Studies of Repeating Sawteeth using M3D-C[1]

Presented by S. C. Jardin

J. Breslau, J. Chen, N. Ferraro[1], I. Krebs[2]

Princeton Plasma Physics Laboratory
[1]General Atomics
[2]IPP, Garching

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We are using M3D-C[1] to solve the MHD equations to compute the self-consistent long-time (transport timescale) behavior of a tokamak discharge subject to:

- loop voltage (I_P controller)
- density source (n_e controller)
- heating source (NB)
- momentum source (NB)
- shaping fields

- resistivity η
- viscosity ν
- thermal conductivity $\kappa_{\|}$ & $\kappa_{\perp}$
- particle diffusivity D
- ion-skin depth $d_i = c/\omega_{pi}$

Standard transport model:

$$\eta = \eta_0 \left(\frac{T_e}{T_{e0}} \right)^{-3/2} \qquad \nu, D = \text{const} \qquad \kappa_{\perp} = \kappa_0 \left(1 + \alpha \left| \nabla T \right|^2 \right) \left(\frac{p}{p_0} \right)^{-1/2} \qquad \kappa_{\|} \simeq 10^5 \kappa_0$$

- Variable heating source, momentum source, and ion-skin depth.
- Initial conditions have $q_0 < 1$, so one sawtooth always occurs.
- Emphasis is on large $S >> 10^6$ and realistic d_i

Note: results presented today have $\alpha = 0$.

Can we quantitatively model full repeating sawtooth cycles?

Code Improvements for efficient 2F nonlinear

- Variable timestep
  - Set a minimum and a maximum GMRES iterations: KSP_MIN and KSP_MAX
  - Reduce Δt by 5% if # of iterations > KSP_MAX
  - Increase Δt by 5% if # of iterations < KSP_MIN
- Implicit treatment of hyper-resistivity terms:

$$\mathbf{A} = R^2\nabla\varphi\times\nabla f + \psi\nabla\varphi - F_0 \ln R\hat{Z}, \qquad \mathbf{B} = \nabla\psi\times\nabla\varphi - \nabla_\perp f' + F\nabla\varphi$$

$$\mathbf{V}_M = \left[\psi,\ F,\ f,\ \Delta^*\psi\right]$$

Magnetic field variables in implicit advance

  - [optional] third term improves numerical stability
  - [optional] 4th term allows implicit hyper-resistivity…reduces GMRES iterations
  - Results with/without this term being compared
- Electron mass in Ohm's law
  - Adds (small) diagonal term in two-fluid advance
  - Results with/without this term being compared

$$f' \equiv \partial f/\partial\varphi$$

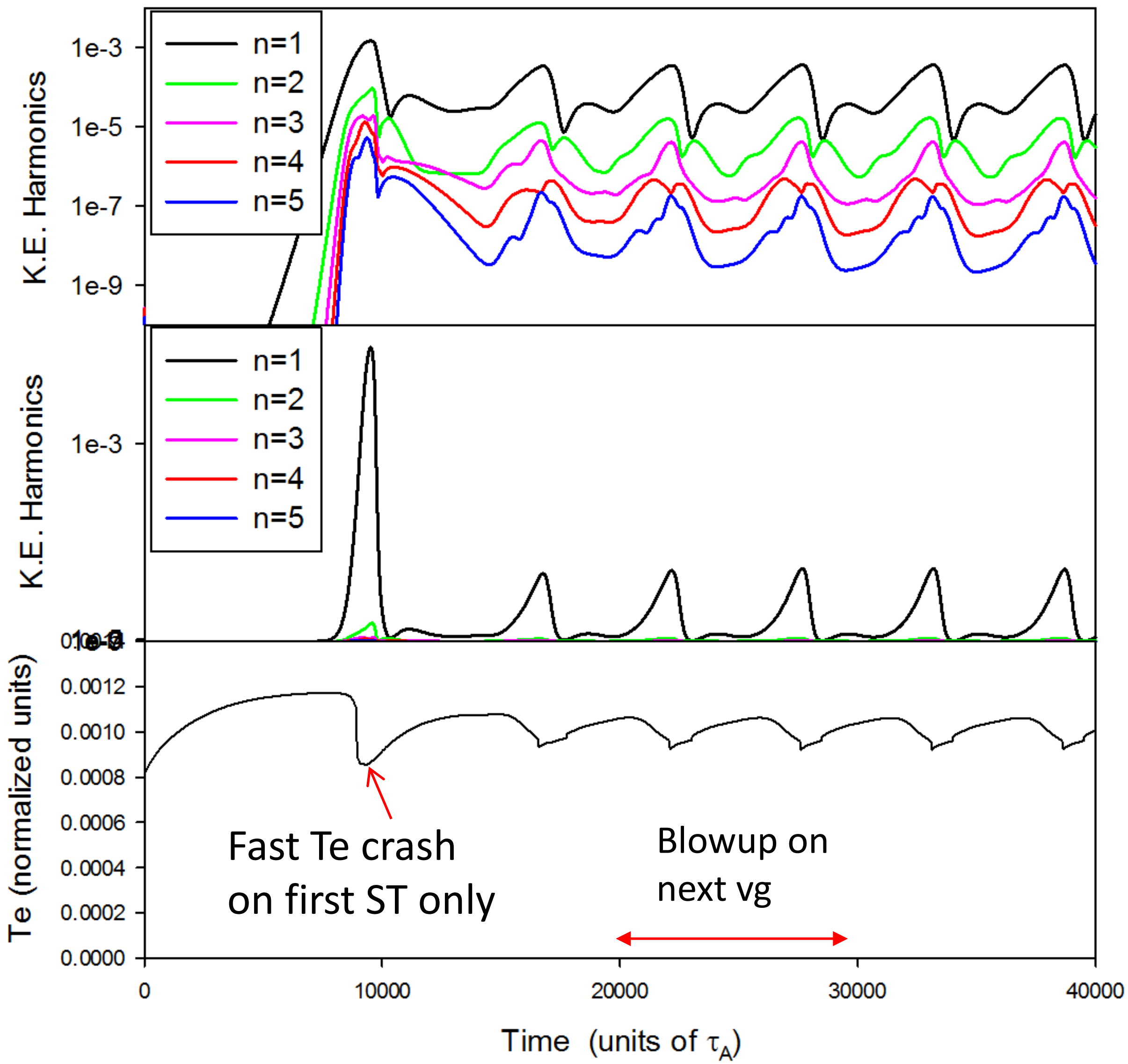
$$F \equiv F_0 + \nabla_\perp^2 f$$

Summary of results:

- low β (< 0.5%) discharges exhibit periodic oscillations
- However
  - do not observe fast Te crash
  - unrealistic scaling with η
  - 2-fluid ($d_i > 0$) effects do not change these conclusions

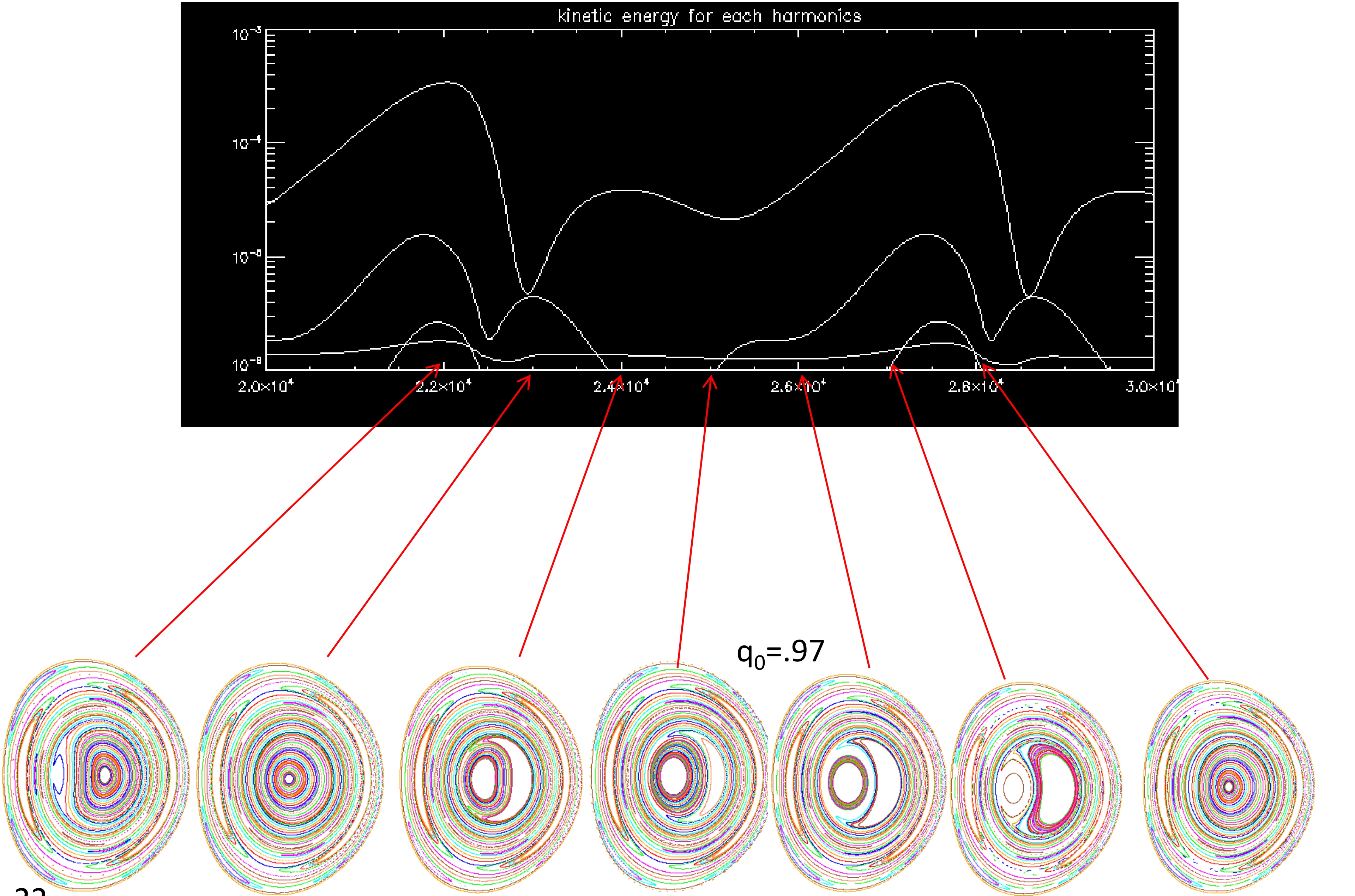
- For higher β discharges, the oscillations die out
  - stationary state is formed with $q_0 = 1 + \varepsilon$ and helical poloidal flow
  - sheared rotation and 2F terms can bring these oscillations back

Typical periodic oscillations S=10^6, β=.001

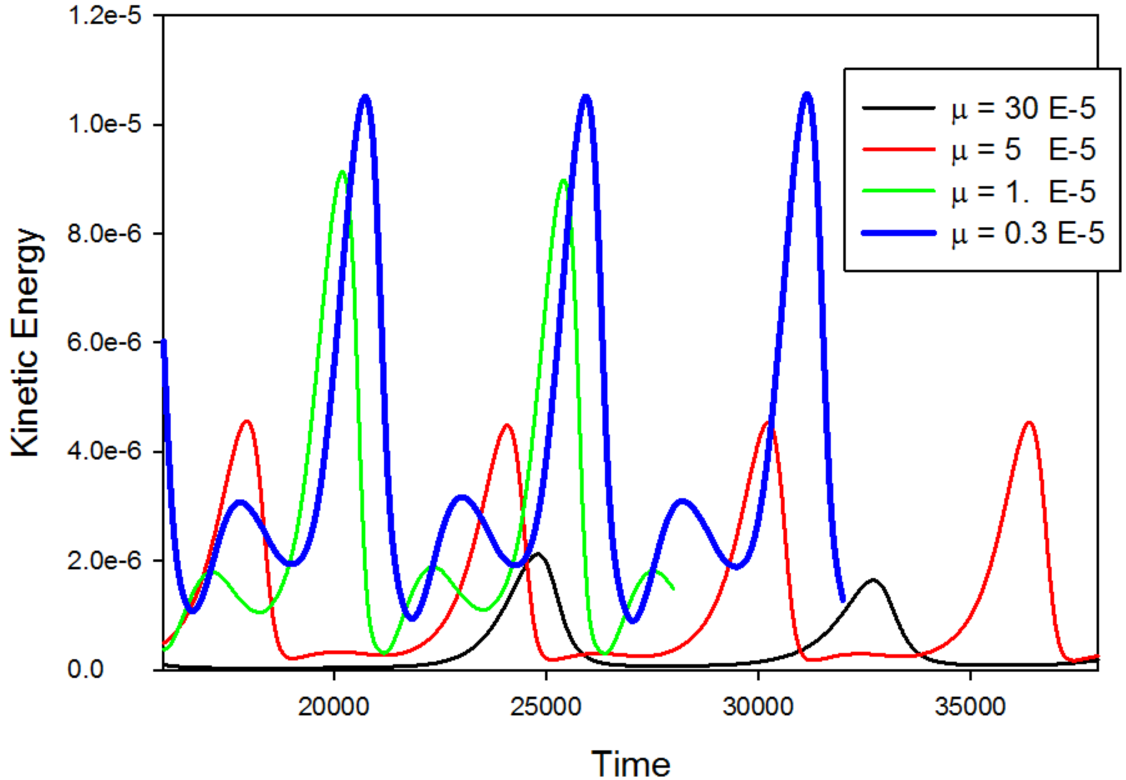


CMOD 16G

Typical Kadomsev-like periodic oscillations S=10^6 β=.001



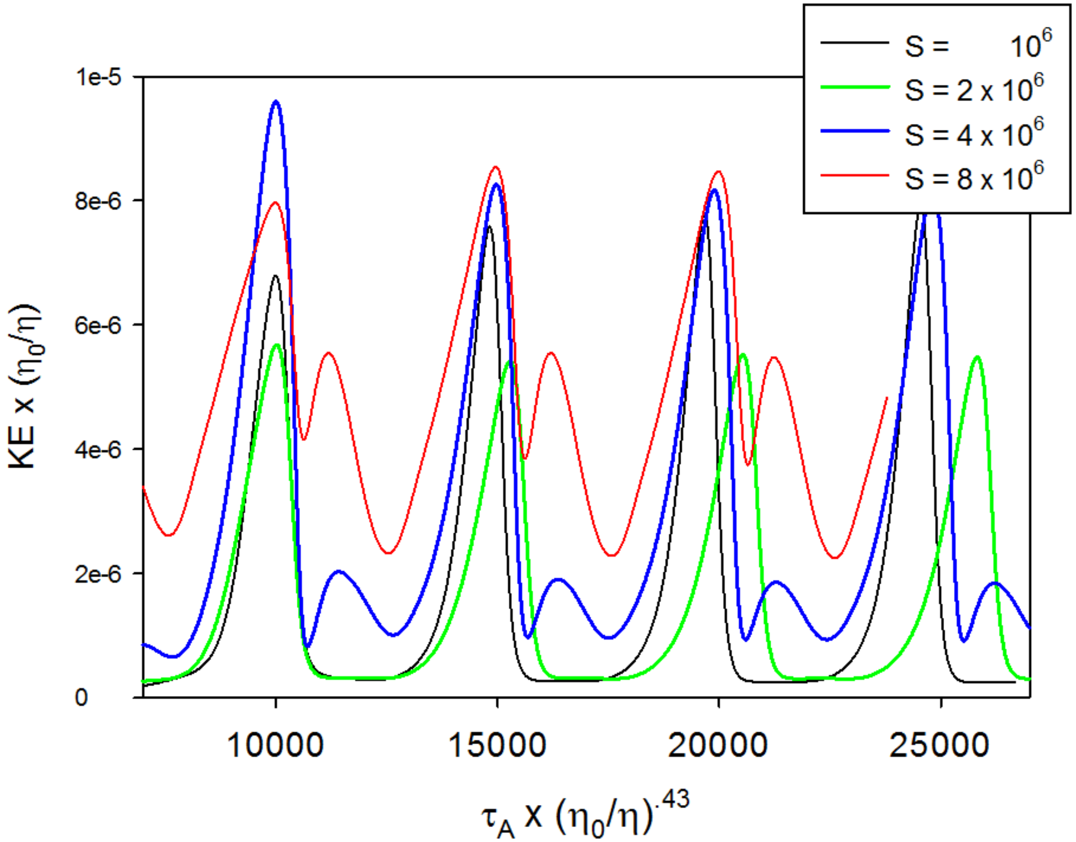
Viscosity Scan at constant S=10^6 and β=.001



- Max KE amplitude increases with μ^{-1} (to a point)
- Period increases with μ
- Lowest μ can have more complex behavior (bouncing)
- Basic character of repeating Kadomsev reconnection unchanged

CMOD08
CMOD10
CMOD26
CMOD30

Resistivity scan: β = .001, no rotation



- Period gets longer as η gets smaller as $\eta^{-0.43}$
- Kinetic energy (and ΔTe) per event decreases as η
- Less like Kadomsev reconnection at highest S values as KE does not decrease to low value between events

- Δq_0 decreases from 0.05 to less that 0.01 as η decreases

CMOD07 μ = 10 E-5
CMOE09 μ = 10
CMOD29 μ = 2.5
CMOD1E μ = 1.0

➔ *Resistive MHD leads to scaling that is inconsistent with experiment at high S >> 10^6 !*

Change in q_0 between sawteeth: $\Delta q_0 = \eta \, \Delta t$

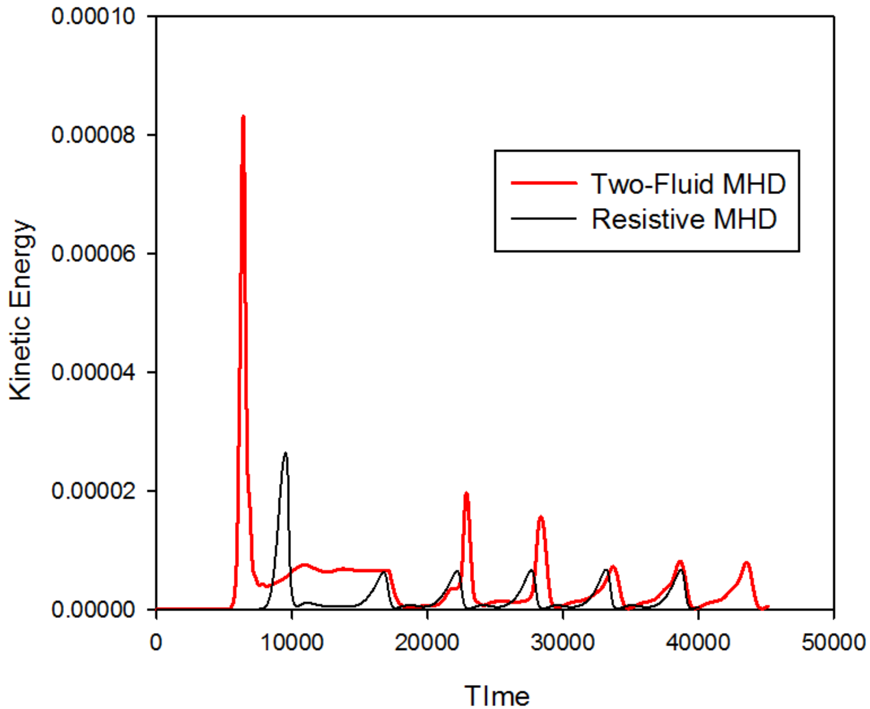
displacement due to resistive MHD: $\xi = \xi_0 e^{\gamma \, \Delta t}$

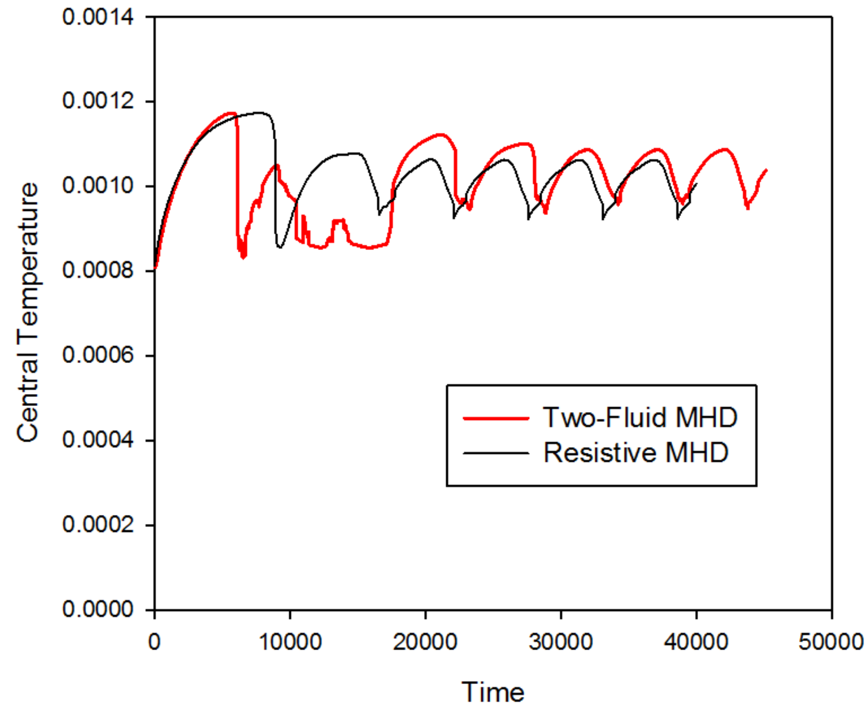
if $\gamma \sim \eta^{1/3}$, time for N e-foldings: $\Delta t = \dfrac{N}{\gamma} \sim \eta^{-1/3}$

$$\Delta q_0 = \eta \, \Delta t \sim \eta^{2/3} \rightarrow 0 \text{ as } \eta \rightarrow 0$$

This seems to rule out Kadomtsev reconnection at high S .
(This conclusion reached by Wesson in 1987)

Comparison of resistive MHD and 2F MHD





- Two simulations with same β=.001 and S=10^6: with and without 2F terms
- Two-fluid terms change the initial behavior, but not the long-time behavior of repeating sawteeth.
- Still no fast Te crash after the first ST

CMOD16G
CMOD25G

Summary of results:

- low β (< 0.5%) discharges exhibit periodic oscillations
- However
  - do not observe fast Te crash
  - unrealistic scaling with η at high S
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- For higher β discharges, the oscillations die out
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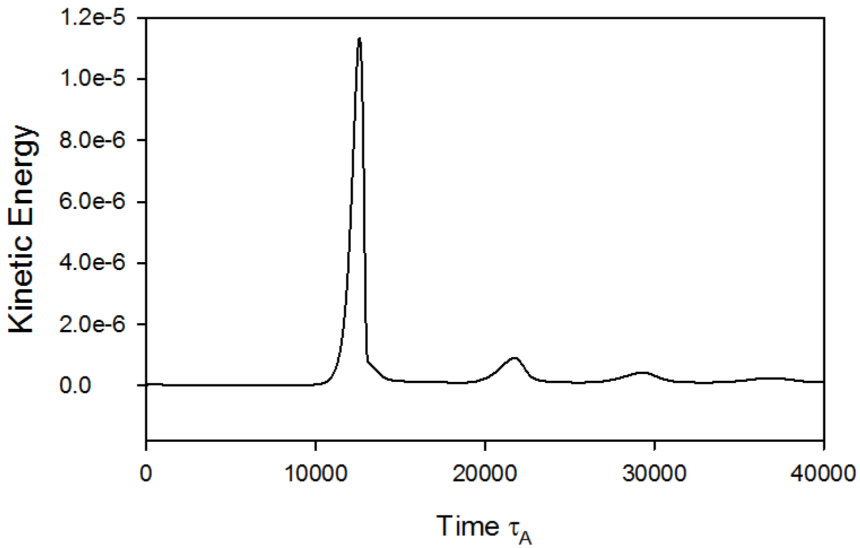
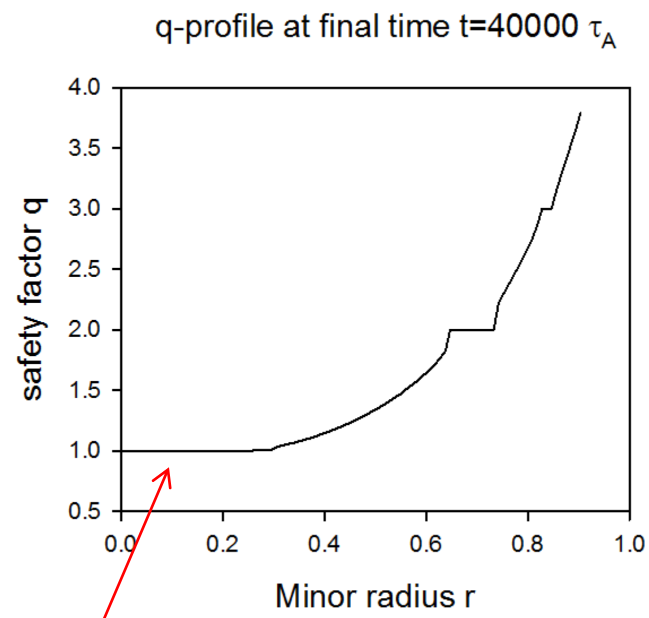
β=0.5%   -- oscillations die out to form stationary state



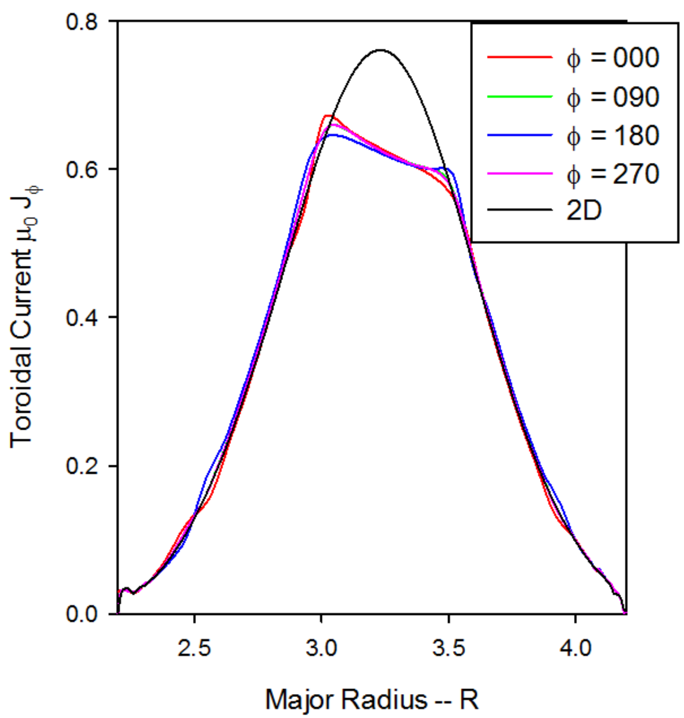


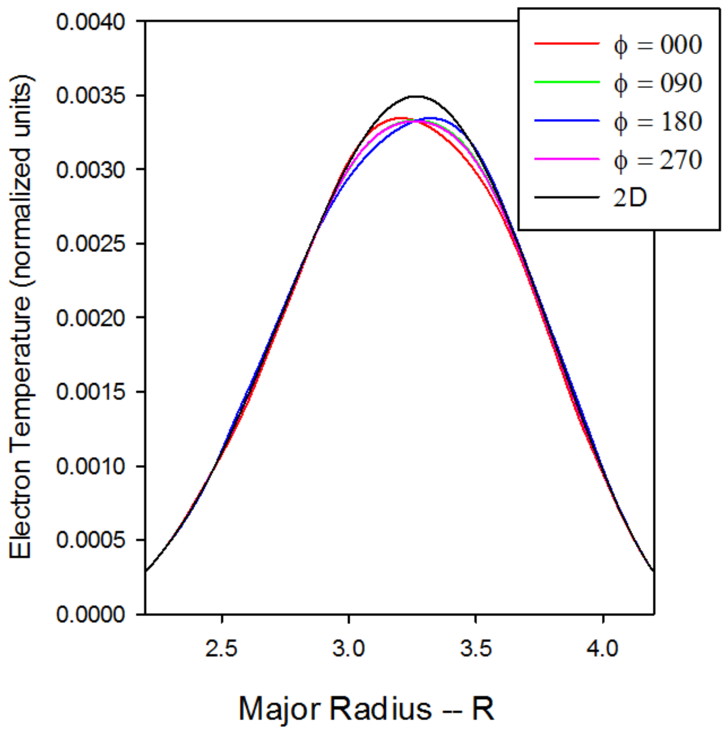
Large region in center with q = 1 + ε

CMOD15 μ=10
Also, see
CMOD02
CMOD11
CMOD35

Note: This is a self-organized state!

Stationary state has flattened current profile

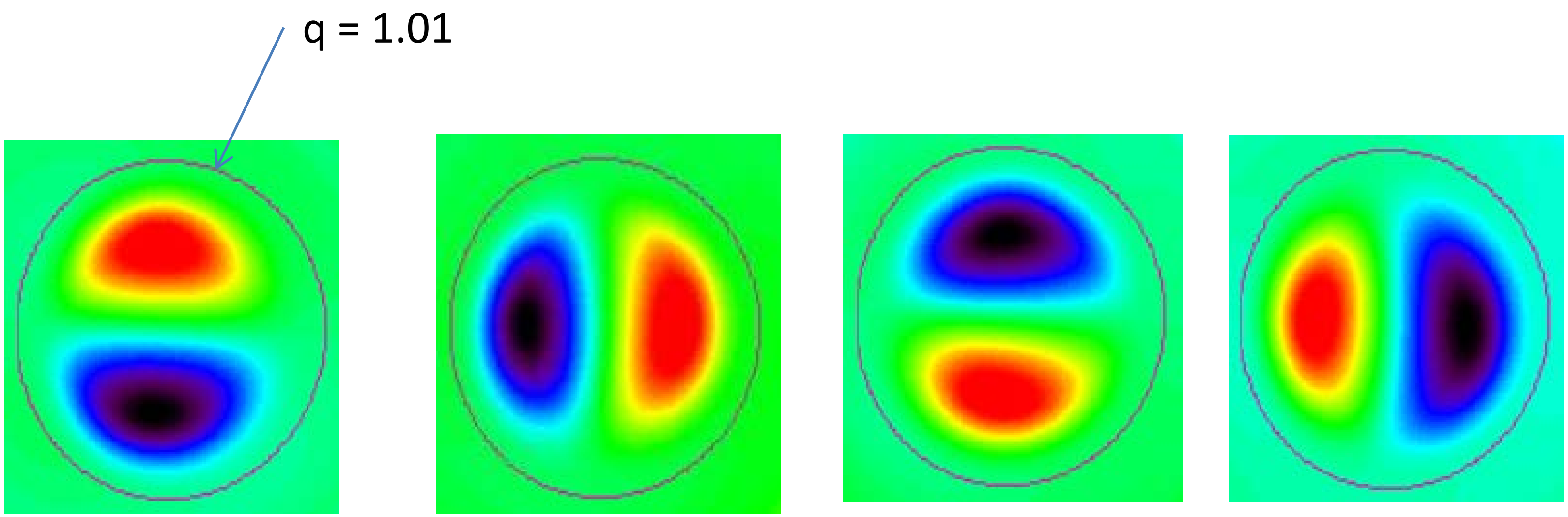




Comparison of current density and Te in 3D case (different angles) and 2D show that current flattening is 3D effect

run15

Central poloidal flow flattens current profile

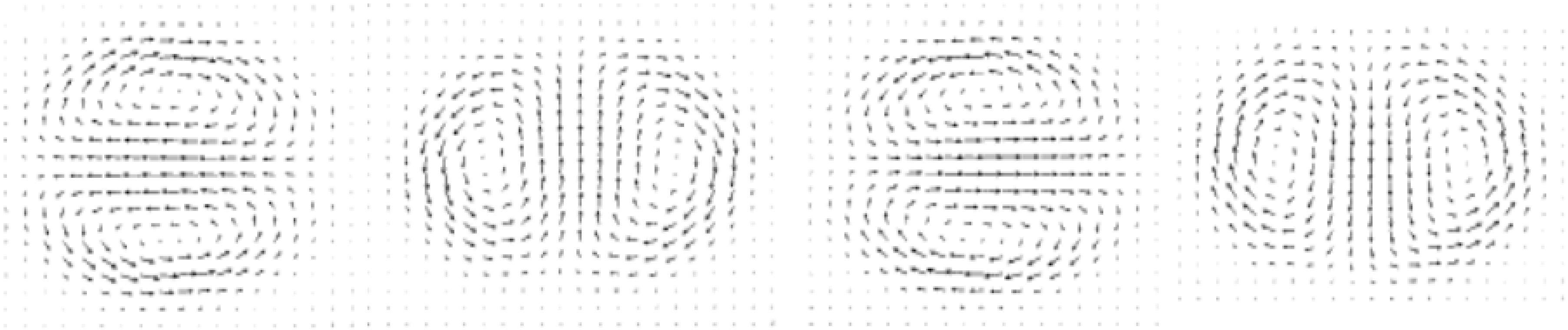


$\varphi = 0^0$ $\varphi = 90^0$ $\varphi = 180^0$ $\varphi = 270^0$

Contours of poloidal velocity stream function at final time

Unstable flattened current and pressure profiles with $q_0 = 1 + \varepsilon$ drive interchange mode which in turn keeps current flat

Hill's vortex like flow pattern in center



$\varphi = 0^0$ $\varphi = 90^0$ $\varphi = 180^0$ $\varphi = 270^0$

This agrees with the "quasi-interchange" model of Wesson. However, it is stationary and not repeating as in experiment.

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- However
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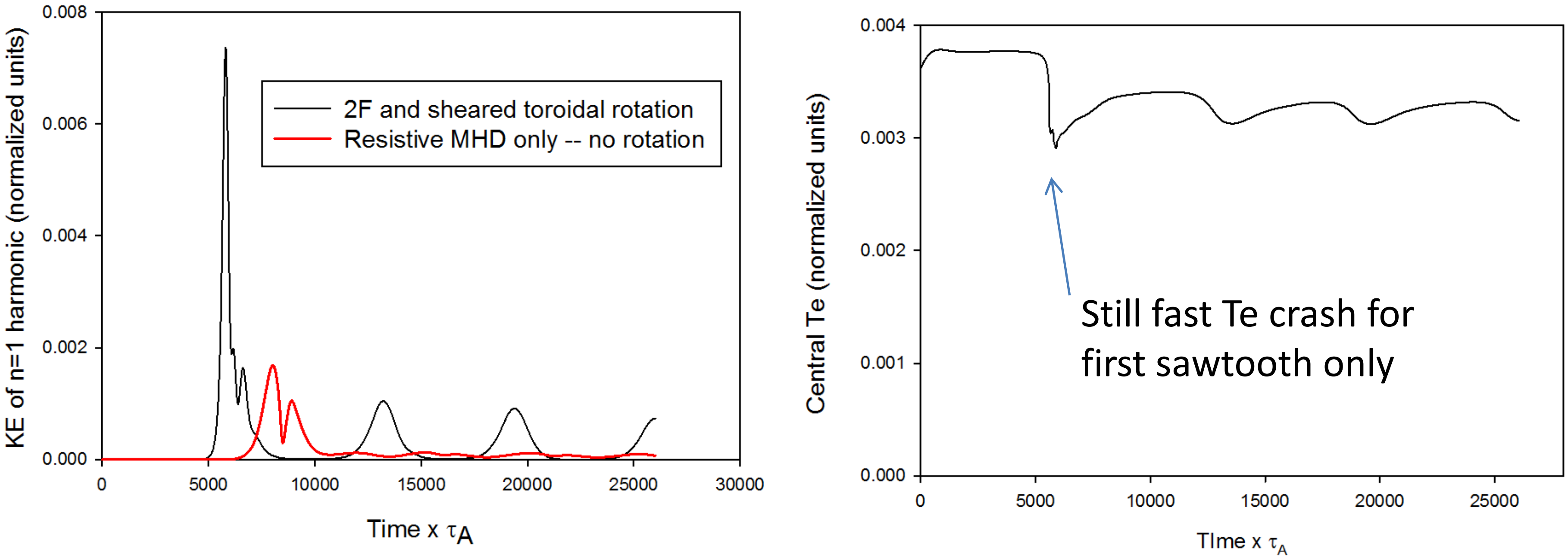
- For higher β discharges, the oscillations die out
  - stationary state is formed with $q_0 = 1 + \varepsilon$ and helical poloidal flow
  - sheared rotation and 2F terms can bring these oscillations back

Sheared rotation tends to cause oscillations to reappear (for some cases)



- Comparison of two runs with same β = 0.5% and same S=10^6
- Two-fluid (TF) terms lead to faster initial crash, but not faster repeated crashes
- In this case, the oscillations eventually died out, and system went back to the stationary state

CMOD37G
CMOD011

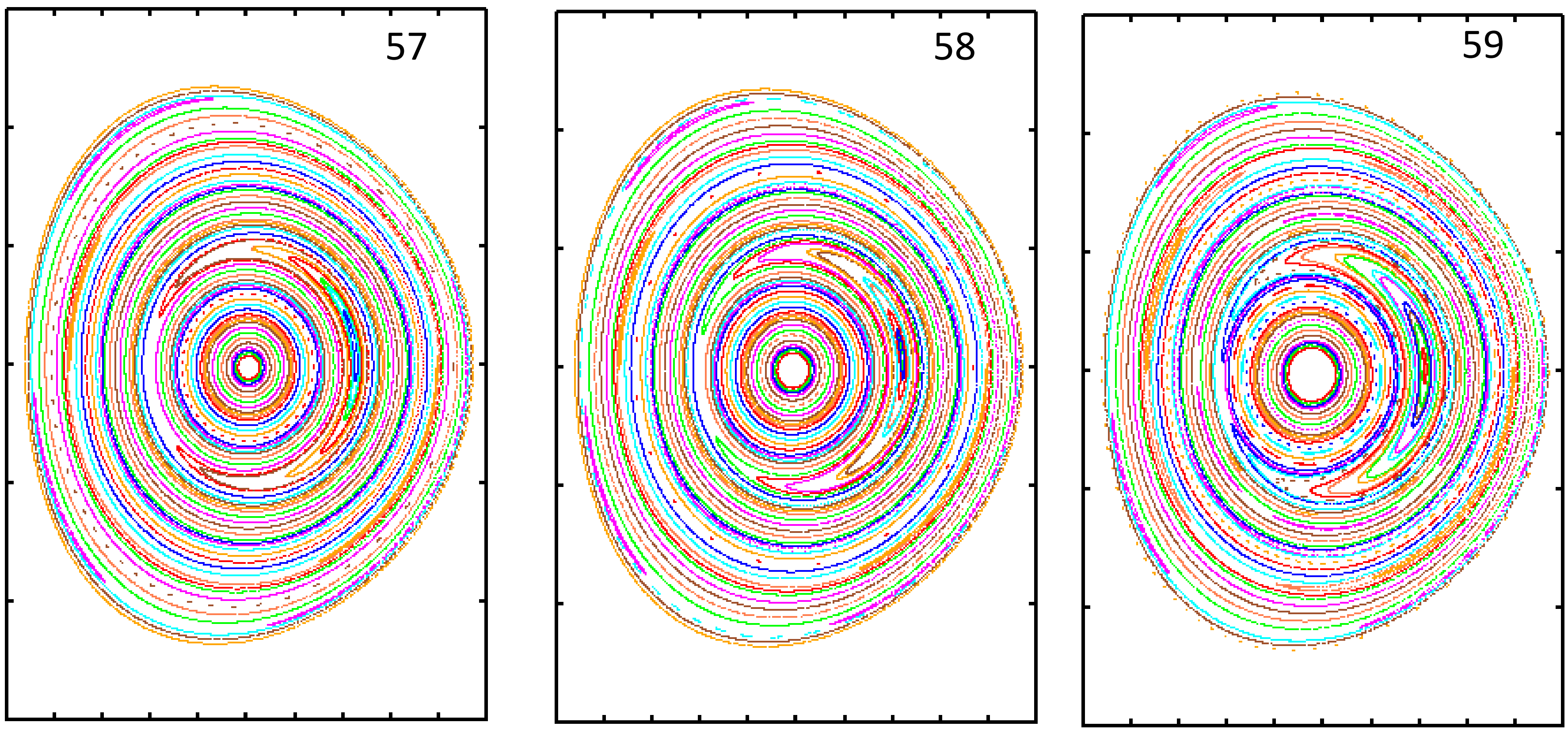
More on Two-Fluid Effects

$$n\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \bullet \mathbf{\Pi}_{GV} + \mu \nabla^2 \mathbf{V}$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + d_i \left(\mathbf{J} \times \mathbf{B} - \nabla p_e \right)$$

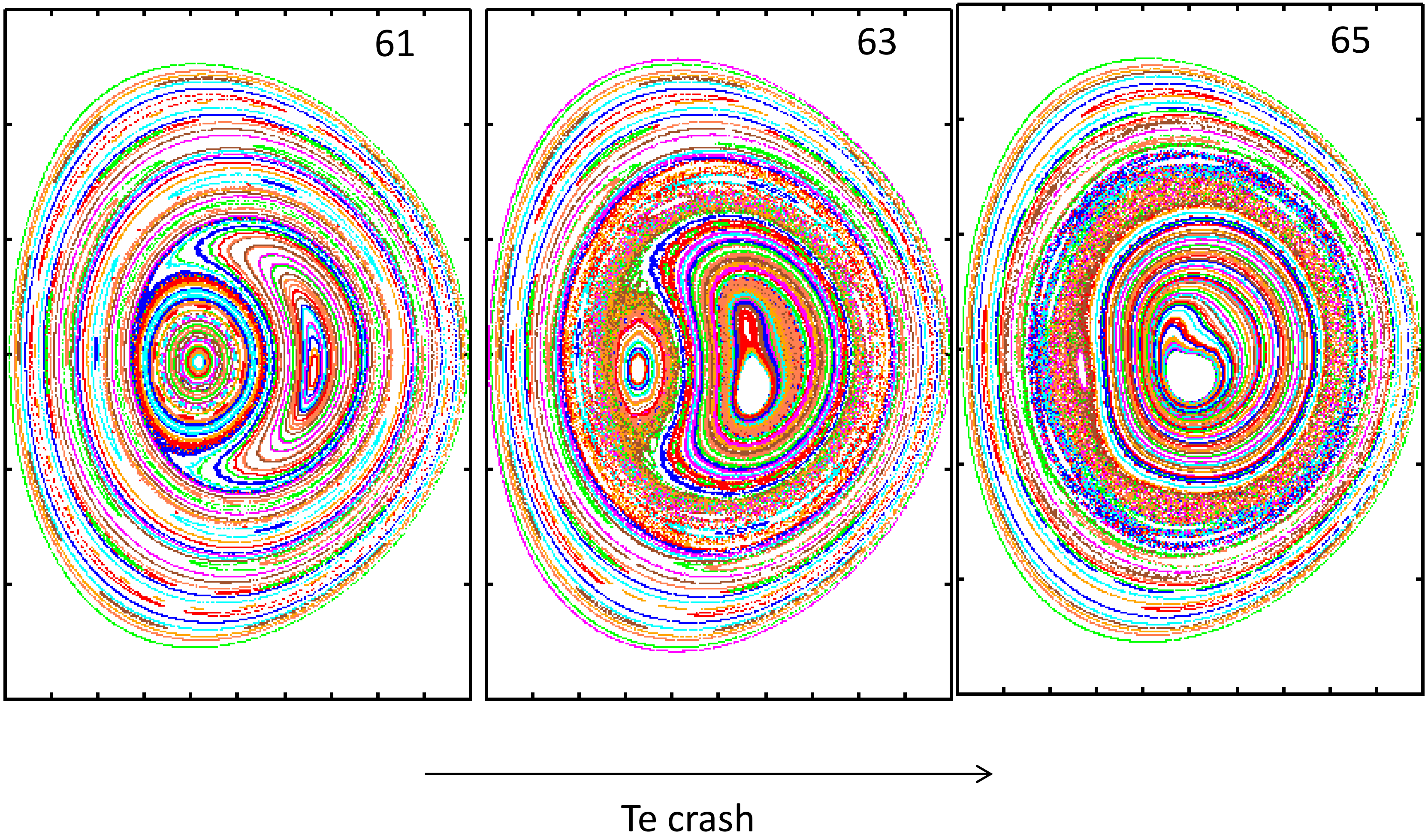
- Full Braginskii gyroviscous tensor
- Keep **J x B** term in Ohm's law
- 8 scalar variables advanced in time
- Ion skin depth: $d_i = \frac{1}{\ell_0}\left[\frac{M_i}{\mu_0 n_0 e^2}\right]^{1/2} = .0227 \frac{1}{\ell_0 [m]} \left[n_0[20]\right]^{-1/2}$

First 2F sawtooth shows distinctive shape



CMOD run25

Surfaces are destroyed in reconnection region during temperature crash



CMOD run25

Future Directions

- Develop better understanding of saturated state with $q=1+\varepsilon$
  - Very similar to "quasi-interchange" model of Wesson
  - How does this flatten current?   Analytic model?
  - How does it flatten temperatures and densities?
  - Reproduce with alternate ψ equation to understand role of Φ

- Would a different transport model lead to a different saturated state ?
  - Or, to repeated temperature crashes ?

- How does behavior depend on d_i (ion skin depth) at large S ?

- Neoclassical effects

- Convergence studies

Alternate form of poloidal flux equation may shed light on the role of the electric potential in sustaining stationary state

$$\dot{\mathbf{A}} = -\mathbf{E} - \nabla\Phi \qquad \mathbf{V} = R^2\nabla U \times \nabla\varphi + \omega R^2\nabla\varphi + R^{-2}\nabla_\perp\chi$$

Alternate form with electric potential:

$$\dot{\psi} = R^2[U,\psi] - R^2(U,f') - R^{-2}(\chi,\psi) - [\chi,f'] - \Phi' + \eta\Delta^*\psi + TF$$

$$\nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = \nabla_\perp \cdot \left[-\frac{F}{R^2}\nabla_\perp U + \frac{\omega}{R^2}\nabla_\perp\psi + \omega\nabla_\perp f' \times \nabla\varphi + \frac{F}{R^4}\nabla_\perp\chi \times \nabla\varphi \right]$$
$$+\nabla_\perp \cdot \eta\left[-\frac{1}{R^2}\nabla F \times \nabla\varphi - \frac{1}{R^2}\nabla f'' \times \nabla\varphi - \frac{1}{R^4}\nabla_\perp\psi' \right] + TF$$

We now solve this form with electric potential eliminated:

$$\nabla_\perp \bullet \frac{1}{R^2}\nabla\dot{\psi} = \nabla_\perp \bullet \frac{1}{R^2}\nabla R^2[U,\psi] - \nabla_\perp \bullet \frac{1}{R^2}\nabla R^2(U,f') + \nabla_\perp \cdot \left[\frac{F}{R^2}\nabla_\perp U\right]' - \nabla_\perp \cdot \left[\frac{\omega}{R^2}\nabla_\perp\psi + \omega\nabla_\perp f' \times \nabla\varphi\right]'$$
$$-\nabla_\perp \bullet \frac{1}{R^2}\nabla R^{-2}(\chi,\psi) - \nabla_\perp \bullet \frac{1}{R^2}\nabla[\chi,f'] - \nabla_\perp \cdot \left[\frac{F}{R^4}\nabla_\perp\chi \times \nabla\varphi\right]'$$
$$+\nabla_\perp \bullet \frac{1}{R^2}\nabla\eta\Delta^*\psi + \nabla_\perp \cdot \left[\frac{\eta}{R^2}\nabla F^* \times \nabla\varphi + \frac{\eta}{R^4}\nabla_\perp\psi'\right]' + TF$$

Summary

- Two types of long time behavior are observed for resistive MHD without momentum source:
  - periodic oscillations at low β
  - stationary states at higher beta

- Resistive MHD (without rotation) predicts non-physical scaling for sawtooth period and amplitude at large S

- Sheared rotation promotes periodic behavior and good surfaces

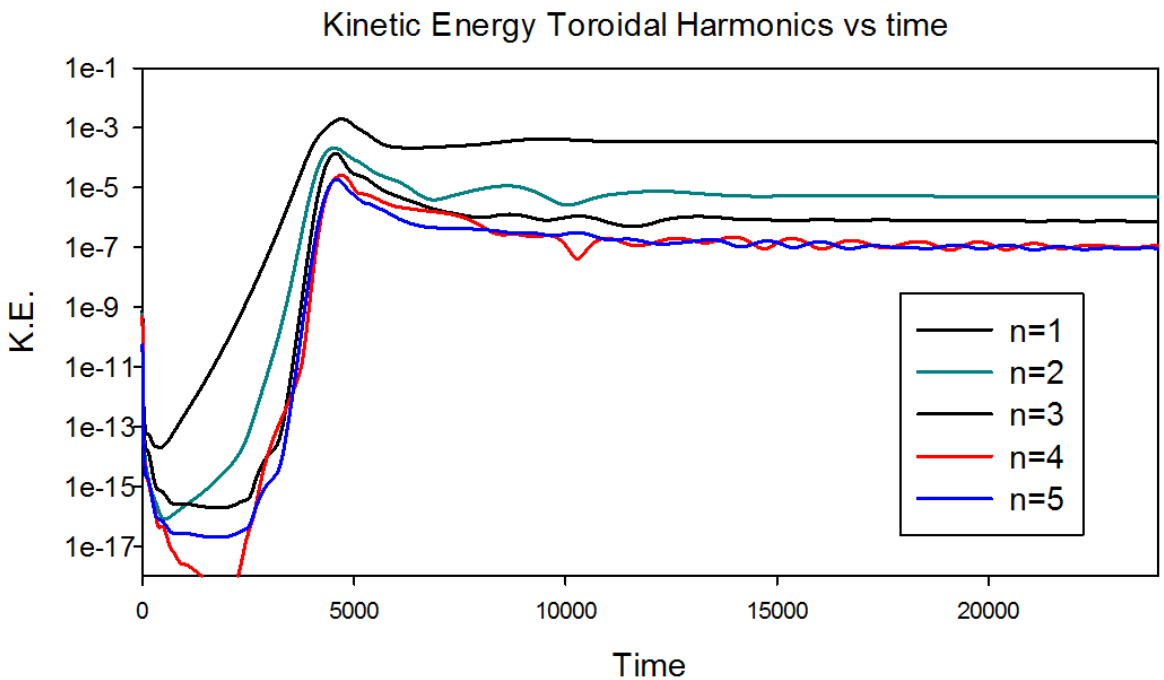
- Two fluid terms lead to:
  - more circular interior surfaces,
  - shorter reconnection layer
  - faster initial reconnection times
  - stochastic layer forms at late times

- However, do not observe fast Te crash for repeating sawteeth
  - something is missing in model ??

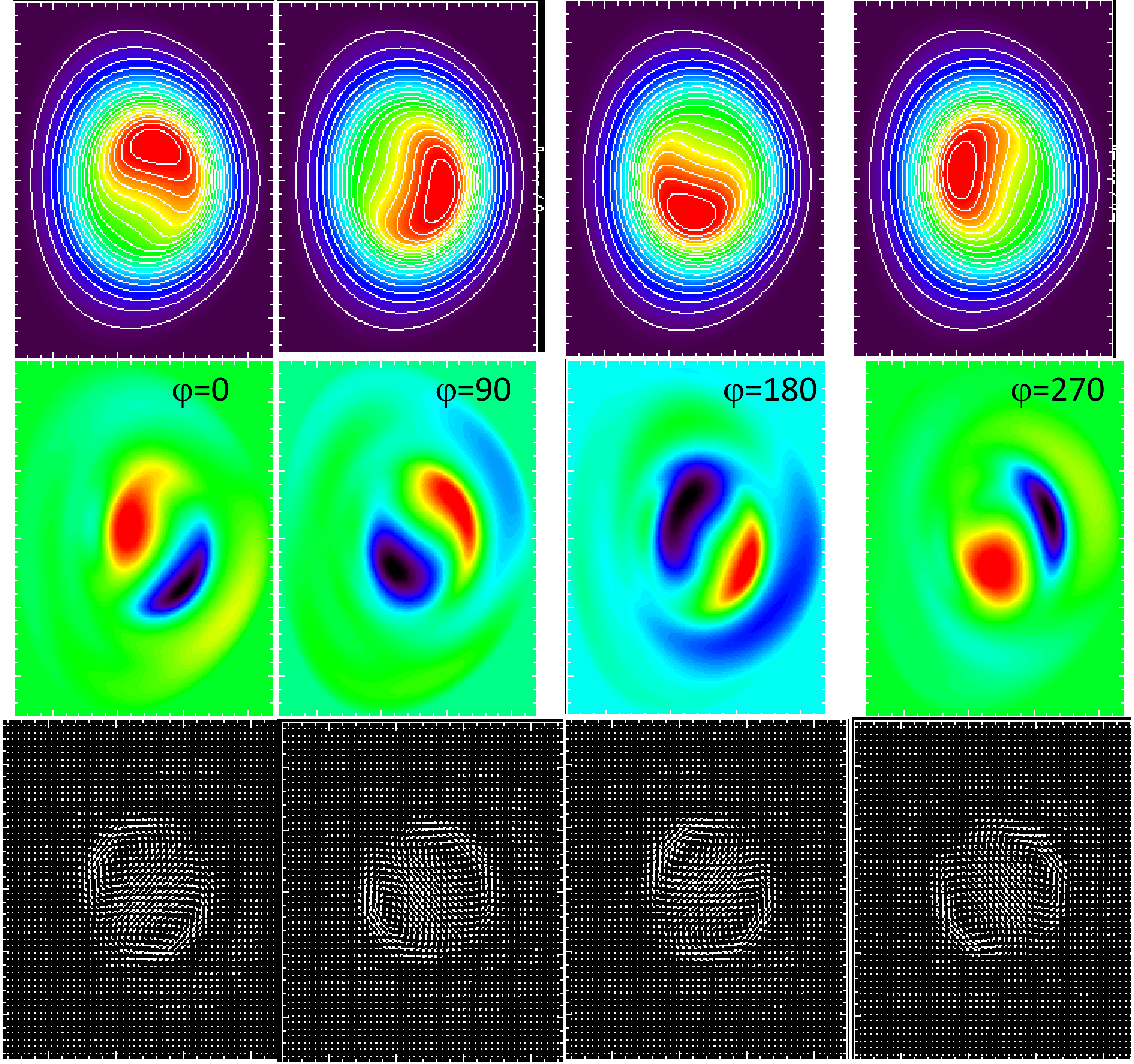
Extra viewgraphs

Stationary Helical State with Flow

In some cases, the sawteeth die out, and the system becomes stationary on all timescales.



T=24000



Electron Temperature

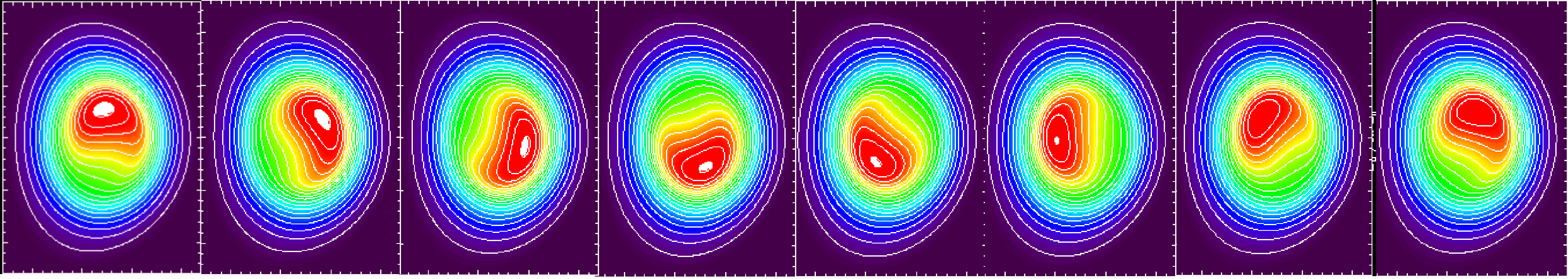
Toroidal Velocity

(Max= .004)

Poloidal Velocity

max=0.00064

CMOD run25

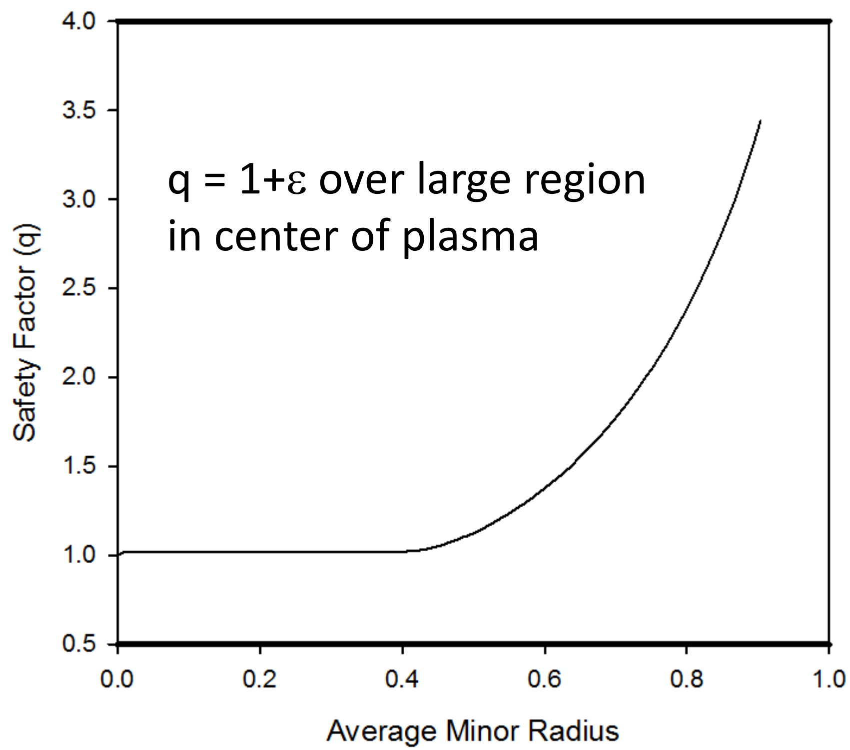


T=17000 T=20000 T=24000

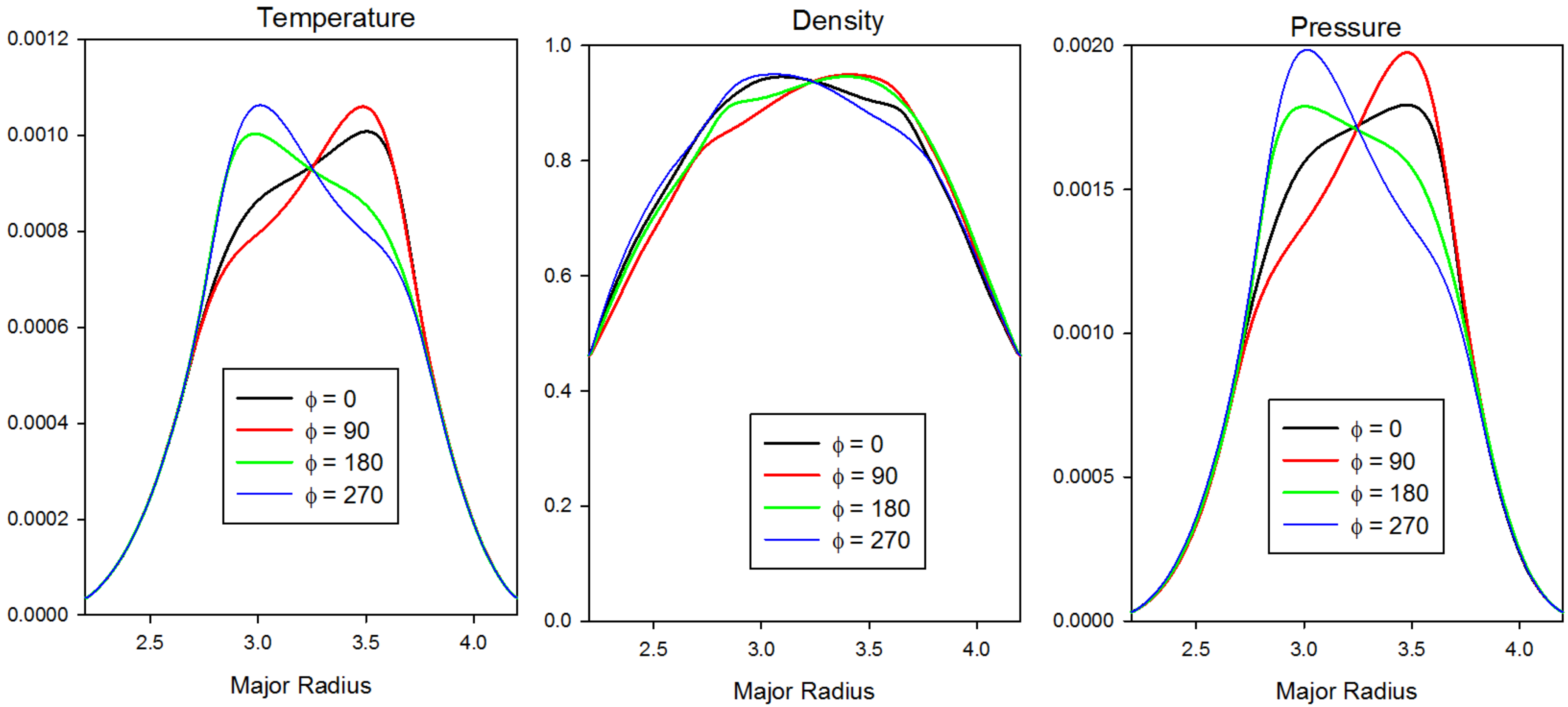
Te at same poloidal location as time evolves -→

Mode is rotating every 7000 τ_A

Surfaces are almost axisymmetric with just a small m=1 island



CMOD run25



Interior to the region where q=1+ε, p, n, and T profiles are not constant on the magnetic surfaces. i.e., $p \neq p(\psi)$

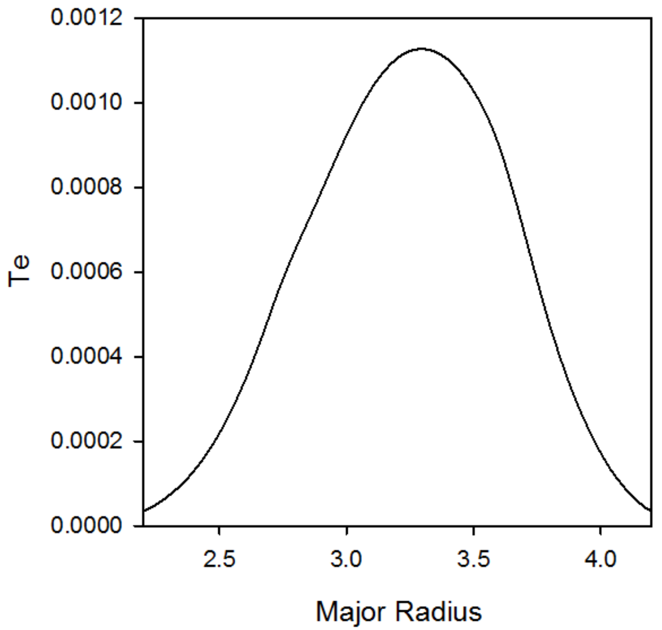
Exterior to the q=1 surface, they are constant on surfaces $p=p(\psi)$

CMOD run25

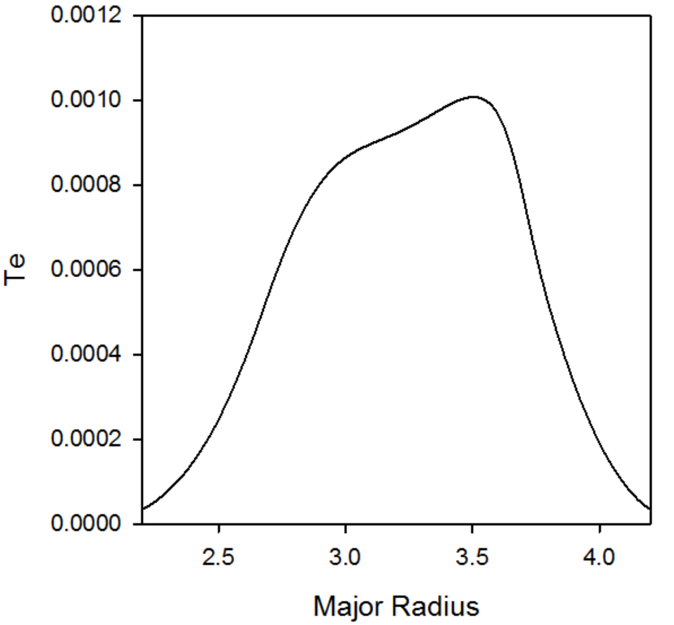
Terms in temperature equation

$$u \bullet \nabla T_e \quad = \quad \eta J^2 + \nabla \bullet \kappa_\perp \nabla T_e$$

Axisymmetric state (before)



Non-axisymmetric state (after)

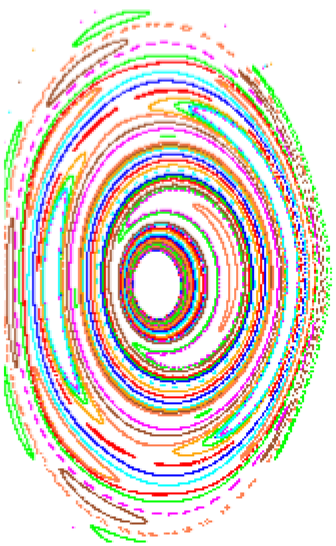


$$1.0\times10^{-8} \simeq 3.7\times10^{-7} - 3.7\times10^{-7}$$

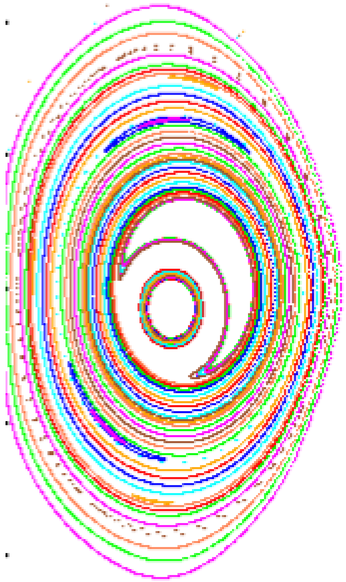
$$3.4\times10^{-7} \simeq 3.7\times10^{-7} - 3.0\times10^{-8}$$

CMOD run25

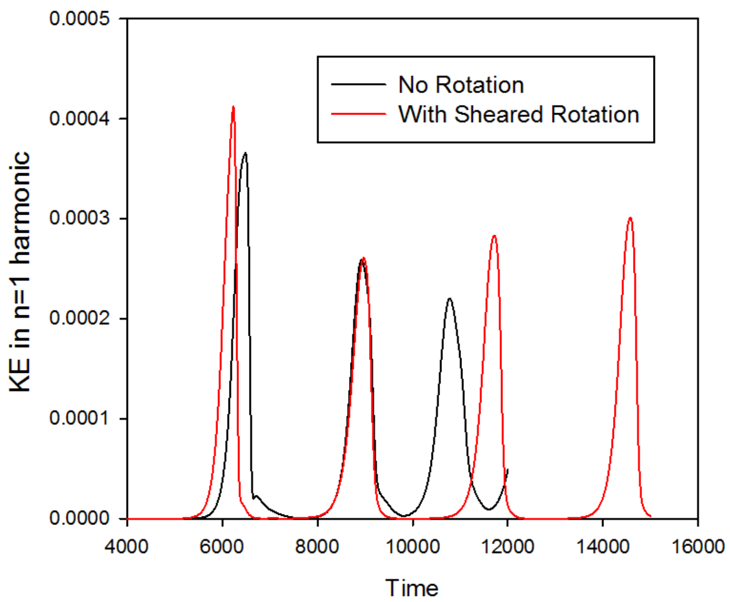
Effect of Sheared Rotation



Without rotation



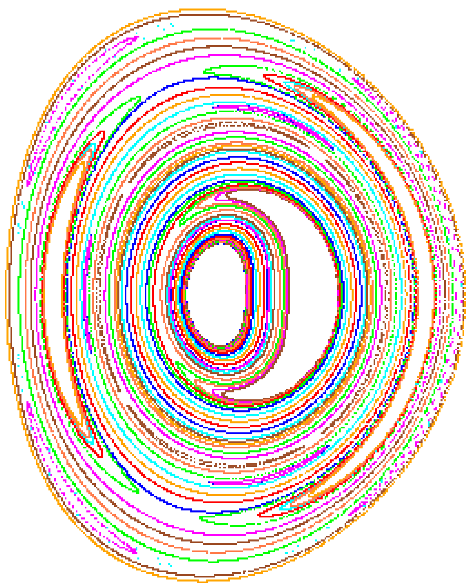
With sheared rotation



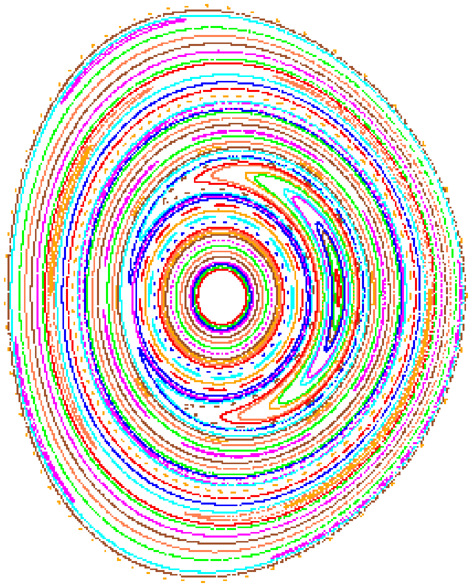
Without sheared rotation in ellipse:

- sawteeth tend to die out
- large magnetic islands form

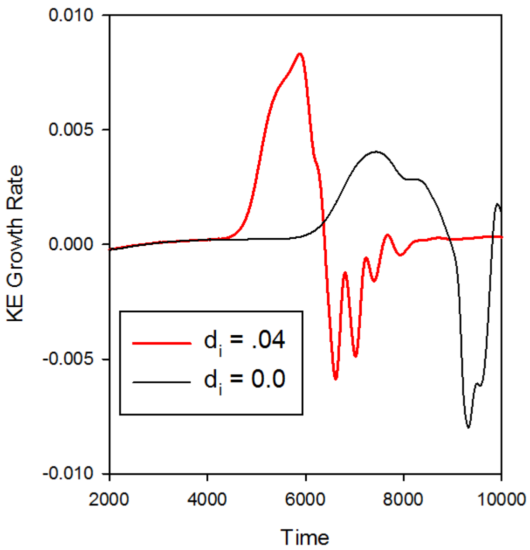
Comparison of surfaces for resistive and two-fluid MHD at similar stage in cycle shows reconnection layer is shorter in two-fluid. Rate increases by ~ 2



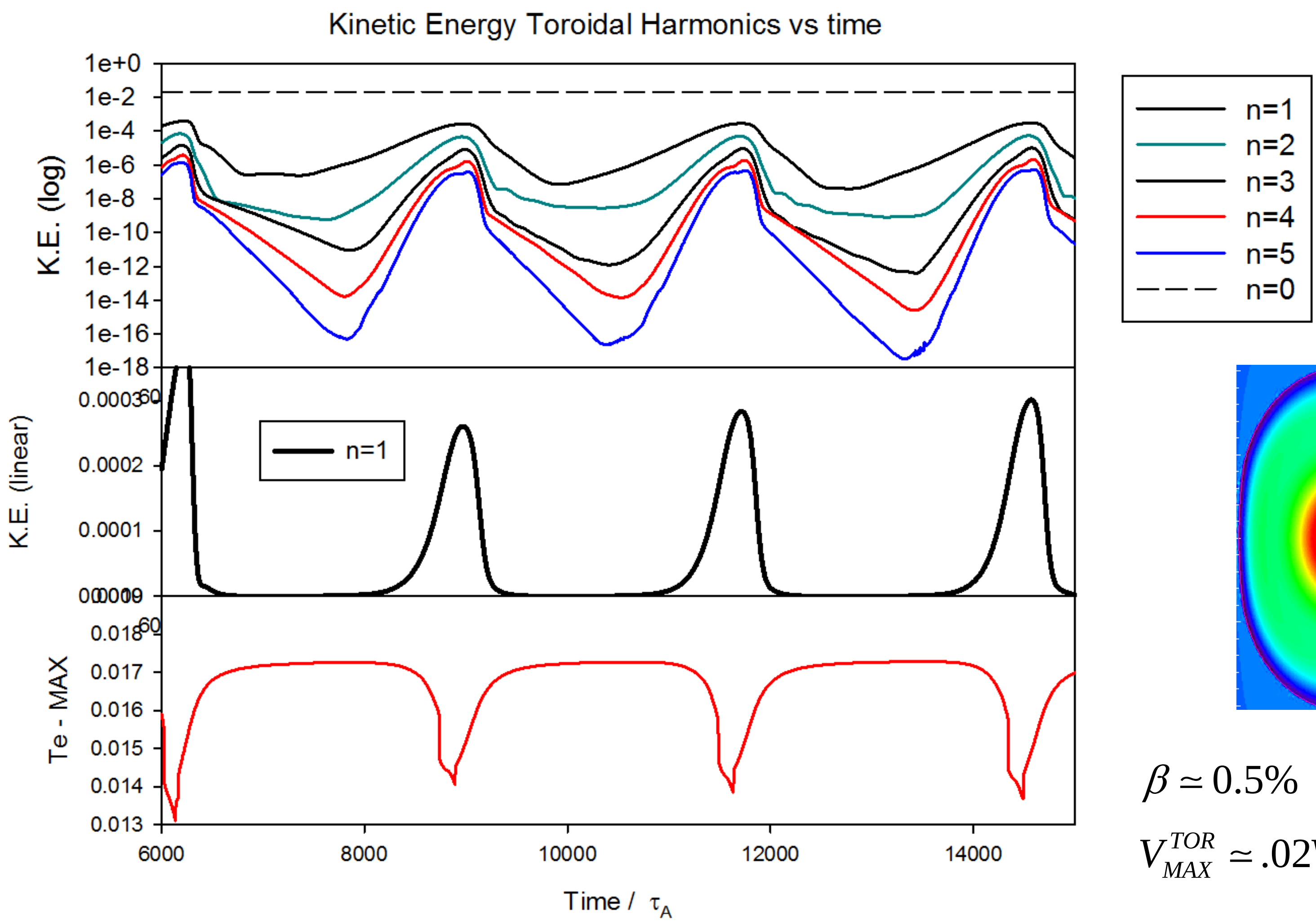
d_i=0

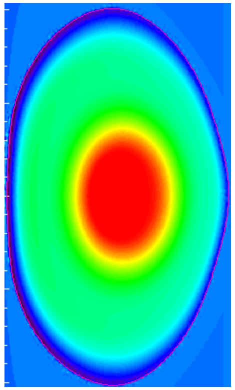


d_i= 0.04



Canonical periodic oscillating discharge



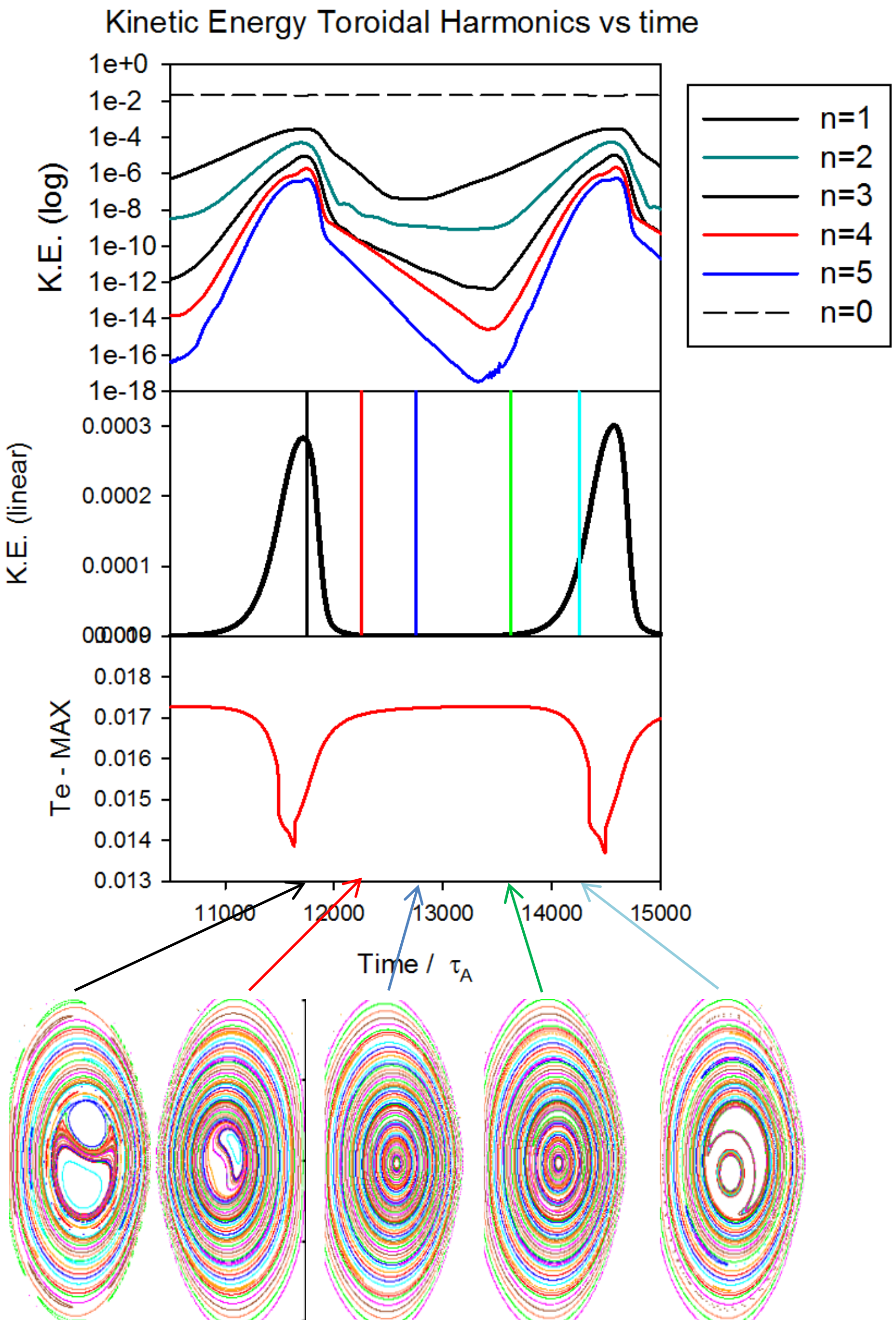


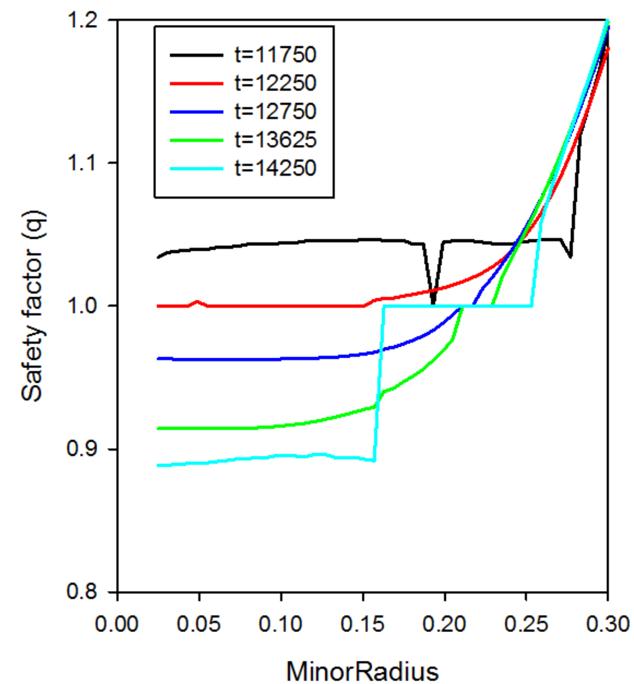
$\beta \simeq 0.5\%$ $\quad q_{edge} \simeq 5.5$

$V_{MAX}^{TOR} \simeq .02 V_{Alfven}$ $\quad S \simeq 10^6$

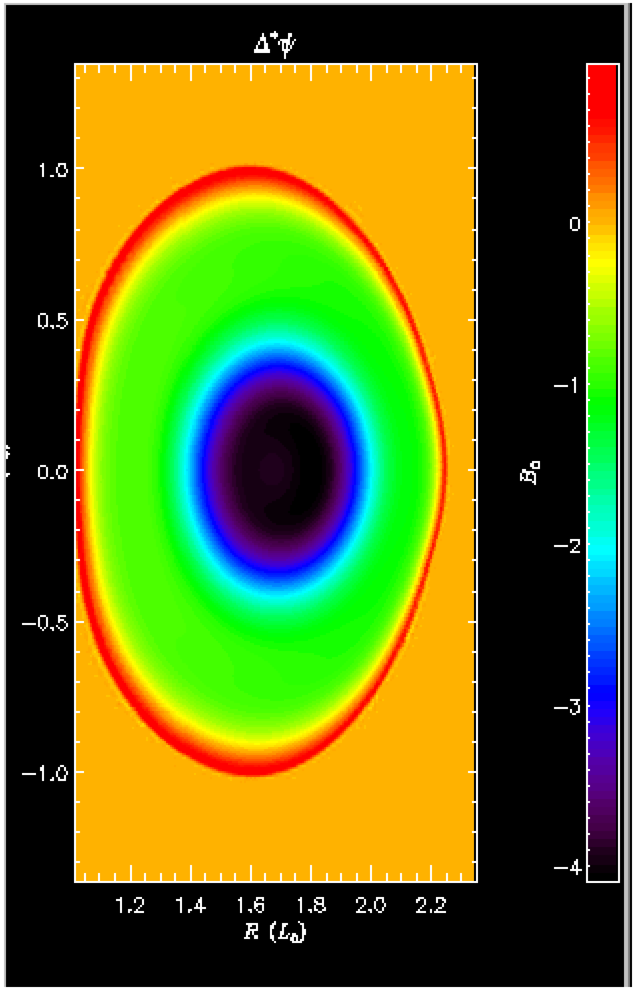
DIII-D 118164- J45

q-profile evolution during single sawtooth cycle

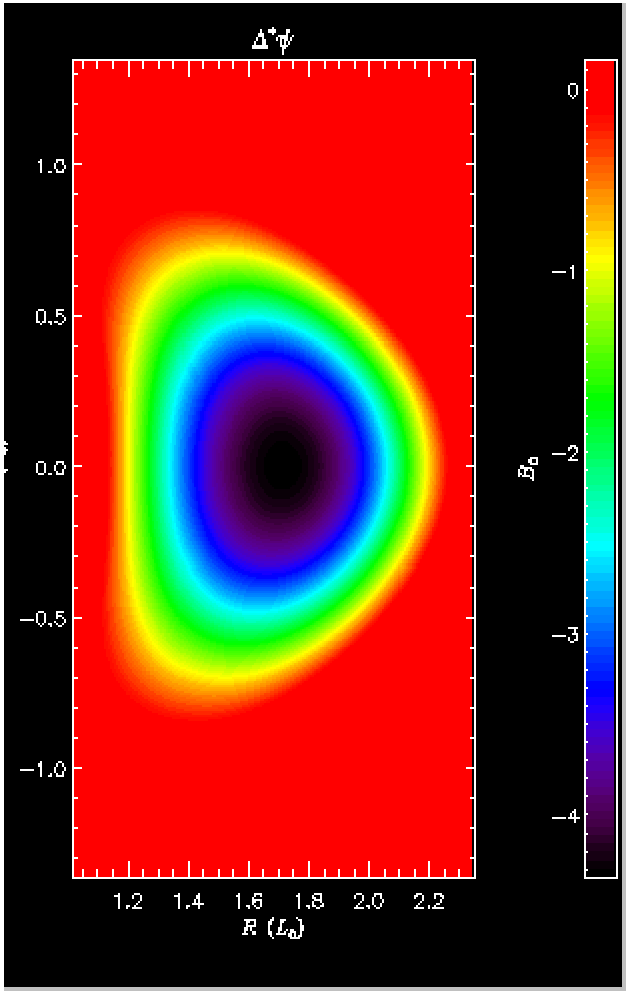




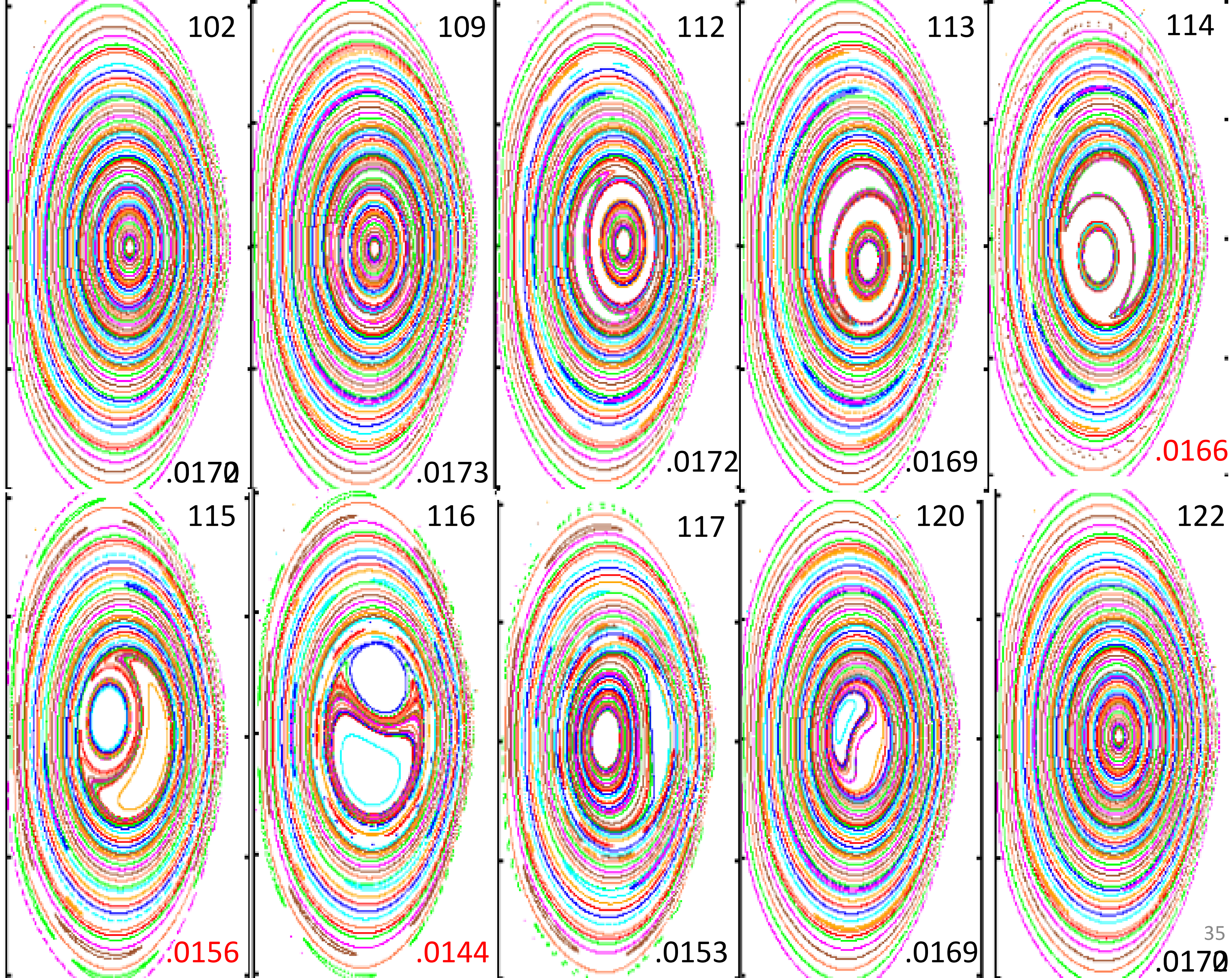
Differences in sawtooth behavior for bean-shaped and elliptical-shaped plasmas has been well documented experimentally (Lazarus, Tobias, …)



DIII-D shot 118164



DIII-D shot 118162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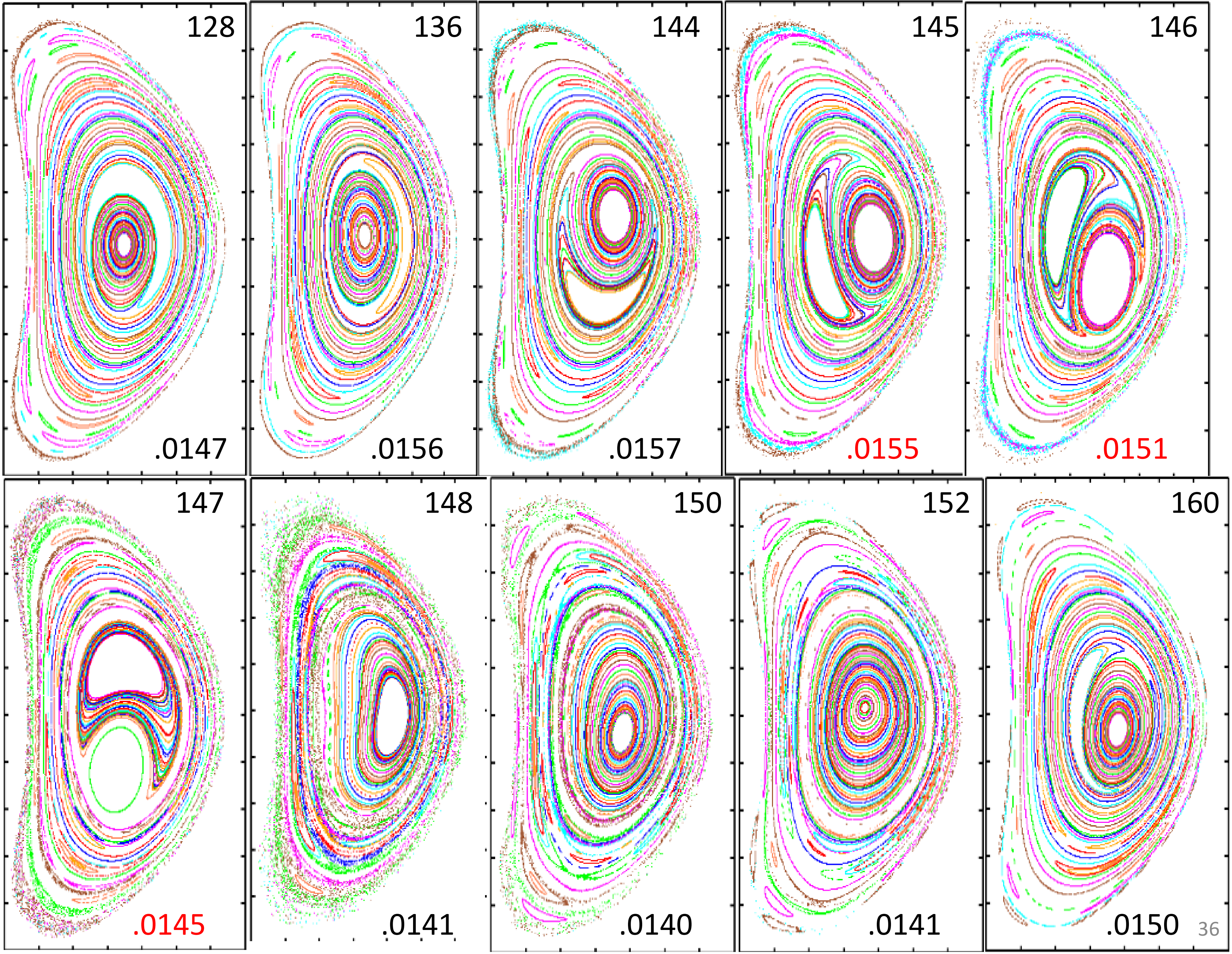

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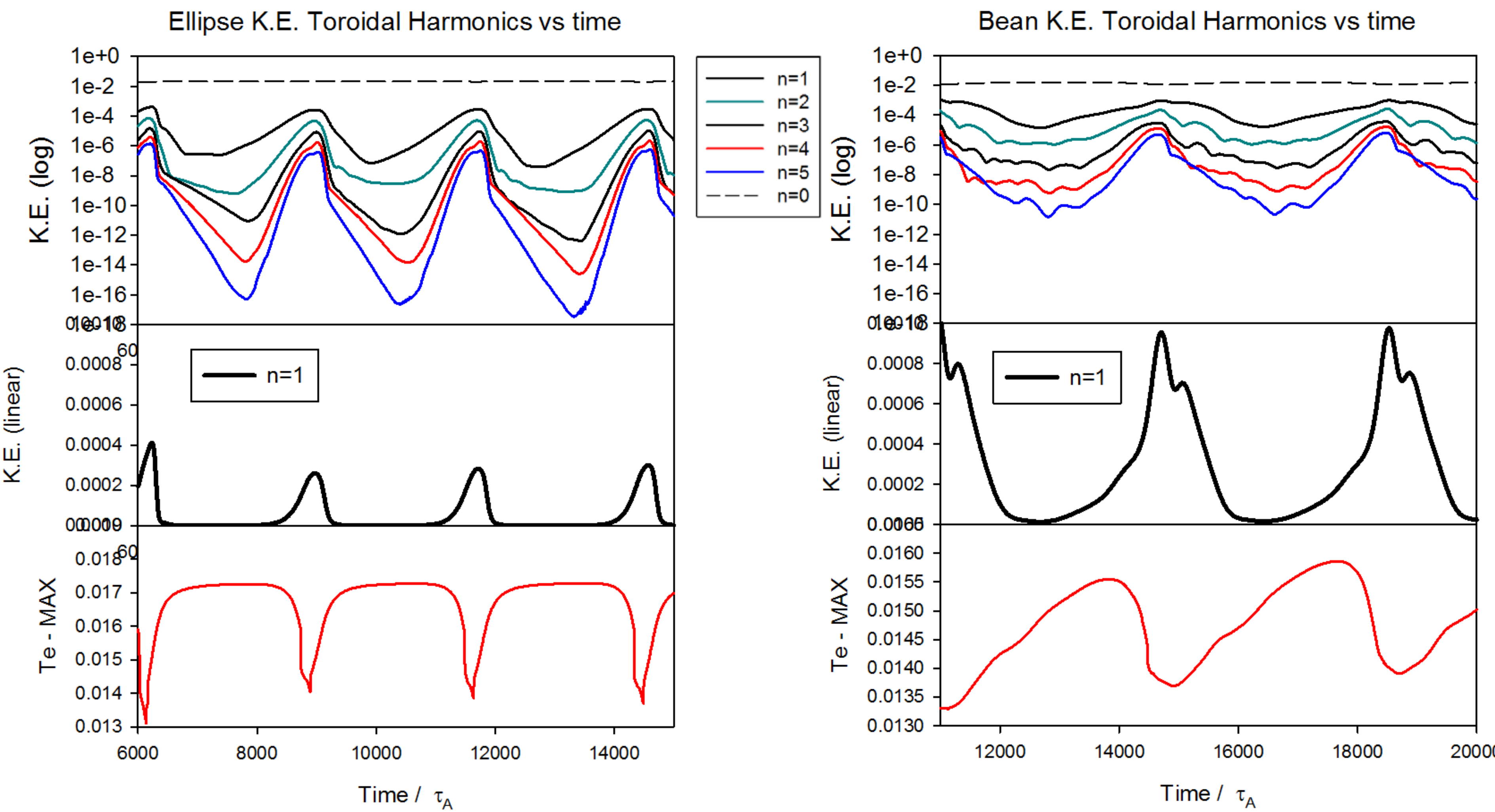
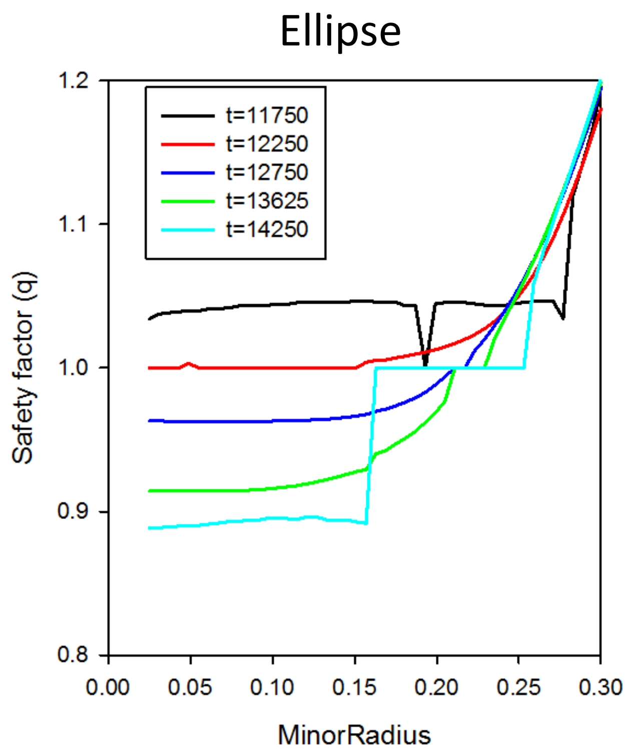
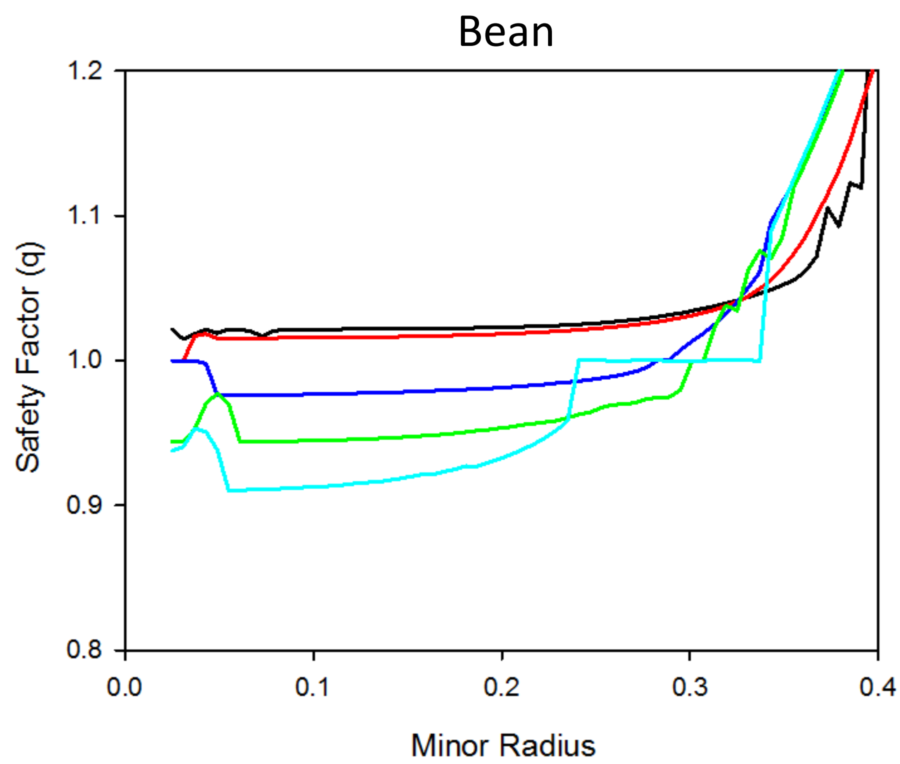
Comparison of Ellipse and Bean



Bean has shorter period, larger amplitude n=1, less decay in energy harmonics between ST.

Comparison of Ellipse and Bean





In Bean: -- q=1 surface extends to a larger radius
-- q(0) does not vary as much during sawtooth cycle