PROGRESS ON COUPLED NEOCLASSICAL-MAGNETOHYDRODYNAMIC SIMULATIONS

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CEMM MEETING
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Hybrid neoclassical-MHD solver

- Necessary for proper simulation of core plasma instabilities, (e.g., neoclassical tearing modes)
- Use existing MHD time-evolution code (e.g., M3D-C^1, NIMROD)
- Developing new code that solves the Ramos drift-kinetic equations in nonaxisymmetric geometries
- Current status of code
  - Ion and electron equations implemented
  - All drives including flows, ∇P, ∇T_e, and ∇T_i
  - Axisymmetric toroidal geometry
  - Assumes T_e=T_i
  - Assumes P & T_s are flux functions

Drift-kinetic equations

$$\frac{\partial \bar{f}_{NMs}}{\partial t} + wy\mathbf{b}\cdot\nabla \bar{f}_{NMs} - \frac{1}{2}w\left(1-y^2\right)\mathbf{b}\cdot\nabla \ln B\frac{\partial \bar{f}_{NMs}}{\partial y} = \langle C_{ss} + C_{ss'}\rangle_\alpha$$

$$+\left\{\frac{wy}{nT_s}\left[\frac{2}{3}\mathbf{b}\cdot\nabla\left(p_{s\|}-p_{s\perp}\right) - \left(p_{s\|}-p_{s\perp}\right)\mathbf{b}\cdot\nabla\ln B - \mathbf{F}_s^{coll}\right]\right.$$

$$+ P_2(y)\frac{w^2}{3v_{ths}^2}\left(\nabla\cdot\mathbf{u}_s - 3\mathbf{b}\cdot\left[\mathbf{b}\cdot\nabla\mathbf{u}_s\right]\right) + \frac{1}{3nT_s}\left(\frac{w^2}{v_{ths}^2}-3\right)\nabla\cdot\left(q_{s\|}\mathbf{b}\right)$$

$$\left. - \frac{\varsigma\left(e_s\right)}{3m_s\Omega_s}\left[\frac{1}{2}P_2(y)\frac{w^2}{v_{ths}^2}\left(\frac{w^2}{v_{ths}^2}-5\right) + \frac{w^4}{v_{ths}^4} - 10\frac{w^2}{v_{ths}^2} + 15\right]\left(\mathbf{b}\times\nabla\ln B\right)\cdot\nabla T_s\right\} f_{Ms}$$

- Evolves difference between full, gyroaveraged distribution function and shifting Maxwellian
- Axisymmetric 4D phase space
  - $\tilde{\psi}$ denotes a flux surface, θ is the poloidal angle
  - w is the total velocity in frame of macroscopic flow
  - $y = \cos\chi$ is cosine of the pitch angle
  - Density, temperatures, and pressures assumed to be flux functions
- Cross-species collisional terms dropped for ions

Collision operators

- Fokker-Planck-Landau form used

$$\begin{aligned}\langle C_{ss}+C_{ss'}\rangle_\alpha =&\nu_{Ds}(w)\mathcal{L}[\bar{f}_{NMs}] - \nu_s f_{Ms}\frac{v_{ths}}{v_{ths'}^2}\frac{\mathbf{b}\cdot\mathbf{J}}{e_s n}\xi_{s'}y \\ &+\frac{\nu_s v_{ths}^3}{w^2}\frac{\partial}{\partial w}\left\{\xi_s\left[w\frac{\partial \bar{f}_{NMs}}{\partial w}+\frac{w^2}{v_{ths}^2}\bar{f}_{NMs}\right]+\xi_{s'}\left[w\frac{\partial \bar{f}_{NMs}}{\partial w}+\frac{m_s w^2}{m_{s'}v_{ths'}^2}\bar{f}_{NMs}\right]\right\} \\ &+\frac{\nu_s v_{ths}}{n}f_{Ms}\left(4\pi v_{ths}^2\bar{f}_{NMs}-\Phi_s[\bar{f}_{NMs}]+\frac{w^2}{v_{ths}^2}\frac{\partial^2\Psi_s[\bar{f}_{NMs}]}{\partial w^2}\right)\end{aligned}$$

where

$$\nu_{Ds}(w)=\frac{\nu_s v_{ths}^3}{w^3}\left[\varphi_s-\xi_s+\varphi_{s'}-\xi_{s'}\right] \qquad \mathcal{L}[f]=\frac{1}{2}\frac{\partial}{\partial y}\left[(1-y^2)\frac{\partial f}{\partial y}\right]$$

$$\varphi_s=\varphi\left(x=\frac{w}{v_{ths}}\right)=\frac{2}{\sqrt{2\pi}}\int_0^x \exp(-t^2/2)dt \qquad \xi_s=\xi\left(x=\frac{w}{v_{ths}}\right)=\frac{1}{x^2}\left[\varphi(x)-\frac{2x}{\sqrt{2\pi}}\exp(-x^2/2)\right]$$

- Poisson equations for the Rosenbluth potentials

$$\frac{d}{dw}\left(w^2\frac{\partial\Phi_s}{\partial w}\right)+\frac{\partial}{\partial y}\left[(1-y^2)\frac{\partial\Phi_s}{\partial y}\right]=-4\pi w^2\bar{f}_{NMs}$$

$$\frac{d}{dw}\left(w^2\frac{\partial\Psi_s}{\partial w}\right)+\frac{\partial}{\partial y}\left[(1-y^2)\frac{\partial\Psi_s}{\partial y}\right]=w^2\Phi_s$$

Hybrid iteration scheme

Evolve DKE(s) to get (possible steady state) distribution function for given magnetic configuration

Take moments to get necessary closures for MHD equations

(e.g., friction force)

Evolve MHD equations to get new magnetic configuration using extended MHD time evolution code

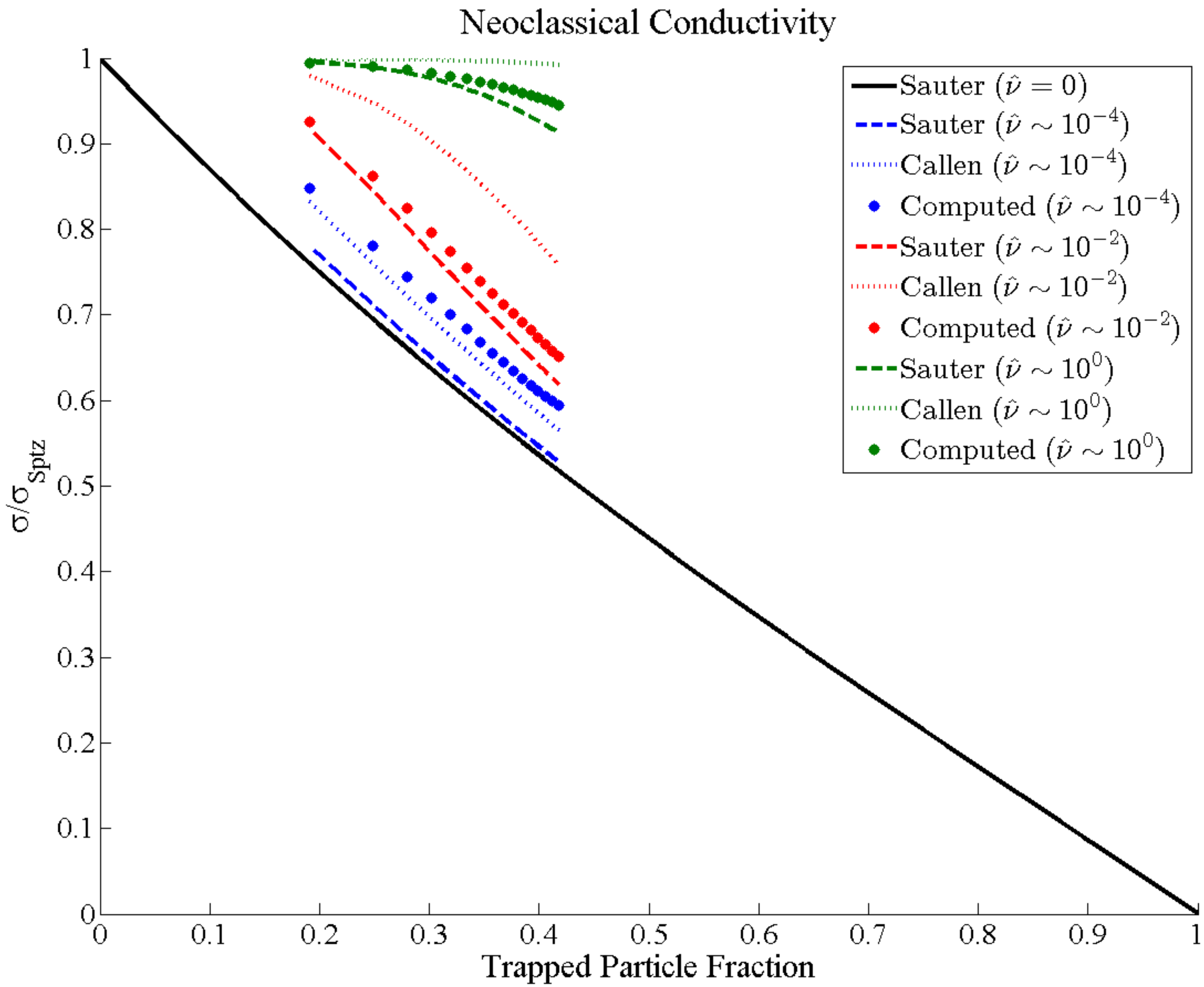
Calculating Sauter-like coefficients

- When run to steady state, we can calculate the neoclassical conductivity and bootstrap current coefficients for an equilibrium
- Must separate inhomogeneous source terms in DKE
- Coefficients given by collisional friction force and pressure anisotropy via parallel Ohm's law

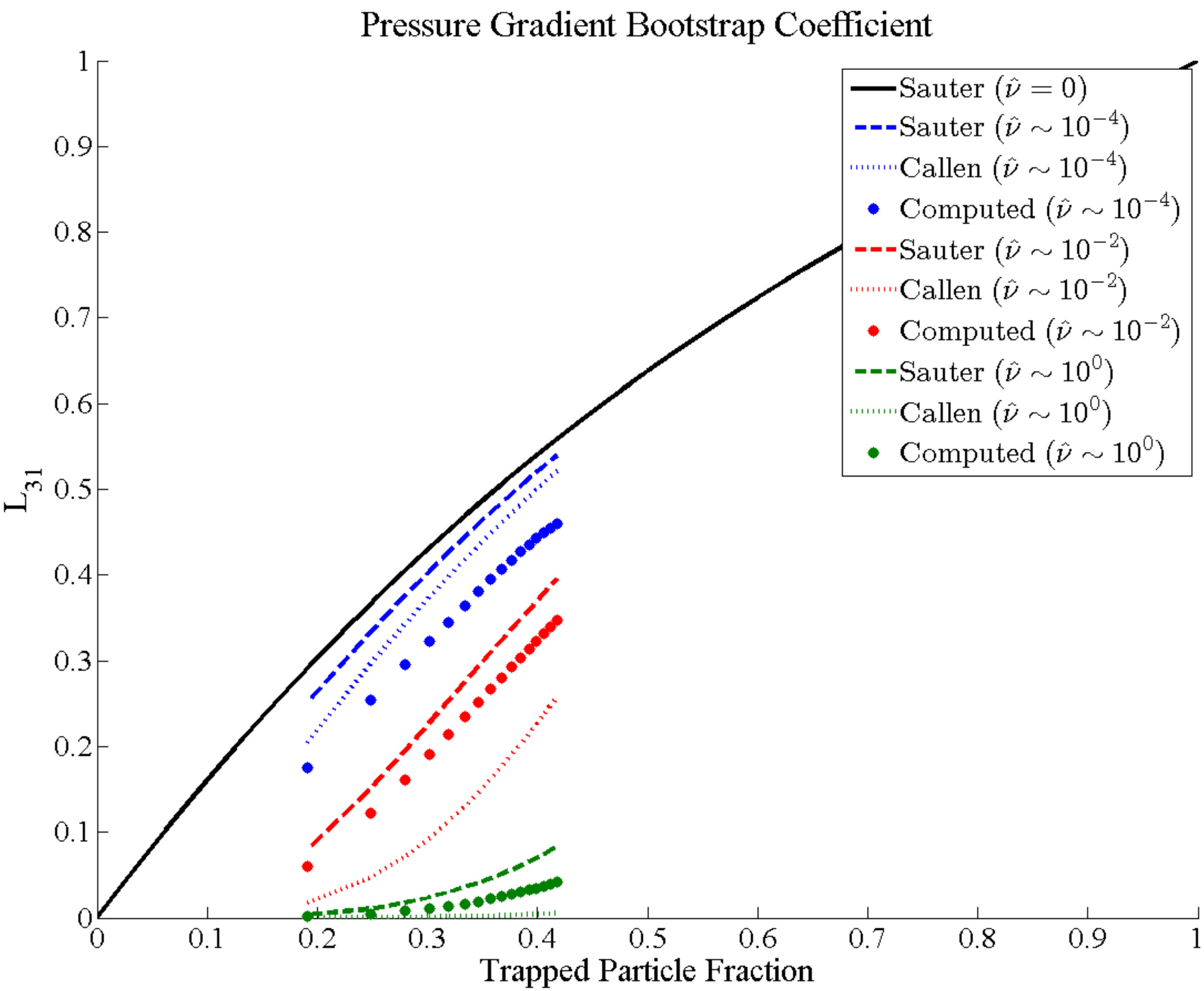
$$\langle \mathbf{J} \cdot \mathbf{B} \rangle = \sigma_{neo} \langle \mathbf{E} \cdot \mathbf{B} \rangle + I \left[\mathcal{L}_{31} \frac{dP}{d\psi} + \mathcal{L}_{32} n \frac{dT_e}{d\psi} + \alpha \mathcal{L}_{34} n \frac{dT_i}{d\psi} \right]$$

$$U_i = \alpha \frac{I}{e \langle B^2 \rangle} \frac{dT_i}{d\psi}$$ where $$\mathbf{u}_i = U_i(\psi)\mathbf{B} + R^2 \left[\frac{d\phi}{d\psi} + \frac{1}{en} \frac{d(nT_i)}{d\psi} \right] \nabla\zeta$$

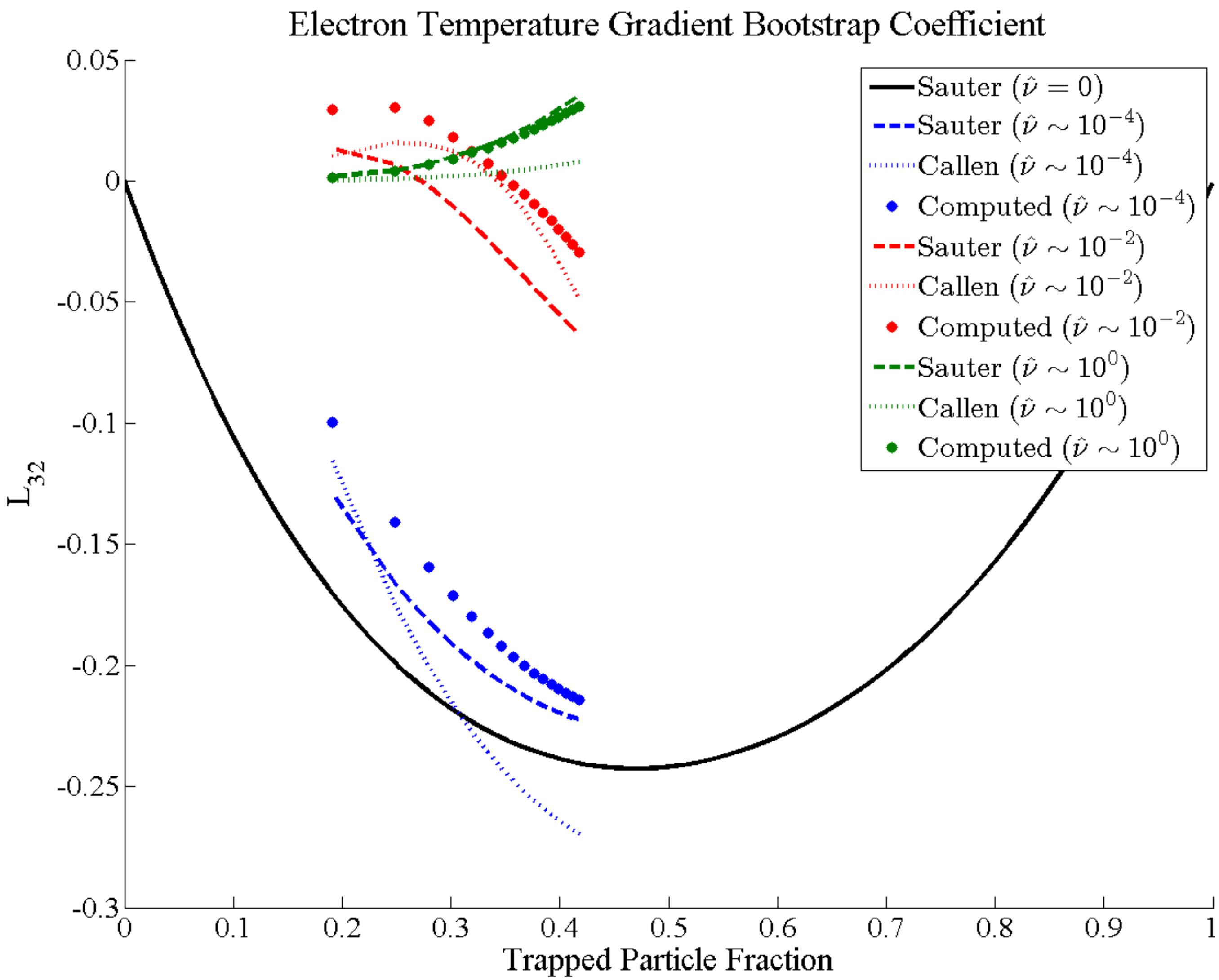
Conductivity Benchmark



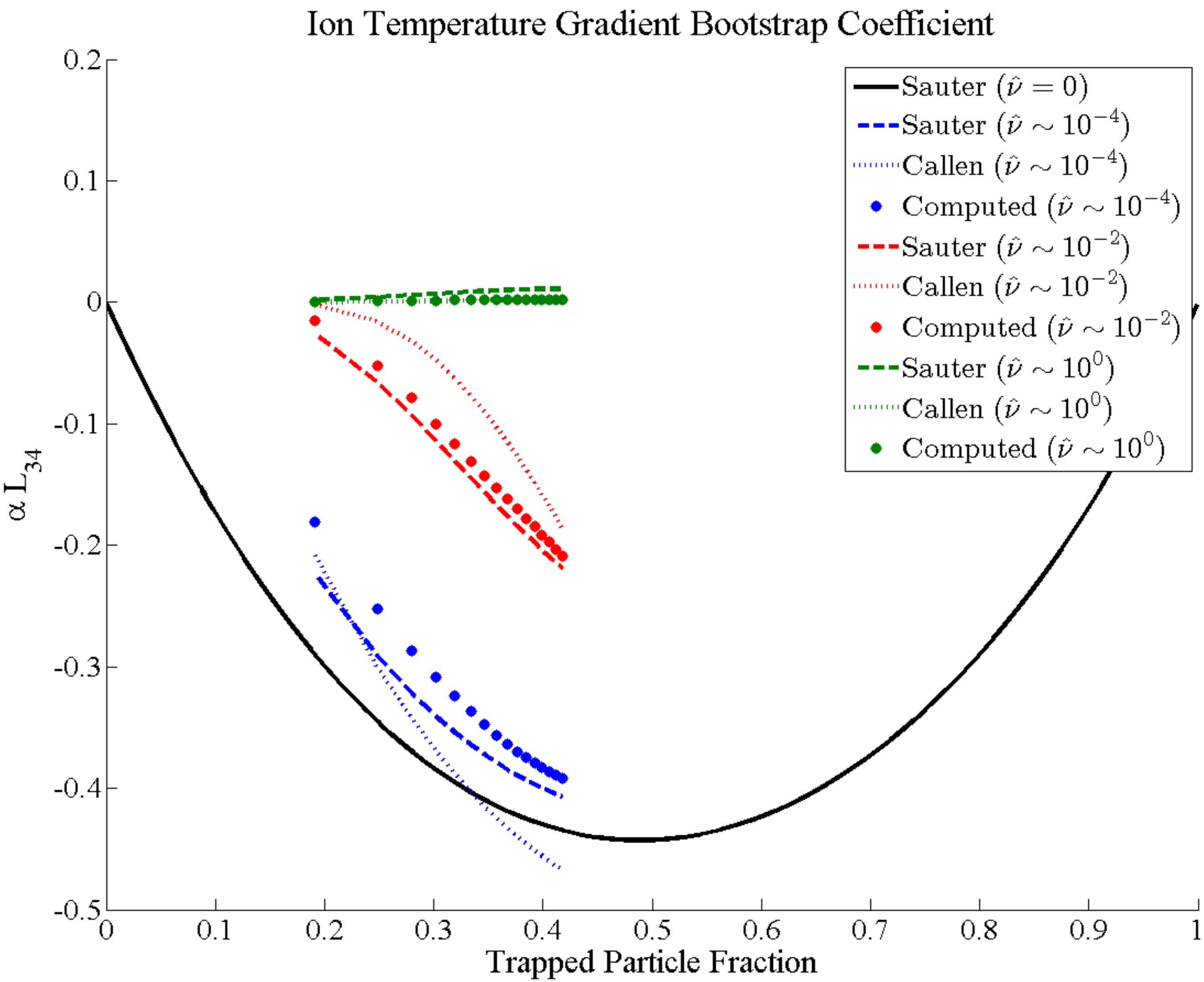
∇P Bootstrap Coefficient Benchmark



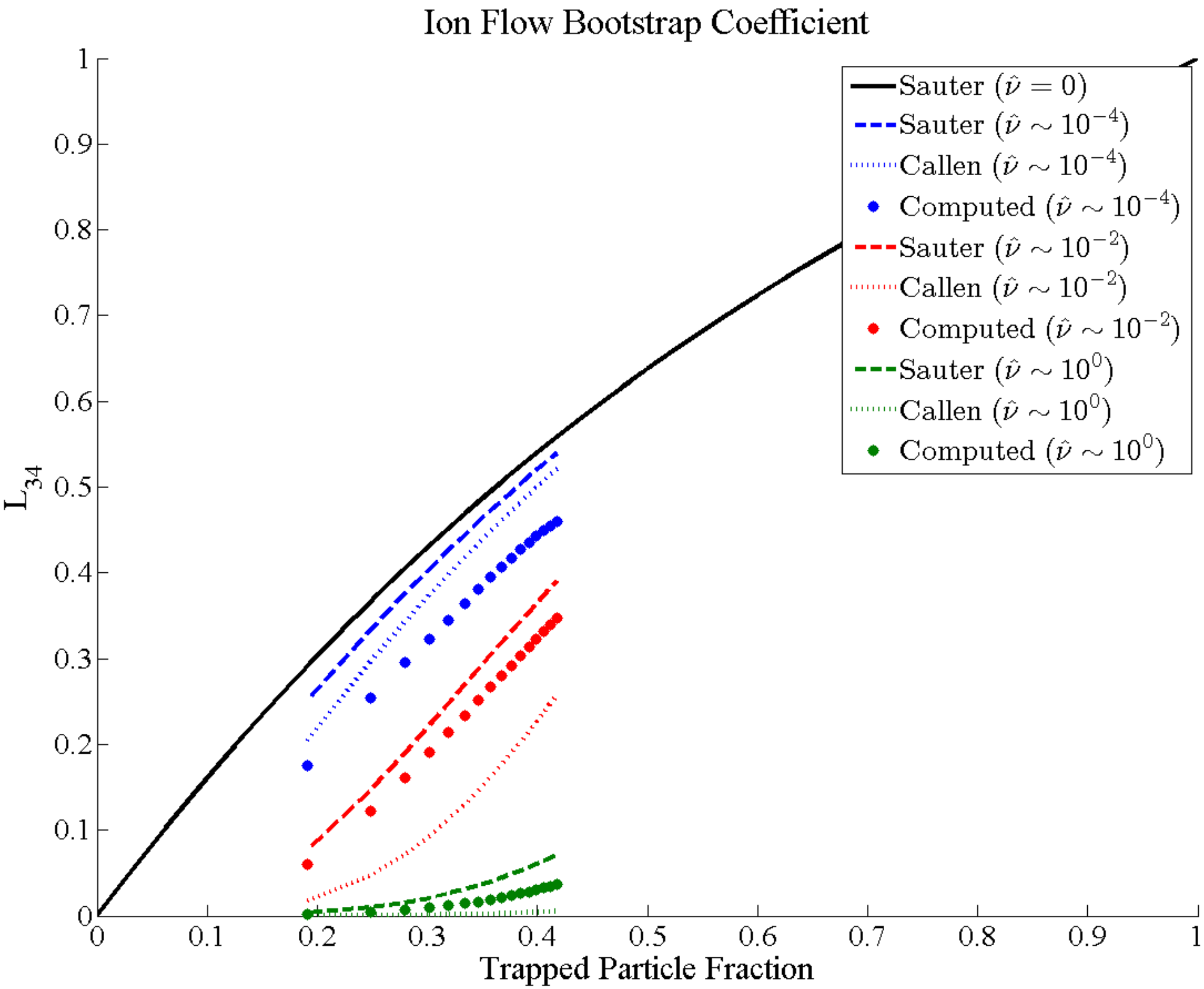
∇T_e Bootstrap Coefficient Benchmark



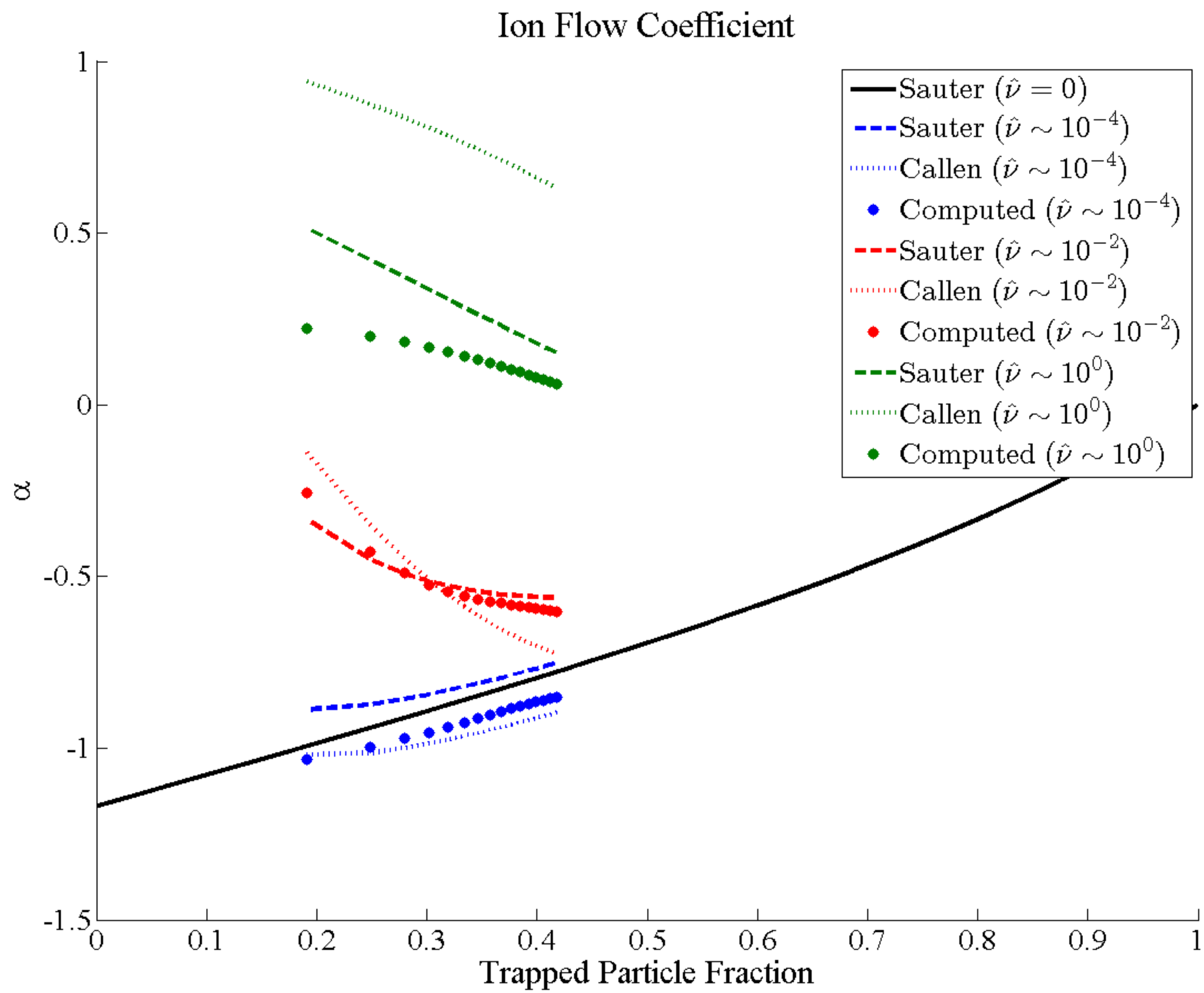
∇T_i Bootstrap Coefficient Benchmark



Ion Flow Bootstrap Coefficient Benchmark



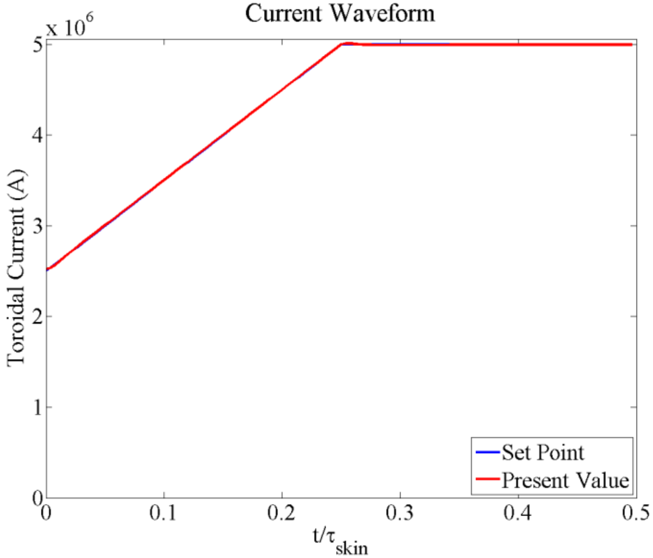
Ion Flow Coefficient Benchmark



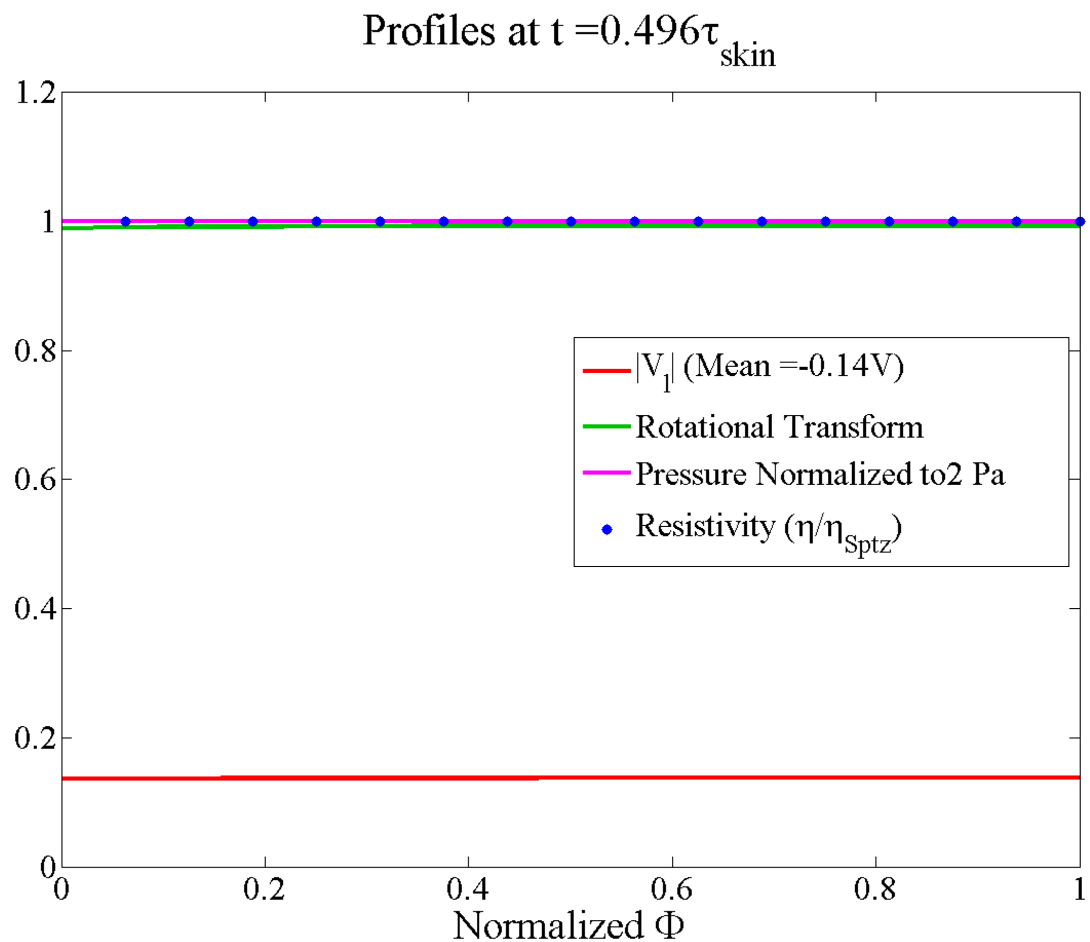
1D MHD Test Solver

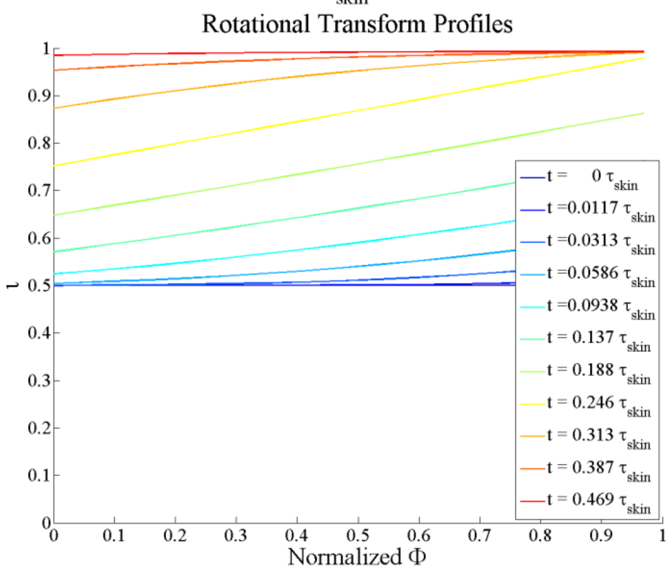
- From $\mathbf{B} = \nabla\psi \times \nabla\zeta + I\nabla\zeta$ $\quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ $\quad \mathbf{E} + \mathbf{u} \times \mathbf{B} = \mathbf{R}$, we can show that $\boxed{\frac{\partial \iota}{\partial t} = -\frac{\partial V_L}{\partial \Phi}}$ where $\Phi = \frac{1}{2\pi}\int \mathbf{B}\cdot\nabla\zeta\, dV$, $\iota = -2\pi\frac{d\psi}{d\Phi}$, and $V_L = -2\pi\frac{\langle \mathbf{B}\cdot\mathbf{R}\rangle}{\langle \mathbf{B}\cdot\nabla\zeta\rangle}$
- Assume a large aspect ratio, expansion equilibrium
- Current controller applies loop voltage at edge
  - All knowledge of resistivity comes through the Ohm's Law
  - For stability: $\mathbf{R} \Rightarrow \mathbf{R}^n + \eta_{Sptz}\left(\mathbf{J}^{n+1} - \mathbf{J}^n\right)$

Evolution with Spitzer resistivity

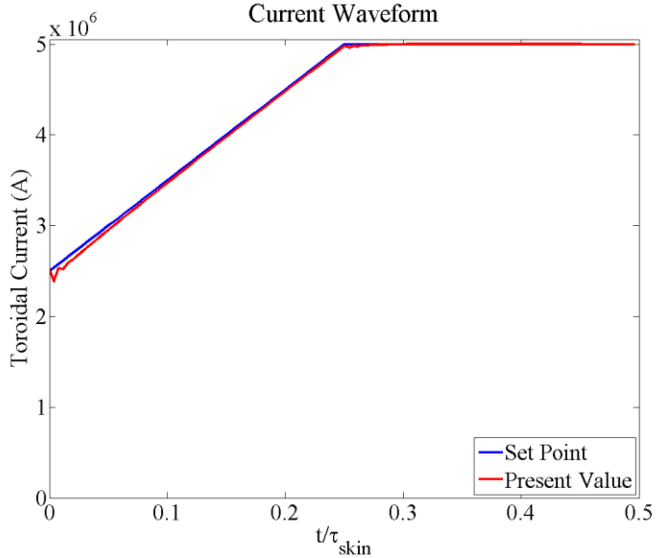


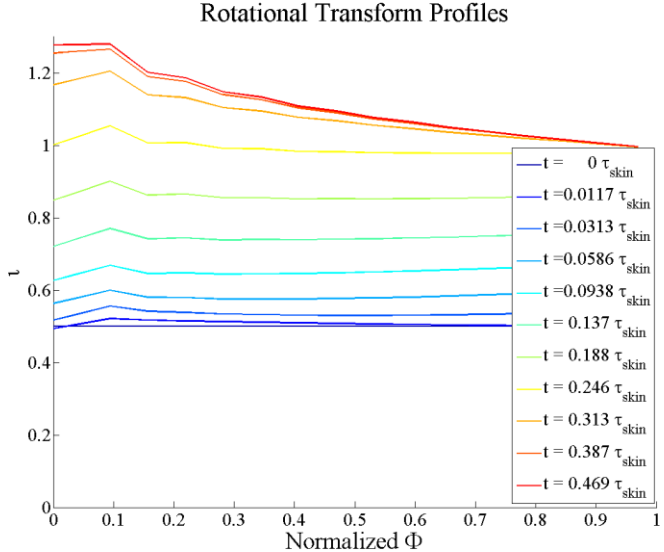
$$\boxed{\langle \mathbf{B} \cdot \mathbf{R} \rangle = \eta_{Sptz} \langle \mathbf{B} \cdot \mathbf{J} \rangle}$$



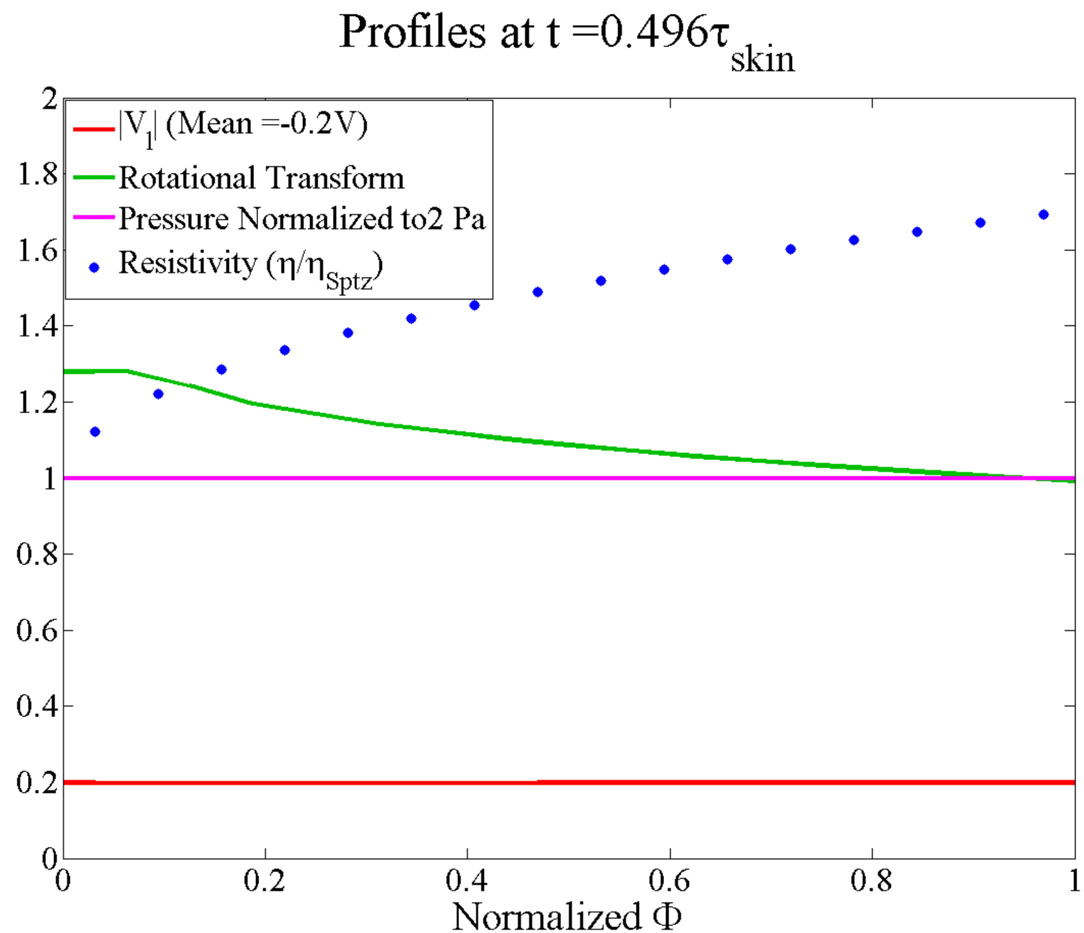


Evolution with DKE solver (no $\frac{dP}{d\psi}$)

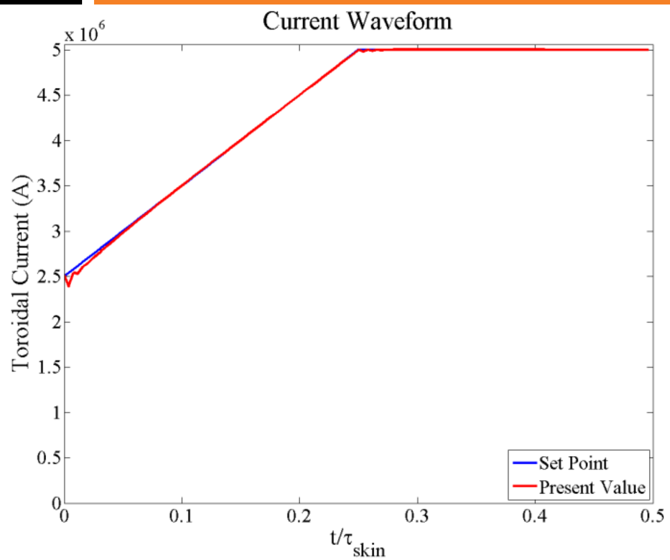


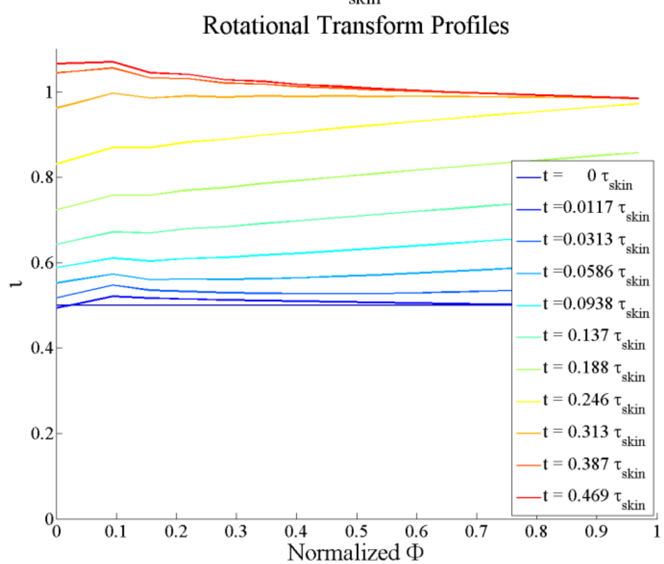


$$\boxed{\langle \mathbf{B}\cdot\mathbf{R}\rangle = \frac{1}{en}\left[\langle \mathbf{B}\cdot\mathbf{F}_e^{coll}\rangle + \langle (p_{e\parallel} - p_{e\perp})\,\mathbf{b}\cdot\nabla B\rangle\right]}$$

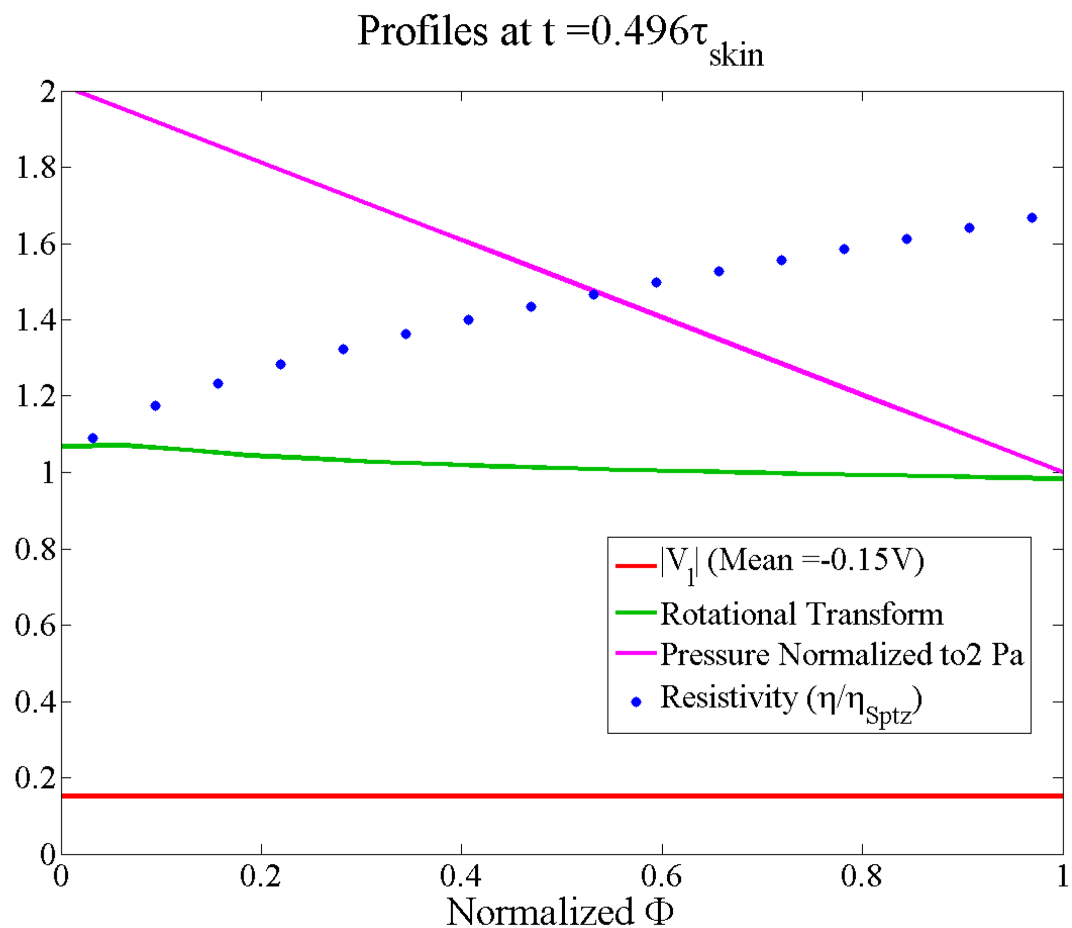


Evolution with DKE solver (with $\frac{dP}{d\psi}$)





$$\boxed{\langle \mathbf{B}\cdot\mathbf{R}\rangle = \frac{1}{en}\left[\langle \mathbf{B}\cdot\mathbf{F}_e^{coll}\rangle + \langle (p_{e\parallel} - p_{e\perp})\,\mathbf{b}\cdot\nabla B\rangle\right]}$$



Future work

- Incorporate ∇T_s bootstrap into MHD simulations
  - May require iteration with temperature equation
- Compare to MHD evolution with Sauter model on different timescales
- Couple to more advanced MHD code, e.g., M3D-C^1
- Investigate alternate representations and extensions to non-axisymmetric geometries
  - Discussing triangular finite elements in (y,θ) with group at Rensselaer Polytechnic Institute