Magnetic self-organization in tokamaks

S. C. Jardin[1]

Nate Ferraro[2], Isabel Krebs[1,3] , Jin Chen[1] ,
Dmytro Meshcheriakov[3] , Stuart Hudson[1] , Fan Zhang[4] ,
Seegyoung Seol[4] , Mark Shephard[4] , Josh Breslau[1] ,
Guo-Yong Fu[1] , Sibylle Guenter[3] , Amitava Bhattacharjee[1]

[1] *Princeton Plasma Physics Laboratory, Princeton, NJ*
[2] *General Atomics, San Diego, CA*
[3] *Institute for Plasma Physics, Garching, Germany*
[4] *Rensselaer Polytechnic Institute SCOREC Center*

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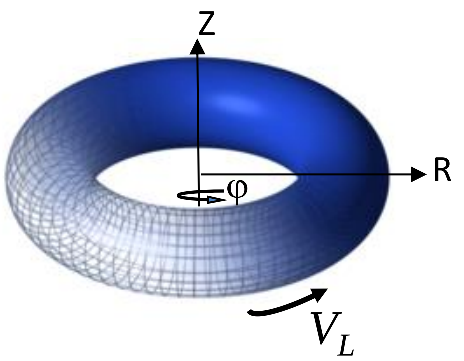
Summary

- Last Fall I presented a talk at the CEMM meeting “explaining” a self-organized stationary state that can occur in tokamaks
- That explanation involved an unstable interchange mode flattening the temperature profile which in turn keeps the current from peaking.
- However, since then we learned that experimentally, (on DIII-D), this stationary state can exist *even when the temperature profile is not flattened and when central current drive is applied!*
- This led us to widen the parameter regime of our investigation, and we have now identified a separate mechanism that keeps the current from peaking…even more dominant than temperature flattening in most regions of parameter space.
- New mechanism is a *nonlinear dynamo*: very robust and normally dominant over the temperature flattening phenomena

3D resistive MHD in torus with source terms

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\mathbf{V}) = S_n$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{B}$$



$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \bullet \nabla \mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{V} \qquad \mu = \mu_0$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \qquad T = p/n \qquad \eta = \eta_0 (T/T_0)^{-3/2}$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\frac{3}{2} p\mathbf{V}\right) = -p\nabla \bullet \mathbf{V} + \underline{\nabla \bullet n\kappa_\perp \nabla T} + \nabla \bullet n\kappa_{\|}\mathbf{bb} \bullet \nabla T + \eta J^2 + \underline{S_e}$$

balance in equ.

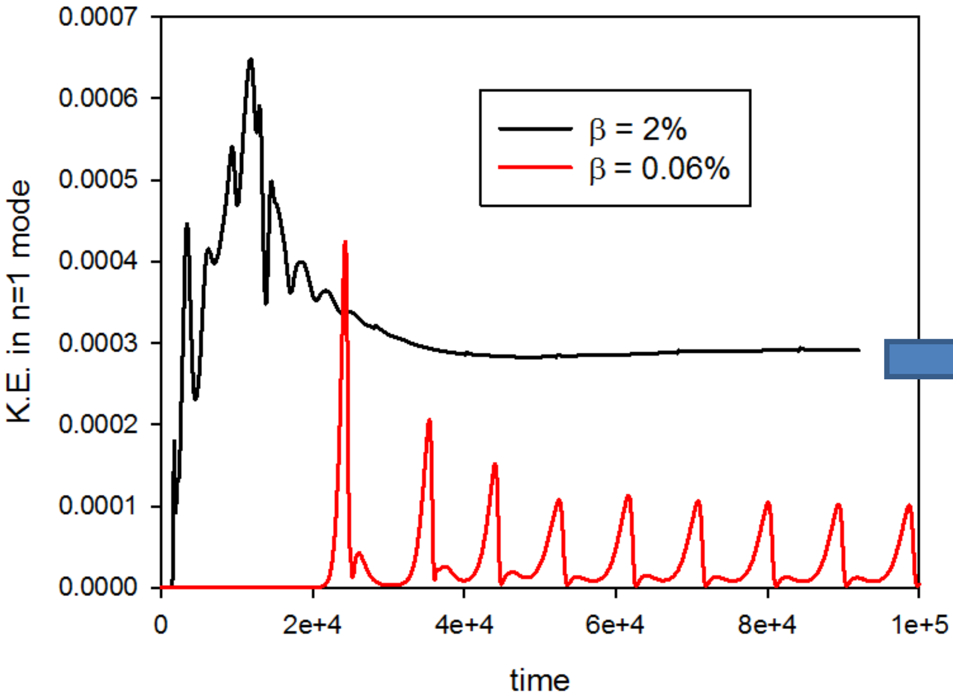
S_n(t) volume source adjusted in control loop to keep total # of particles constant
V_L(t) applied at boundary in control loop to keep total plasma toroidal current constant

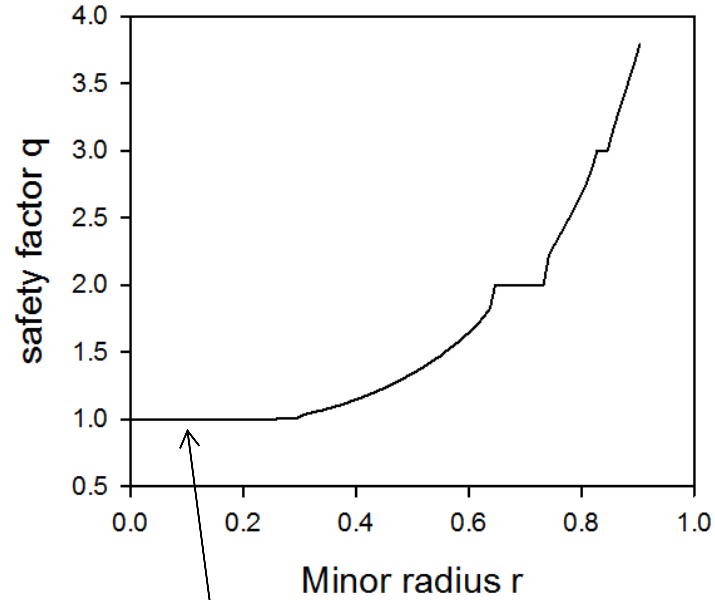
Series of runs with same $\beta = \mu_0 p/B^2$ ~ 2% but differing $\kappa_{\perp 0}$ and S_e (proportional)

$\kappa_\perp \sim \kappa_{\perp 0} (T/T_0)^{-1/2}$ $\quad \kappa_{\|} \sim 10^5 \times \kappa_\perp$ $\quad \eta_0 \sim 10^{-6}$ $\quad \kappa_{\perp 0} = 18\eta_0,\ 36\eta_0,\ 72\eta_0,\ 144\eta_0$

(4 3D cases will be presented + 1 2D)

$\beta \equiv \mu_0\, p/B^2$ =2% behavior much different from low β





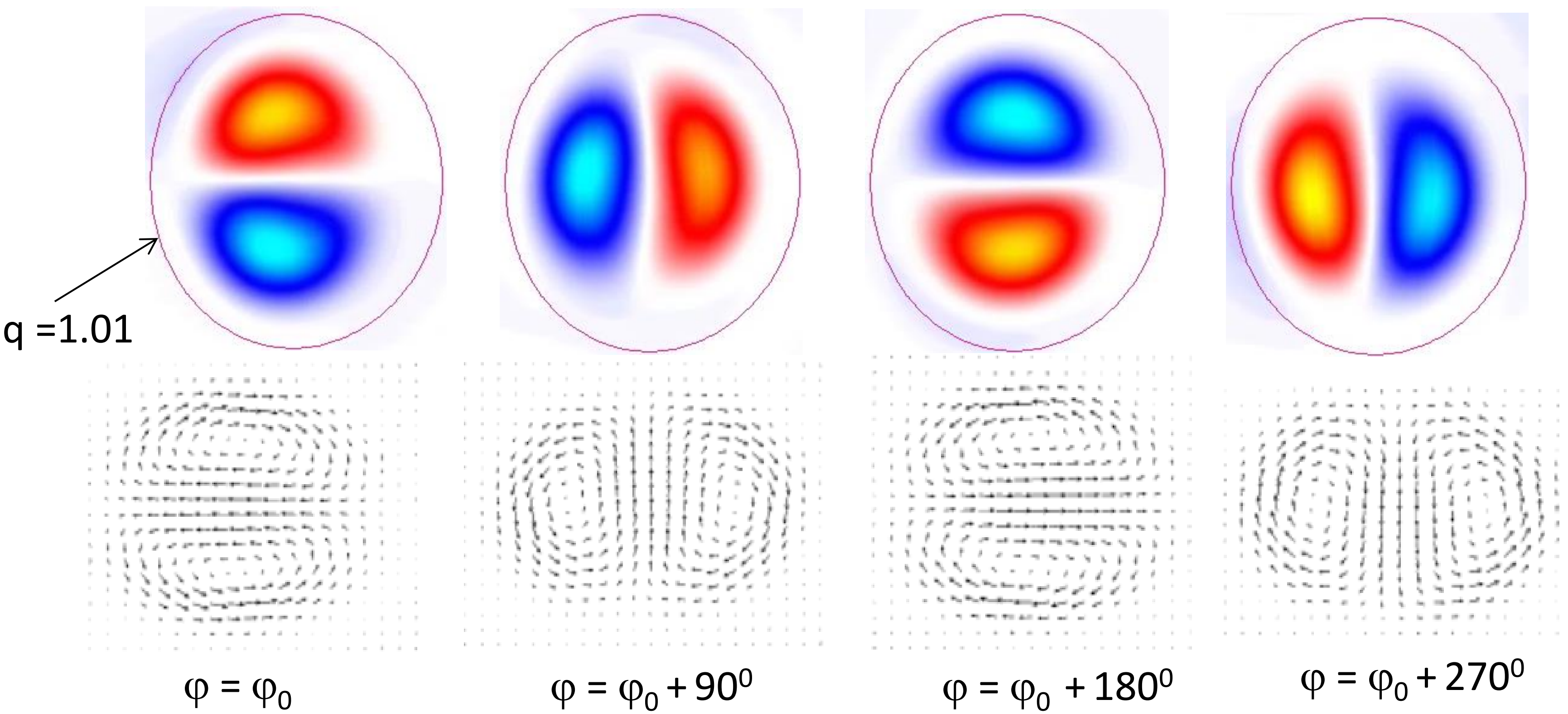
Large region in center with q = 1

- At low-β, plasma kinetic energy (and T_{e0} and q_0) undergo periodic oscillations where current peaks, reconnection occurs and process repeats (sawteeth)

- At 2% β, plasma goes into a stationary state with large helical flow patterns and ultra-low magnetic shear with q=1 in center

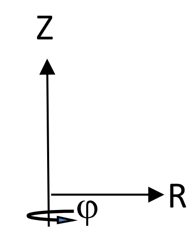
$$q = \frac{\#\ \text{of toroidal transits}}{\#\ \text{of poloidal transits}}$$

Stationary helical flow pattern persists driven by unstable interchange mode

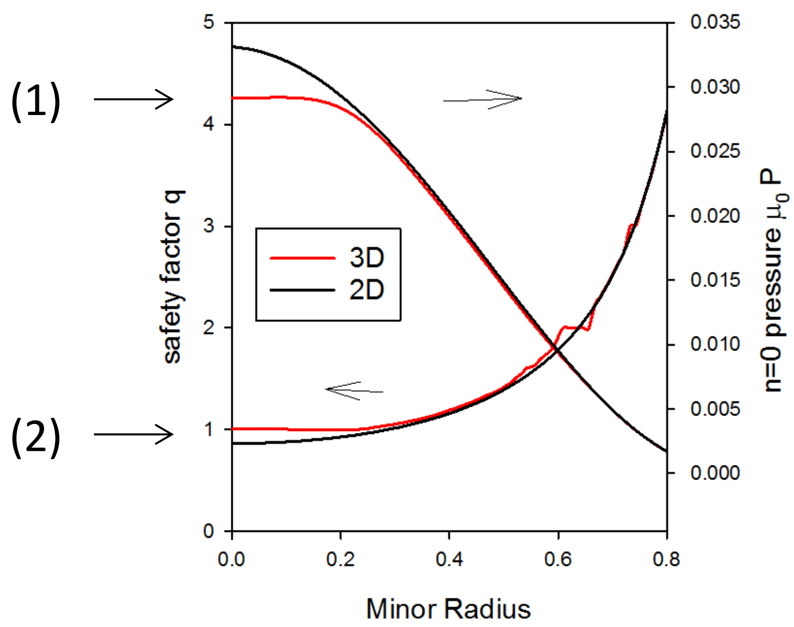


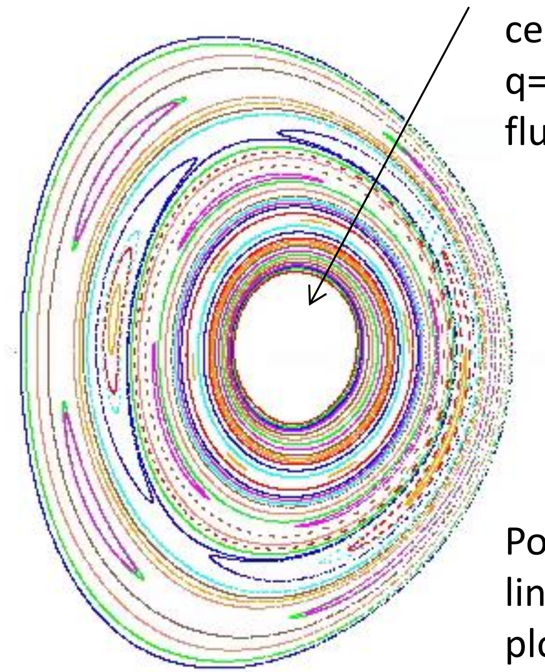
Plotted on top is poloidal velocity stream function U where $\mathbf{V}_{1,1} = R^2 \nabla U \times \nabla \varphi$

On bottom are vectors of poloidal velocity $\mathbf{V}_{1,1}$



Comparison of profiles from 2D and 3D calculation shows 2 differences





Volume in center with q=1 and no flux surfaces

Poincare “field line mapping” plot of 3D final configuration

(1) Central pressure is flattened in 3D calculation compared to 2D

(2) Central q-profiles is less than 1 in 2D, equal to 1 in 3D

Why doesn't q_0 continue to decrease in 3D run*?

In a stationary state,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = 0 \quad \Rightarrow \quad \mathbf{E} = -\nabla \Phi + \frac{V_L}{2\pi} \nabla \varphi \qquad (1)$$

Generalized Ohm's law:

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \qquad (2)$$

In the stationary state, (1)+(2) becomes:

$$\nabla \Phi - \mathbf{V} \times \mathbf{B} = -\eta \mathbf{J} + \frac{V_L}{2\pi} \nabla \varphi \qquad (3)$$

$\hat{\varphi}\bullet$

$\mathbf{B}\bullet$

If we dot $\mathbf{B}$ into Eq. (3):

$$\mathbf{B}\bullet\nabla \Phi = \nabla\bullet(\mathbf{B}\Phi) = -\eta \mathbf{B}\bullet\mathbf{J} + \frac{V_L}{2\pi} \mathbf{B}\bullet\nabla \varphi \qquad (4)$$

If magnetic surfaces exist, and we surface average (4), we get the well-known condition that the surface averaged current is completely determined by the resistivity profile:

$$\eta \langle \mathbf{J}\bullet\mathbf{B} \rangle = \frac{V_L}{2\pi} \langle \mathbf{B}\bullet\nabla \varphi \rangle$$

This is satisfied exactly in 2D stationary states

V_L is a constant (applied loop voltage)

φ is the toroidal angle

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φ is the toroidal angle

But, it is not satisfied in 3D in the central region! Why not?

Basic Physics of self-organized stationary discharge with q_0=1

(1) Central ultra-low shear (ULS) region with q=1 (and no flux surfaces)

(2) Drives interchange instability producing $\mathbf{V}_{1,1}$

(3) Produces electric potential
$$\nabla\Phi_{1,1} \simeq \mathbf{V}_{1,1} \times \mathbf{B}$$

(4) Also perturbs temperature:
$$\frac{2}{3}\nabla\bullet\left[n\kappa_{\perp}\nabla T_{1,1}\right] \simeq n\mathbf{V}_{1,1}\bullet\nabla T$$

(5) Also perturbs magnetic field:
$$\eta\mathbf{J}_{1,1} = \mathbf{V}_{1,1} \times \mathbf{B} - \eta_{1,1}\mathbf{J}$$
$$\nabla \times \mathbf{B}_{1,1} = \mu_0 \mathbf{J}_{1,1}$$

(6) 3+5 produce effective enhanced loop voltage in central region:
$$V_{n=0} = \mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1} + \cdots$$

(7) 2+4 produce velocity field that tends to flatten temperature profile
$$\delta T_{n=0} \simeq -\mathbf{V}_{1,1}\bullet\nabla T_{1,1}$$

(8)Axisymmetric (n=0) parallel Ohm's law is modified in two ways that broaden current
$$\eta\mathbf{J}\bullet\mathbf{B}_{n=0} + V_{n=0} = \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

resistivity profile is flattened

positive in center, flattening current

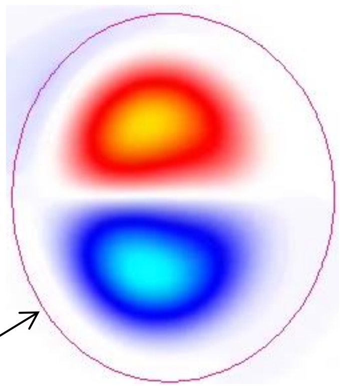
(9) No longer constrained to $\eta\langle\mathbf{J}\bullet\mathbf{B}\rangle = \text{const.}$ central region approaches minimum energy Taylor State $\mathbf{J} = \lambda\mathbf{B}$ ➔ ULS

- These perturbations will also produce other islands via mode coupling, but they do not play a central role

(2) Ultra-flat q profile drives interchange instability

Plotted is U on one toroidal plane (φ=0) from a 3D simulation where:

$$\mathbf{V}_{1,1} = R^2 \nabla U \times \nabla \varphi$$

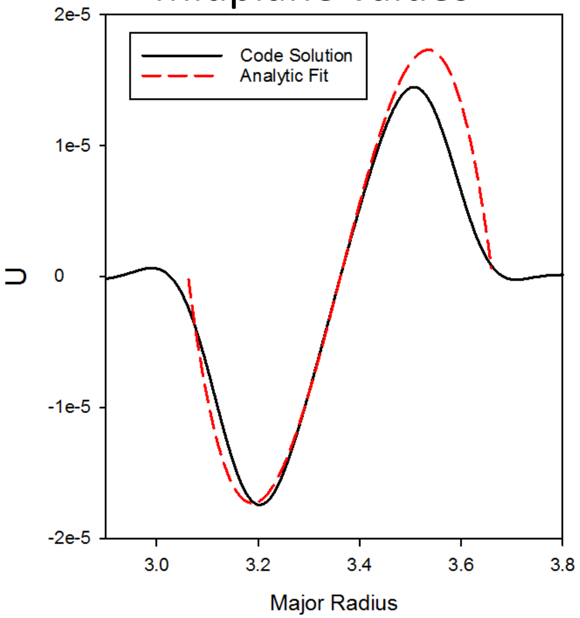


q =1.01

Compare with the unstable eigenfunction found in [1]

$$U(r,\theta,\varphi) = U_0 r[1-(r/r_1)^2]\sin(\theta-\varphi)$$

midplane values



Shape of stationary nonlinear code velocity stream function agrees well with linear eigenfunction.

Almost the same for all values of κ_0 -- but amplitude depends on β

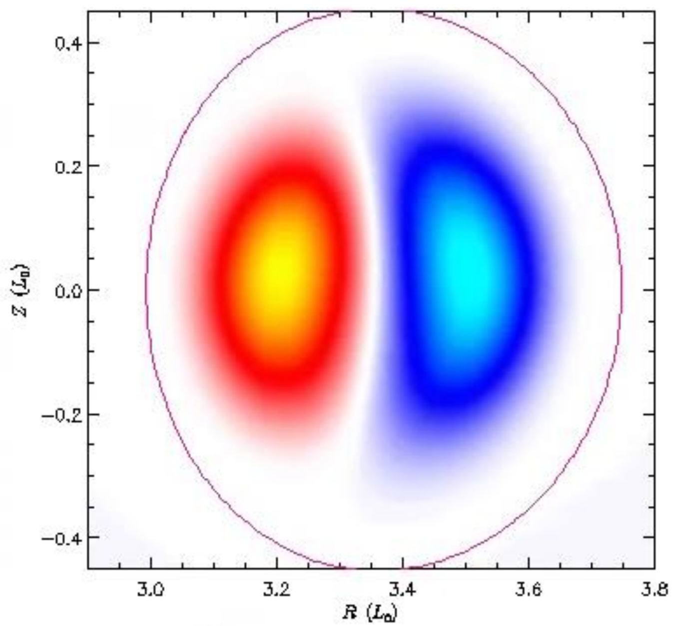
[1] Hastie and Hender, NF **28** (1988) p. 585 "Toroidal internal kink stability in tokamaks with ultra flat q profiles"

(3) Driven flow from interchange produces electric potential

$$\nabla\Phi - \mathbf{V}\times\mathbf{B} = -\eta\mathbf{J} + \frac{V_L}{2\pi}\nabla\varphi$$

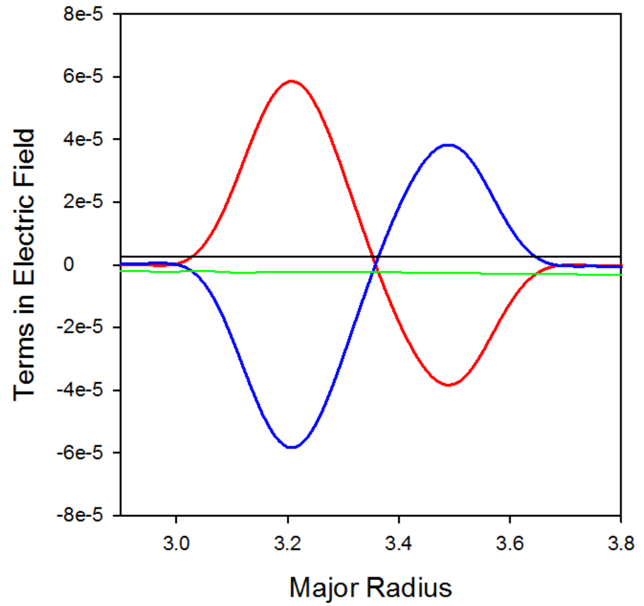
These 2 large terms must almost cancel

potential Φ at one toroidal plane



$\varphi = 0$

Mid-plane values of individual terms making up toroidal electric field (color coded) at one toroidal location



Terms on either side of equal sign mostly cancel (but not exactly)

(4) Velocity field also perturbs temperature and pressure.

But much larger asymmetry in low κ cases than high κ

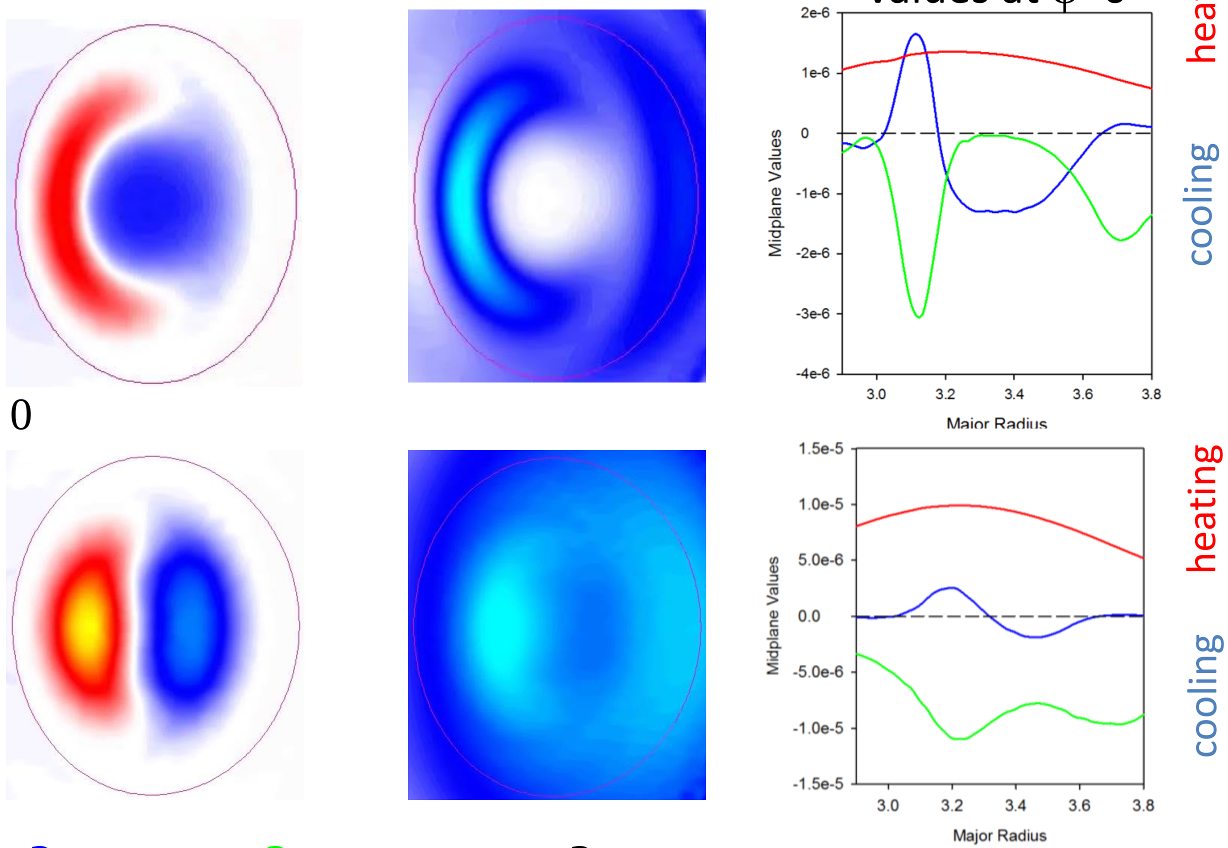
$n\mathbf{V}\bullet\nabla T + \frac{2}{3}nT\nabla\bullet\mathbf{V}$ $\qquad$ $-\frac{2}{3}\nabla\bullet n\kappa_{\perp}\nabla T$

Mid-plane values at φ=0

low κ

$\varphi = 0$

high κ



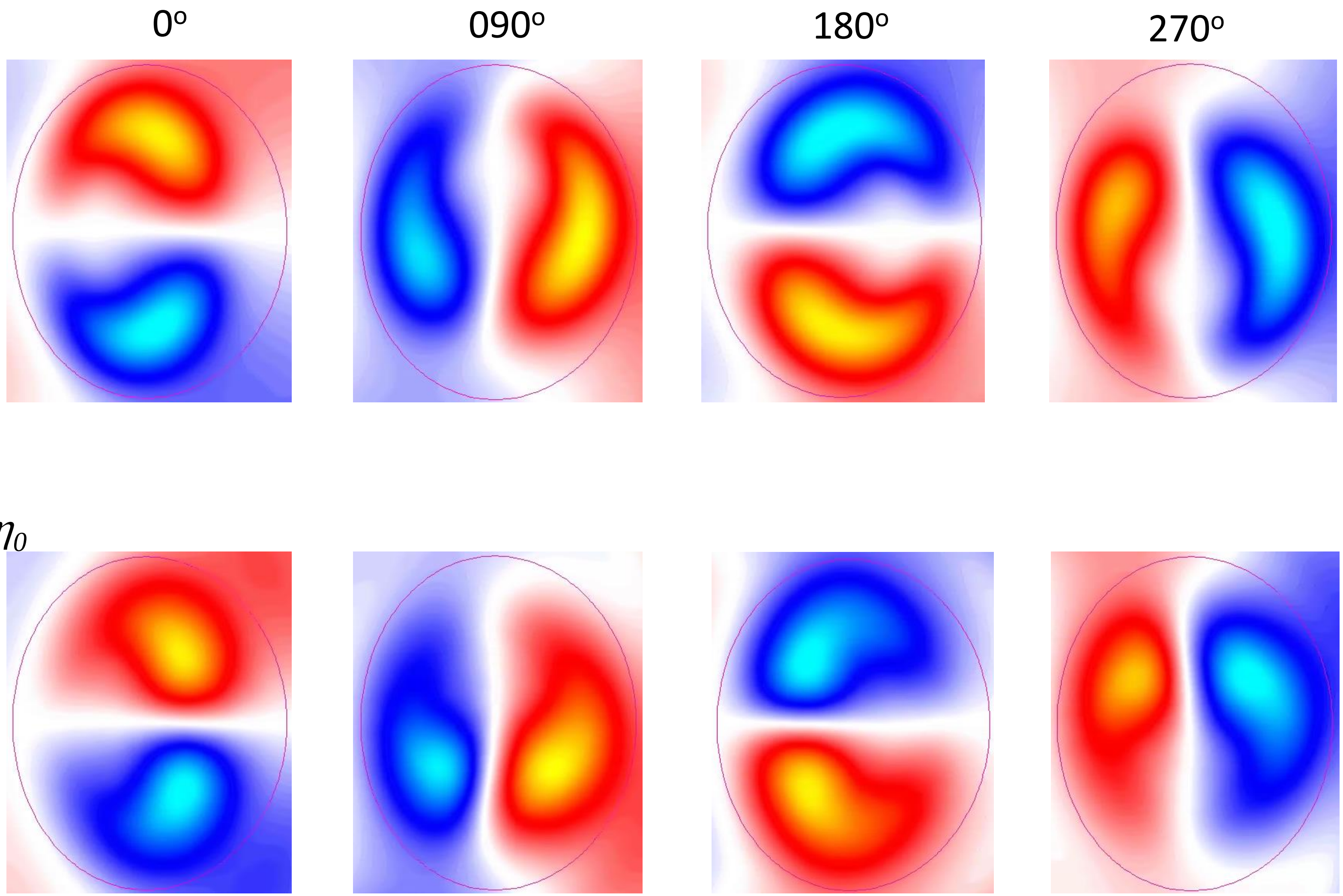
$$\frac{\partial T}{\partial t} + n\mathbf{V}\bullet\nabla T + \frac{2}{3}nT\nabla\bullet\mathbf{V} - \frac{2}{3}\nabla\bullet n\kappa_{\perp}\bullet\nabla T + \frac{2}{3}\nabla\bullet\mathbf{q}_{\parallel} = \eta J^2 + S_e$$

(1,1) components balance in 3D $\qquad$ These balance in 2D

Toroidal derivative of pressure profile at 4 locations

Low $\kappa = 18\eta_0$
$\delta p \sim \pm 0.0020$

High $\kappa = 144\eta_0$
$\delta p \sim \pm 0.0009$



Pressure profile develops a strong (1,1) component. It is of a similar form but about twice as large for the low κ case as for the high κ case

(5) The toroidal magnetic field is perturbed by the perturbed pressure

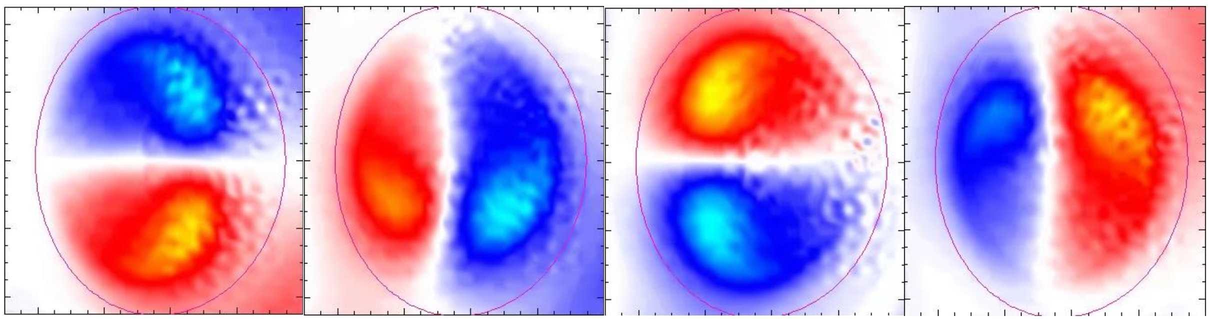
$$\delta\left(p+\frac{1}{2}B_T^2\right)=\delta p+\frac{1}{R^2}F\ \delta F\sim 0 \quad\Rightarrow\quad \delta F\sim-\delta p\frac{R^2}{F}$$

$$\mathbf{B}=\nabla\psi\times\nabla\varphi-\nabla_\perp\frac{\partial f}{\partial\varphi}+F\nabla\varphi$$

$$F=F_0+R^2\nabla_\perp^2 f$$

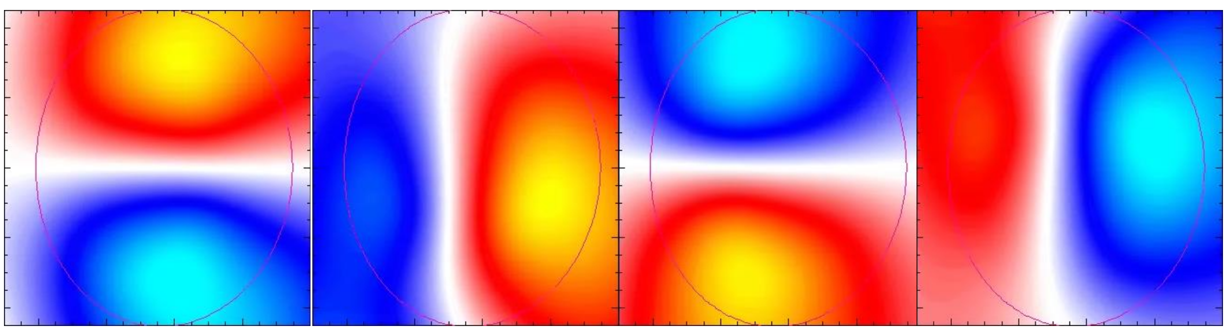
φ = 0° | φ = 90° | φ = 180° | φ = 270°

$\dfrac{\partial F}{\partial\varphi}$



Perturbed pressure causes a (1,1) perturbation in the toroidal field.

$\dfrac{\partial f}{\partial\varphi}$



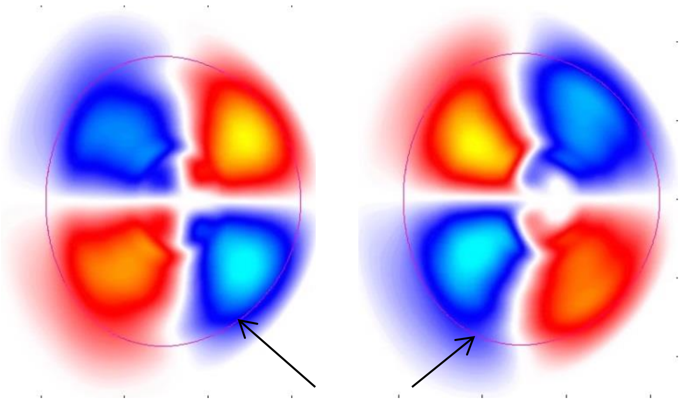
Because div **B**=0 this causes a (1,1) poloidal field component

The poloidal flux Ψ is also perturbed.

$$\mathbf{B} = \nabla\psi \times \nabla\varphi - \nabla_{\perp}\frac{\partial f}{\partial\varphi} + F\nabla\varphi$$

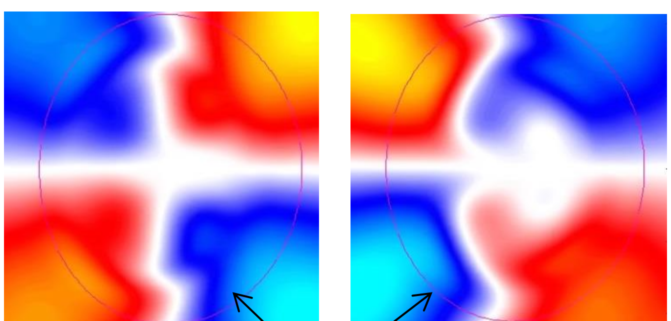
$$F = F_0 + R^2\nabla_{\perp}^2 f$$

0 °   180°



q = 2

Close up



q = 1

Poloidal flux ψ is also perturbed:

- It is dominantly (m,n) = (2,1) due to the (2,1) island
- However, it also has a (1,1) component that plays an essential role.
- The (1,1) component combines with the (1,1) velocity to produce a (0,0) voltage.

$$\boxed{\frac{\partial\psi}{\partial t} = \underbrace{\left[-\mathbf{V}\bullet\nabla\psi - \frac{\partial\Phi}{\partial\varphi}\right]}_{R\hat{\varphi}\bullet\left[\mathbf{V}\times\mathbf{B} - \nabla\Phi\right]} + \eta\Delta^*\psi = V_L/2\pi} \qquad (*)$$

$$(*)_{1,1} \Rightarrow \quad \eta_{0,0}\Delta^*\psi_{1,1} = \left[\mathbf{V}\bullet\nabla\psi + \frac{\partial\Phi}{\partial\varphi}\right]_{1,1} - \eta_{1,1}\Delta^*\psi_{0,0}$$

$$(*)_{n=0} \Rightarrow \quad \eta_{n=0}\Delta^*\psi_{n=0} = \underbrace{\left[\mathbf{V}_{1,1}\bullet\nabla\psi_{1,1}\right]_{n=0}}_{-V_0} + V_L/2\pi$$

Effective toroidal “loop voltage”

(6) Terms in parallel Ohm's law

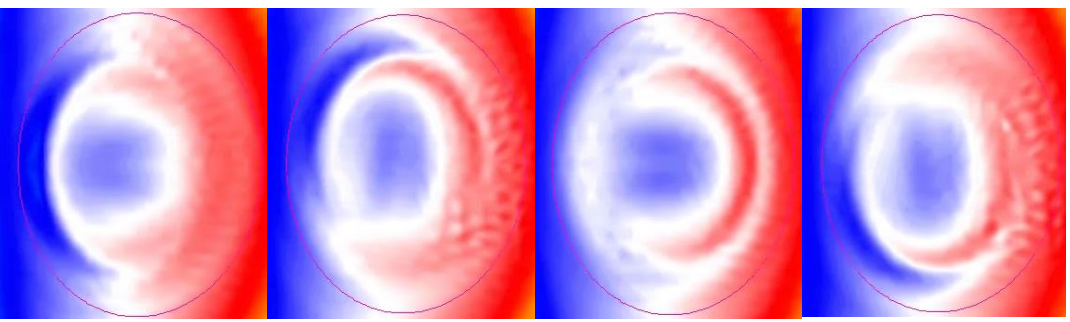
$$\mathbf{B}\bullet\left[\nabla\Phi - \mathbf{V}\times\mathbf{B} + \eta\mathbf{J} = \frac{V_L}{2\pi}\nabla\varphi\right]$$

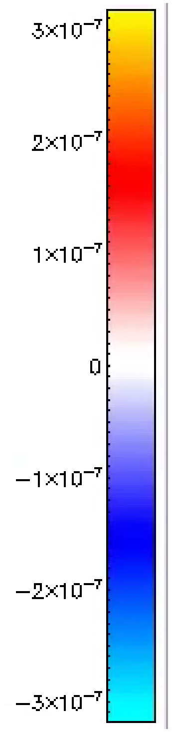
Similarly, $\mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1}$ term leads to an effective voltage along the field in center

$$\Rightarrow \qquad \eta\mathbf{J}\bullet\mathbf{B} = -\mathbf{B}\bullet\nabla\Phi + \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

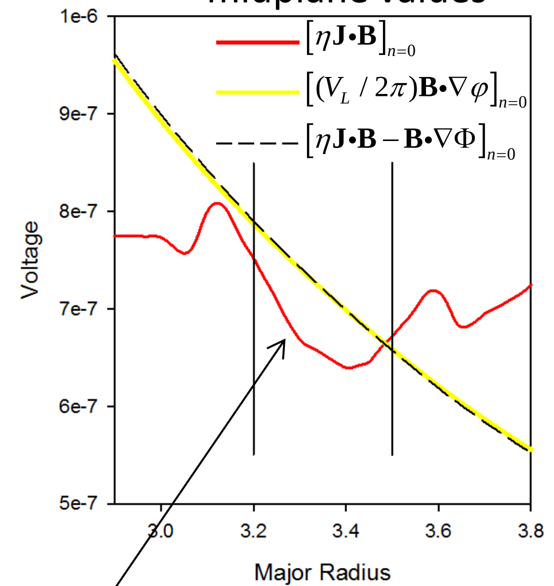
$$-\mathbf{B}\bullet\nabla\Phi$$

φ = 000 ° φ = 090 ° φ = 180 ° φ = 270 °





Toroidally averaged midplane values



White is zero, blue is negative. Note that in center region is negative at all toroidal locations indicating a (0,0) component generated non-linearly

$$[\mathbf{B}\bullet\nabla\Phi]_{n=0} \sim 7\times10^{-8}$$

Region in center where

$$[\eta\mathbf{J}\bullet\mathbf{B} \quad < \quad (V_L/2\pi)\mathbf{B}\bullet\nabla\varphi]_{n=0}$$

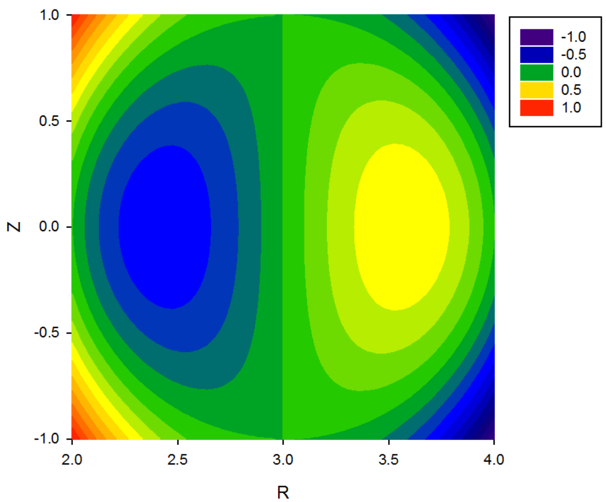
allowing q_0 to stay at 1.

(6b) How can $\mathbf{B}\bullet\nabla\Phi$ have a non-zero toroidal average in a volume?

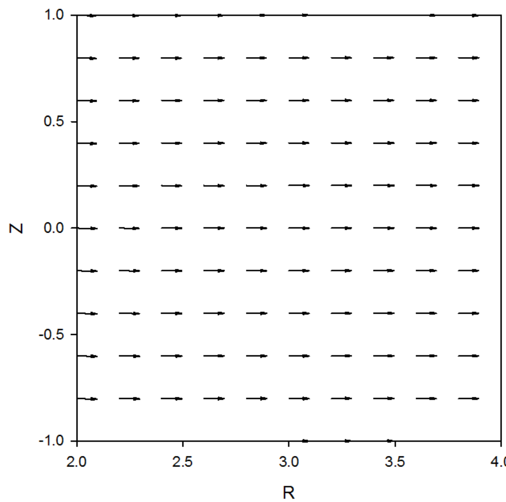
$$\mathbf{B}\bullet\nabla\Phi = \nabla\bullet\left(\mathbf{B}\Phi\right)\overset{?}{=}0$$

Now suppose $\tilde{\mathbf{B}}$ is a small (1,1) field component resonant with Φ

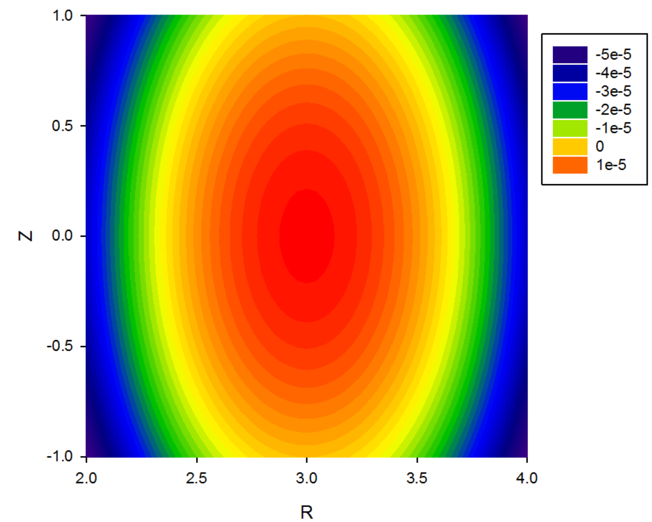
Potential Φ



Perturbed field $\tilde{\mathbf{B}}$



$\left\langle \tilde{\mathbf{B}}\bullet\nabla\Phi \right\rangle$



$$\Phi = \Phi_0 r\left(1-r^2\right)\cos\left(\theta-\varphi\right)$$

$$\tilde{\mathbf{B}} = \varepsilon\cos(\theta-\varphi)\hat{r} - \varepsilon\sin(\theta-\varphi)\hat{\theta}$$

$$\left\langle \mathbf{B}\bullet\nabla\Phi \right\rangle = \varepsilon\Phi_0 \left\langle \begin{array}{l} (1-3r^2)\cos^2\left(\theta-\varphi\right) \\ +\left(1-r^2\right)\sin^2\left(\theta-\varphi\right) \end{array} \right\rangle$$

positive definate for r sufficiently small

Resonant field perturbation produces an effective voltage along perturbed field!

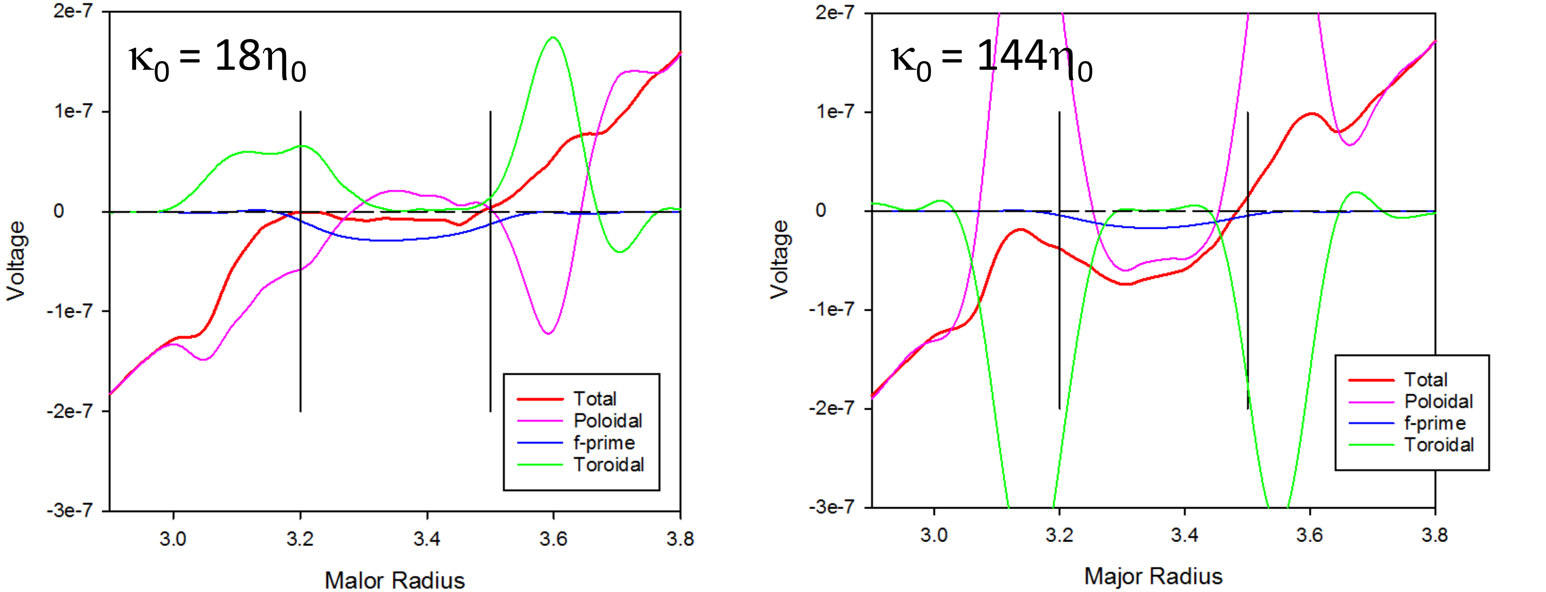
(6c) Magnetic field components that contribute to [B •∇Φ]$_{n=0}$

$$\mathbf{B}\bullet\nabla\Phi = \nabla\psi\times\nabla\varphi\bullet\nabla\Phi - \nabla_{\perp}\frac{\partial f}{\partial\varphi}\bullet\nabla\Phi + F\nabla\varphi\bullet\nabla\Phi$$

$$\mathbf{B} = \nabla\psi\times\nabla\varphi - \nabla_{\perp}\frac{\partial f}{\partial\varphi} + F\nabla\varphi$$

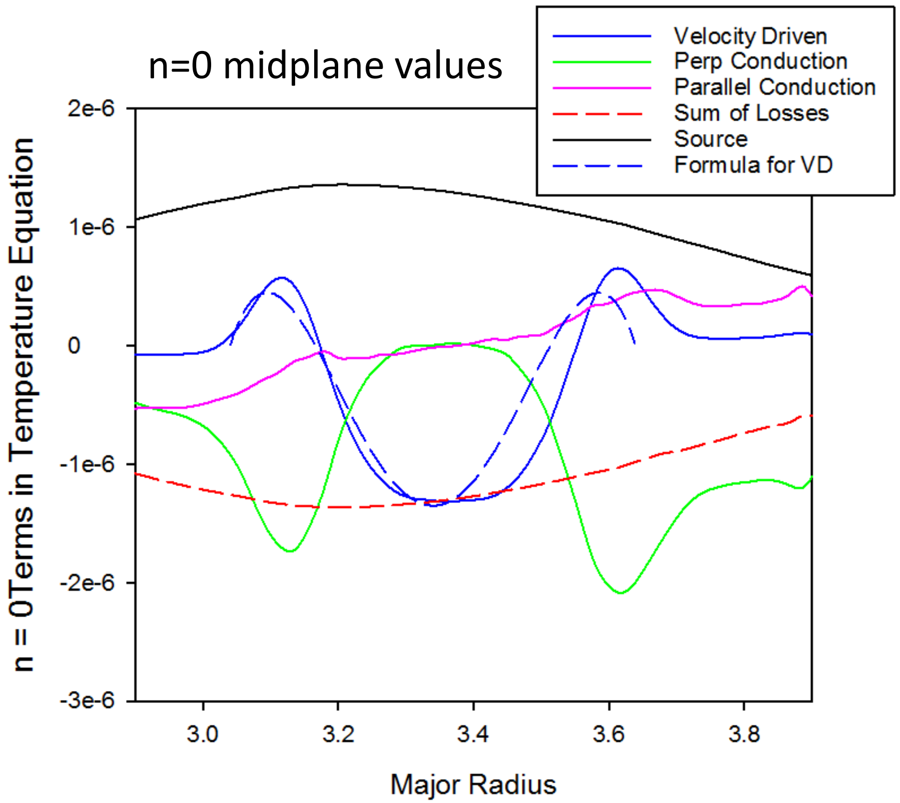
$$F = F_0 + R^2\nabla_{\perp}^2 f$$

(color coded) midplane values of toroidally averaged (n=0) voltage arising from the different components of the perturbed magnetic field



The part due to the resonant *poloidal field* can adjust to promote current peaking at low κ and prevent peaking at high κ, keeping q_0=1 in all cases.

(7) Perturbed temperature and velocity flatten n=0 temperature



The partial flattening of the temperature profile due to the axisymmetric (n=0) component of $n\,\mathbf{V}_{1,1}\bullet\nabla T_{1,1}$ agrees with analysis

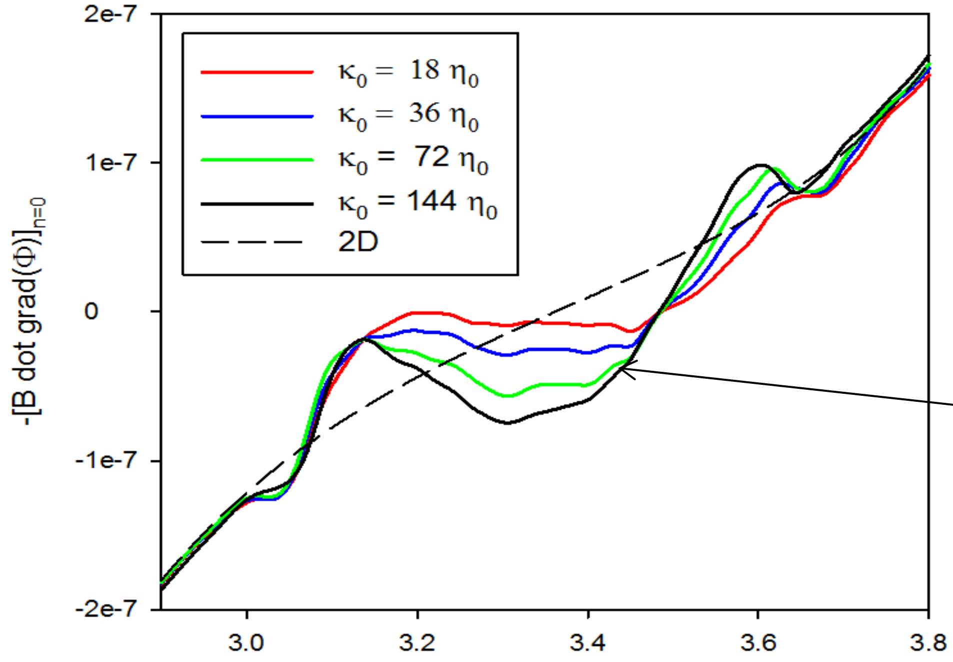
$$n\mathbf{V}\bullet\nabla T+\frac{2}{3}nT\nabla\bullet\mathbf{V}+\frac{2}{3}\nabla\bullet\mathbf{q}_{\perp}+\frac{2}{3}\nabla\bullet\mathbf{q}_{\parallel}=\eta J^{2}+S_{e}$$

$$n_0\mathbf{V}^{(1)}\bullet\nabla T^{(1)}=\frac{2}{3}S\left[1-\left(r/r_1\right)^2\right]\left[1-3\left(r/r_1\right)^2\right]$$

(8) Parallel Ohm's law is modified in two ways that broaden current

$$\left(\eta \mathbf{J}\bullet\mathbf{B}\right)_{n=0} = -\left(\mathbf{B}\bullet\nabla\Phi\right)_{n=0} + \frac{V_L}{2\pi}\left(\mathbf{B}\bullet\nabla\varphi\right)_{n=0}$$

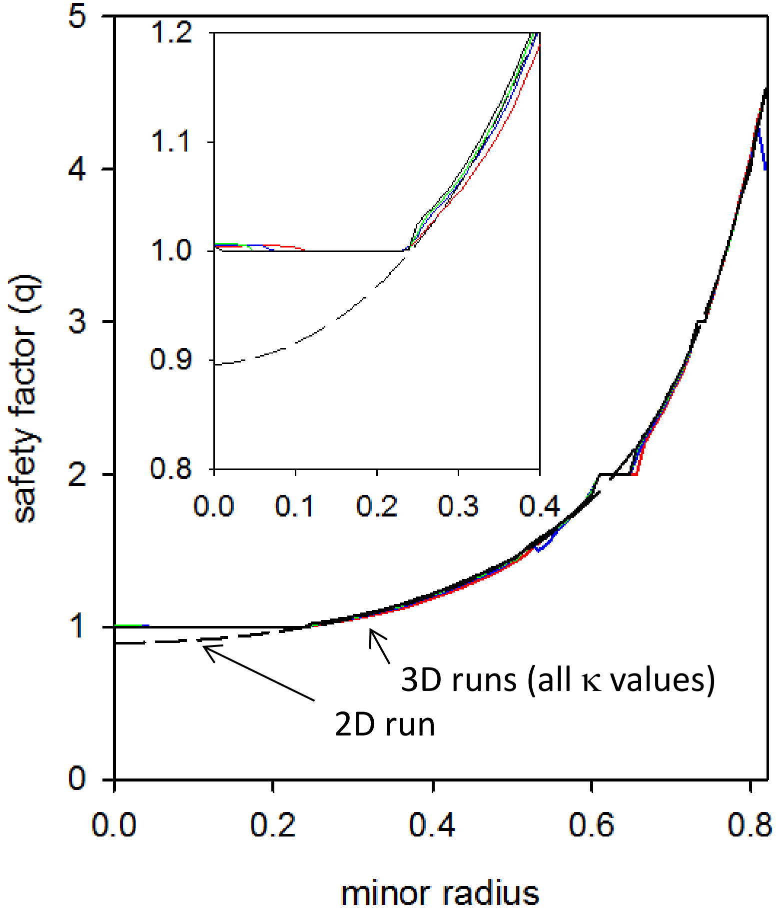
$$-\left(\mathbf{B}\bullet\nabla\Phi\right)_{n=0}$$



- Similarly, nonlinear processes from $\mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1}$ produce an effective n=0 toroidal voltage in the center (as needed) to keep q=1 in central volume.
- Dashed line is 2D result

(9) No longer constrained to $\eta\langle \mathbf{J}\bullet\mathbf{B}\rangle = \mathrm{const}$, central regions in all 3D runs approach minimum energy Taylor State with q=1

Final q-profile in 2D and 3D runs



The nonlinear drive that keeps the current from peaking gets stronger as q→1 from above

This feedback mechanism results in an ultra-flat q-profile in center with $q_0 = 1 + \varepsilon$ (where $\varepsilon \ll 1$)

Remaining questions:

- For which range of parameters does this occur?
- Effect of sheared rotation?
- Effect of 2-fluid terms?
- Relation to experiments in ASDEX-U and DIII-D?

Extra Viewgraphs

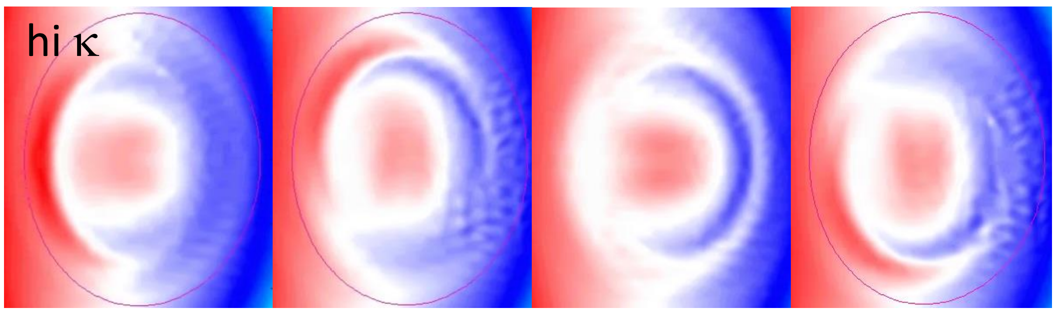
(6a) Effective enhanced toroidal voltage in central region

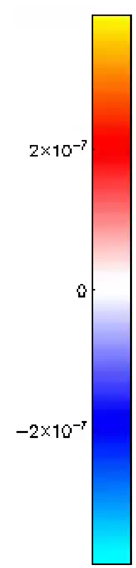
Because $\mathbf{V} \times \mathbf{B} \neq \nabla\Phi$ exactly, there is a residual $\mathbf{V}_{1,1} \times \mathbf{B}_{1,1}$ part that leads to a n=0 voltage in center

$$\hat{\varphi} \cdot \left[\nabla\Phi - \mathbf{V} \times \mathbf{B} + \eta \mathbf{J} = \frac{V_L}{2\pi} \nabla \varphi \right]$$

$$\hat{\varphi} \cdot \left[\nabla\Phi - \mathbf{V} \times \mathbf{B} \right]$$

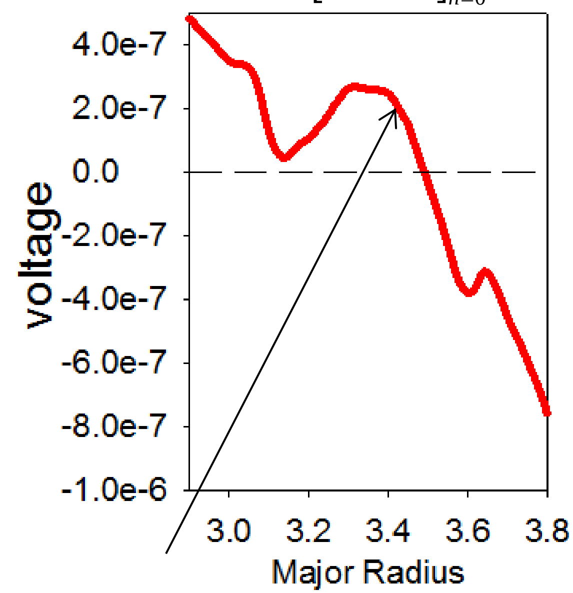
φ = 0° φ = 90° φ = 180° φ = 270°





Toroidally averaged midplane value

$$\hat{\varphi} \cdot \left[-\mathbf{V} \times \mathbf{B} \right]_{n=0}$$



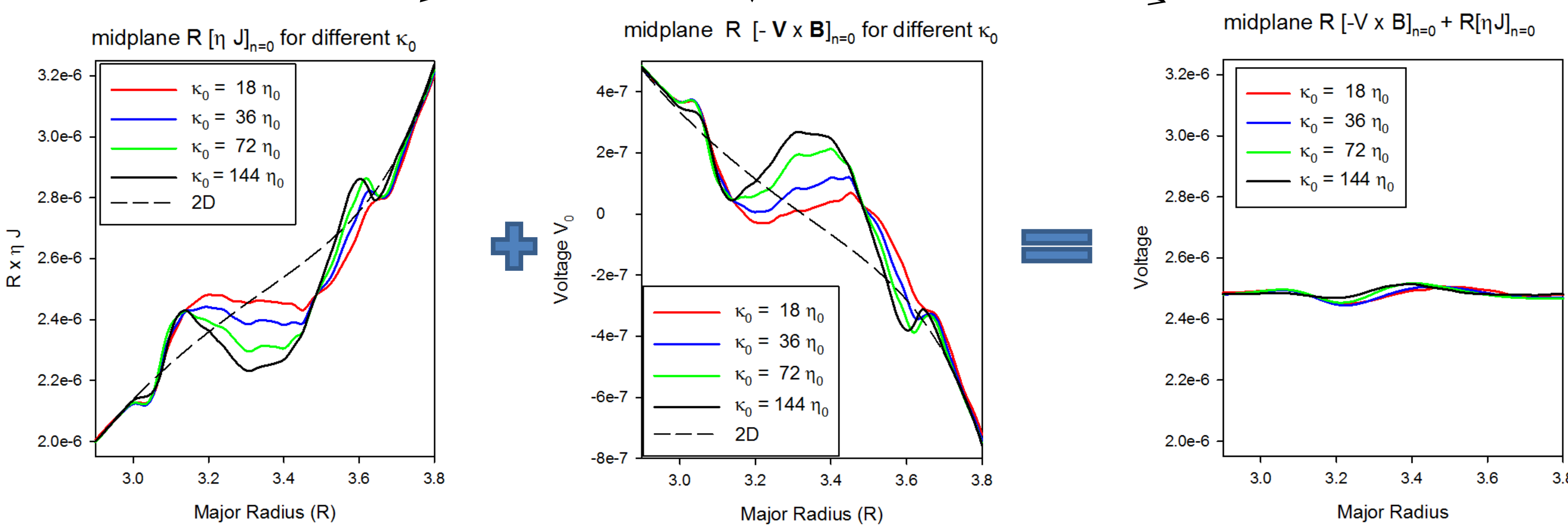
White is zero. Red is positive. Blue is negative.
Note the red in center at all toroidal locations.

A non-zero (positive) n=0 piece in center allows $\eta \mathbf{J}$ to be smaller there.

➔ **Counters current peaking, keeping q_0=1**

(8a) Toroidal Ohm's law is modified in two ways that broaden current

$$\eta_{n=0}\Delta^{*}\psi_{n=0}+\left[-\mathbf{V}_{1,1}\bullet\nabla\psi_{1,1}\right]_{n=0}=V_L/2\pi$$



Compared to the 2D case (dashed line), we see that all 3D cases have an effective n=0 toroidal voltage coming from the nonlinear term $\mathbf{V}_{1,1}\bullet\nabla\psi_{1,1}$. This adjusts the current profile to keep q=1 in central volume.

Basic Physics of self-organized stationary discharge with q_0=1

(1) Central ultra-low shear (ULS) region with q=1 (and no flux surfaces)

(2) Drives interchange instability producing $\mathbf{V}_{1,1}$

(3) Produces electric potential

$$\nabla\Phi_{1,1} \simeq \mathbf{V}_{1,1} \times \mathbf{B}$$

(4) Also perturbs temperature:

$$\frac{2}{3}\nabla\bullet\left[n\kappa_{\perp}\nabla T_{1,1}\right] \simeq n\mathbf{V}_{1,1}\bullet\nabla T$$

(5) Also perturbs magnetic field:

$$\eta\mathbf{J}_{1,1} = \mathbf{V}_{1,1} \times \mathbf{B} - \eta_{1,1}\mathbf{J}$$

$$\nabla \times \mathbf{B}_{1,1} = \mu_0\mathbf{J}_{1,1}$$

(6) 3+5 produce effective enhanced loop voltage in central region:

$$V_{n=0} = \mathbf{B}_{1,1}\bullet\nabla\Phi_{1,1} + \cdots$$

(7) 2+4 produce velocity field that tends to flatten temperature profile

$$\delta T_{n=0} \simeq -\mathbf{V}_{1,1}\bullet\nabla T_{1,1}$$

(8)Axisymmetric (n=0) parallel Ohm's law is modified in two ways that broaden current

$$\eta\mathbf{J}\bullet\mathbf{B}_{n=0} + V_{n=0} = \frac{V_L}{2\pi}\mathbf{B}\bullet\nabla\varphi$$

resistivity profile is flattened

positive in center, flattening current

(9) No longer constrained to $\eta\langle\mathbf{J}\bullet\mathbf{B}\rangle = \text{const.}$ central region approaches minimum energy Taylor State $\mathbf{J} = \lambda\mathbf{B}$ ➔ ULS

- These perturbations will also produce other islands via mode coupling, but they do not play a central role