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CEMM Meeting 2016

Non-linear dynamics of compound sawteeth in tokamak core plasmas

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- Compound sawteeth in experiments
- XTOR-2F simulations of compound sawteeth
- Internal kink mode stability during the sawtooth ramp phase

- Radial displacement of hot core due to the internal kink mode

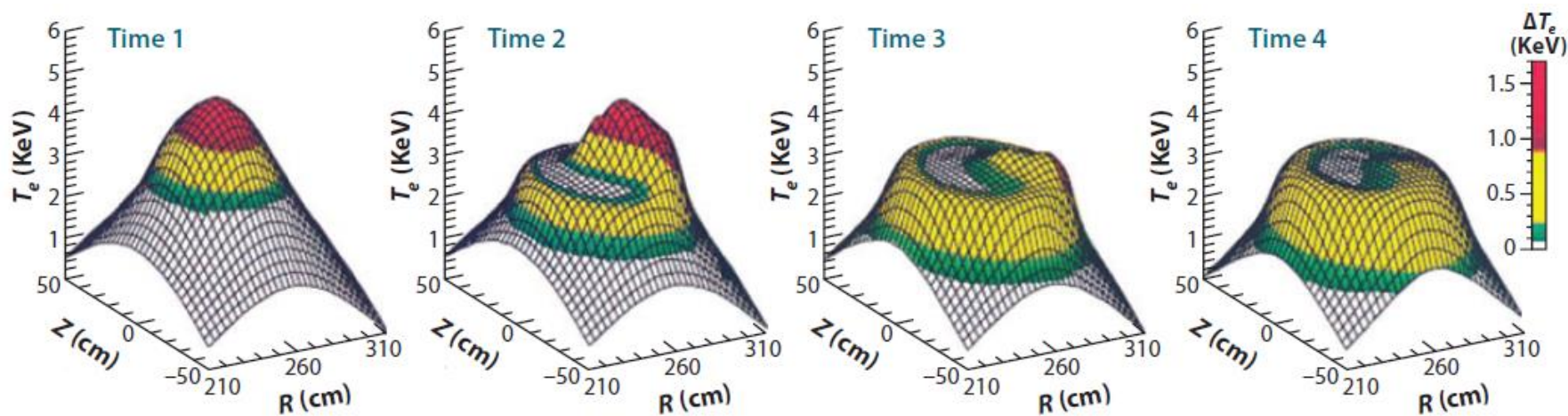
- Flattened temperature profile after sawtooth crashes

- Reconnection of magnetic surfaces near the $q = 1$ surface

B.B. Kadomtsev, *Sov. J. Plasma Phys.* **1** 389 (1976) ; A.Y. Aydemir, *Phys. Rev. Lett.* **59** 649 (1987) ;
F. L. Waelbroeck, *Phys. Fluids B* **1** 2372 (1989) ; D. Biskamp and J. F. Drake, *Phys. Rev. Lett.* **73** 971 (1994) ; etc...

- Ongoing efforts to model and understand sawtooth physics

A.Y. Aydemir, *Phys. Fluids B* **4** 3469 (1992) ; J.A. Breslau *et al.*, *Commun. Comput. Phys.* **4** 647 (2008) ;
F. Halpern *et al.*, *Phys. Plasmas* **18** 102501 (2011) ; T. Nicolas *et al.*, *Phys. Plasmas* **21** 112305 (2012) ; etc...



E. G. Zweibel and M. Yamada, *Annu. Rev. Astron. Astrophys.* **47** 291 (2009)

- Compound sawtooth : “partial crash” visible on measurements (e.g. T_e, n_e, SXR)

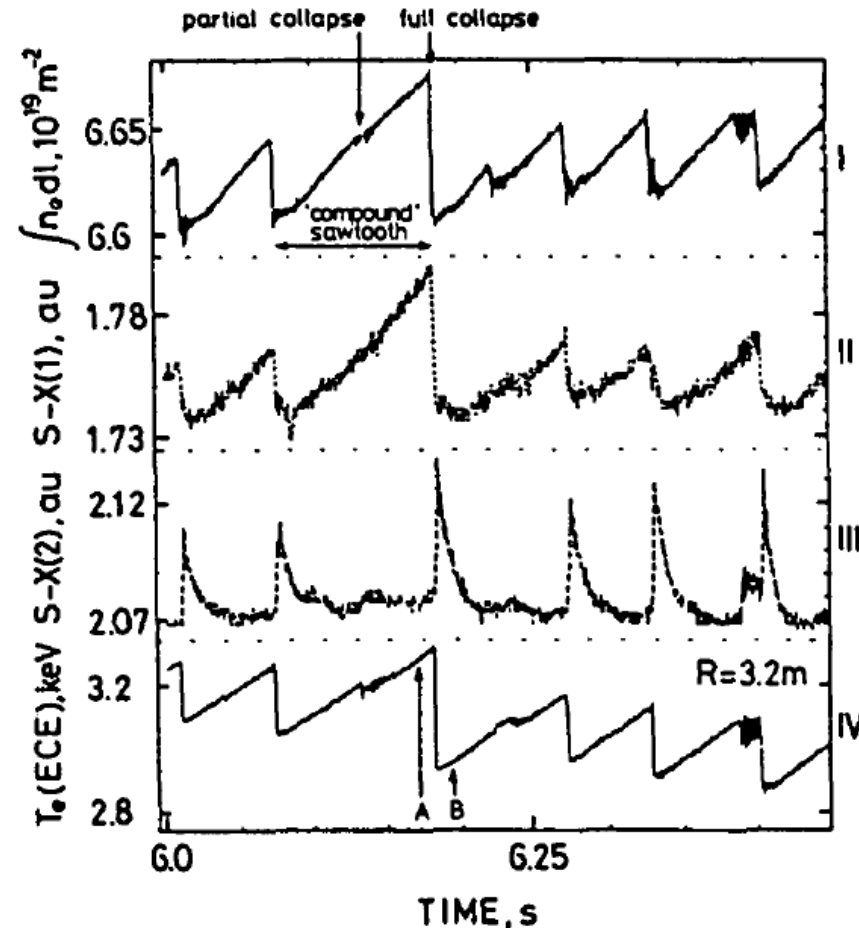
- Otherwise, “ordinary sawtooth”

- Compound sawteeth are frequently observed in experiments,

W. Pfeiffer, *Nucl. Fus.* **25** 673 (1985) ;
G. Taylor *et al.*, *Nucl. Fus.* **26** 339 (1986) ;
S.B. Kim, *Nucl. Fus.* **26** 1251 (1986) ; etc..

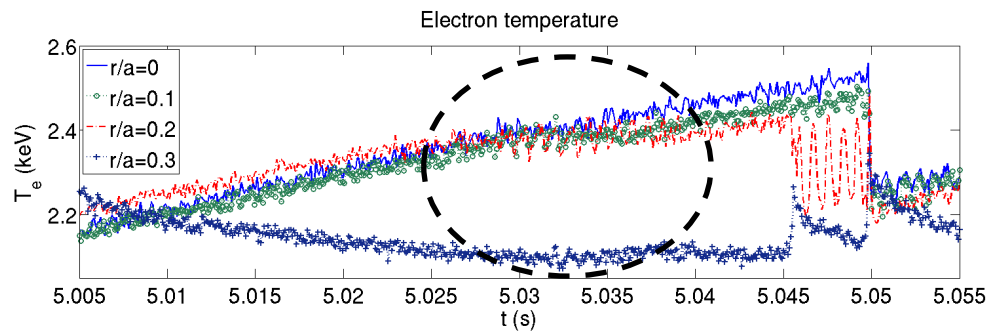
→ The dynamics of partial crashes differs from ordinary crashes

- Validation of MHD models
Effects on plasma confinement ?
Impurity accumulation ? etc..



D. J. Campbell *et al.*, *Nucl. Fus.* **26** 1085 (1986)

- **2D SXR tomographies** in TdeV and EAST show that the hot core is radially displaced & rotates poloidally, **but** not fully expelled out of the $q = 1$ surface



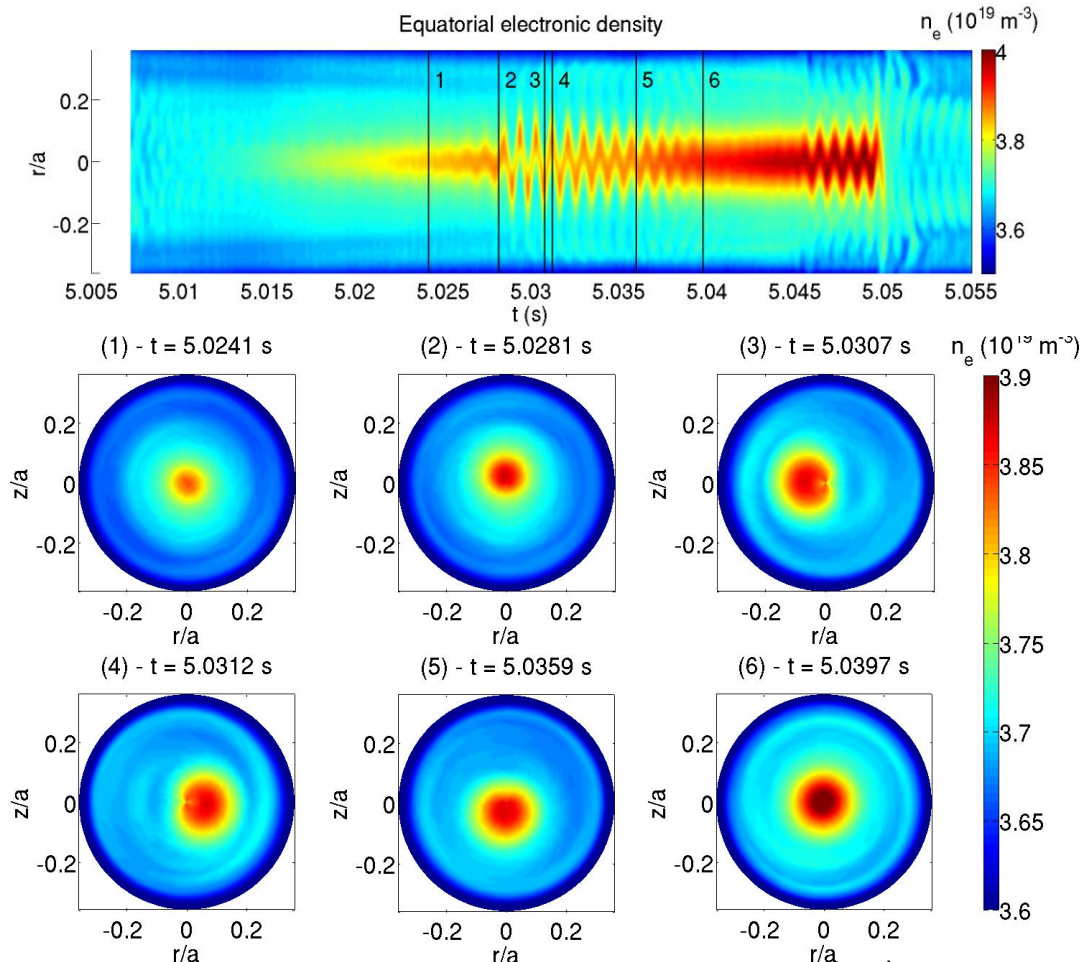
C. Janicki *et al.*, *Nucl. Fus.* **30** 950 (1990)

L.-Q. Xu *et al.*, *Chin. Phys. B* **23** 085201 (2014)

Hot core is preserved and rotates poloidally during partial crashes

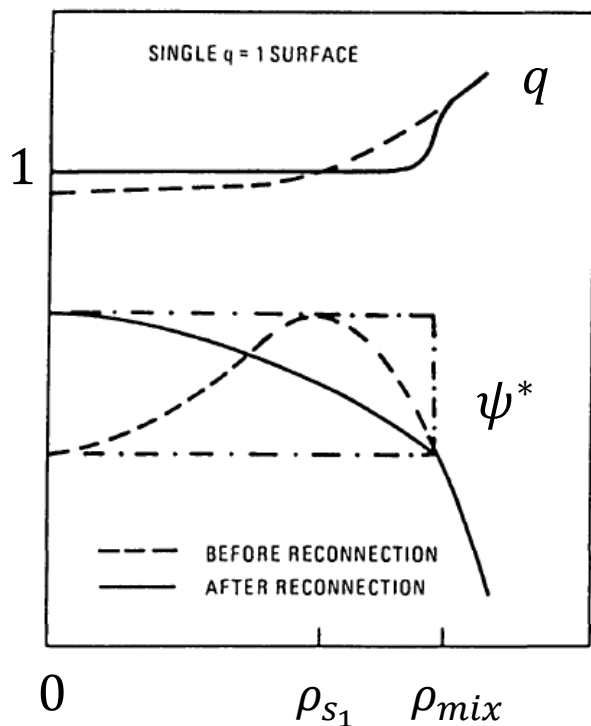
- 2D SXR tomographies in TdeV and EAST show that the hot core is radially displaced & rotates poloidally, **but** not fully expelled out of the $q = 1$ surface

C. Janicki *et al.*, *Nucl. Fus.* **30** 950 (1990)
L.-Q. Xu *et al.*, *Chin. Phys. B* **23** 085201 (2014)

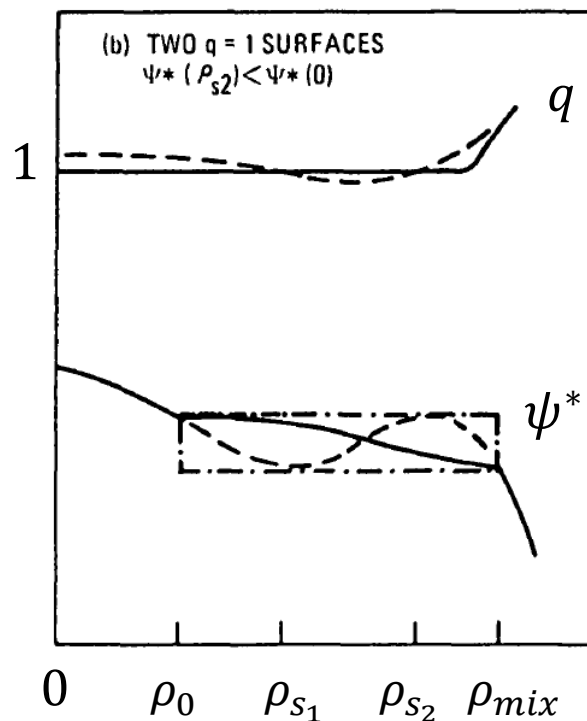


Also observed in reflectometry measurements in Tore Supra #44634

- Considering **multiple** $q = 1$ surfaces, partial reconnection located off the magnetic axis V.V. Parail and G.V. Pereverzev, *Sov. J. Plasma Phys.* 6 14 (1980)
- Need more explanation about the onset conditions for compound sawtooth



W. Pfeiffer, *Nucl. Fus.* 25 673 (1985)



- Compound sawteeth in experiments
- Internal kink mode stability during the sawtooth ramp phase
- XTOR-2F simulations of compound sawteeth

■ The XTOR-2F code models **nonlinear** and **two-fluid full 3D MHD**

H. Lütjens and J.-F. Luciani, *J. Comput. Phys.* **229** 8130 (2010)

$$\partial_t N + \nabla \cdot (N\mathbf{V}) + \frac{\nabla P_i}{e} \cdot \nabla \times \frac{\mathbf{B}}{B^2} = \nabla \cdot (D\nabla N - \mathbf{V}_p N) + S_N$$

Density equation

$$Nm_i(\partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla)\mathbf{V} + (\mathbf{V}_i^* \cdot \nabla)\mathbf{V}_\perp) = \mathbf{J} \times \mathbf{B} - \nabla P + \mu \nabla^2 (\mathbf{V} + \mathbf{V}_i^*)$$

Momentum equation

$$\begin{aligned} \partial_t P + \mathbf{V} \cdot \nabla P + \gamma P \nabla \cdot \mathbf{V} + \frac{\gamma}{e} (T \nabla P_i + P_i \nabla T_i + P_e \nabla T_e) \cdot \mathbf{E} \\ = (\gamma - 1) [\nabla \cdot (N_i \chi_\perp \nabla_\perp T) + \nabla \cdot (N_i \chi_\parallel \nabla_\parallel T \mathbf{b})] + S_H \end{aligned}$$

Pressure equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nabla \times \left(\frac{\nabla_\parallel P_e}{Ne} \mathbf{b} \right) - \nabla \times \eta \mathbf{J}$$

Ohm's law & Faraday's law

with $\mathbf{V} = \mathbf{V}_{\parallel,i} + \mathbf{V}_E$ fluid velocity, $\mathbf{V}_i^* = (\mathbf{B} \times \nabla P_i) / NeB^2$ ion diamagnetic velocity

$N = N_i = N_e$, $P = P_e + P_i$, μ plasma viscosity

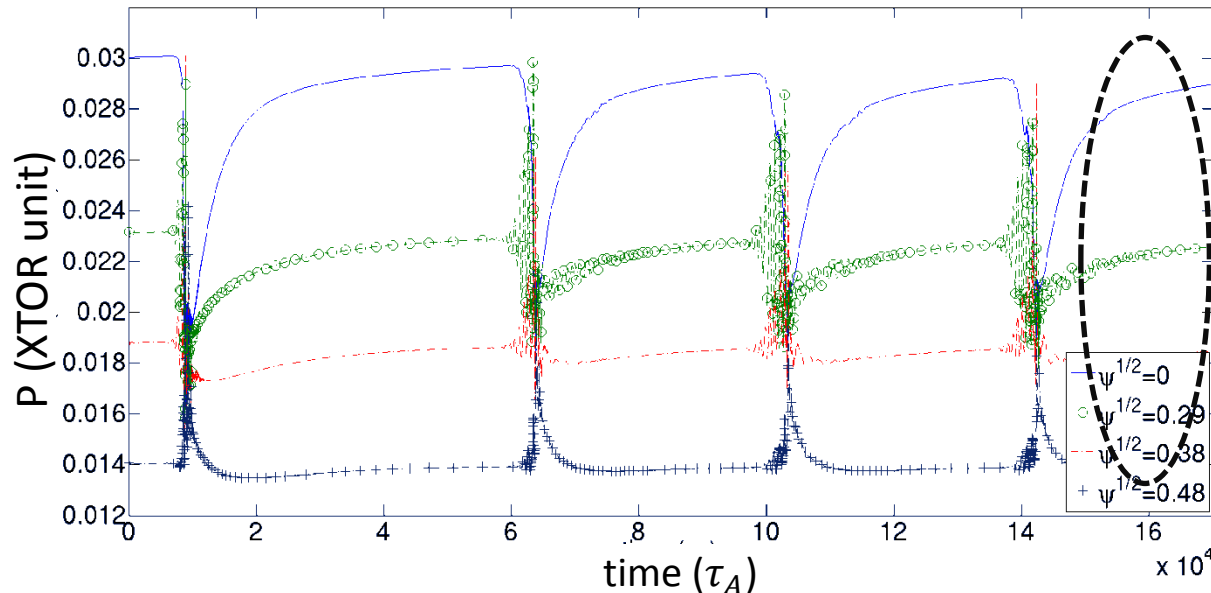
■ Resolution of the system with a **Newton-Krylov** solver, **fully implicit** scheme

Flat q -profile during sawtooth ramp phase : the internal kink mode is unstable

- q -profile flattened after sawtooth crash and remains **flat** during ramp phase
- The internal kink mode is unstable for **low-sheared** q -profiles, even when $q \gtrsim 1$
(shear $\hat{s} = r/q \times dq/dr$)
- Numerical study on linear stability in accordance with analytical predictions
- However, **no partial crash** (i.e. “ordinary sawtooth”)

R. J. Hastie *et al.*, *Phys. Fluids* **30** 1756 (1987)

H. J. de Blank and T. J. Schep, *Phys. Fluids B* **3** 1136 (1991)

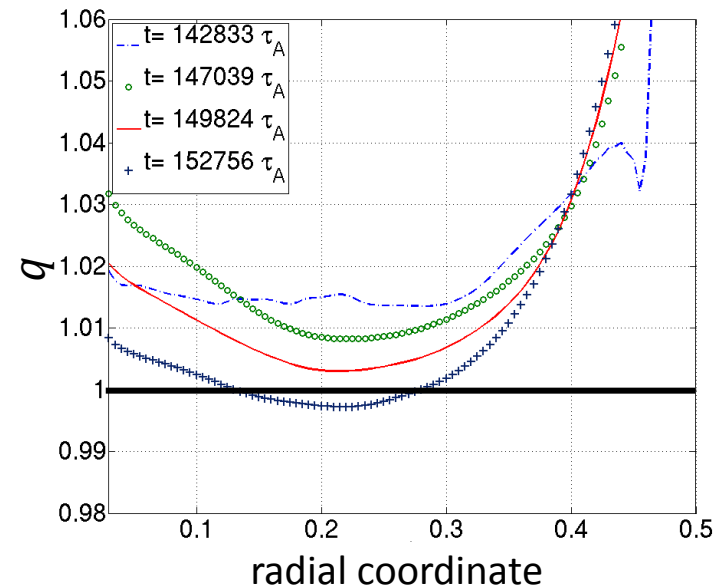
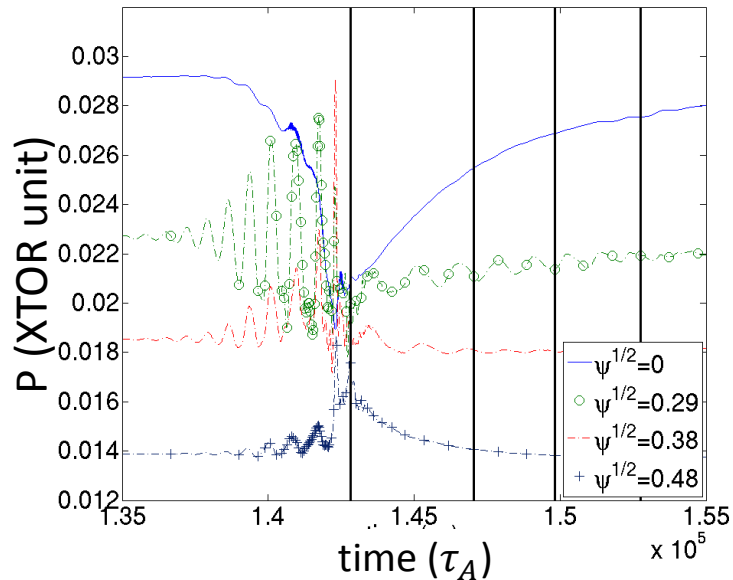


Flat q -profile during sawtooth ramp phase : the internal kink mode is unstable

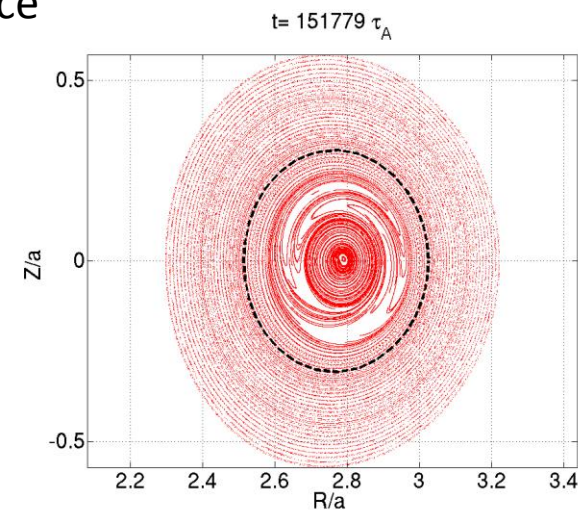
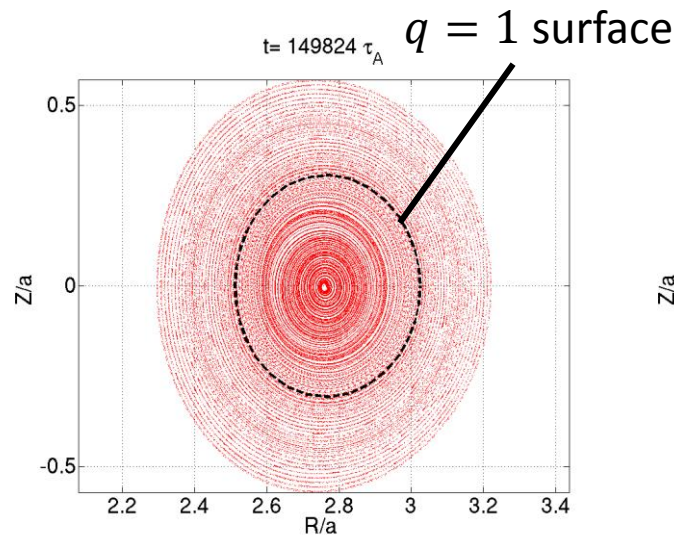
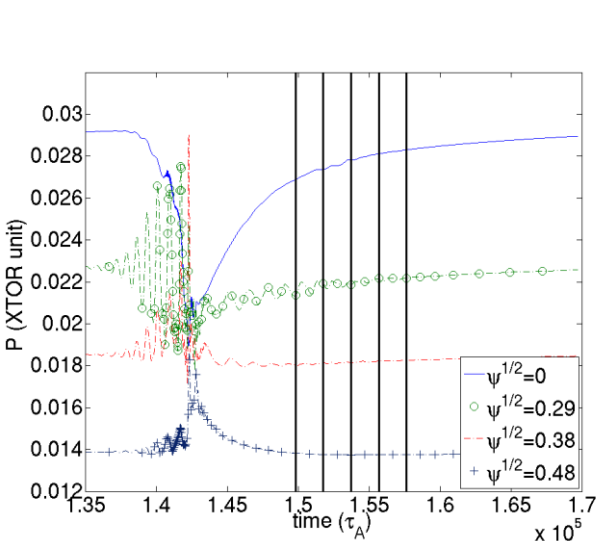
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R. J. Hastie *et al.*, *Phys. Fluids* **30** 1756 (1987)

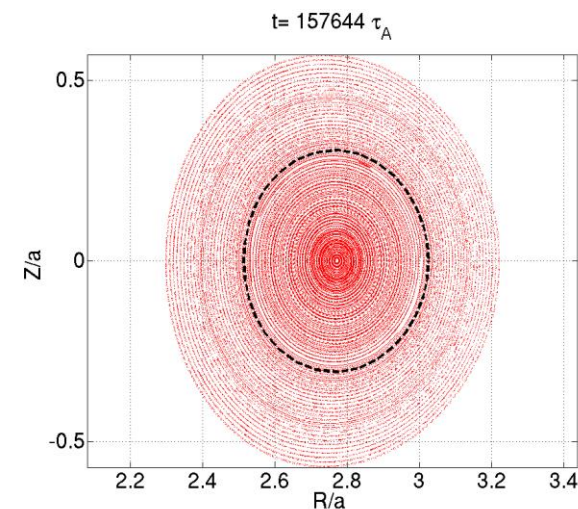
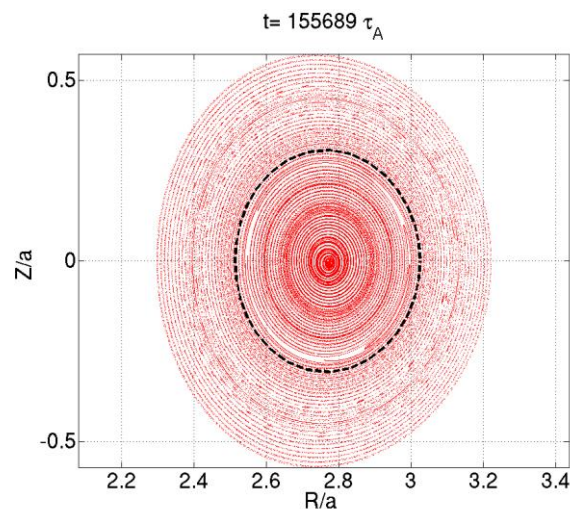
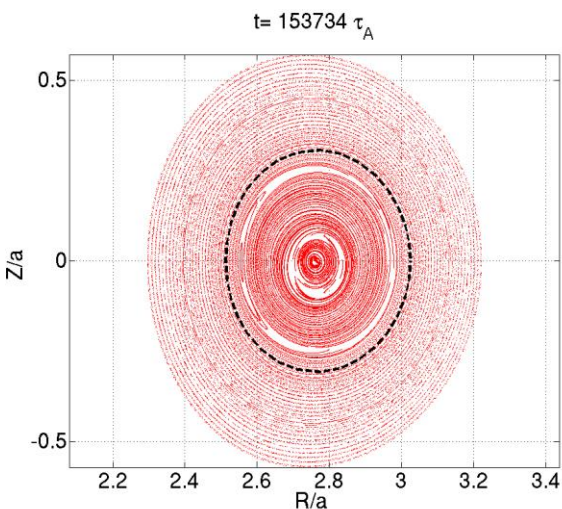
H. J. de Blank and T. J. Schep, *Phys. Fluids B* **3** 1136 (1991)



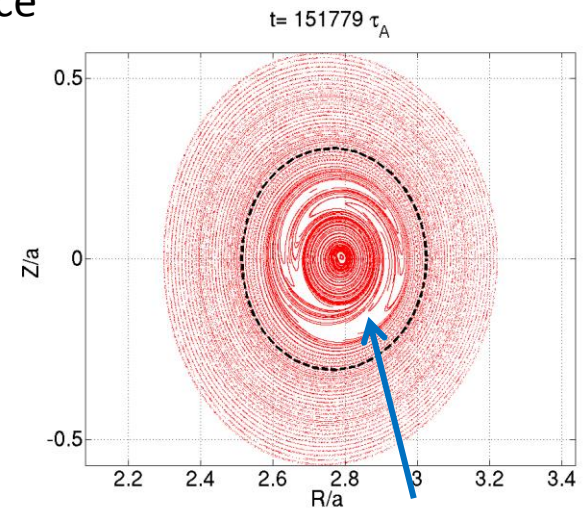
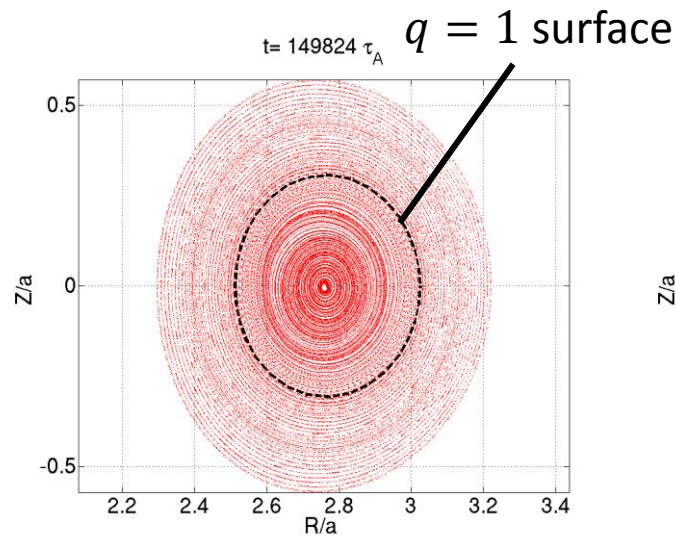
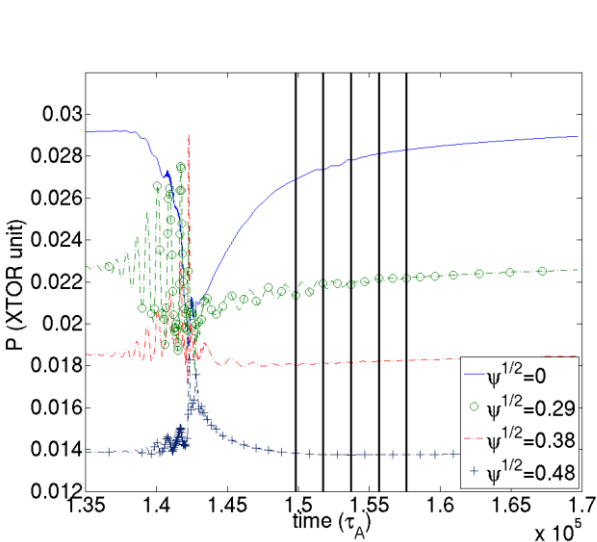
No partial crash, but instability visible in the Poincaré plots



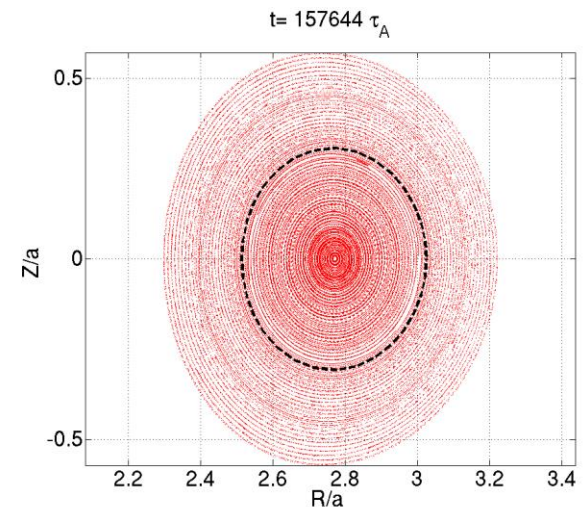
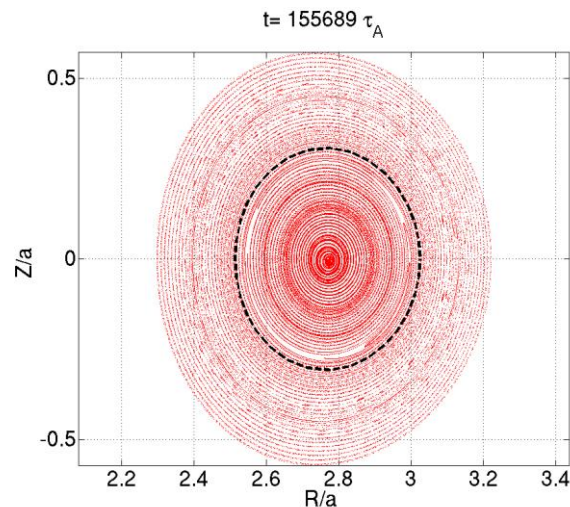
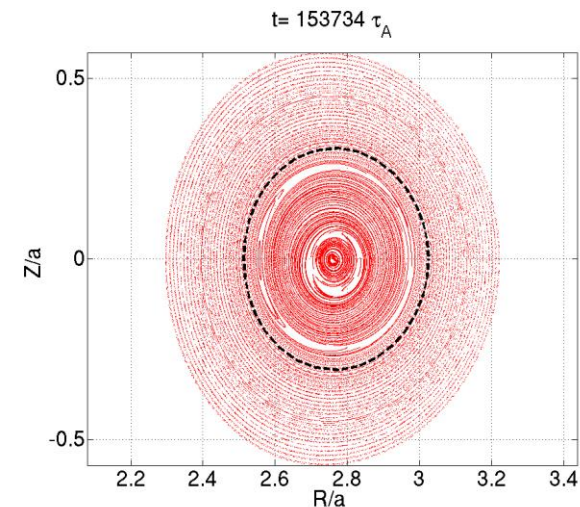
Poincaré plots : track field lines and allow to access magnetic topology



No partial crash, but instability visible in the Poincaré plots



However, no radial displacement of the hot core



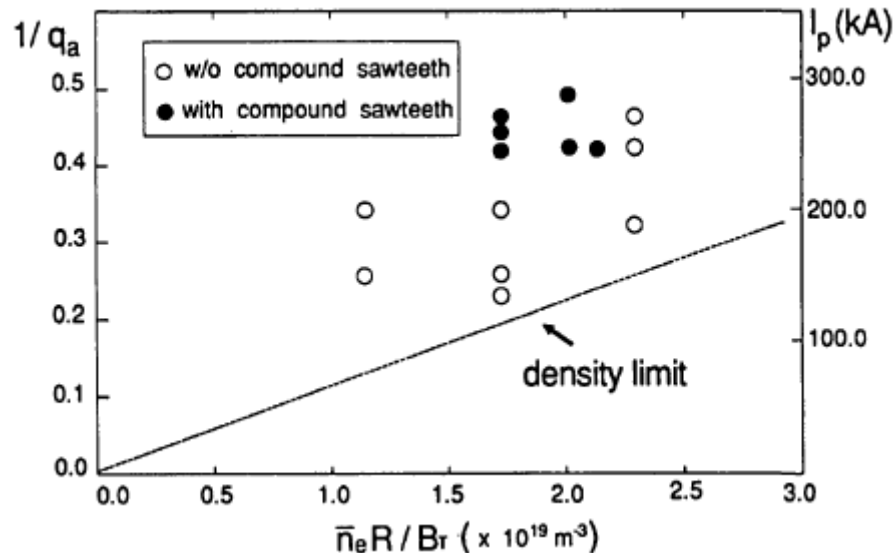
- Compound sawteeth in experiments
- Internal kink mode stability during the sawtooth ramp phase
- XTOR-2F simulations of compound sawteeth

- Simulation of compound sawteeth in an **elliptic cross section**
- Scan on parameters (κ , r_s/a) destabilizing the internal kink mode

H. Lütjens *et al.*, *Nucl. Fus.* **32** 1625 (1992)

A. Martynov *et al.*, *Plasma Phys. Control. Fusion* **47** 1743 (2005)

A. D. Turnbull and F. Troyon, *Nucl. Fus.* **29** 1887 (1989)



C. Janicki *et al.*, *Nucl. Fus.* **30** 950 (1990)

Compound ST simulations with XTOR-2F by destabilizing the internal kink mode

- Simulation of compound sawteeth in an **elliptic cross section**

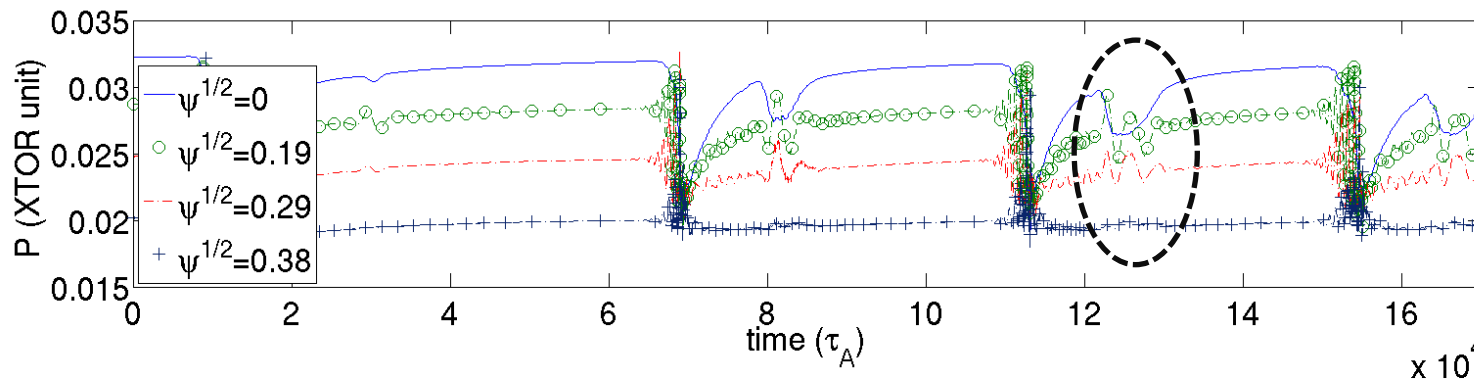
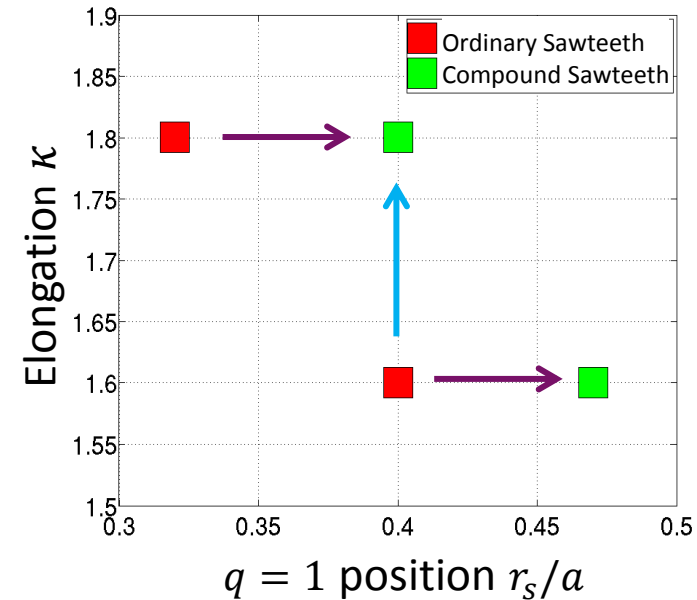
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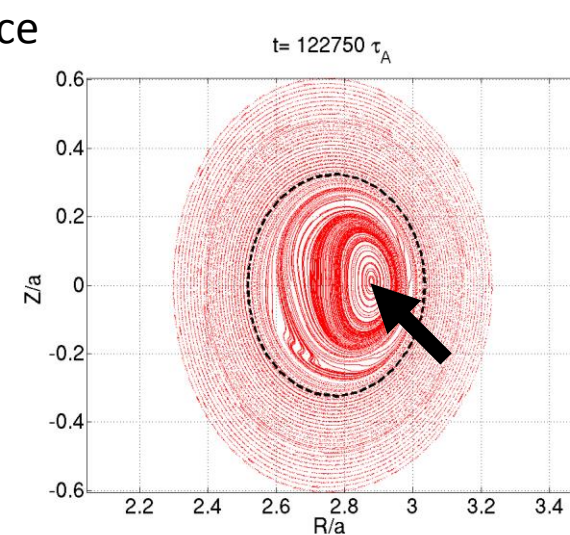
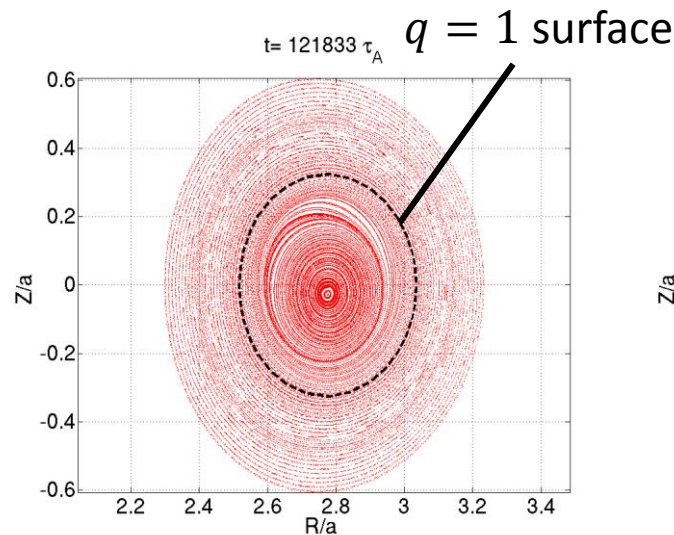
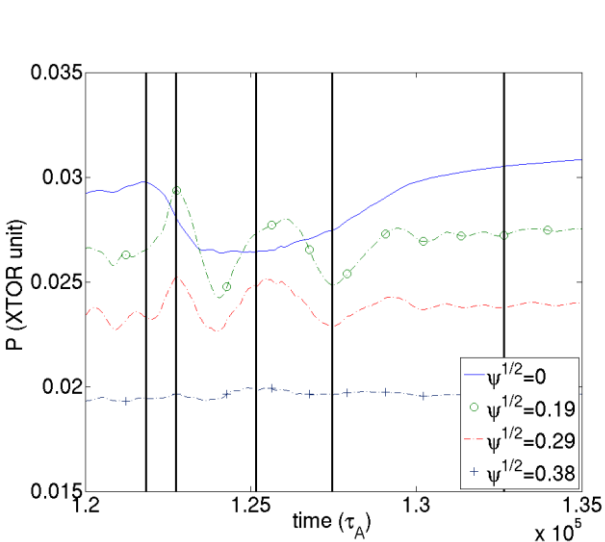
H. Lütjens *et al.*, *Nucl. Fus.* **32** 1625 (1992)

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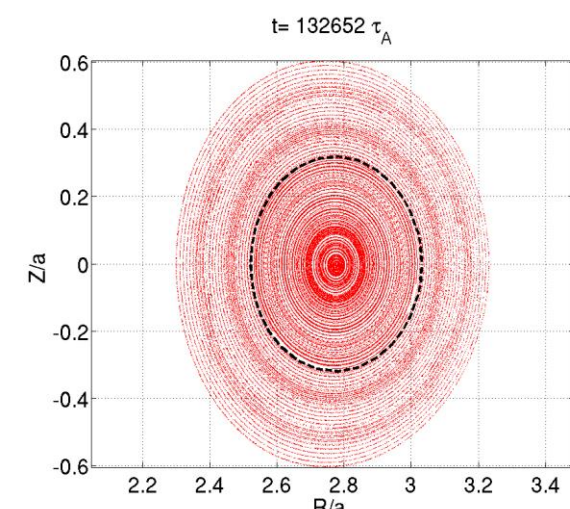
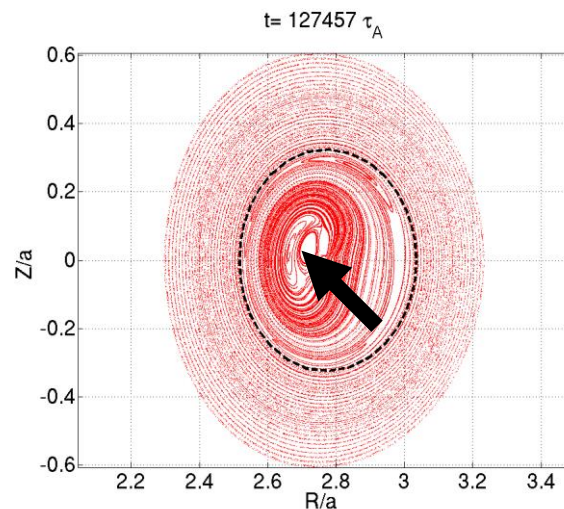
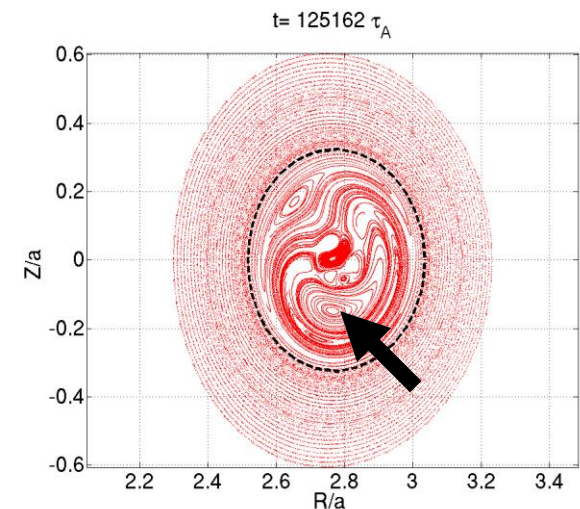
A. D. Turnbull and F. Troyon, *Nucl. Fus.* **29** 1887 (1989)

- The scan of two parameters gives the effects on the internal kink mode as expected

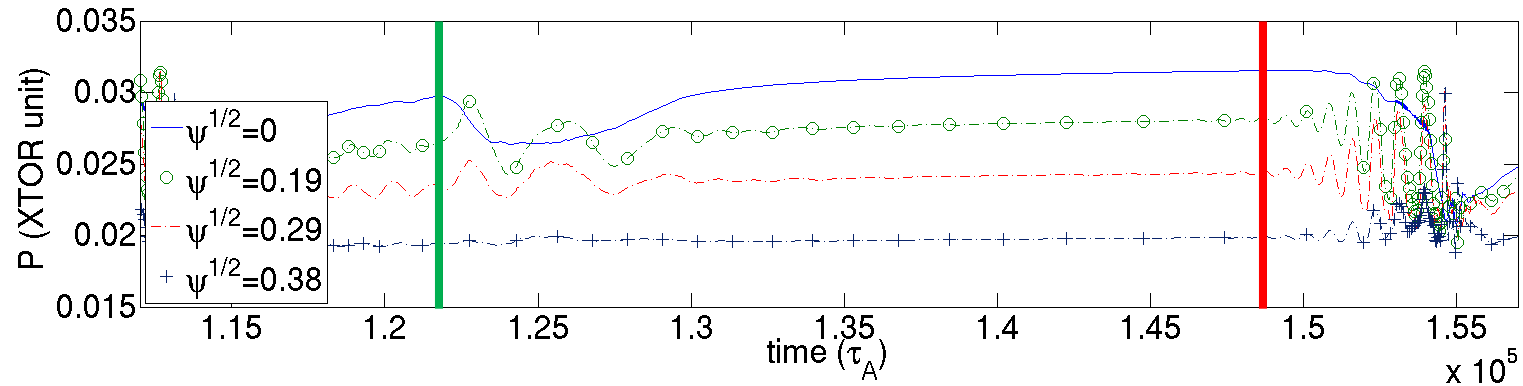




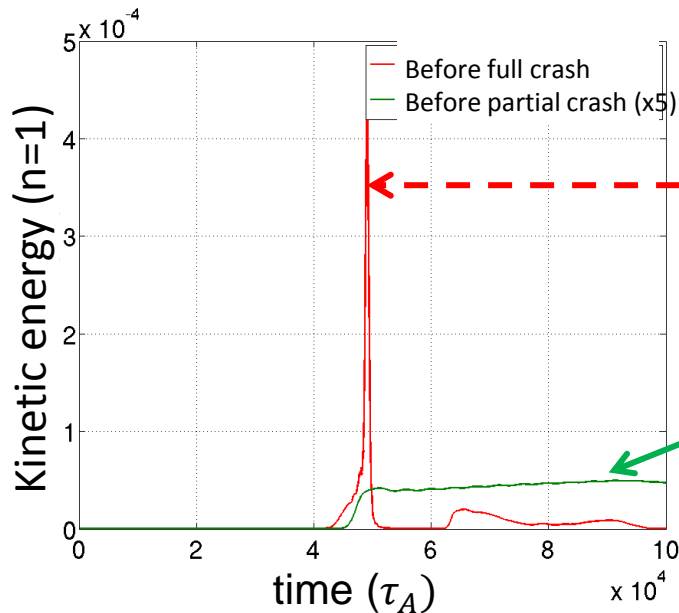
Displaced hot core remains and rotates during partial crash



The partial crash and the ordinary ST crash are of a different nature



Restart from new equilibria reconstructed at times of interest



Typical behaviour of an ordinary sawtooth crash

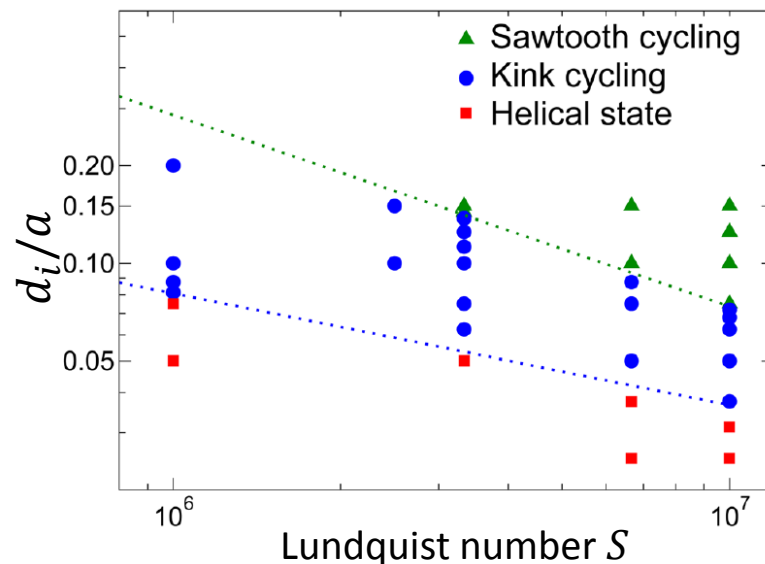
Growth rate : $\gamma_{ordinary} > \gamma_{partial}$

Non-linear saturation of kinetic energy :
Signature of a **saturated helical state**

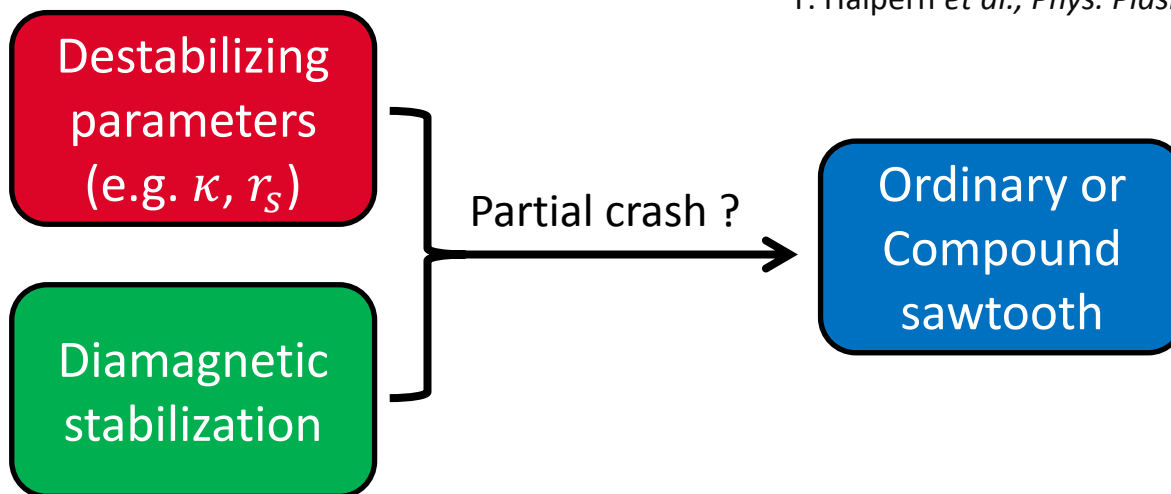
F. Halpern *et al.*, *Plasma Phys. Control. Fusion* **53** 015011 (2011)

F. Halpern *et al.*, *Phys. Plasmas* **18** 102501 (2011)

- Saturated helical state obtained for **weak diamagnetic effect**
- Internal kink mode is shown to be **unstable** due to flat q -profiles



F. Halpern *et al.*, *Phys. Plasmas* **18** 102501 (2011)



G. Ara *et al.*, *Ann. Phys.* **112** 443 (1978)

- ☑ The **partial crash** is characterized by the **internal kink mode** with a **displaced hot core**, which is **not fully expelled** out of the $q = 1$ surface.
- ☑ The internal kink mode during sawtooth **ramp phase** is **unstable**, due to **low-sheared q -profiles**
- ☑ Depending on the internal kink mode **growth rate** during the ramp phase, ordinary or compound sawtooth is obtained
 - So far, no compound sawtooth simulations in circular geometry, β_p scan
 - Threshold in κ , r_s/a ? Fraction of compound ST among ordinary ST ?
 - Competition between unstable internal kink mode and diamagnetic stabilization during sawtooth ramp phase ?

Additional Slides

- Toroidal MHD equilibrium computed by the CHEASE code

H. Lütjens *et al.*, *Comp. Phys. Comm.* **97** 219 (1996)

- Finite difference method in **radial** direction and **pseudo-spectral** method in **poloidal** and **toroidal** directions

H. Lütjens and J.-F. Luciani, *J. Comput. Phys.* **229** 8130 (2010)

Automatically adjusted timesteps

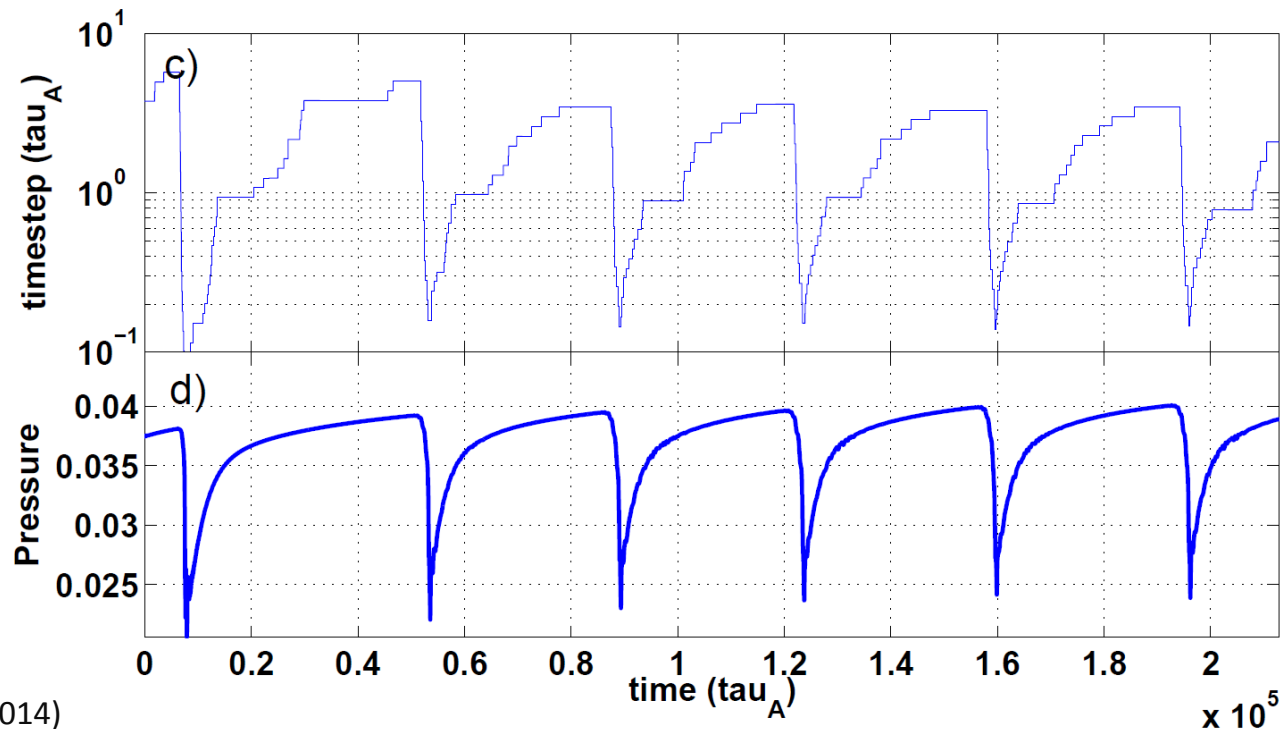
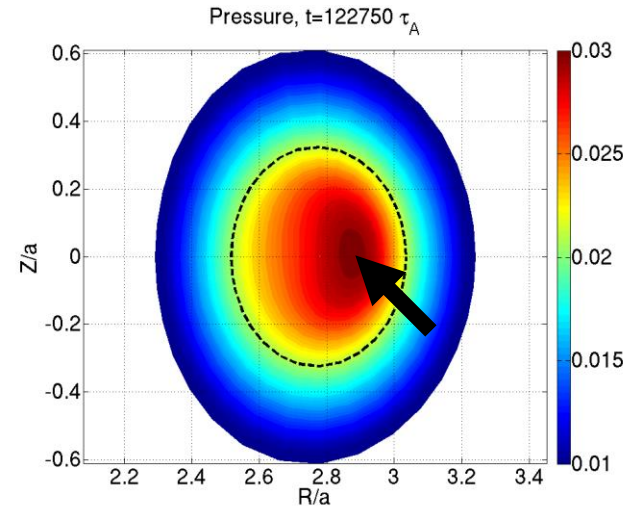
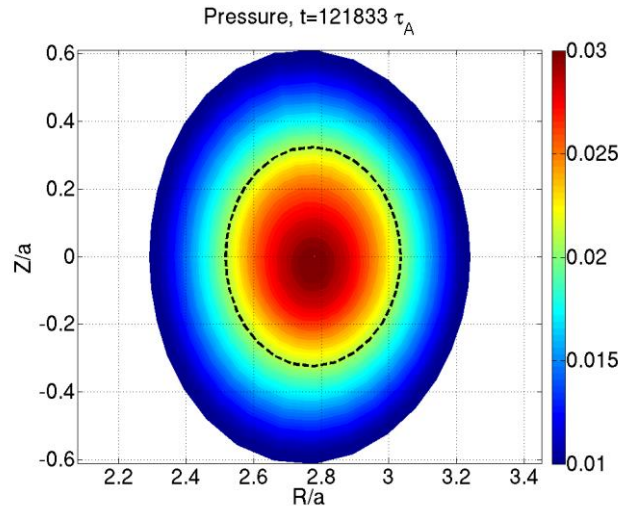
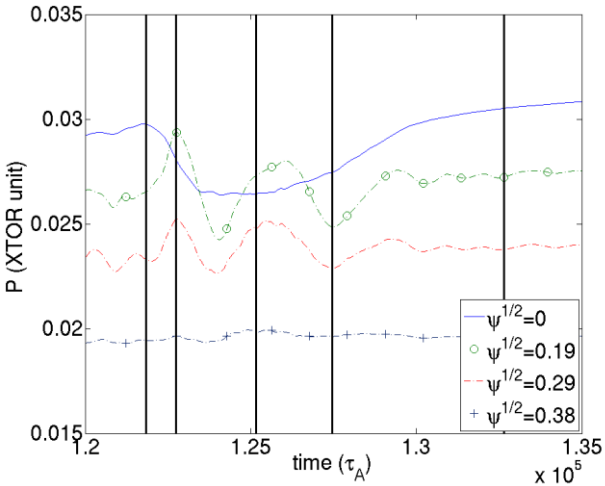
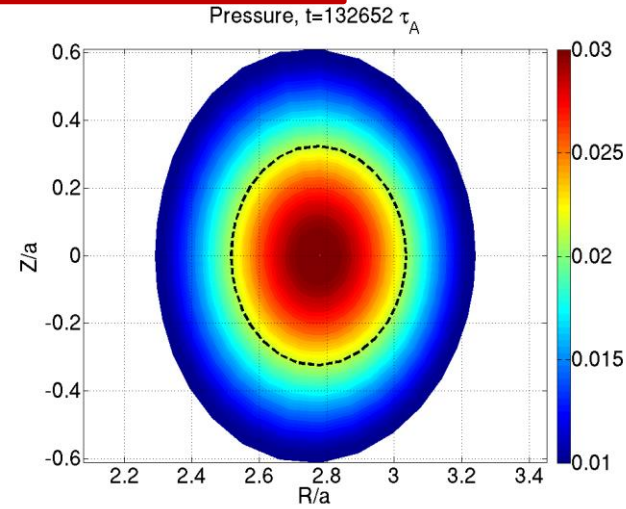
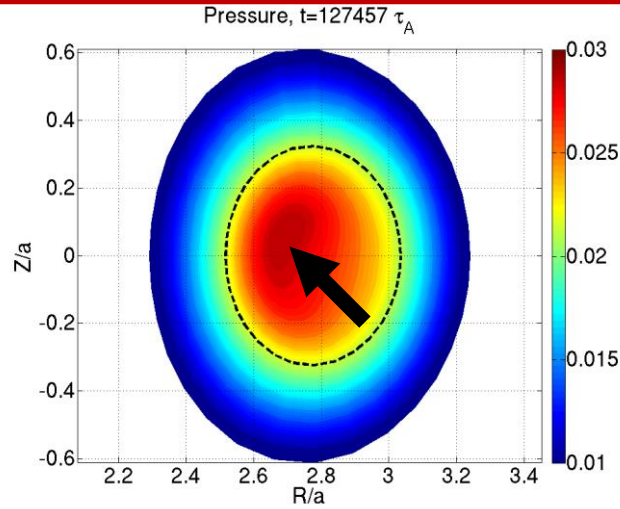
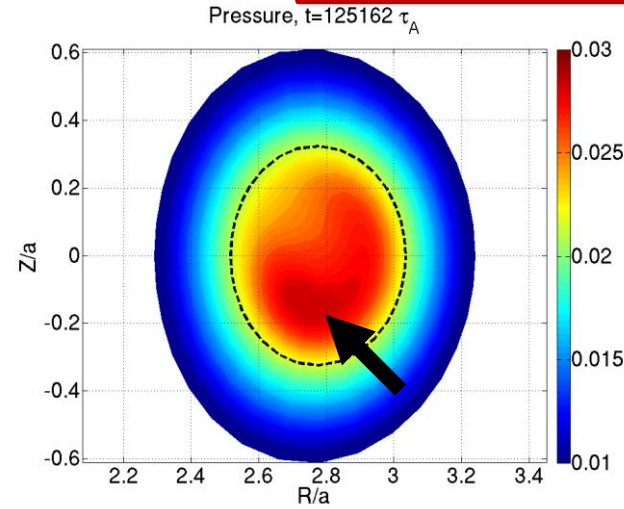
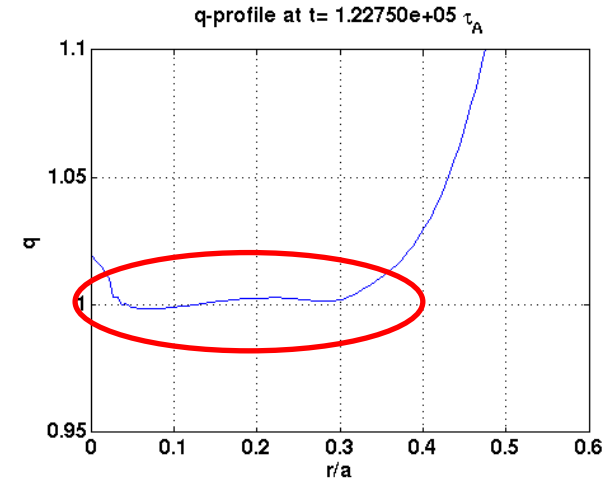
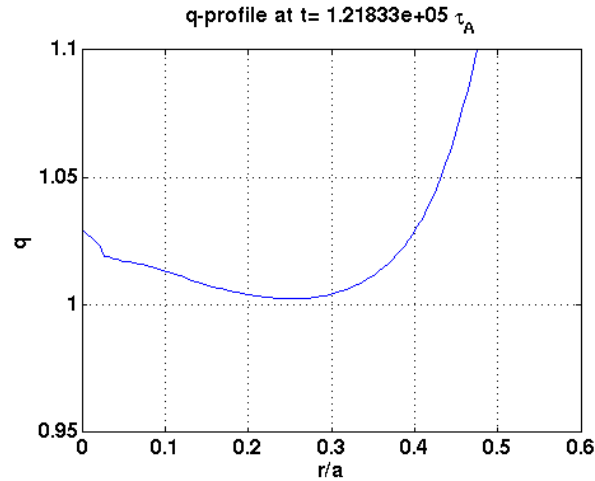
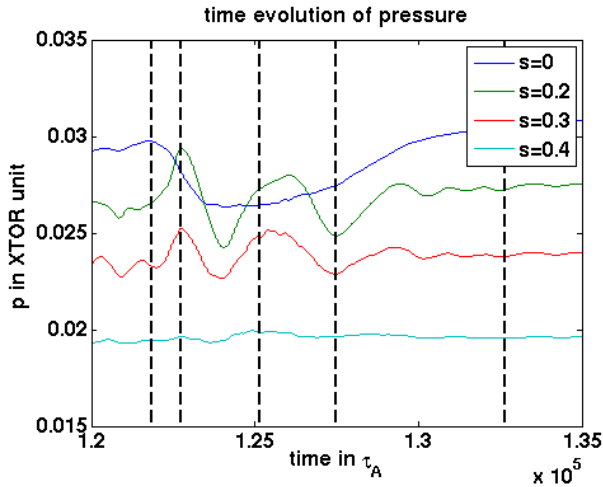


Figure from PhD thesis of T. Nicolas (2014)

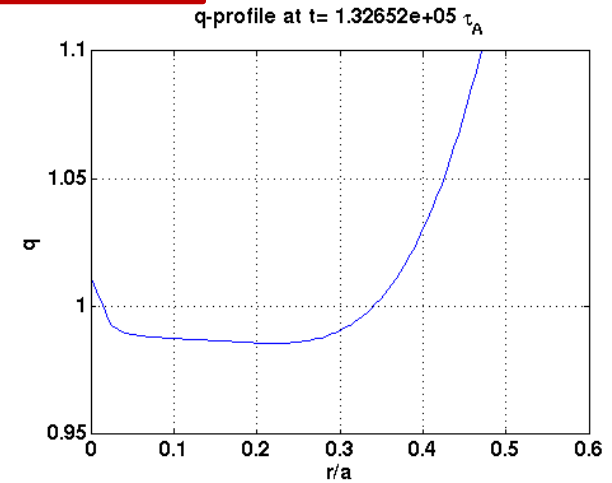
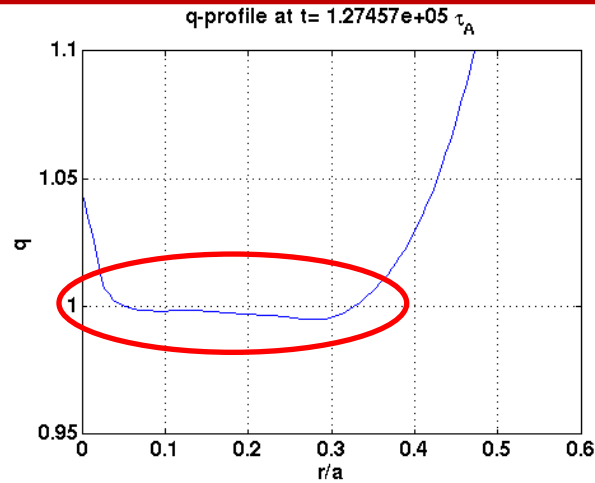
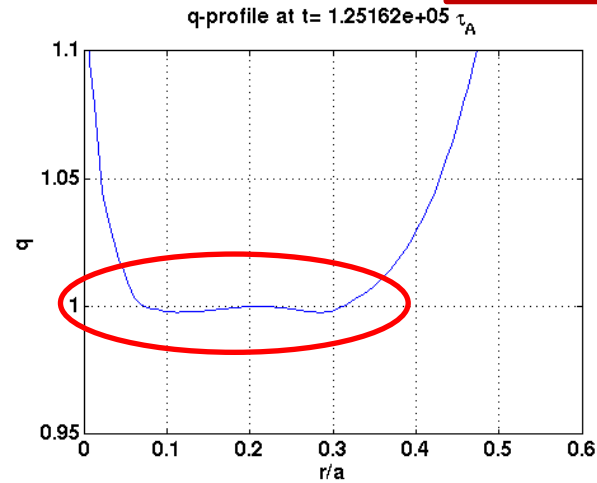


Displaced hot core remains and rotates during partial crash



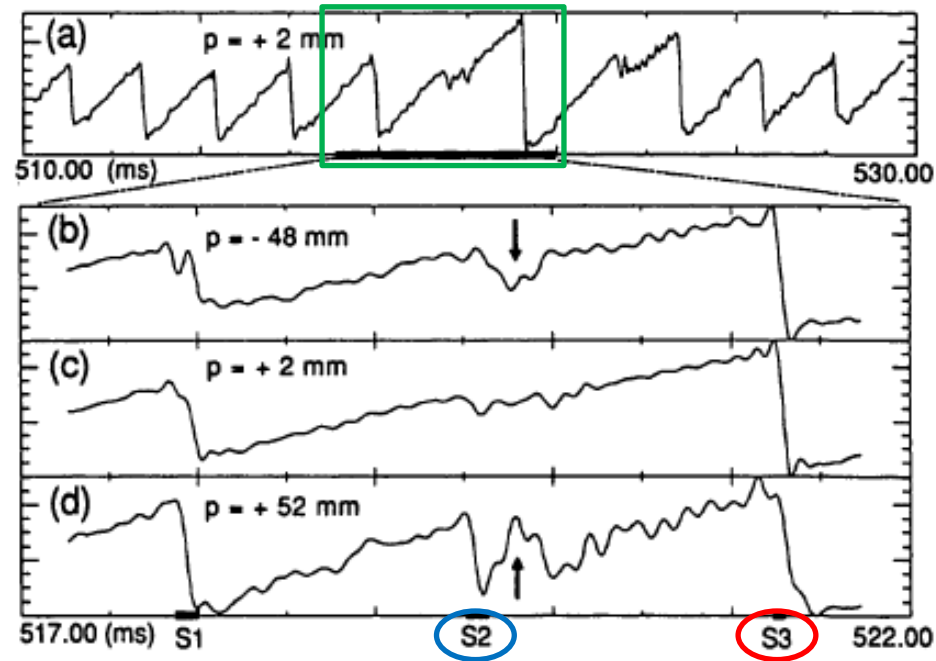


Multiple $q = 1$ surfaces during partial crash



- Fast Soft X-Ray integration often used to diagnose sawtooth crashes in tokamaks
- Compound sawteeth observed with **2D Soft X-Ray tomographies** in Tokamak de Varennes (discharge #6059, Ohmic heating)

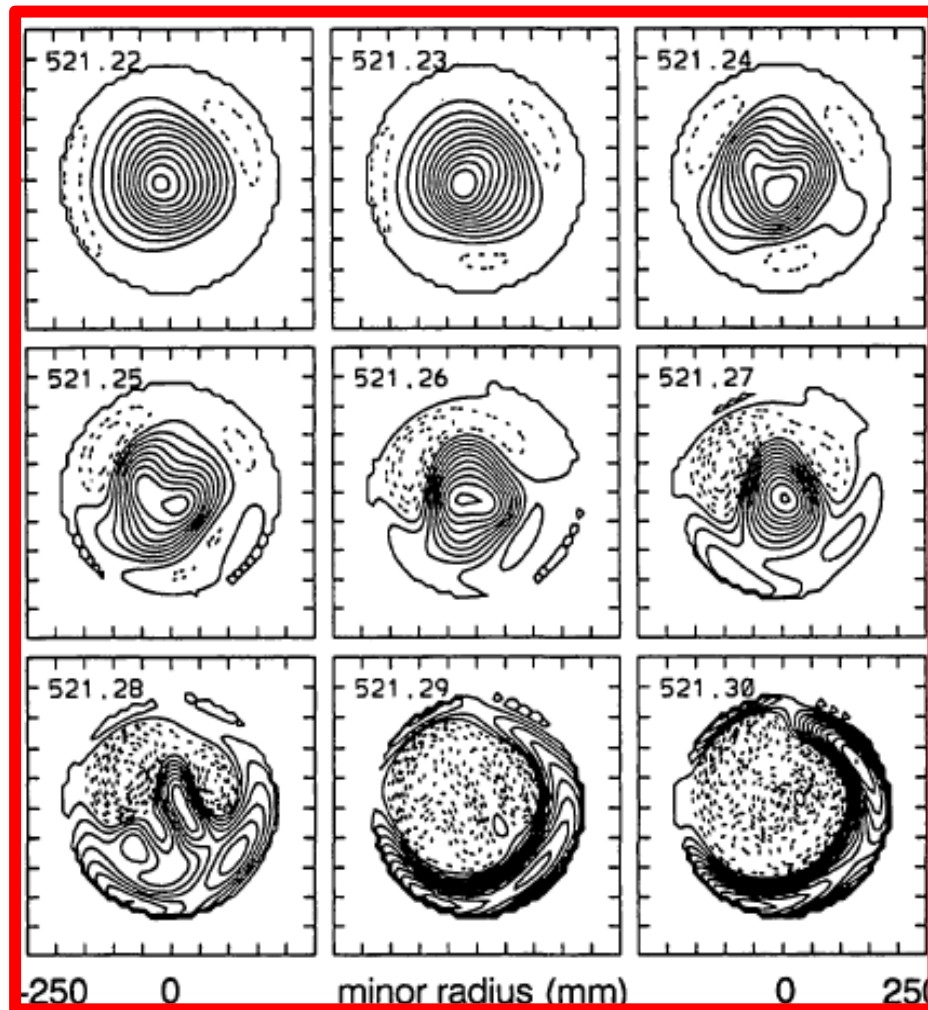
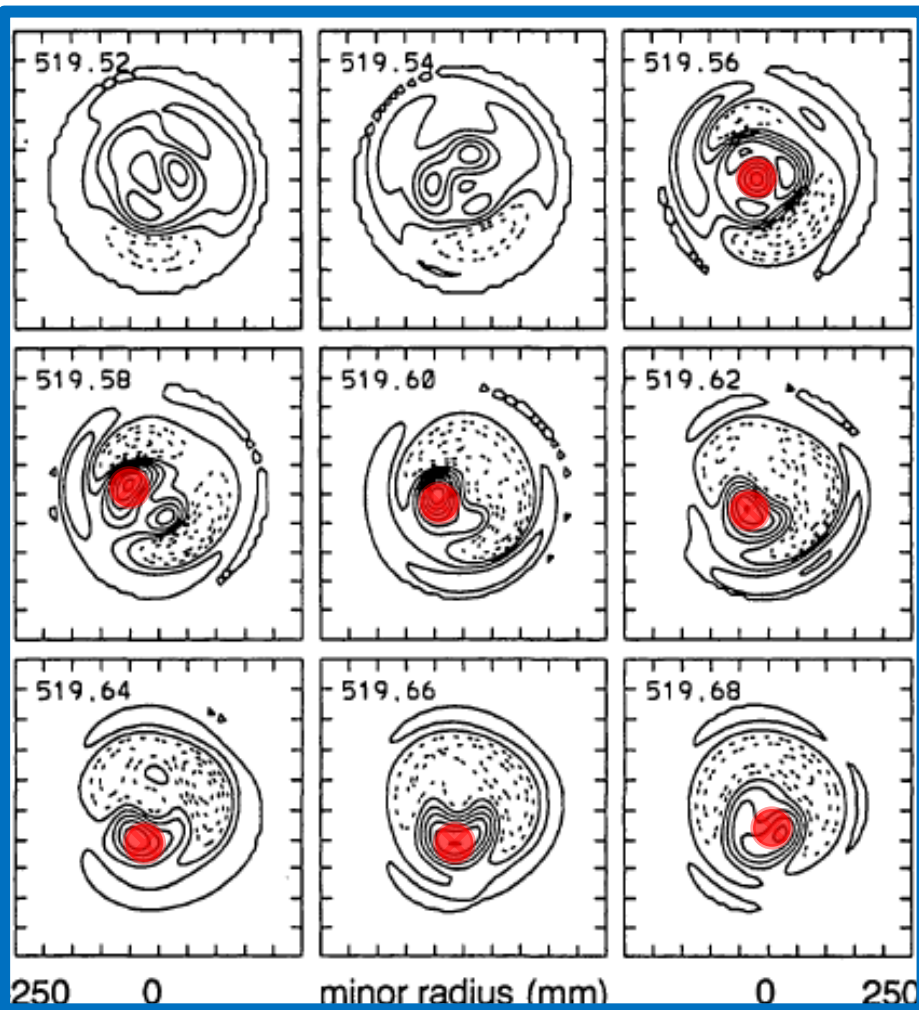
C. Janicki et al., *NF Letters* **30** 950 (1990)



- Similar observations with SXR diagnostic in other tokamaks (e.g. EAST)

L. Xu et al., *Chin. Phys. B* **23** 085201 (2014)

Hot core is preserved and rotates poloidally during partial crashes



C. Janicki et al., *NF Letters* 30 950 (1990)

During partial crash (**blue**), hot core is preserved in central region and **rotates**

- $n = 1$ mode stability **before** and **after** a partial crash :

internal kink mode is **unstable**

→ **core displacement**

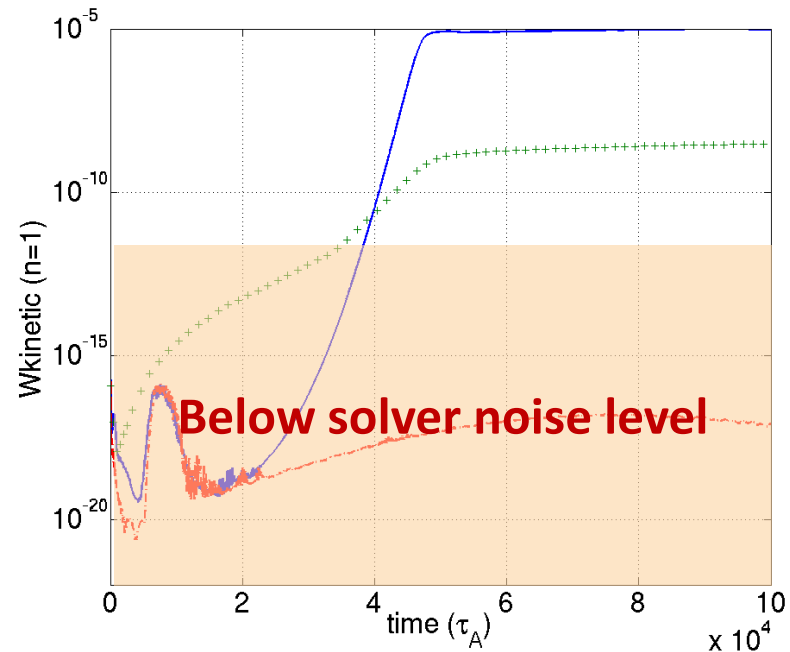
- Even for the case **without** partial crash, **unstable (1, 1) mode during ramp phase**

- q -profile remains **flat** during ramp phase

→ internal kink mode **unstable**

R. J. Hastie *et al.*, *Phys. Fluids* **30** 1756 (1987)

H. J. de Blank and T. J. Schep, *Phys. Fluids B* **3** 1136 (1991)



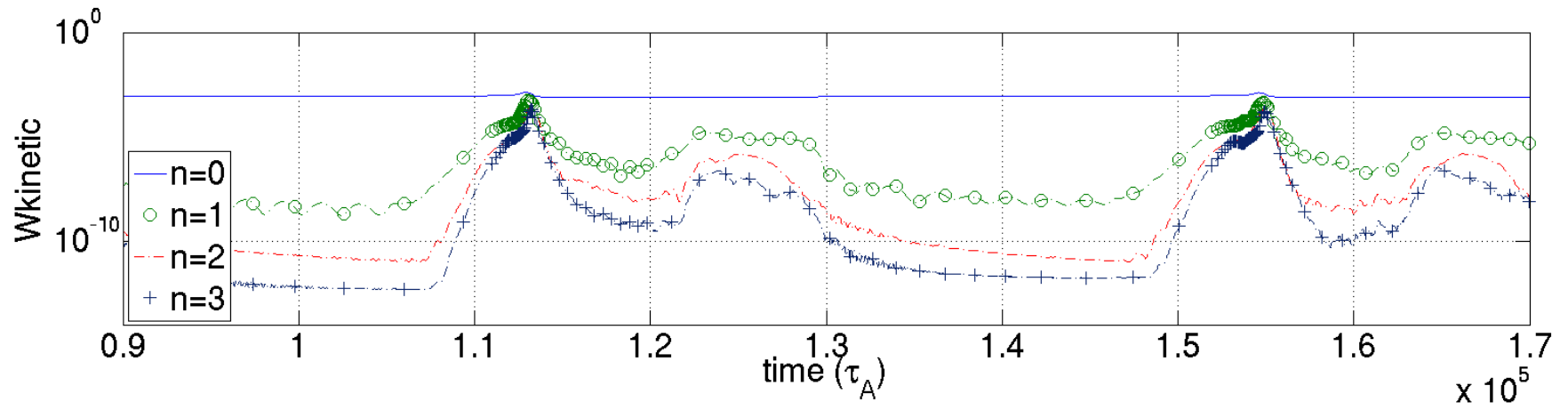
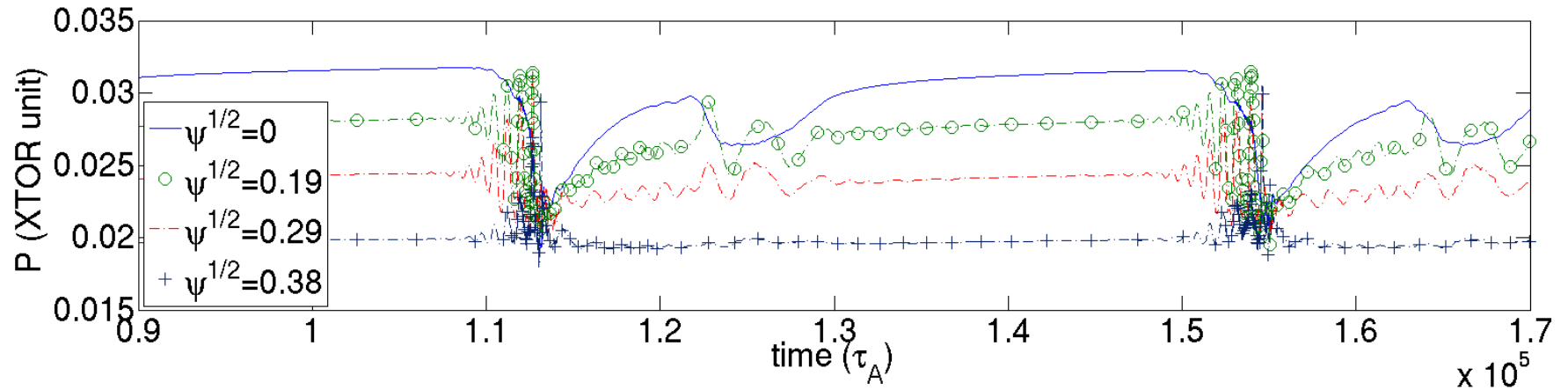
- Non-linear saturation of $n = 1$ mode kinetic energy

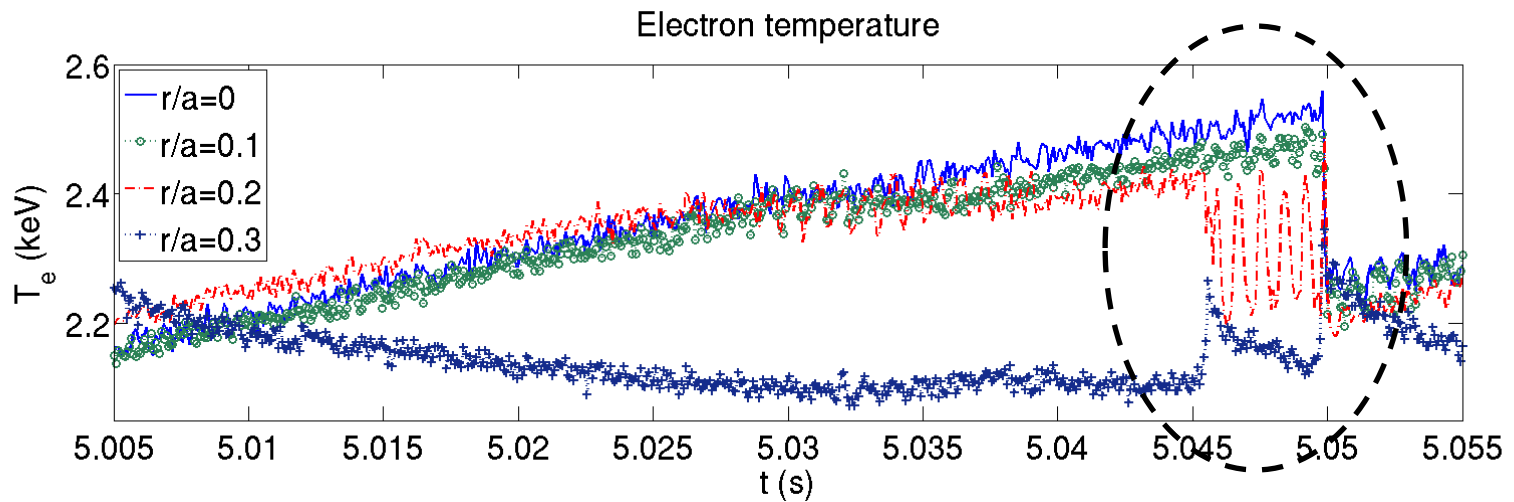
→ Signature of saturated helical state

F. Halpern *et al.*, *Plasma Phys. Control. Fusion* **53** 015011 (2011)

F. Halpern *et al.*, *Phys. Plasmas* **18** 102501 (2011)

Evolution of the kinetic energy during compound sawteeth





V.S. Udintsev *et al.*, *Plasma Phys. Control. Fusion* **47** 1111 (2005)