NIMROD MODELING OF SAWTOOTH MODES

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Sawtooth basics

Normal sawtooth mode

-Plasma has q(0) > 1, peaked current density on axis

-Ohmic heating introduced (e.g. 80 keV neutral beam)

-Plasma near axis preferentially heated (higher J) ➔ decreased core resistivity (~ $T^{-3/2}$) ➔ further current peaking, decreased q(0)

-(1,1) internal kink instability triggered when q(0) < 1, which rearranges magnetic flux and flattens temperature profile

-Cycle repeats

DIII-D shot #96043

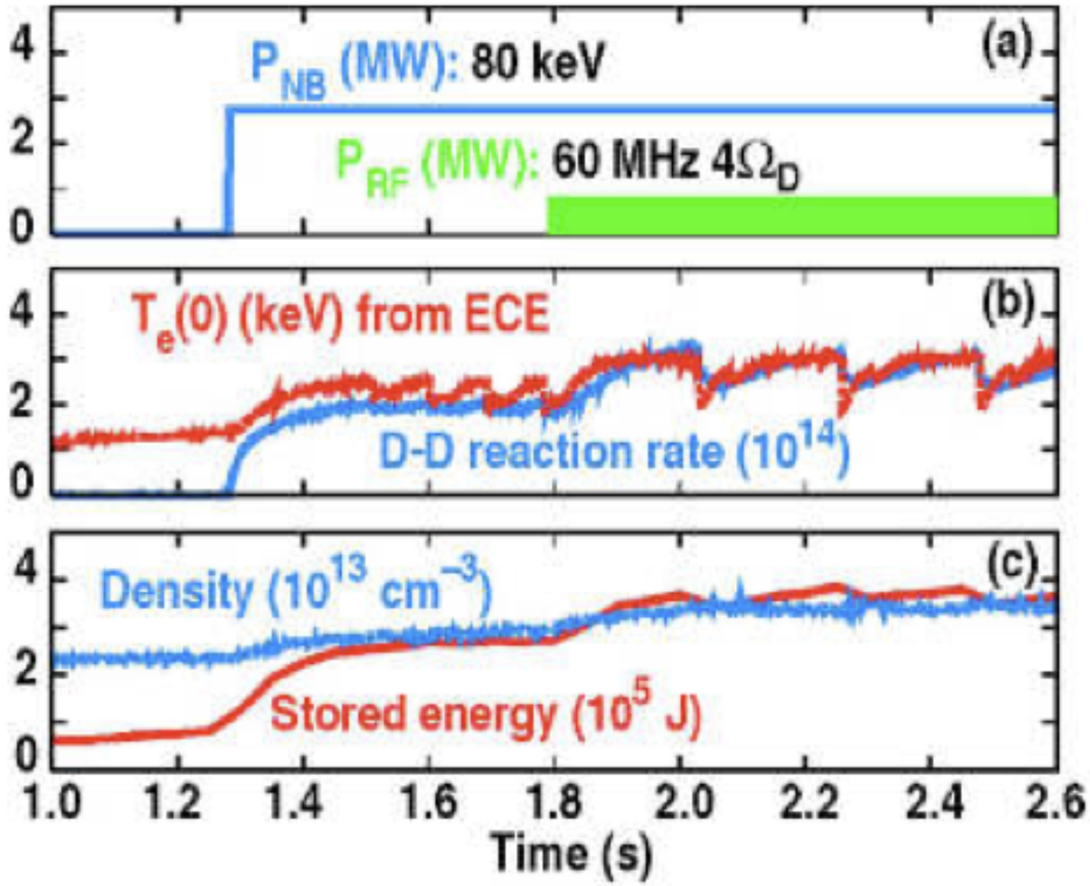


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Giant sawtooth basics

Giant sawtooth mode

-Energetic particle population (e.g. induced by RF heating, or fusion reactions) alters stability of internal kink mode

-Higher temperatures and stored energies achievable even with q(0) < 1

-Terminates like a normal sawtooth crash, but with larger amplitude

-Potential trigger for ELMs, NTMs, large heat transfer to vessel wall

"slow leak" description
"soft β limit"

DIII-D shot #96043

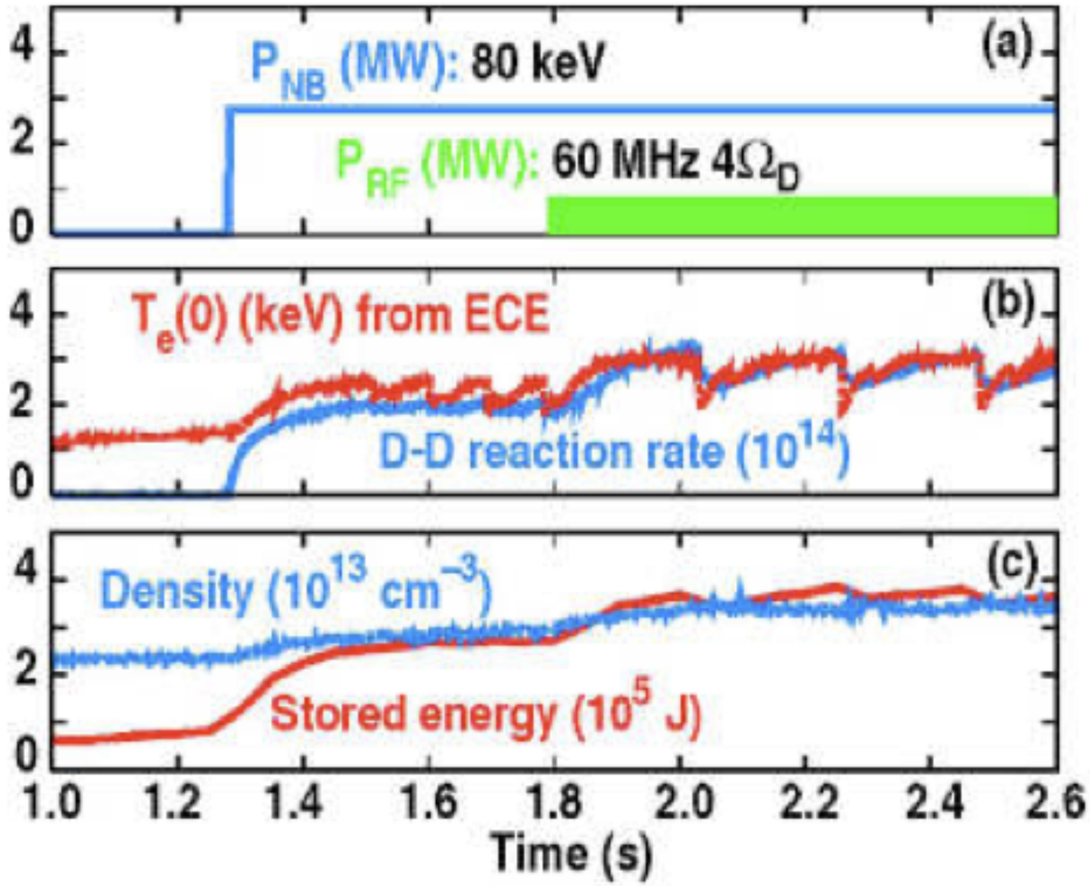


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Hot-particle sawtooth stabilization in NIMROD: computational approaches

Momentum equation has an extra term:

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho(\vec{V} \cdot \nabla)\vec{V} = \vec{J} \times \vec{B} - \nabla \cdot \overleftrightarrow{P} - \nabla \cdot \overleftrightarrow{P}_{hot}$$

Continuum kinetic:

$\overleftrightarrow{P}_{hot}$ from moments of continuum solution to drift-kinetic equation (E. Held)

Kinetic PIC:

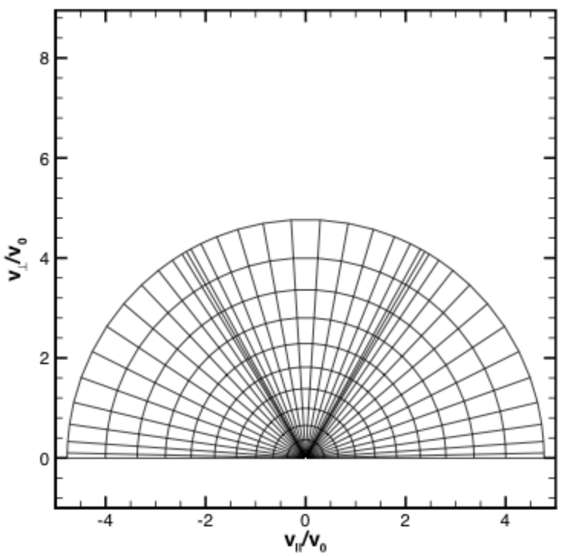
$\overleftrightarrow{P}_{hot}$ from moments of PIC distribution, evolving according to drift-kinetic equation (C. Kim, D. Schnack, T. Jenkins)

$$\overleftrightarrow{P} \rightarrow (1-\beta_{frac})\overleftrightarrow{P}$$

$\overleftrightarrow{P}_{hot} = \beta_{frac}\overleftrightarrow{P}$ comes from energetic particles, via T_i (energetic particles have low density and high temperature; $n_{hot} << n$ but $P_{hot} \sim P$)

Continuum kinetic formulation

Write the drift-kinetic equation as



$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \left[\nabla f - \frac{1-\xi^2}{2\xi} \nabla \ln B \frac{\partial f}{\partial \xi} - \frac{s}{2} \nabla \ln T_0 \frac{\partial f}{\partial s}\right] - C(f) +$$

$$\frac{1-\xi^2}{2\xi}\left[-\xi^2 \frac{\mathbf{b}}{B} \cdot \frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + \xi^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial \xi} +$$

$$\frac{s}{2}\left[-(1-\xi^2) \frac{\mathbf{b}}{B} \cdot \frac{\partial \mathbf{B}}{\partial t} + \frac{q}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + (1+\xi^2) \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial s} = 0$$

formulated in pitch angle and normalized speed coordinates

$$\xi = v_{||}/v \qquad s = v/v_0$$

and with drift velocity

$$\mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{T_0 s^2}{q B^2}\left[\left(1+\xi^2\right) \mathbf{b} \times \nabla B + 2\xi^2 \mu_0 \mathbf{J}_\perp + (1-\xi^2)\mu_0 \mathbf{J}_{||}\right] + \frac{m v_0 s \xi}{q B^2} \mathbf{b} \times \frac{\partial \mathbf{B}}{\partial t}$$

Discretize velocity space, use NIMROD's finite-element machinery to evolve hot-particle distribution on the velocity grid

Moments of hot-particle distribution form the hot-particle pressure tensor

Kinetic PIC formulation

Drift-kinetic equation for hot ions:

$$\frac{\partial f_\alpha}{\partial t} + \vec{u}\cdot\vec{\nabla} f_\alpha + a\frac{\partial f_\alpha}{\partial v_\parallel} = C(f_\alpha)$$

Change of variables: evolve parallel particle motion instead of pitch angle and speed

$$\vec{u} = v_\parallel \hat{b} + \frac{\vec{E}\times\vec{B}}{B^2} + \frac{v_\parallel}{\Omega_\alpha}\hat{b}\times\frac{\partial \hat{b}}{\partial t} + \frac{\mu}{q_\alpha}\left(\frac{\hat{b}\times\vec{\nabla}B}{B} + \hat{b}\hat{b}\cdot\left(\vec{\nabla}\times\hat{b}\right)\right) + \frac{v_\parallel^2}{\Omega_\alpha}\left(\vec{\nabla}\times\hat{b} - \hat{b}\hat{b}\cdot\left(\vec{\nabla}\times\hat{b}\right)\right)$$

$$a = \frac{q_\alpha}{m_\alpha}\hat{b}\cdot\vec{E} - \frac{\mu\hat{b}\cdot\vec{\nabla}B}{m_\alpha}$$

collisions determine equilibrium hot-particle distribution function (slowing-down distribution)

present implementation

Standard delta-f approach:

$$f_\alpha = f_{\alpha 0} + \delta f_\alpha$$
$$\vec{u} = \vec{u}_0 + \delta\vec{u}$$
$$a = a_0 + \delta a$$

$$\frac{d}{dt}\left(\frac{\delta f_\alpha}{f_{\alpha 0}}\right) = -\frac{\delta\vec{u}\cdot\vec{\nabla}f_{\alpha 0}}{f_{\alpha 0}} - \frac{\delta a}{f_{\alpha 0}}\frac{\partial f_{\alpha 0}}{\partial v_\parallel}$$

weight equation

Moments of hot-particle PIC distribution form the hot-particle pressure tensor

Hot-particle distribution function, as presently implemented in NIMROD

Current form of hot-particle pressure tensor contribution: slowing-down distribution

$$f_\alpha = \frac{P_0 \exp(P_\zeta / \psi_n)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$$

P_ζ canonical toroidal momentum

ψ_n normalized poloidal flux

ε_c critical slowing-down energy

$$P_\zeta = \frac{v_{\|} R B_\zeta}{\Omega} - \psi_{pol}$$

a function of equilibrium field quantities

particle orbits – linear response to equilibrium fields, with particle weight carrying all amplitude effects

NIMROD reads the DIII-D equilibrium files

DIII-D shot #96043

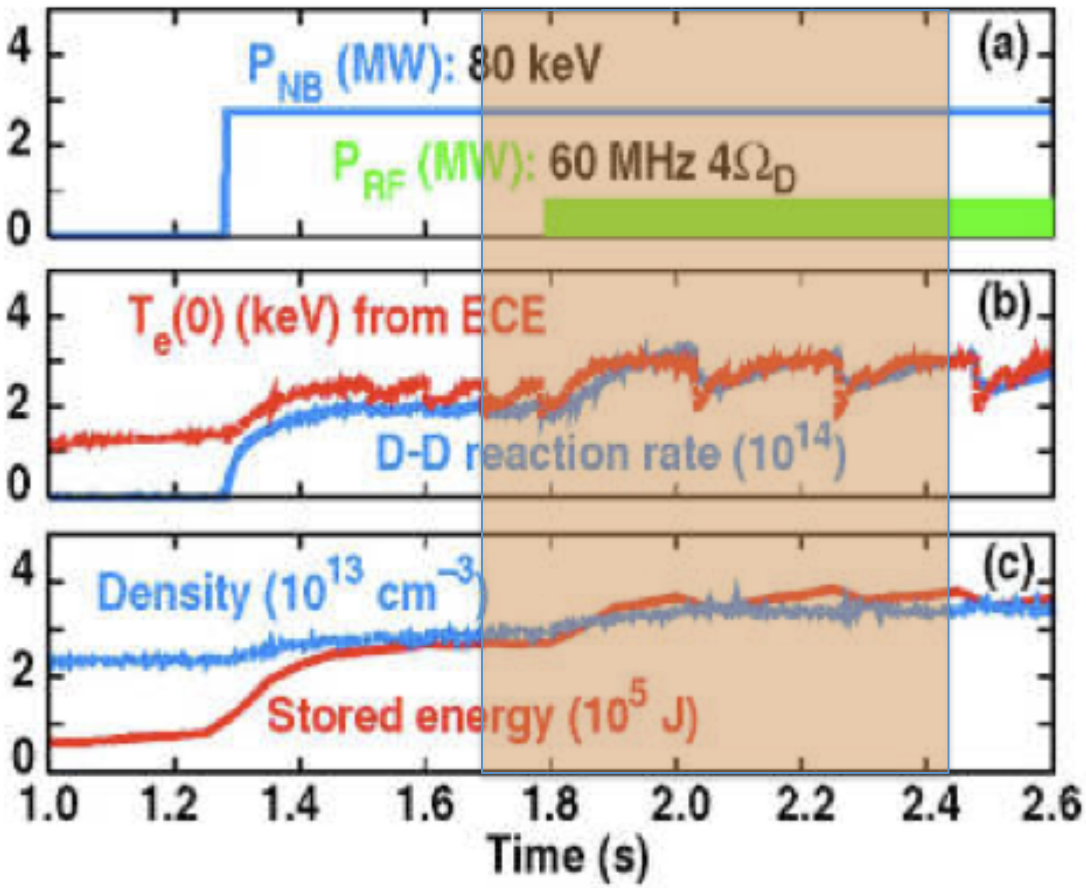


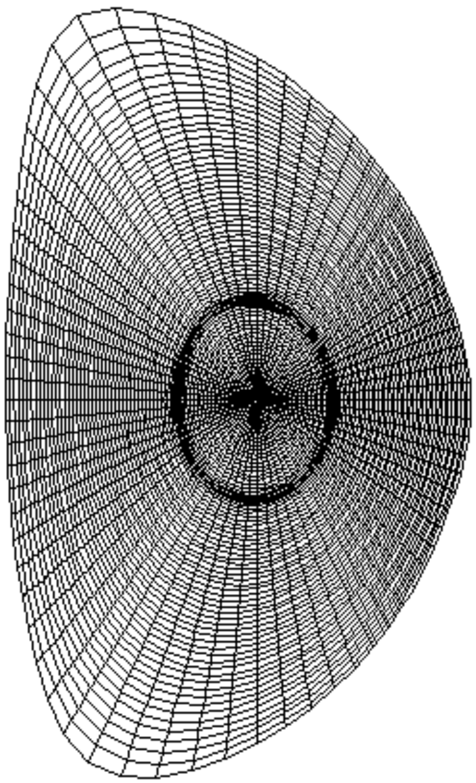
Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

- 40 EFIT files (provided by Alan Turnbull) from within the orange box, covering both conventional and giant sawtooth cycles.

- Issues – preserve total current from EFIT, get correct orientation of J.B (important for particle drifts)

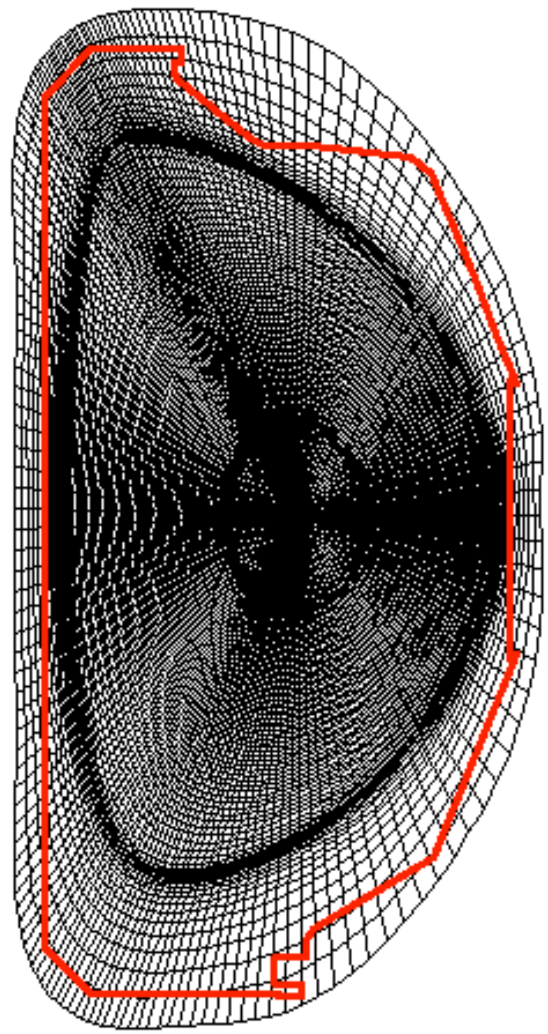
Improvements to equilibrium GS solutions, and sanity checks

growth rate = $3.13 \times 10^4\ s^{-1}$



Conducting wall is stabilizing

growth rate = $3.28 \times 10^4\ s^{-1}$



•Improvements to Grad-Shafranov solves near separatrix

Hot particles can also modify the equilibrium

Conventional Grad-Shafranov:

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}_p$$

$$\vec{J}_p\times\vec{B}=\vec{\nabla}p_p$$

$$\vec{\nabla}\cdot\vec{B}=0$$

+axisymmetry: yield

$$\Delta^*\psi=-\mu_0 R^2 p_p' - FF'$$

$$F=F(\psi)=RB_\theta(R,Z)$$

$$p_p=p_p(\psi)$$

$$\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\theta+RB_\theta(R,Z)\vec{\nabla}\theta$$

Modified Grad-Shafranov [see E. V. Belova et al., *Phys. Plasmas* **10**, 3240 (2003)]:

$$\vec{\nabla}\times\vec{B}=\mu_0(\vec{J}_p+\vec{J}_h)$$

$$\vec{J}_p\times\vec{B}=\vec{\nabla}p_p$$

$$\vec{J}_h\times\vec{B}=\vec{\nabla}p_h$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\cdot\vec{J}_h=0$$

+axisymmetry: yield

$$\Delta^*\psi=-\mu_0 R^2 p_p' - HH' - \mu_0 GH' + \mu_0 RJ_{h\theta}(R,Z)$$

$$H=H(\psi)=RB_\theta(R,Z)-G(R,Z)$$

$$p_p=p_p(\psi)$$

$$\vec{J}_h=\vec{\nabla}G\times\vec{\nabla}\theta+R\vec{J}_h(R,Z)\vec{\nabla}\theta$$

$$\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\theta+RB_\theta(R,Z)\vec{\nabla}\theta$$

We are exploring whether a more general equilibrium solve is needed.

Hot-particle stabilization of sawtooth modes at high Lundquist number

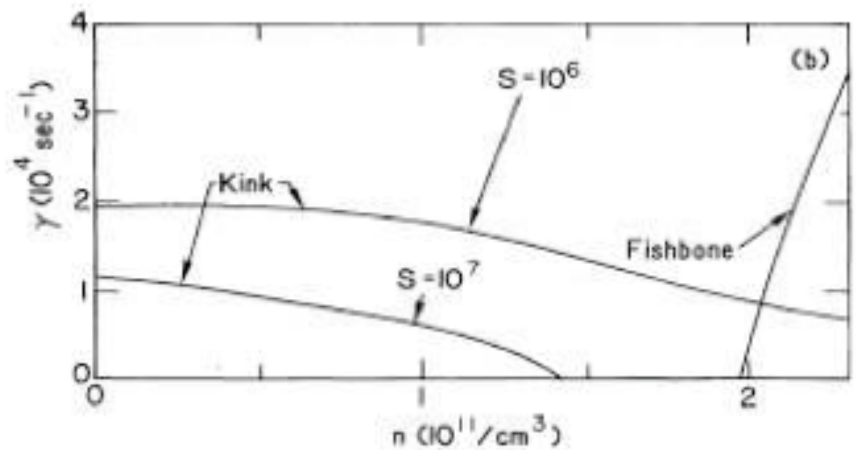
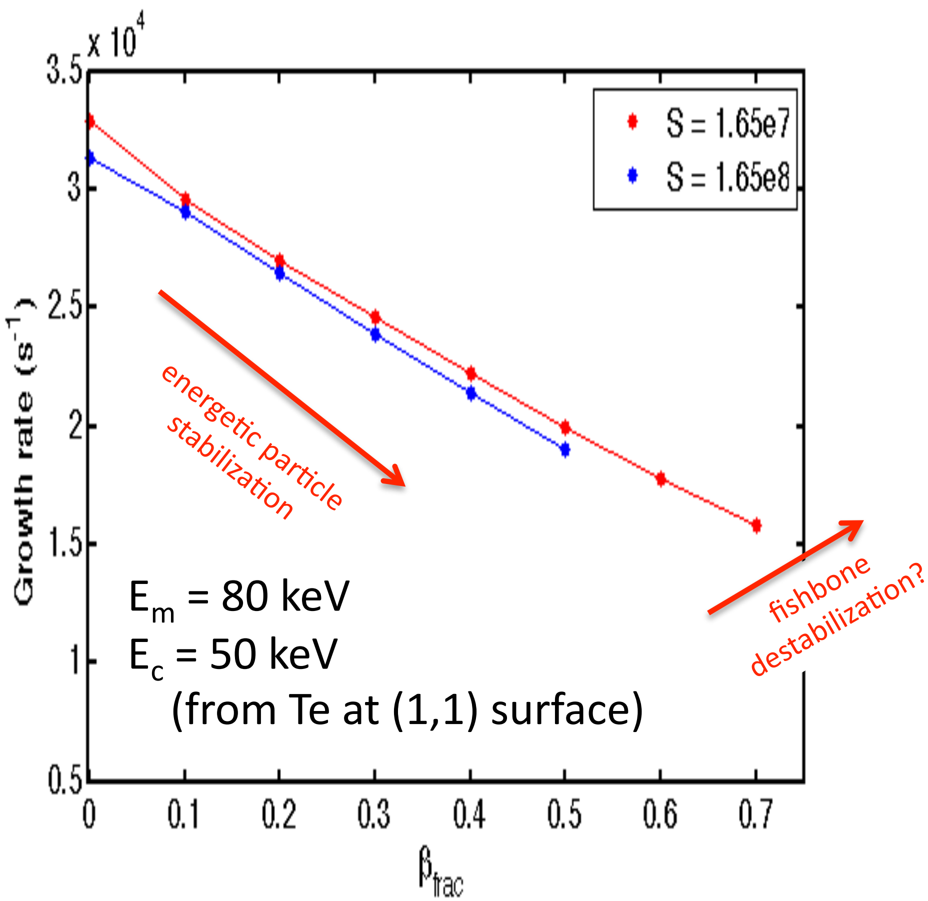


Figure from R. B. White et al., PRL **62**, 539 (1989).

•For a fixed hot-particle density, higher Lundquist number is stabilizing
•Fishbone destabilization is expected at higher densities

Recent results do not show fishbone destabilization (still checking these)



E_m = peak energy of slowing-down distribution function

- Energetic particle kink stabilization
- Stabilization (3rd adiabatic invariant – toroidal precession of energetic trapped particles modifies MHD) requires

  $\omega_{pd}/\gamma_R >> 1$

  (growth slow compared to precession) and thus high Lundquist number S – we also see this stabilization effect

Can we run at high enough energies and Lundquist numbers to achieve full stabilization? (particle population in phase space)

How much does the form of the hot-particle distribution function matter?

Sawtooth stability

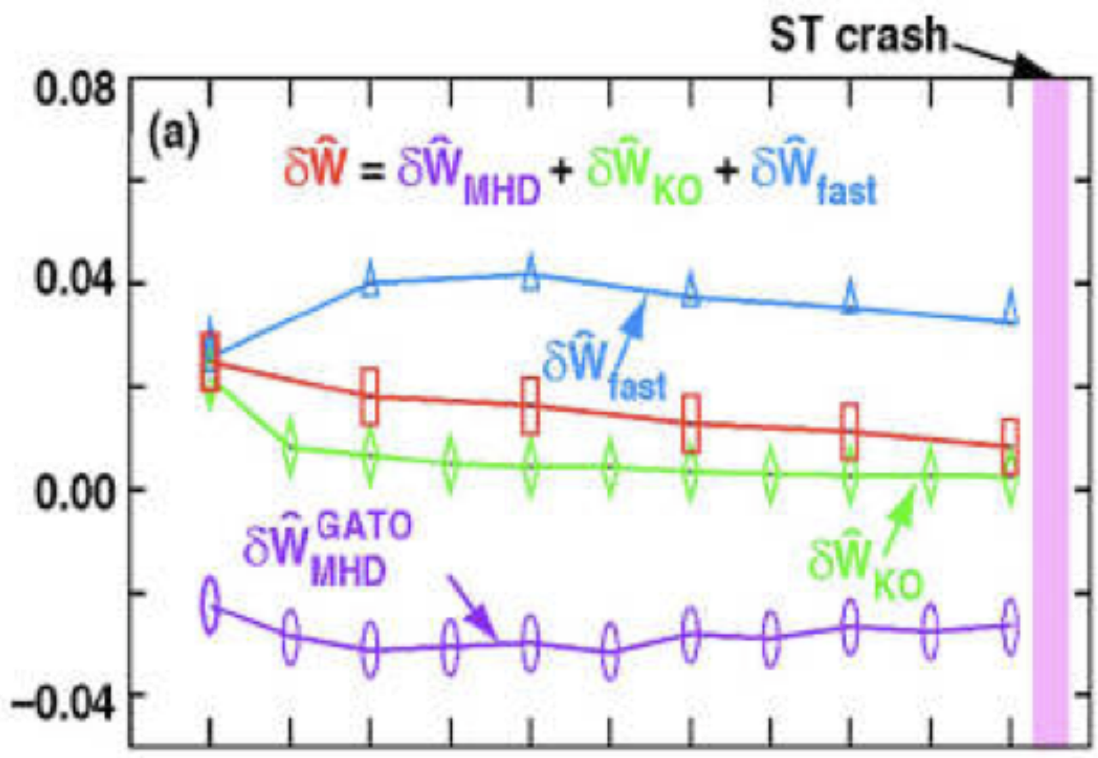


Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).

Hot-particle effects

Bulk pressure tensor effects

Ideal MHD effects

Total stability parameter

- Does ideal MHD + hot-particle kinetics explain everything?
- Role of two-fluid effects?

Plan of action going forward

- Continue exploring the extent to which present model can accurately characterize the MHD (and 2-fluid) behavior of linear sawtooth onset – improve model as needed
- Near-term goal – ensure self-consistency between PIC and continuum approaches, in collaboration with Eric.
- Examine the effect of more general hot-particle distribution functions, higher S values
- Exercise different combinations of physics components – MHD, 2-fluid, parallel closure, particles (all of them important for this work at some level)
- Code performance improvements for development milestone
- Eventual milestone – DIII-D shot 96043 modeling of hot-particle induced giant sawtooth stabilization.

Project milestones (CEMM)

	Year 4	Year 5
Sawtooth	•Apply continuum closure models for energetic and thermal ions to the Giant Sawtooth problem (Tech-X).	•Continue linear modeling of sawtooth stabilization in DIII-D shot 96043 (Tech-X). •Demonstrate nonlinear evolution of sawtooth with continuum kinetic closures and extended MHD Ohm's law (Tech-X/USU).
Model development - continuum kinetic (with Eric Held)	•Improve parallel scaling of kinetic closures (USU).	•Demonstrate applicability by applying to a 3D coupled problem (USU/Tech-X)
Model development - kinetic PIC	•Begin new particle parallelization development for NIMROD (Tech-X).	•Complete, test, and apply the new particle parallelization in NIMROD (Tech-X).