MHD velocity boundary conditions and halo current

H. Strauss, *HRS Fusion*

CEMM / Sherwood Meeting 2014

Outline

- velocity boundary conditions
  - numerical comparison of Dirichlet, Neumann, DEBS, and Robin
  - sheath compatible MHD boundary conditions
- halo current in disruptions
  - Relation of toroidal variation of toroidal current to vertical displacement
  - Halo vs. Hiro
  - relative sign of perturbed plasma current and wall current

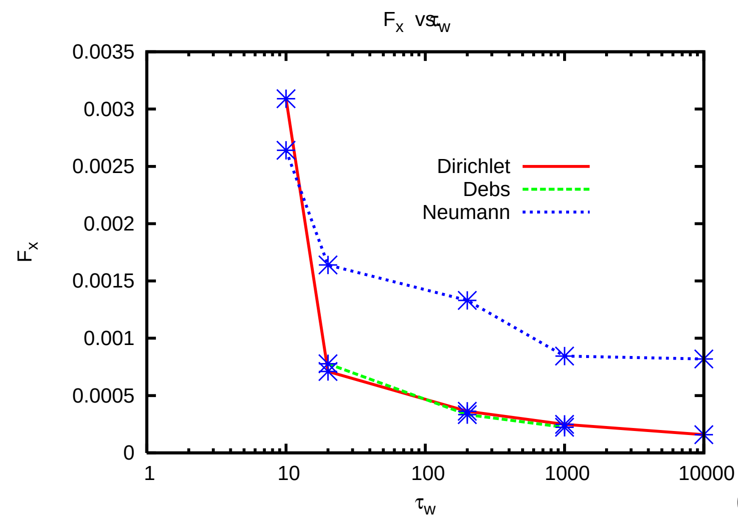
H. Strauss, Velocity boundary conditions at a tokamak resistive wall, Phys. Plasmas **21**, 032506 (2014).

Comparison of velocity boundary conditions on sideways force F_x in disruption simulations

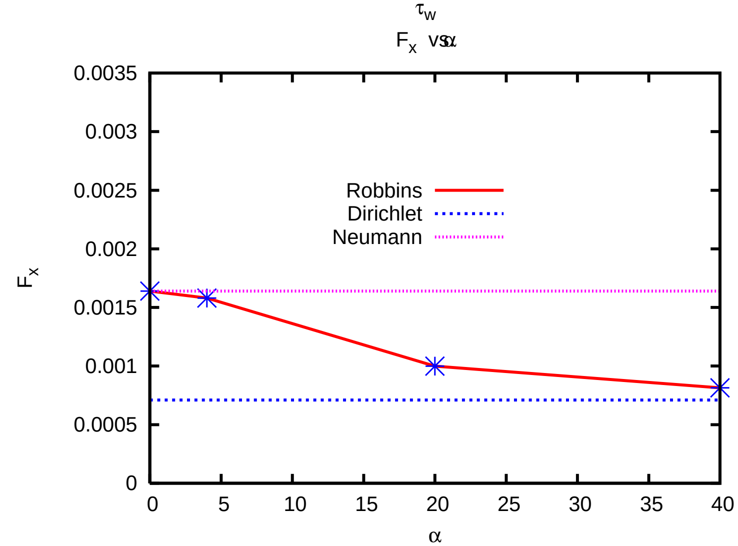
- Dirichlet: $v_n = 0$, rigid wall
- Neumann: $\partial v_n / \partial n = 0$, absorbing wall
- Robin $\partial v_n / \partial n + \alpha v_n = 0$, compromise
- DEBS $v_n = \mathbf{E} \times \mathbf{B} / B^2$, $\mathbf{E} = \eta_{wall} \mathbf{J}_{wall}$

(a) $F_x(\tau_{wall})$ for Neumann, Dirichlet and DEBs velocity boundary conditions. The force is qualitatively similar, suggesting that boundary conditions are not the dominant effect. DEBS $\approx$ Dirichlet when $\tau_{wall} >> \tau_A$.

(b) $F_x(\alpha)$ for Robin boundary condition with $\tau_{wall} = 20\tau_A$. Neumann for $\alpha = 0$, Dirichlet for $\alpha \to \infty$.



(a)



(b)

sheath compatible boundary conditions

The velocity is

$$\mathbf{v} = \mathbf{v}_\perp + v_\| \frac{\mathbf{B}}{B}, \quad \mathbf{v}_\perp = c\frac{\mathbf{E}\times\mathbf{B}}{B^2}, \quad \mathbf{E} = \nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \tag{1}$$

The sheath potential accelerates ions to the sound velocity [Stangeby, 2000] $c_s = (T_e/M_i)^{1/2}$ at which they strike the wall. The parallel velocity $v_\|$ does not affect the magnetic field, hence does not affect halo current or wall force. Near the wall $c_s/v_A \approx 10^{-2}$, so $v_\| \approx 0$ is a reasonable approximation.

The electrostatic potential Φ at the sheath entrance is approximately [Stangeby, 2000]

$$\Phi \approx 3\frac{T_e}{e}. \tag{2}$$

The perpendicular velocity normal to the wall, from (1),(2), is approximately

$$v_{\perp n} = k_\perp \frac{c}{B}\Phi \approx 3k_\perp \rho_s c_s = 3r\omega_* = \mathcal{O}(\rho_s) \tag{3}$$

where ρ_s is the gyroradius using c_s. For modes with MHD scale length, $k_\perp \rho_s \ll 1$, so

$$v_{\perp n} = 0 \tag{4}$$

is a good approximation.

$\partial \mathbf{A}/\partial t$ contribution to $\mathbf{E}$

The sheath is described by the usual electrostatic approximation, in the case of interest $k_\perp \Delta << 1$, where Δ is the sheath thickness. This has been shown for radio - frequency electromagnetic waves [D'Ippolito,2006] and it is easily verified for MHD.

The magnetic field in leading order does not vary in the sheath; $(1/c)(\partial A_\parallel/\partial t)$ varies on the $k_\perp^{-1}$ length scale. Otherwise magnetic perturbations of $\mathcal{O}(\Delta^{-1})$ would be produced. Similarly b_n is constant.

The $\mathbf{b}$ component of (1) can be integrated to give $E_\parallel = \nabla_\parallel \tilde{\Phi}$ where

$$\tilde{\Phi} = \Phi - (1/c)(\partial A_\parallel/\partial t)s, \tag{5}$$

and s is a local coordinate such that $b_n \partial s/\partial n = 1$, and $s = \mathcal{O}(\Delta)$. Effectively $(1/c)(\partial A_\parallel/\partial t)$ is absorbed into Φ by a gauge transformation.

Then a standard analysis [Stangeby, 2008] yields $n_e(\tilde{\Phi}), n_i(\tilde{\Phi})$.

The parallel electron momentum equation is

$$E_\parallel = -\nabla_\parallel \tilde{\Phi} \approx -\frac{T_e}{e n_e} \nabla_\parallel n_e, \quad n_e \approx n_0 \frac{e\tilde{\Phi}}{T_e} \tag{6}$$

The ions also satisfy electrostatic momentum balance.

From Maxwell's equations, eliminating the displacement current using the Coulomb gauge $\nabla \cdot \mathbf{A} \approx \nabla \cdot \mathbf{A}_\perp = 0$, yields the standard equation for $\tilde{\Phi}$,

$$\nabla^2 \tilde{\Phi} \approx \frac{\partial^2 \tilde{\Phi}}{\partial n^2} = 4\pi e[n_i(\tilde{\Phi}) - n_e(\tilde{\Phi})]. \tag{7}$$

The Coulomb gauge condition can be approximately satisfied by choosing

$$\mathbf{A}_\perp = \nabla \chi \times \mathbf{B}, \tag{8}$$

which is similar to the M3D representation. Then $B^2 = \mathbf{B} \cdot \nabla \times \mathbf{A}_\perp = -\nabla \cdot (B^2 \nabla \chi) - \mathbf{J} \times \mathbf{B} \cdot \nabla \chi$.

This requires that $\partial \chi / \partial n = \mathcal{O}(\Delta)$, or else B^2 is $\mathcal{O}(\Delta^{-1})$. Combining (1), (5), and (8) gives the result that $B v_{\perp n}/c = \partial \Phi / \partial l - \partial^2 \chi / \partial n \partial t = \mathcal{O}(\Delta)$, where l is a poloidal coordinate tangent to the wall.

Note that if $\Phi = (\Delta / c) \partial A_\parallel / \partial t$, then $v_n = k_\parallel \Delta (B_n / B) v_A = \mathcal{O}(\Delta)$, taking $\partial / \partial t = k_\parallel v_A$, with $k_\parallel << k_\perp$.

In the MHD limit in which Δ and ρ_s are neglected, this implies that $v_{\perp n}$ satisfies a Dirichlet boundary condition,

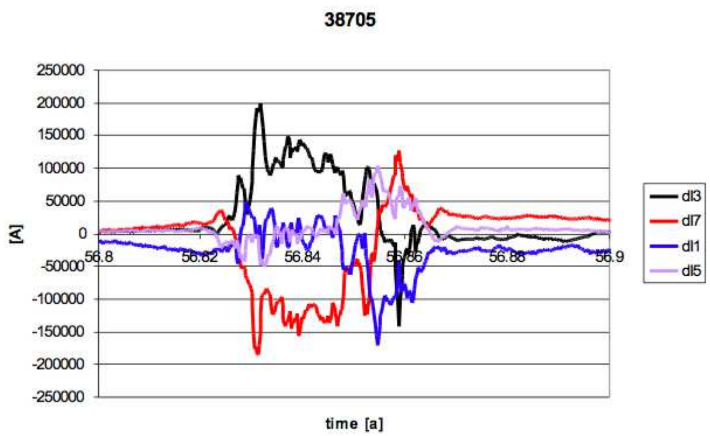
$$v_{\perp n} = 0 \tag{9}$$

and the total normal velocity is

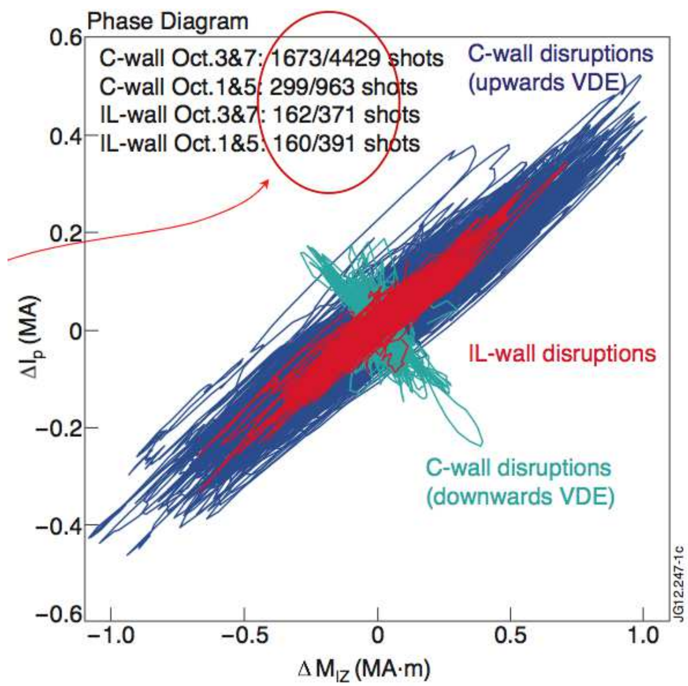
$$v_n = c_s |b_n| \tag{10}$$

directed from the plasma into the wall.

Halo current



(a)



(b)

Toroidal variation of toroidal current

It was found in JET that during disruptions, the toroidal current varied with toroidal angle. It was found that $\Delta I_\phi / I_\phi \approx 0.08$, where ΔI_ϕ is the amplitude of the $n = 1$ variation.

(a) Current $I_\phi(\phi, t)$ measured in quadrants of JET, showing $n = 1$ toroidal variation.

(b) Toroidal current variation ΔI_ϕ vs. ΔM_{IZ}, vertical moment of the current: Hiro current?

L. E. Zakharov, Phys. Plasmas **15** 062507 (2008).

S. N. Gerasimov, ITPA meeting, Abingdon (2013).

The ratio of toroidal current perturbation $\tilde{I}_\phi$ to vertical current moment M_{IZ} is proportional to the VDE displacement ξ_{VDE}.

H. R. Strauss, R. Paccagnella, J. Breslau, Phys. Plasmas (2010) **17**, 082505.

Relation of toroidal current perturbation to vertical current moment

This is an update of the previous result and further numerical verification.

The vertical current moment, which is the perturbed current multiplied by $z = r \sin\theta$, is

$$\tilde{M}_{IZ} = \int_0^a \tilde{J}_\phi r^2 \sin\theta dr d\theta = -\oint \frac{\partial \tilde{\psi}}{\partial r} a^2 \sin\theta d\theta \tag{11}$$

in a circular cross section where the boundary is $r = a$, noting that

$$\tilde{J}_\phi = -\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \tilde{\psi}}{\partial r} - \frac{1}{r^2}\frac{\partial^2 \tilde{\psi}}{\partial \theta^2} \tag{12}$$

integrating by parts, and assuming that the wall is a good conductor, so that $\tilde{\psi} \approx 0$ at $r = a$. The wall is assumed slightly resistive in order that the VDE can be unstable, but the resistive wall penetration time τ_{wall} is assumed long compared to the growth time of the modes.

The toroidal current is

$$\tilde{I}_\phi = \int_0^a \tilde{J}_\phi r dr d\theta = -\oint \frac{\partial \tilde{\psi}}{\partial r} a d\theta \tag{13}$$

Note that (11) and (13) differ by a factor of $\sin\theta$.

Assuming that the region inside the wall, $r < a$, is entirely filled with plasma, the poloidal flux change produced by a displacement potential Φ is

$$\delta\psi = \nabla\Phi \times \nabla\psi \cdot \widehat{\phi} \tag{14}$$

for an axially symmetric potential Φ. The VDE displacement potential has the form $\Phi = \xi_{VDE}(r)\cos\theta$. Iterating $\widetilde{\psi} = \psi_1 + \psi_2 + \psi_3 + \ldots$ and taking the radial derivative, imposing a rigid wall boundary condition $\xi_{VDE}(a) = 0$, gives

$$\psi'_{k+1} = \frac{\xi'_{VDE}}{r}\left(\frac{\partial}{\partial\theta}(\psi'_k \cos\theta) + 2\psi'_k \sin\theta\right) \tag{15}$$

where the prime denotes a radial derivative. Summing (15) over k and integrating over θ, using (11),(13) gives

$$\tilde{I}_\phi = 2\frac{\xi'_{VDE}}{a^2}\tilde{M}_{IZ}. \tag{16}$$

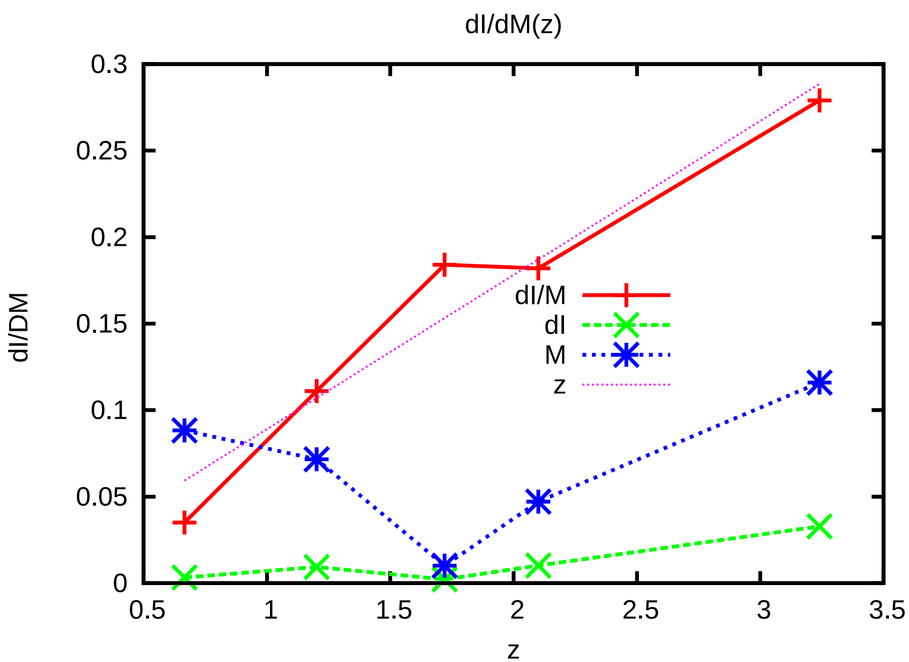
In principle ψ' can consist of an arbitrary sum of (m, n) modes.

For an upward VDE, with $\xi_{VDE}(r) < 0$, but $\xi_{VDE}(a) = 0$, it is necessary that $\xi'_{VDE}(a) > 0$. For an upward VDE, the ratio $\tilde{I}_\phi/\tilde{M}_{IZ}$ is positive, and similarly for a downward VDE, the ratio is negative.

To show this, calculate the displacement in the vertical direction, $d_y = \nabla(r\sin\theta) \times \nabla(\xi_{VDE}\cos\theta) \cdot \widehat{\phi} = -\xi_{VDE}\sin^2\theta - r\xi'_{VDE}\cos^2\theta$. Near the top and bottom of the plasma, $d_y \approx -\xi_{VDE}$, indicating that an upward VDE requires negative ξ_{VDE}, and positive ξ'_{VDE}.

numerical simulations

Plot of $\tilde{I}_\phi/\tilde{M}_{IZ}$ as a function of vertical displacement. The FEAT15MA equilibrium was modified by setting toroidal current and pressure to zero outside the $q = 2$ surface, keeping the total toroidal current and pressure constant, modeling MGI induced disruptions.



Here

$$\frac{\tilde{I}_\phi}{\tilde{M}_{IZ}} = \frac{\oint \tilde{I}_\phi \tilde{M}_{IZ} d\phi}{\oint \tilde{M}_{IZ}^2 d\phi}$$

For $z > 1.6$, the plasma edge $q < 2$ and the $(1, 1)$ is dominant, evidently similar to JET. For $z < 1.6$, $q = 2$ at the plasma edge and the $(2, 1)$ is dominant.

For $z < 1.6$, current does not touch the wall so it can't be Hiro current.

sign of $n > 0$ toroidally varying wall current

Opposite sign of toroidal varying of toroidal current and wall current is not dependent on Hiro current. Integrating $\nabla \cdot J = 0$ over the plasma

$$\frac{\partial \tilde{I}_{\phi}^{wall}}{\partial \phi} = -I_{halo3D}$$

where

$$I_{halo3D} = \int \tilde{J}_n R dl.$$

Then integrate $\nabla \cdot J = 0$ over the thin wall of thickness δ_{wall} surrounding the plasma. In the wall,

$$\frac{\partial \tilde{I}_{\phi}^{wall}}{\partial \phi} = I_{halo3D}$$

where

$$\tilde{I}_{\phi}^{wall} = \delta_{wall} \int \tilde{J}_{\phi}^{wall} dl.$$

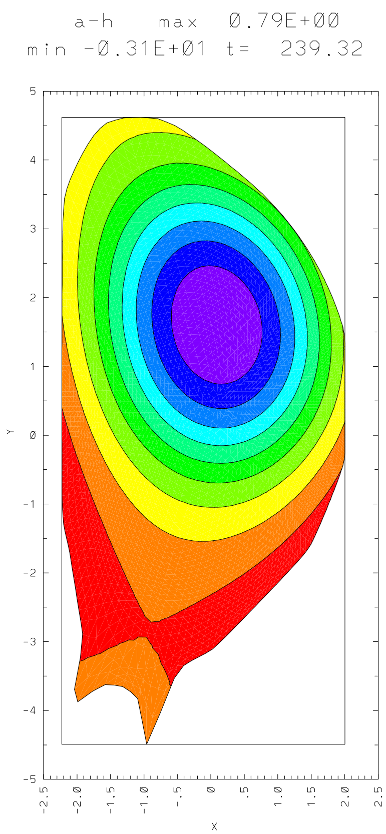
It follows immediately that

$$\tilde{I}_{\phi} + \tilde{I}_{\phi}^{wall} = 0,$$

the toroidally varying toroidal current in the wall is equal and opposite to the toroidally varying toroidal current in the plasma.

$n = 0$ case

In the $n = 0$ case, the wall current is produced by magnetic flux pushed into the wall. It is also opposite the sign of the plasma current.



We can use the previous displacement model, except now consider the equilibrium $(0, 0)$ and mode $(1, 0)$,

$$\psi_1' = \frac{\xi_{VDE}'}{r} \psi_0' \sin\theta$$

The magnetic flux at the wall satisfies

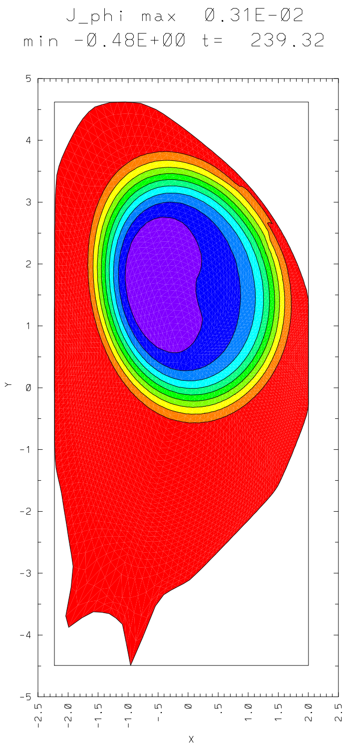
$$\gamma_{VDE} \psi_1 = -\eta_{wall} J_\phi^{wall}$$

where γ_{VDE} is the VDE growth rate and the wall current is

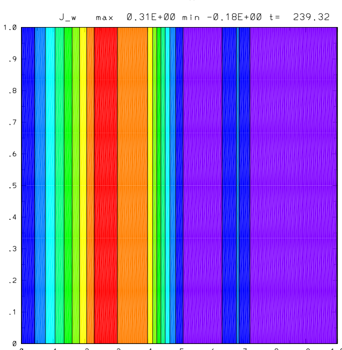
$$J_\phi^{wall} = \frac{1}{\delta_{wall}} (\psi_1' - \psi_v') \tag{17}$$

where ψ_v is the vacuum poloidal flux $\psi_v = \psi_1 a / r$, and

$$\psi_v' = -\frac{\psi_1}{r} \tag{18}$$



(a)



(b)

Combining,

$$(1+\gamma_{VDE}\tau_{wall})\psi_1 = -a\psi_1' \tag{19}$$

where $\tau_{wall} = \delta_{wall} a/\eta_{wall}$, which gives

$$J_\phi^{wall} = -\frac{\gamma_{VDE}\tau_{wall}}{1+\gamma_{VDE}\tau_{wall}}\frac{\psi_1'}{\delta_{wall}} \tag{20}$$

Note that the VDE growth rate is $\gamma_{VDE}\tau_{wall} = c_0 \approx 1$. Defining $I_\phi^{wall} = \pi\delta_{wall} a J_\phi^{wall}$, with $I_\phi = -2\pi a^2 \psi_0'$

$$I_\phi^{wall} = -\frac{c_0}{1+c_0} a\xi_{VDE}' I_\phi \sin\theta \tag{21}$$

The toroidal current density J_ϕ is plotted in (a), and is negative. The wall current J_ϕ^{wall} is plotted in (b). The horizontal axis is $\theta/(2\pi)$, where θ is the angle about $(R,Z) = (0,0)$, and the vertical axis is $\phi/(2\pi)$. J_ϕ^{wall} is maximum and positive at $\theta = \pi/2$, at the top of the wall, and minimum and negative at $\theta = 1.4\pi$, near the bottom.

Conclusions

- Wall force is not dominated by v_n boundary condition
  - most important is $\gamma\tau_{wall}$
- Sheath boundary condition is compatible with Dirichlet
  - plasma absorption in wall is via $v_{\|}$
- relation of toroidal variation of toroidal current was revisited
  - verified that toroidal current variation is proportional to VDE displacement
  - It can not be caused by Hiro current when toroidal current does not touch wall.
  - $n > 0$ wall current is opposite in sign to the toroidally varying plasma current.
  - $n = 0$ wall current closest to the plasma is opposite in sign to plasma current.