U^{\text{`}}). \qquad (1)
\end{aligned}$$

where the "radial" and "poloidal" derivatives are $f' \equiv \nabla r \cdot \nabla_\perp f$ and $f^{\text{`}} \equiv \nabla r \times \nabla_\perp f \cdot \hat{\boldsymbol{\phi}}$ for generalized radial and poloidal coordinates.

The equation for the compressibility shows that linearly the main terms are related to χ, not U ($\hat{y} \equiv \nabla_\perp^2 \chi$)

$$\begin{aligned}
\frac{\partial \hat{y}}{\partial t} &= -\frac{1}{\rho}\left[\nabla_\perp \cdot \frac{J_\phi}{R} \nabla_\perp \psi - \nabla_\perp^2 \left(p + \frac{I^2}{2}\right)\right] - \nabla_\perp^2 \frac{v_\perp^2}{2} \\
&\quad + \nabla_\perp \cdot (w \nabla_\perp U) - \nabla_\perp \chi \times \nabla_\perp w \cdot \hat{\boldsymbol{\phi}} + \mu \nabla^2 \hat{y}. \qquad (2)
\end{aligned}$$

Linear mode compressibility

Linear analysis

If $\tilde{B}_{\|} = 0$, can show that the compressibility

$$\nabla \cdot \mathbf{v}_{\perp} = -2\epsilon \mathbf{v}_{\perp} \cdot \boldsymbol{\kappa} \simeq -2\epsilon(\partial U/\partial Z). \tag{1}$$

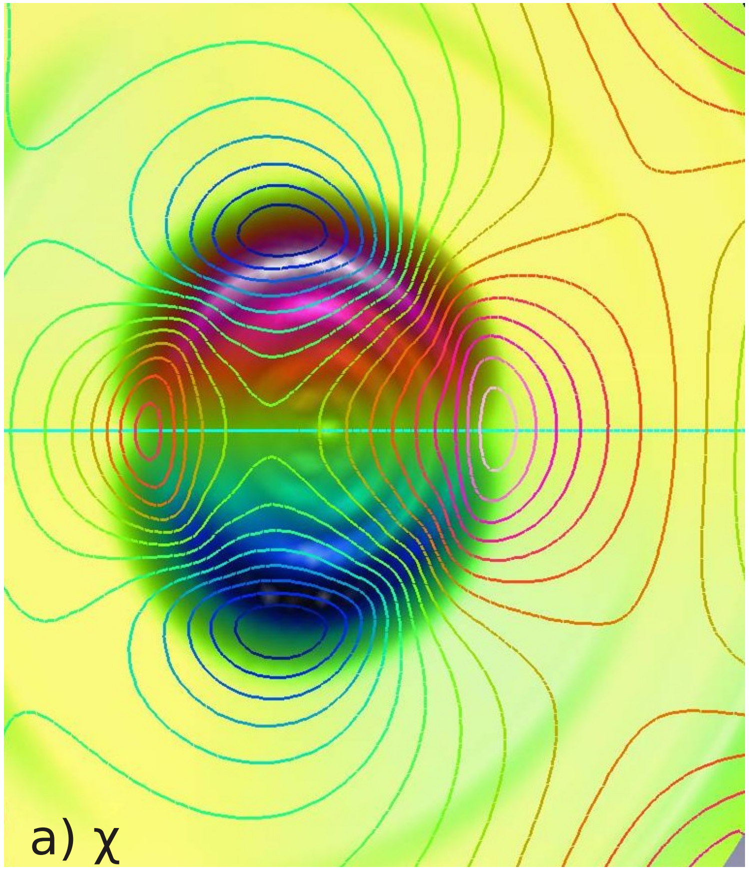
Leads to components $v_{\phi}^{m=0}$ and $v_{\phi}^{m=2}$ representing the "sound wave" terms. The $m \neq 1$ v_{ϕ} and $\nabla \cdot \boldsymbol{\xi}_{\perp}$ contribute to δW and contribute to a linear growth rate scaling $\gamma \sim \epsilon\beta_p$.

If $\tilde{B}_{\|} \neq 0$, $\chi^{m=2}$ enters through $y = \nabla \cdot \nabla_{\perp}\chi$ (note $\chi^{m=1} < \chi^{m=2}$). This introduces the "compressional Alfven wave" coupling of v_{ϕ}, p, I. All the CLAR equations couple to create a 2nd order PDE for χ.

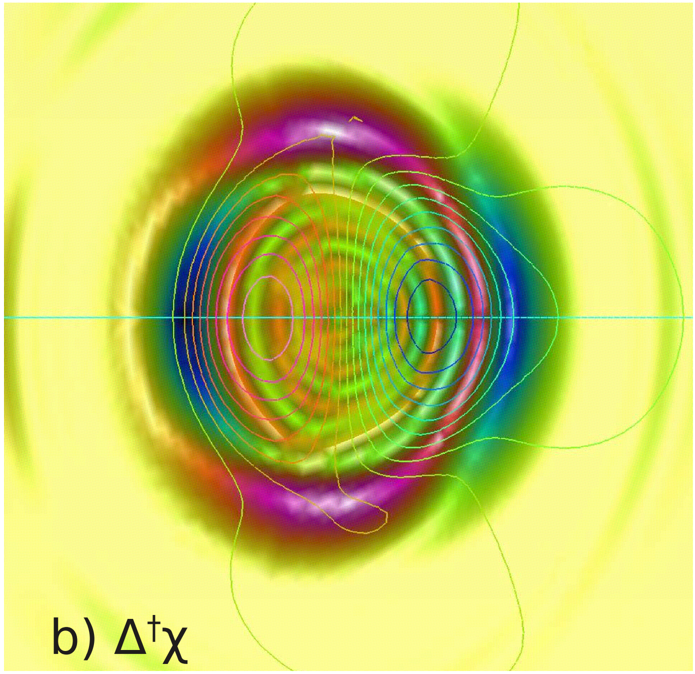
Analysis simpler than Wahlberg '99 - expands around the actual magnetic axis instead of unshifted circular equilibrium.

Nonlinearly, the higher m's localize the layer poloidally near the X-point and prevent the elongated Sweet-Parker layer from developing.

Compressible χ component is m=2, n=1



a) χ



b) $\Delta^{\dagger}\chi$

- a) Lines show χ, shaded red/blue U, poloidal stream function. m=1 U gives main radial displacement over $0<q<1$, $v_r = \gamma \xi^1_r \simeq U^1/r$.
- b) Red,blue shading shows $y=\nabla \cdot \nabla_\perp \chi$ is large and predominantly m=2 at q=1. Contour lines are ψ.

MHD vs Large aspect ratio and RMHD

Perpendicular ($\perp$ to ϕ) momentum equation, neglecting viscosity,

$$\rho(\partial \mathbf{v}_\perp / \partial t) = -\rho(\mathbf{v} \cdot \nabla)\mathbf{v}_\perp + (\rho v_\phi^2 / R)\hat{\mathbf{R}} + (\mathbf{J} \times \mathbf{B} - \nabla p)_\perp = \mathbf{M}. \quad (1)$$

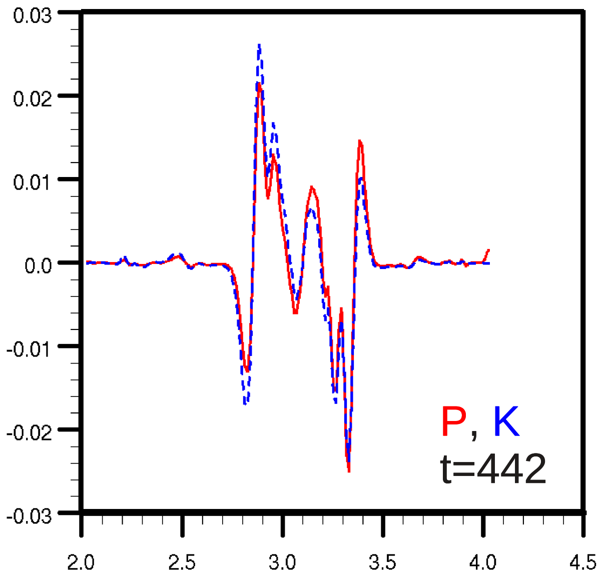
Left hand side:

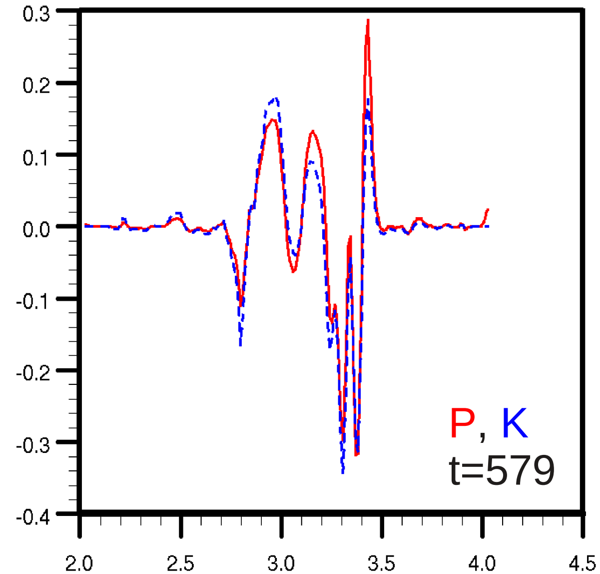
$$\begin{array}{rrcll}
\text{LargeAspectRatio} & \mathbf{M}_L & = & \mathbf{P} - \mathbf{K} & (2) \\
 & \mathbf{P} & = & \nabla_\perp (p + I^2/2) + \rho \nabla_\perp (v_\perp^2/2) & (3) \\
 & \mathbf{K} & = & (J_\phi / R) \nabla_\perp \psi & (4) \\
\text{RMHD (Biskamp 91)} & \mathbf{M}_B & = & \nabla_\perp (p + \rho v_\perp^2/2 + B^2/2). & (5)
\end{array}$$

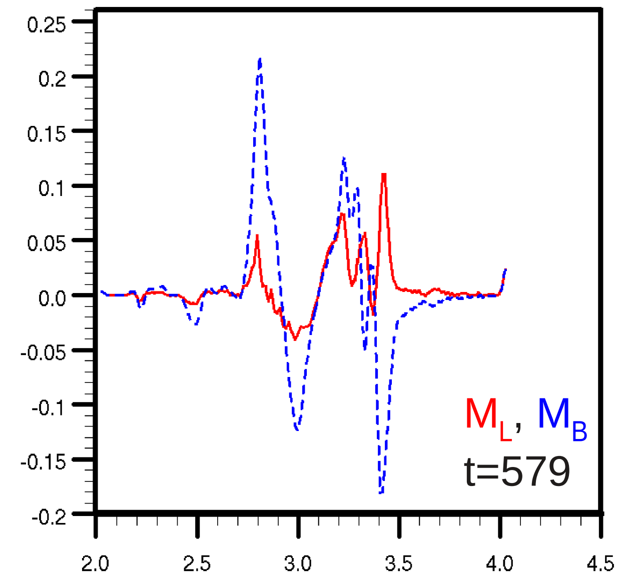
Full MHD (radial component):

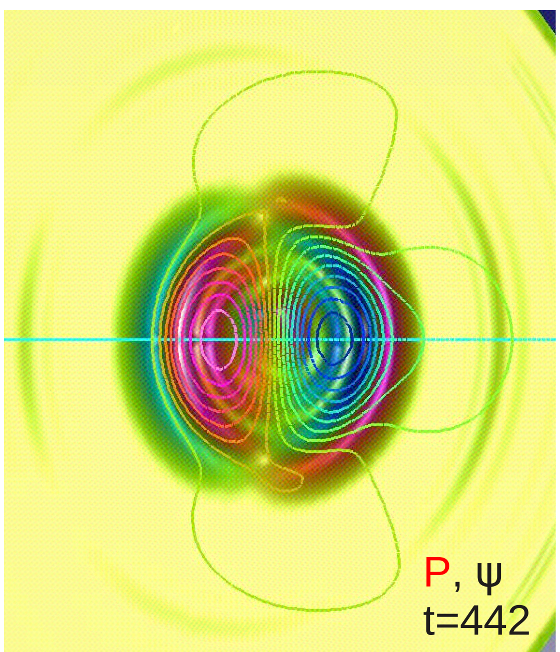
$$\begin{aligned}
M_r \quad = \quad & p' + \rho(v_\perp^2/2)' + (R_o/R)^2 (I^2/2)' - (J_\phi/R)(\psi' - F^{\backprime}) \\
& + \rho(R/R_o) w((R/R_o)U' - \chi^{\backprime}) - \rho(v_\phi^2/R)(\nabla r \cdot \nabla R) \\
& + \rho(v_\phi/R)\left((\partial\chi/\partial\phi)' + (R/R_o)(\partial U/\partial\phi)^{\backprime}\right) \\
& - (R_o/R)^2 (I/R)\left((\partial\psi/\partial\phi)^{\backprime} + (\partial F/\partial\phi)'\right) \\
& - \mu\left(\nabla^2 \chi' + \nabla^2 U^{\backprime}\right). \qquad (6)
\end{aligned}$$

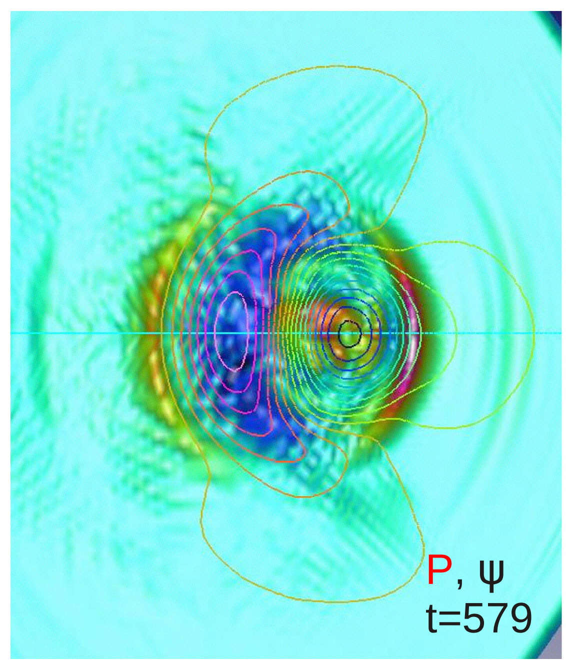
Strong local cancellation occurs in the LAR (and full MHD) perpendicular momentum terms M





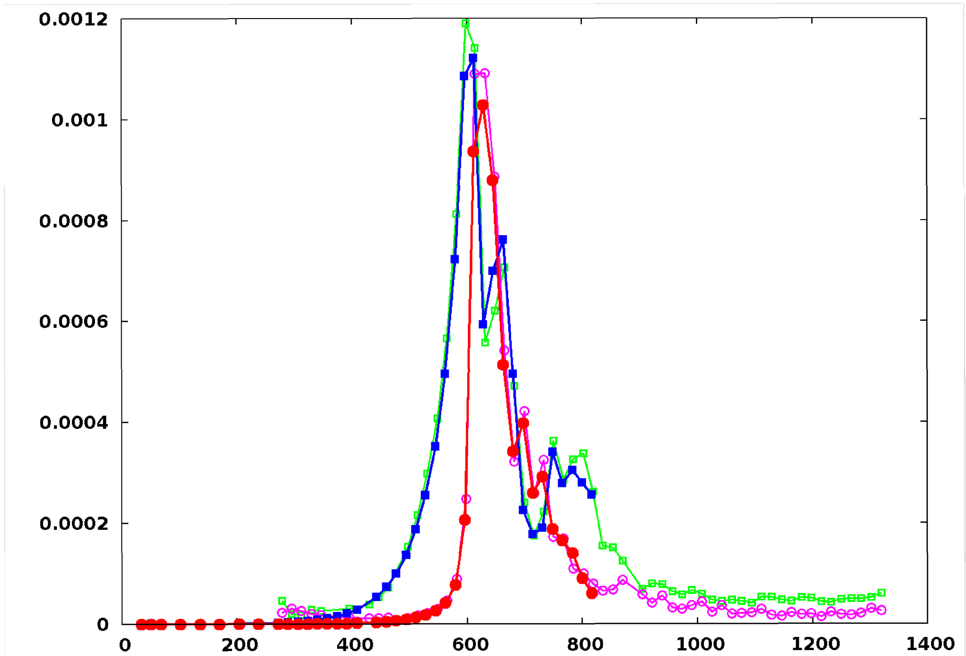


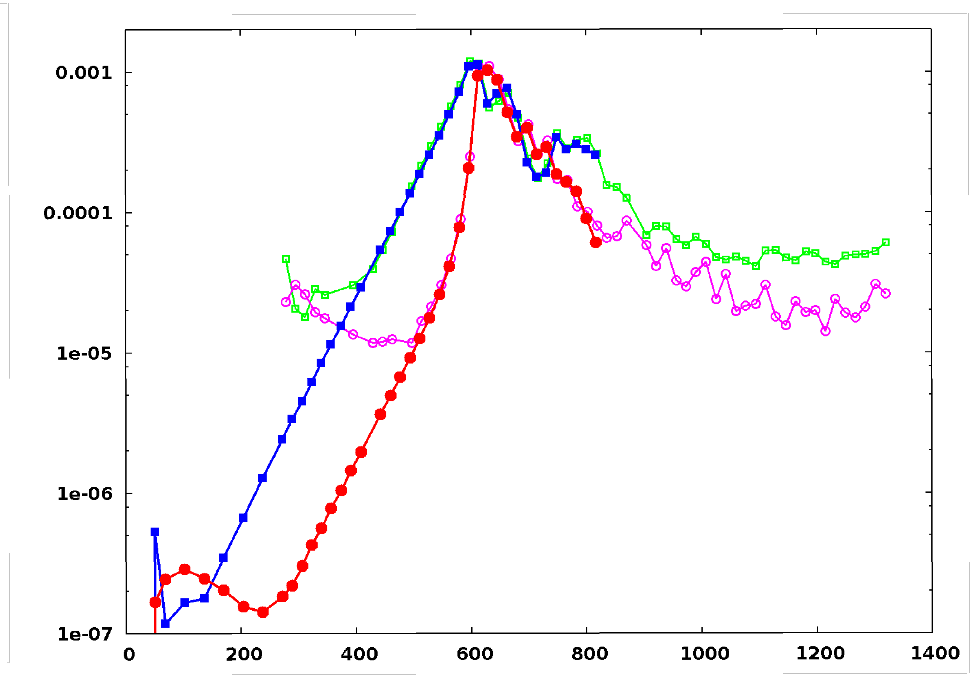




- LAR momentum terms P,K,M shown at early and late times
- Early t=442 has no or very small island; t=579 has island W~r_1/2.
- RMHD M_B always has different shape than M or M_L

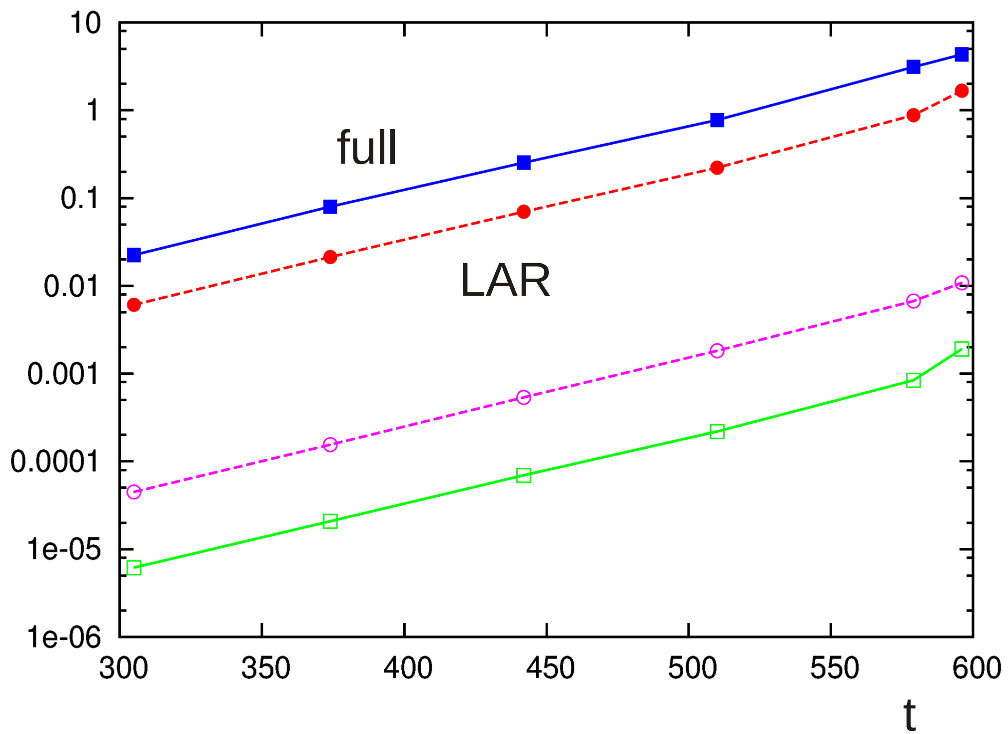
Time history shows late stage fast crash

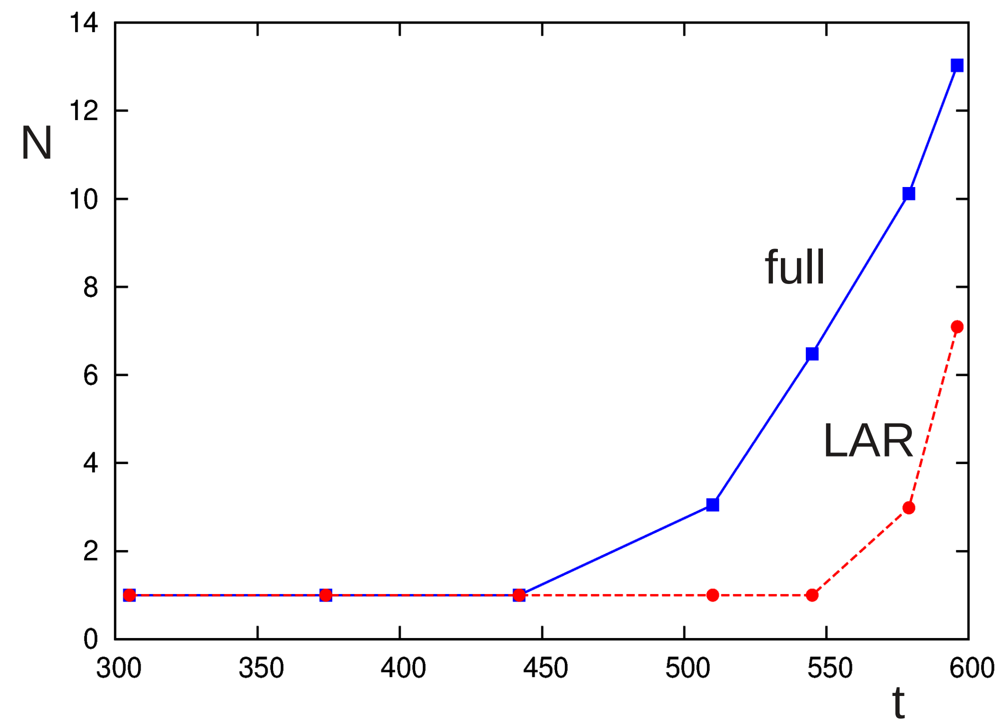




- a) Time history of natural and density-triggered crashes at $S=10^6$. Red is U, blue ψ in L2 norm $\|f\|_2=(\int d^3v\ |f^2| \ /\ \int d^3v)^{1/2}$. Triggered crash is rigidly displaced in time to overlay peak of natural crash (pink/green). Time in τ_A.
- b) Log plot of same.
- Central temperature and 1/1 ψ are completely lost at peak amplitude. Central density is lost over next 200-250 τ_A.

Full MHD develops more higher harmonics faster





- a) Ratio of n≥2 harmonics to the n=1 (L2 norms) in time for M_r (solid top curve) and M_{Lr} (dashed top curve). n=1 value shown in lower two curves.
- Harmonic number N for which at least ½ the total L2 amplitude lies in harmonics n≥N. Full MHD M_r (blue) develops high harmonics much faster than the LAR M_L terms (red), at small island width. (N=2 time cannot be computed).

1/1 helical ion density snakes

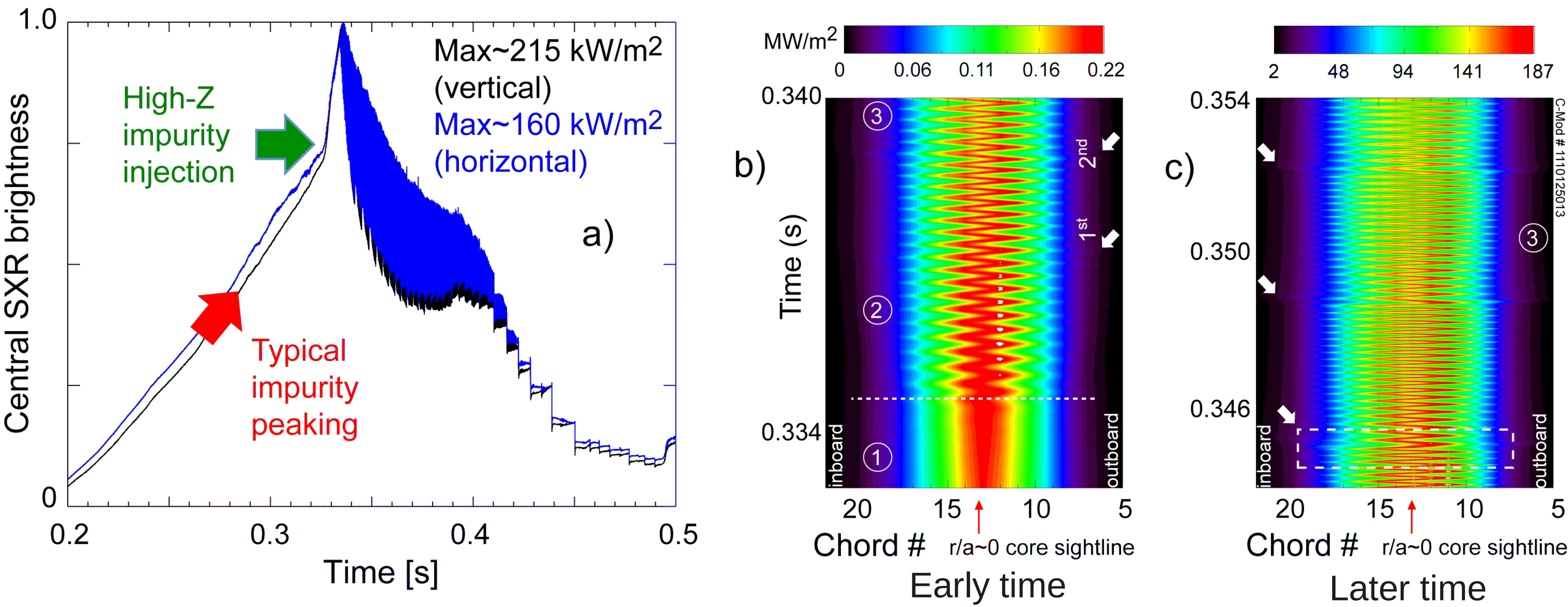
- Snakes are common long-lived helical concentrations of ion density around magnetic rational surfaces, most often at q=1
  - Typically coexist with periodic sawtooth oscillations
  - Variation in ion type, background plasma, formation
- New high resolution observations on Alcator C-Mod show details of heavy-impurity snake formation and interaction with sawtooth oscillations. Simulated with M3D – first results:

  L. Delgado-Aparicio, L. Sugiyama, et al., PRL 2012

  L. Delgado-Aparicio, L. Sugiyama, et al., NF 2013

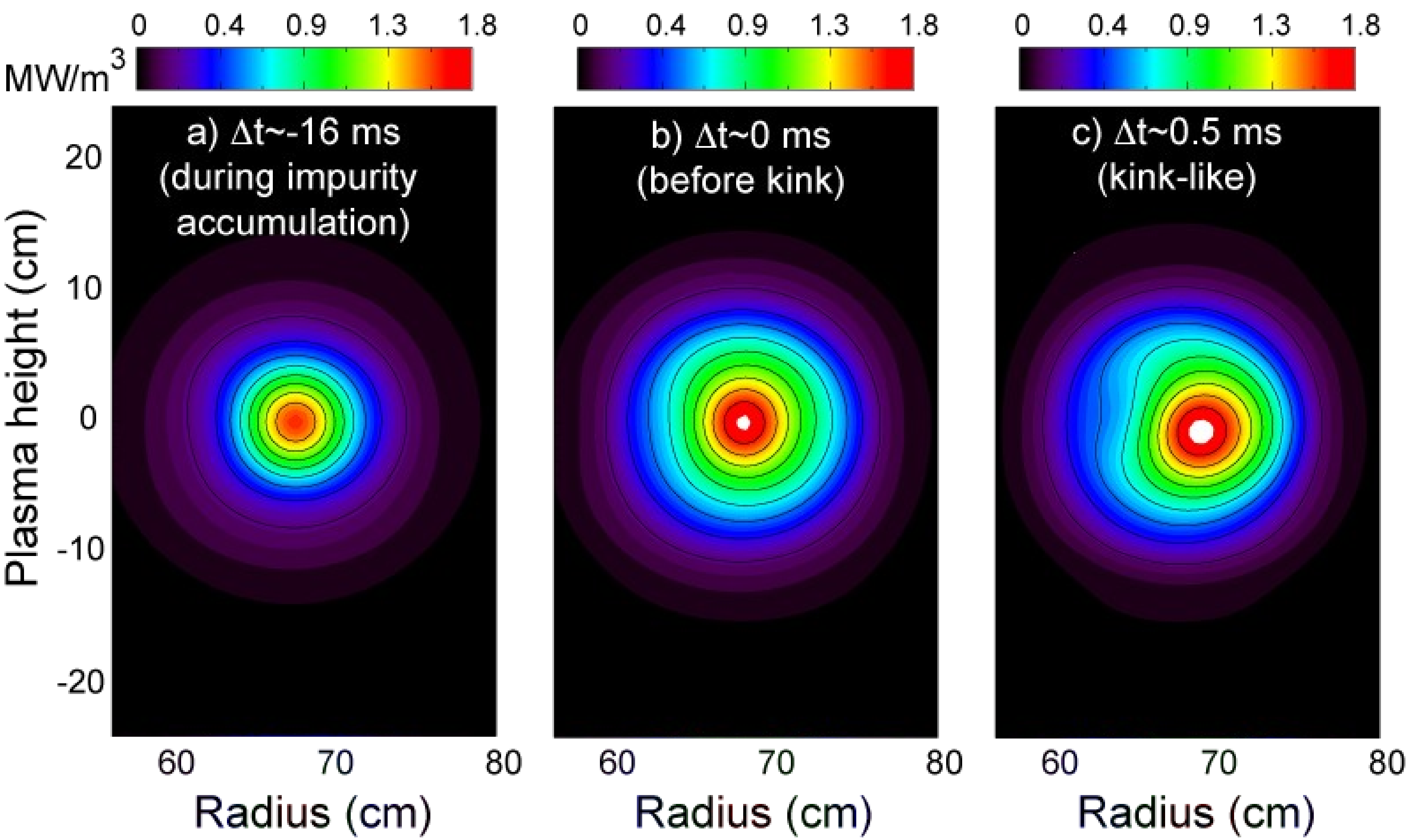
  L. Sugiyama, PoP 2013.
- Important for ITER
  - q=1 radius nearly a/2
  - Tungsten Z=74 vs C-Mod molybdenum Z=45 (32 main charge state in snake) plus diagnostic ions Ar, etc $\rightarrow$ larger δn_e

Long-lived (1,1) snake modes in C-Mod are routinely observed on a number of diagnostics



- Ohmic current ramp-up phase or early in the current flattop.
  - High-T_{edge} at startup increases Mo impurity erosion from wall/limiter
  - Impurity density pinch leads to on-axis impurity peaking (axisymmetric)
- 3 stages: Initial central impurity peaking → Broad 1/1 central kink → 1/1 crescent with sawtooth oscillation of core. (Sawtooth crashes shown by arrows.)

Alcator C-Mod early snake: Formation of broad 1/1 kink structure



Circular

Shifted axisym. core

Rotating kink-like perturbation

← Sawtooth-free →

Mo accumulates in plasma core

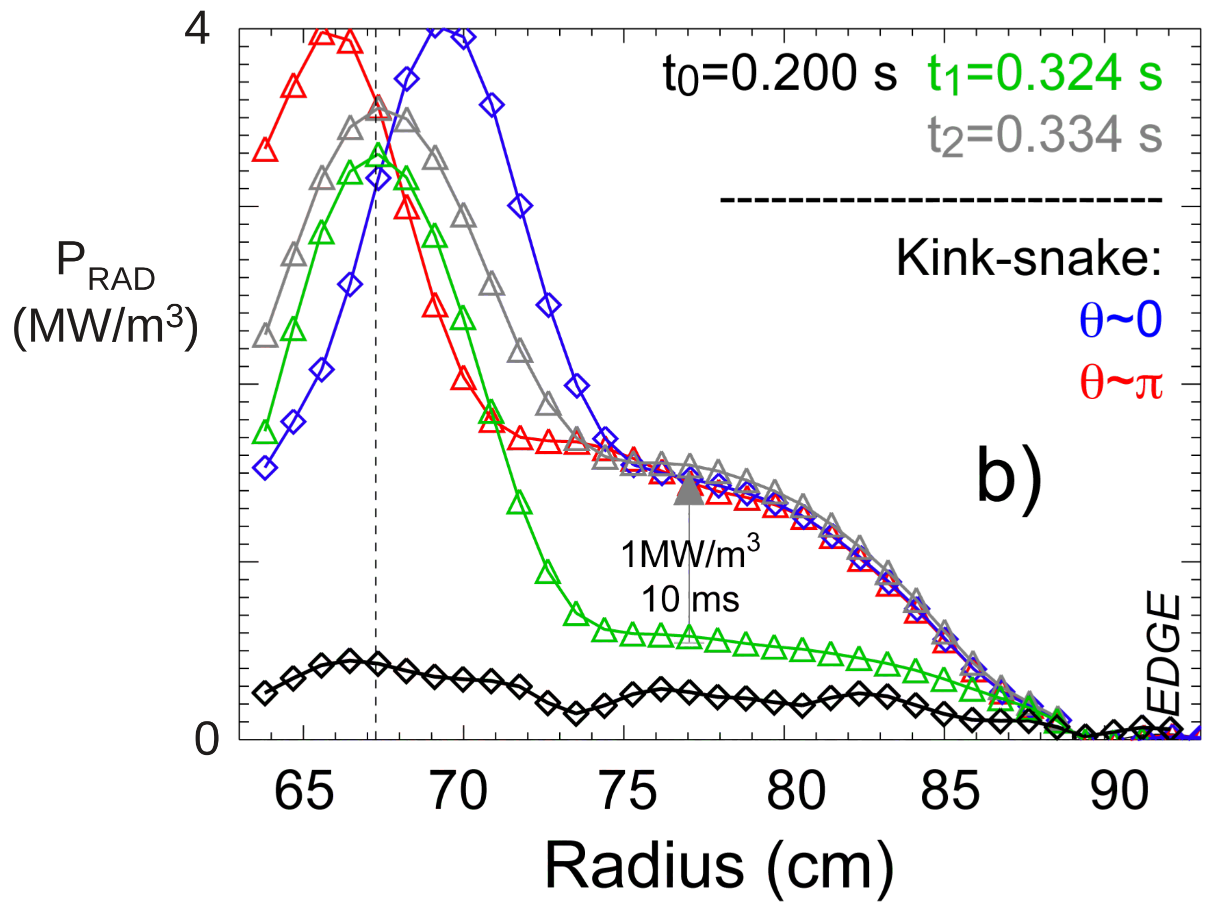
1/1 kink forms

Sawtooth-free early kink-like state lasts from few to hundreds of ms (Continuing impurity-ion pinch?)

First few crashes appear to cause transition to crescent snake inside q<1 (how?)

High-resolution AXUV arrays show growing impurity density kink during snake formation

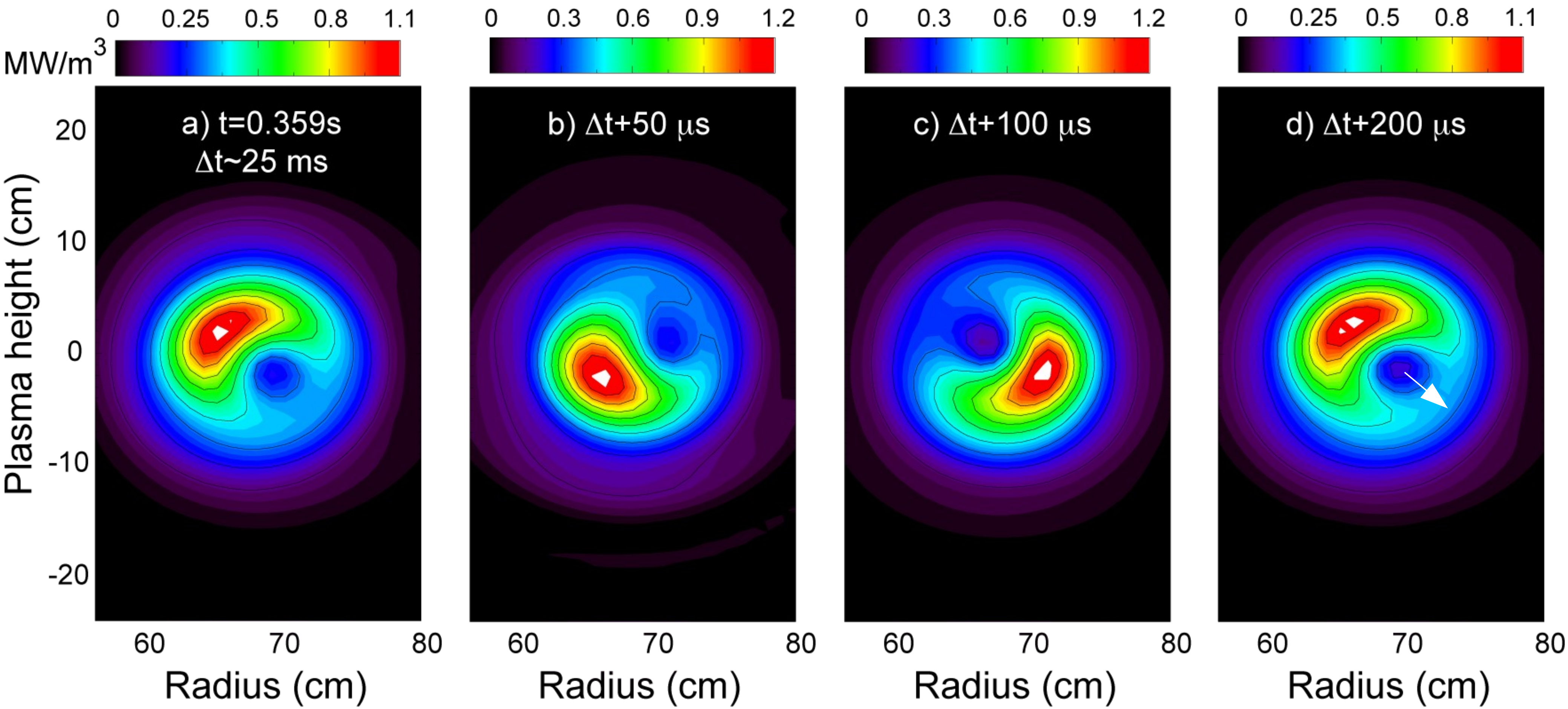
Snake formation: growing density kink



- AXUV P_{rad} measurements show n_{Mo}.
- Allows SXR signal to be identified with impurity density, without T_e contamination (unlike most snake SXR measurements).

Later stage: Crescent-shaped 1/1 “magnetic island”

LDA12: Alcator C-Mod



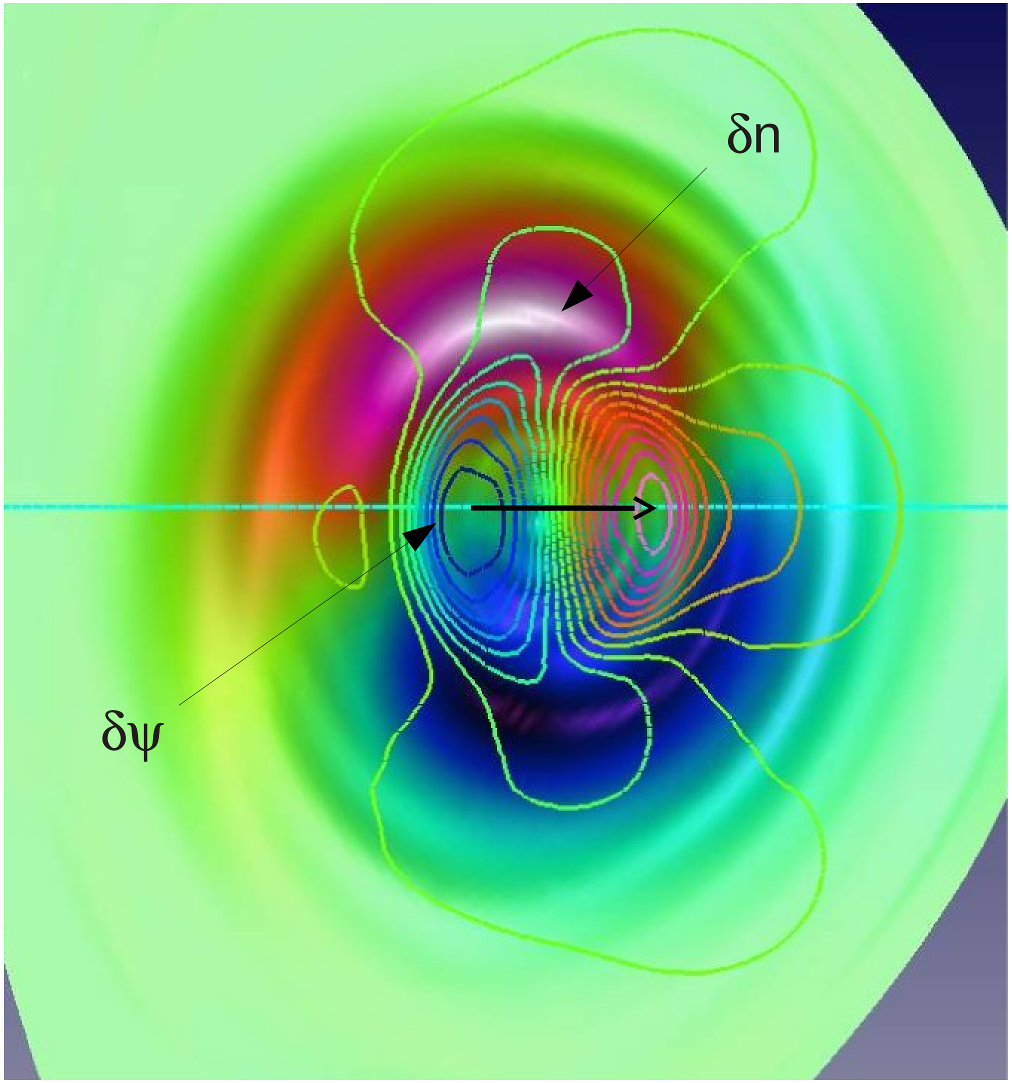
- 1/1 helical structure rotates toroidally with background toroidal rotation (electron diamagnetic direction)
- Periodic partial sawteeth: Dark circular core (low n_{Mo}, higher T) rapidly moves outward to q≈1, then returns inward more slowly to center.

M3D simulation results: $Z\delta n_Z=\delta n_e=\delta n_{MHD}$

- Snake is a nonlinear dynamic density state.
- Density evolution is crucial.
- Toroidal rotation is important.
- Two δn components: $q\simeq 1$, $q<1$
  - Quasi-steady state helical density concentration around q=1 layer (including outside) with small helical δp, since δT evolves to compensate δn. Either helicity $\theta \pm \varphi$. Sustained if small-moderate local q-shear. Similar to W. Cooper's static 3D helical-core equilibria, but self-formed.
  - Helical density at q=1 drives a new nonlinear 1/1 internal-kink type perturbation over q<1, very slowly growing. δn anti-aligned to δp.
  - q<1 kink motion is perpendicular to density concentration at q=1, so a sawtooth would not destroy the snake.
    - 1/1 mode also has convective motion aligned with the kink p,ψ. Here, driven by v_φ rotation, larger than kink motion.
- Best fit to early C-Mod impurity snake has applied-density helicity aligned with **B** and background toroidal rotation of magnitude similar to experiment. Snake rotates with plasma.

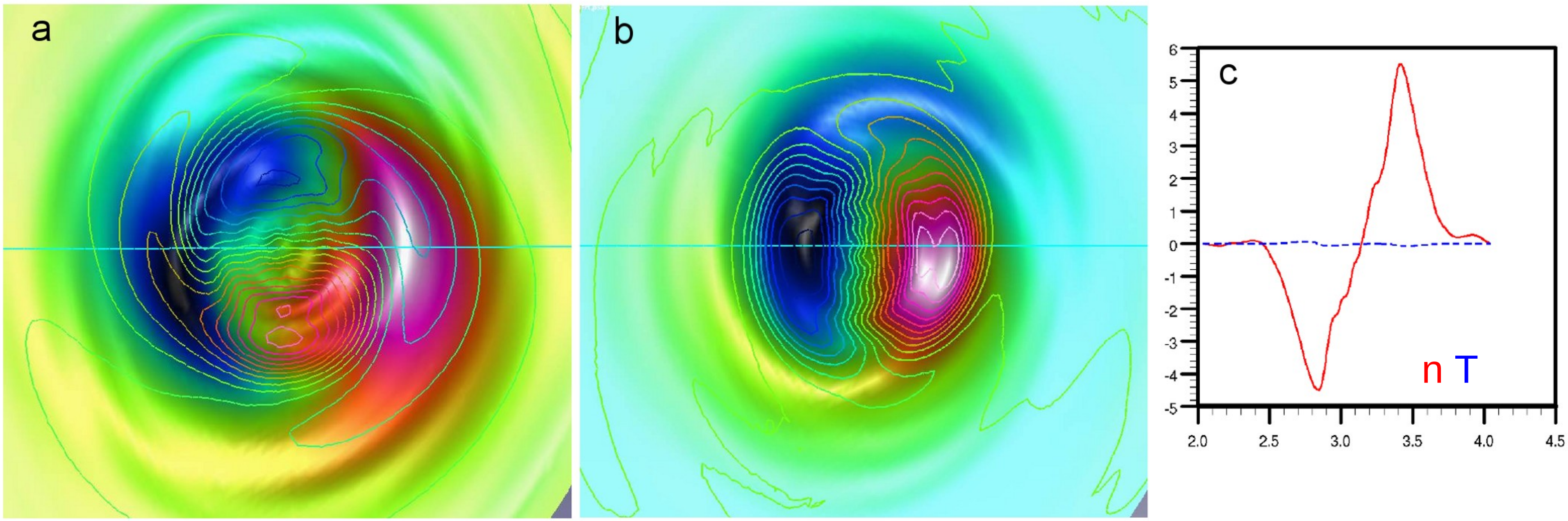
New nonlinear 1/1 mode resembles early stage of C-Mod impurity snake

- Nonlinear MHD simulations with toroidal rotation and separate temperature and density evolution find a new nonlinear, slowly growing m/n=1/1 kink-like mode over $q \lesssim 1$ compatible with C-Mod early snake.
- Quasi-steady-state helical density perturbation peaks near q=1 and extends outside $q \gtrsim 1$.
- Helical temperature $\delta T \simeq -\delta n$ tends to minimize the perturbed pressure gradient (ie, free energy) at $q \simeq 1$, somewhat at q<1.
- No initial magnetic island; forms slowly
- 1/1 kink motion is perpendicular to main density near q=1 so density snake would not be affected by the sawtooth crash.
- Background toroidal rotation important



details in Sugiyama, PoP 2013

Helical δT tends to become anti-aligned with δn



- n, T, p for parallel density helicity and toroidal rotation

  a) At q≃1, helical n=1 δT (contour lines) is anti-aligned with δn (red/blue shaded). Inside q<1, δn is small. (Shown in plane φ=π/2 where density peak on q=1 lies on outboard midplane.)
  c) Midplane profiles for a).
  b) δT (shaded) is almost aligned with δp (lines) and δψ (at φ≈0).

- Mode rotates toroidally with background rotation.

Summary

- Compressibility and finite aspect ratio corrections in the *m*=1, *n*=1 MHD internal kink mode remove many inconsistencies with sawtooth experimental observations
  - Nonlinear MHD: ``X" reconnection region, not Sweet-Parker layer! Fast crash phase with fast onset; rate nearly independent of η. Two-fluid acceleration not needed.
  - Large aspect ratio expansion breaks down at small $r_1/R \simeq 1/10 \Rightarrow$ Important implication for plasma edge instabilities!
- Compressibility ↔ evolving *dn/dt*
  - New type of nonlinear 1/1 mode with a finite size density perturbation at *q*=1 resembles the early stage of heavy-impurity ion snakes in Alcator C-Mod
- 1/1 work continuing - also 1/1 electron fishbones
- Future: Small amount of high-Z impurities can be important for MHD instabilities (charge density δn_e,δp or P_{rad} cooling/η). ITER will have tungsten with Z=74!

Other News – M3D, NERSC

- NERSC Edison/Hopper: problems porting M3D to Edison
  - Potential problem with PETSc 3.3 and M3D: strange run-time memory errors (Josh Breslau and me, independent upgrades); working with NERSC consultants and PETSc group
  - PETSc 3.1 works fine (Hopper) but only 3.3 on Edison
  - Edison: only cray-compiler works with mixed Fortran/C and PETSc, but problems with M3D
- NERSC visualization
  - New NX server doesn't support full Mac screen resolution on laptop Retina display.
  - If you miss Euclid, the old dedicated viz machine, tell NERSC now!
- Many changes being considered at NERSC – your input wanted!