TECH-X
SIMULATIONS EMPOWERING YOUR INNOVATIONS

NIMROD
Non-Ideal MHD with Rotation
Open Discussion Project

CEMM

Effect of scrape-off layer currents on reconstructed tokamak equilibrium

CEMM meeting April 3rd 2016

J. R. King

With contributions from

R. Groebner (General Atomics); J. Hanson and J. Hebert (Auburn);
S. Kruger (Tech-X); Carl Sovinec (UW-Madison)

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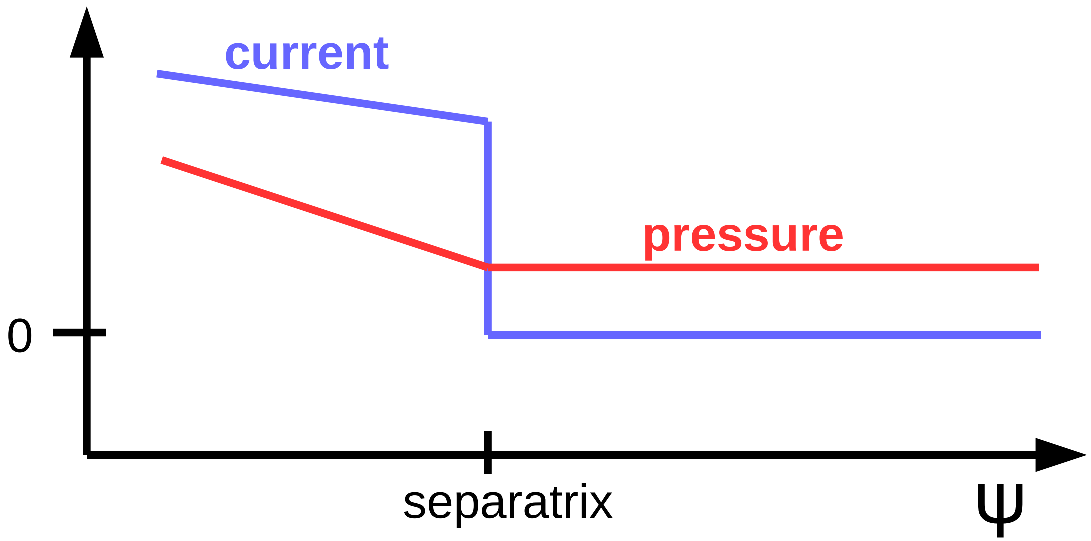
Motivation

- Edge modeling with experimental reconstructions can be corrupted by edge current discontinuity
- This discontinuity can be eliminated by self-consistently resolving the Grad-Shafranov (GS) equation with scrape-off layer (SOL) n/T profile gradients
- How does including these gradients affect the underlying equilibrium?

On NIMROD's GS solver see [Howell and Sovinec, CPC 2014]

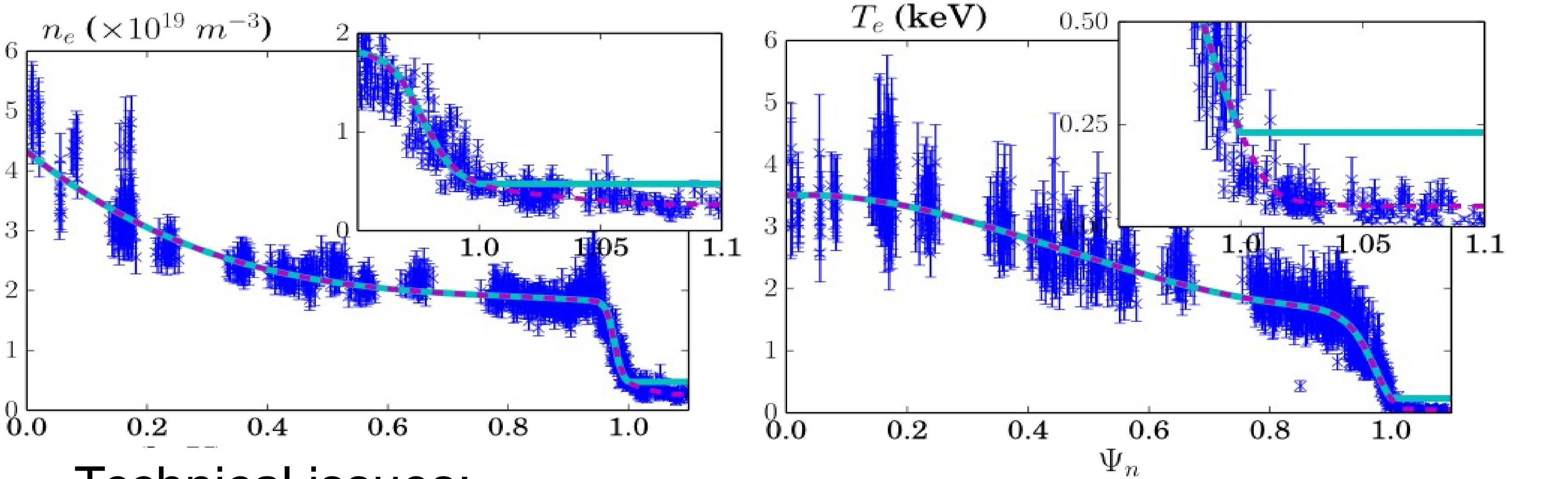
Reconstructions typically contain discontinuous current profiles across the separatrix

- The pressure is assumed to be constant outside the separatrix
- Current discontinuity is problematic for GS re-solves and nonlinear edge studies



Can include SOL region with currents

- The experimental reconstruction doesn't set the gradient of thermodynamic quantities to zero on the LCFS because <u>they aren't measured to be zero</u>

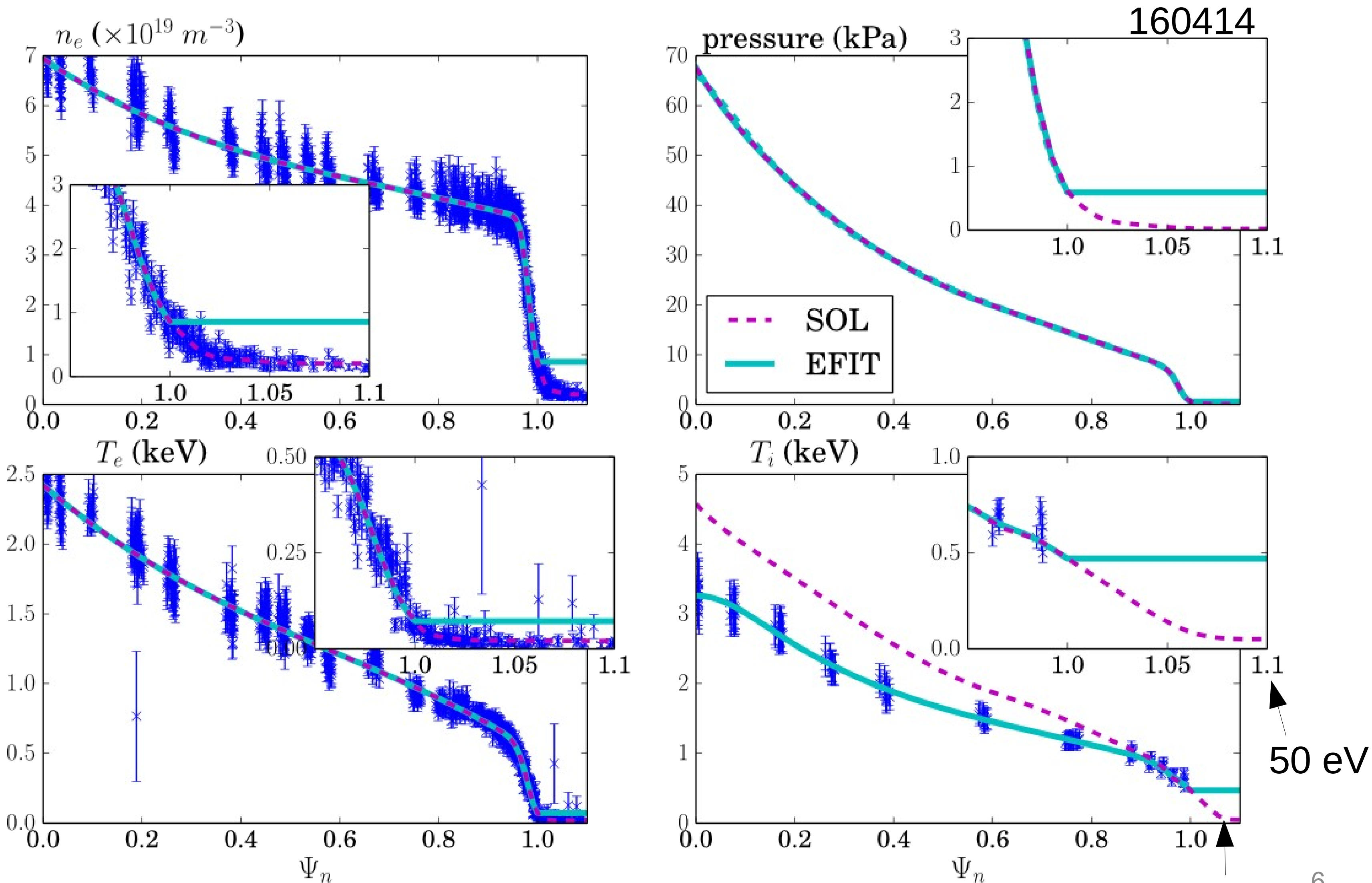


- Technical issues:
  - EFIT profiles only extend to LCFS
  - How do we extrapolate while minimizing free parameters?
  - Result should be as close to possible to known measurements

We study two reconstruction: high and low current in SOL.

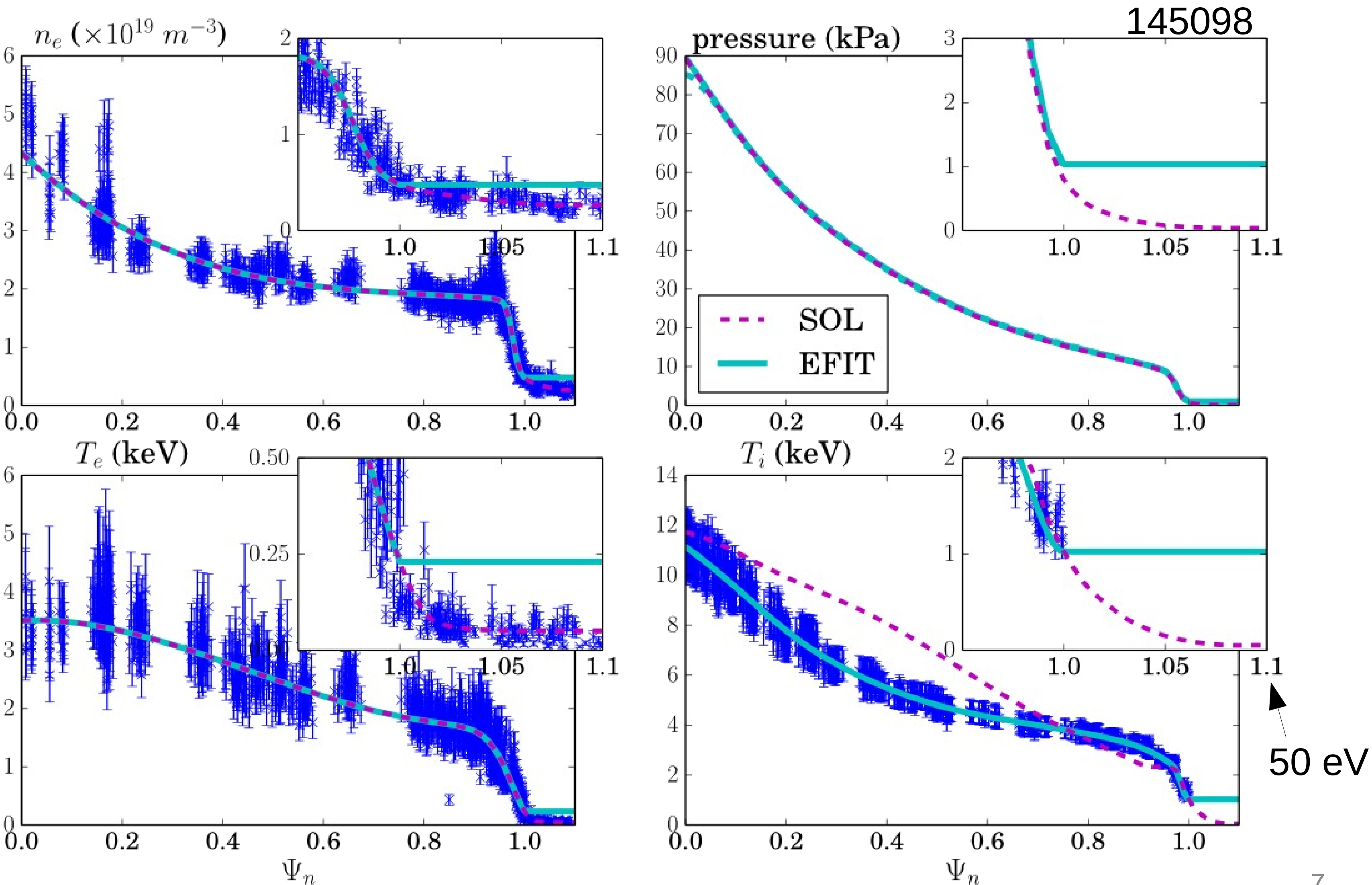
- Helps isolate effects of SOL current from GS re-solve
- Small current: 160414 @ 3025 ms
  - Inter-ELM reconstruction from H-mode shot with ELM pellet pacing.
  - Time selected is last 20% of pellet-trigger inter-ELM period
- Large current: 145098 @ 1800ms
  - QH-mode shot with large edge current
  - Near peeling boundary
  - Reversed-current discharge (flips sign of J_T and Ψ)

Low-current case – use measured SOL profiles except for T_i



CER data outside LCFS affected by trapped ions inside LCFS

High-current case – use measured SOL profiles except for T_i



CER data outside LCFS affected by trapped ions inside LCFS

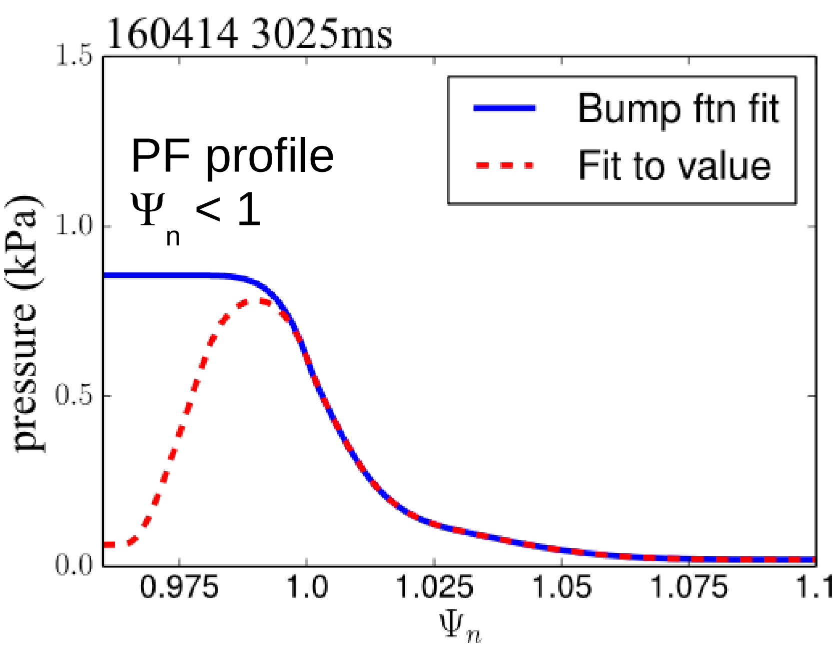
Compare each equilibria with four different cases

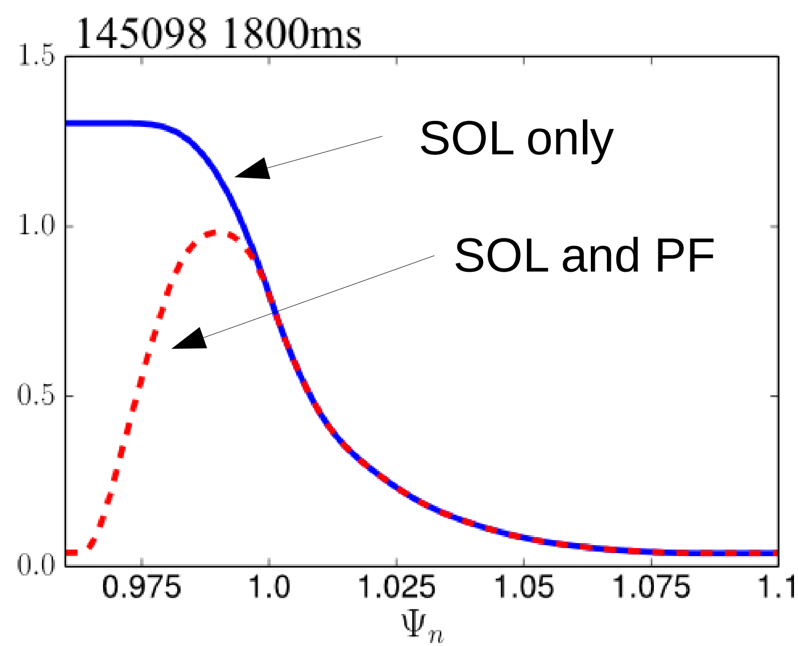
→ 8 total cases

Compare 4 different scenarios:

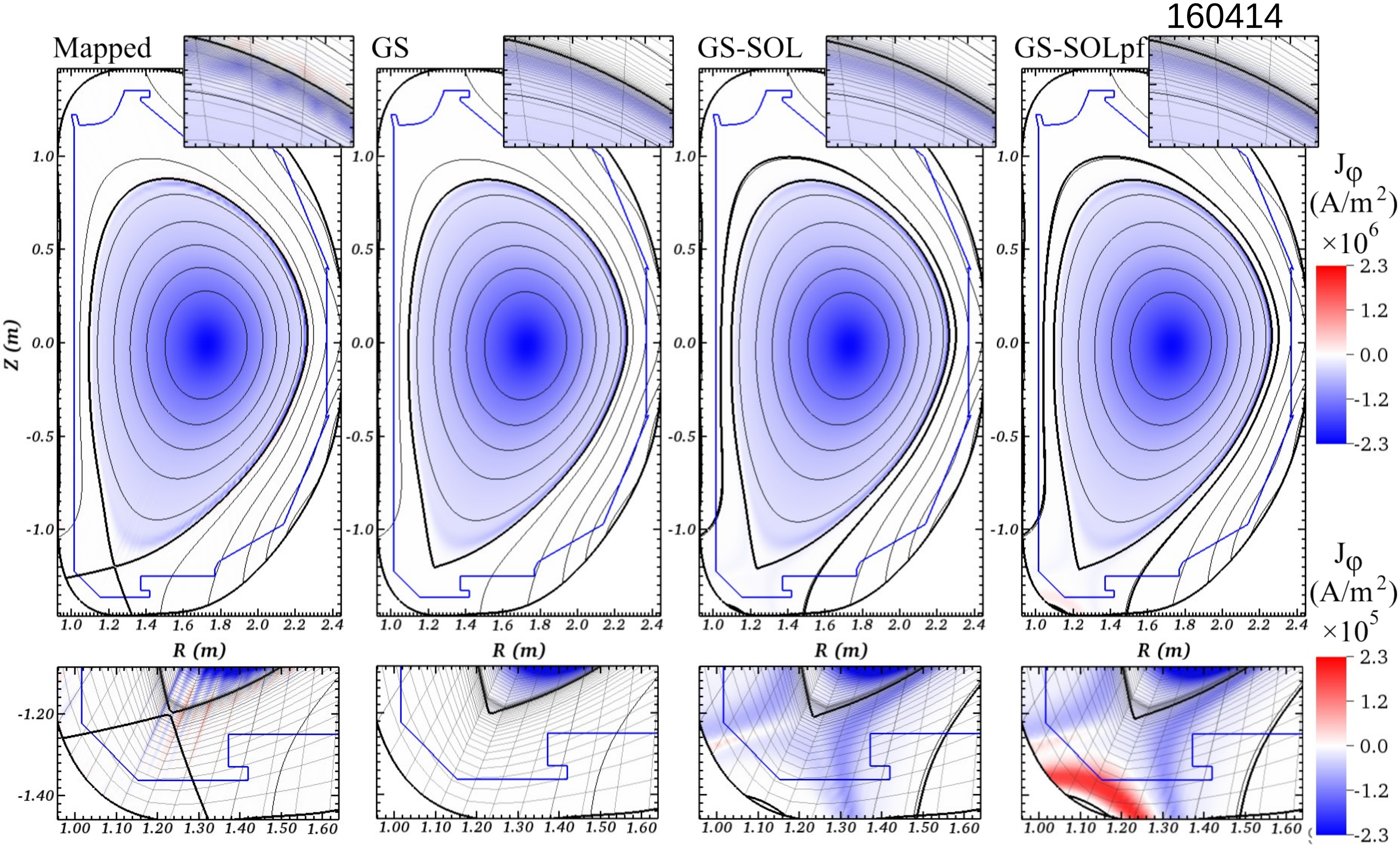
- Mapped
- GS re-solve, no SOL
- GS re-solve with SOL
- GS re-solve with SOL and PF

SOL cases maintain fixed current during re-solve
→ minor effect on result

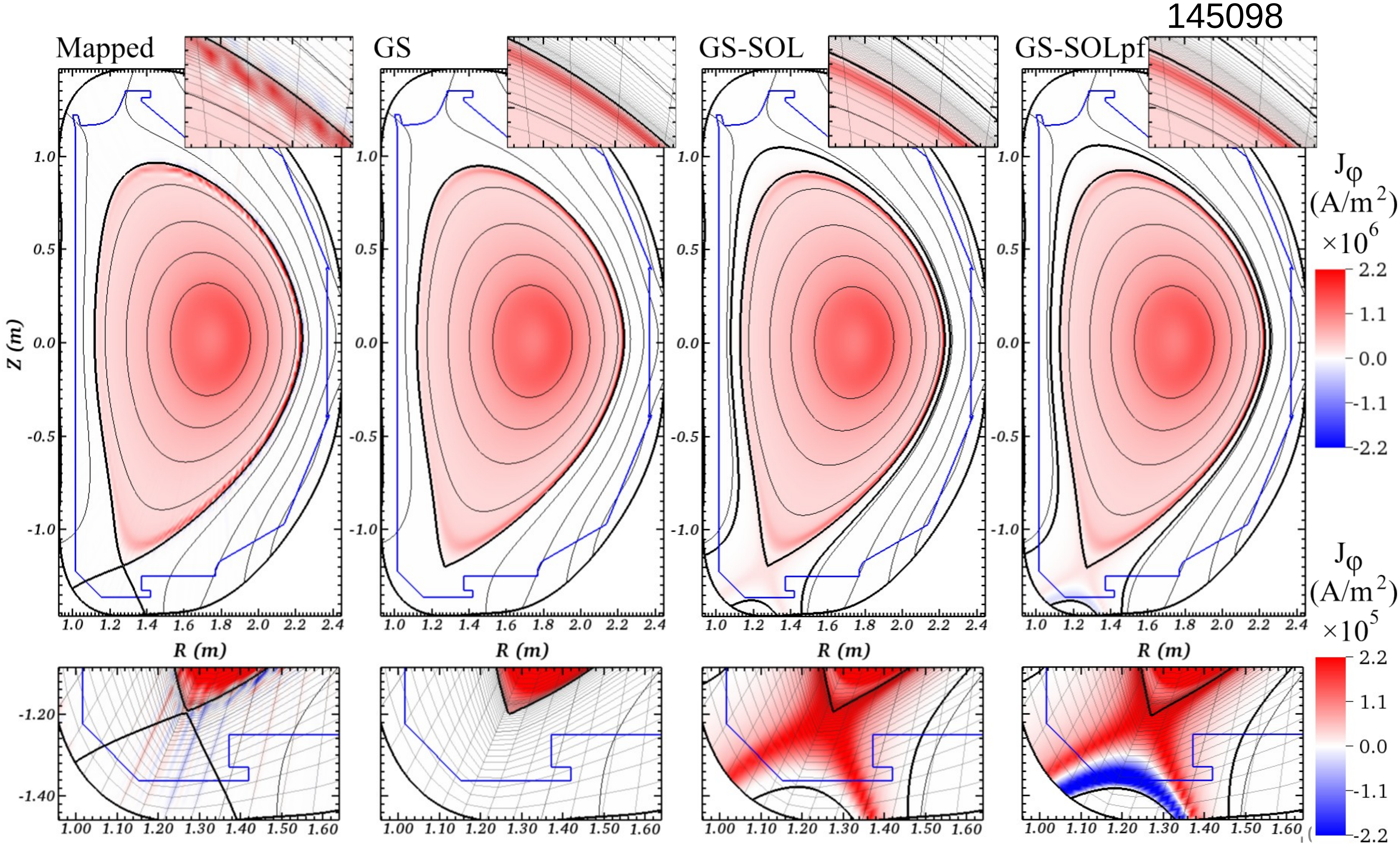




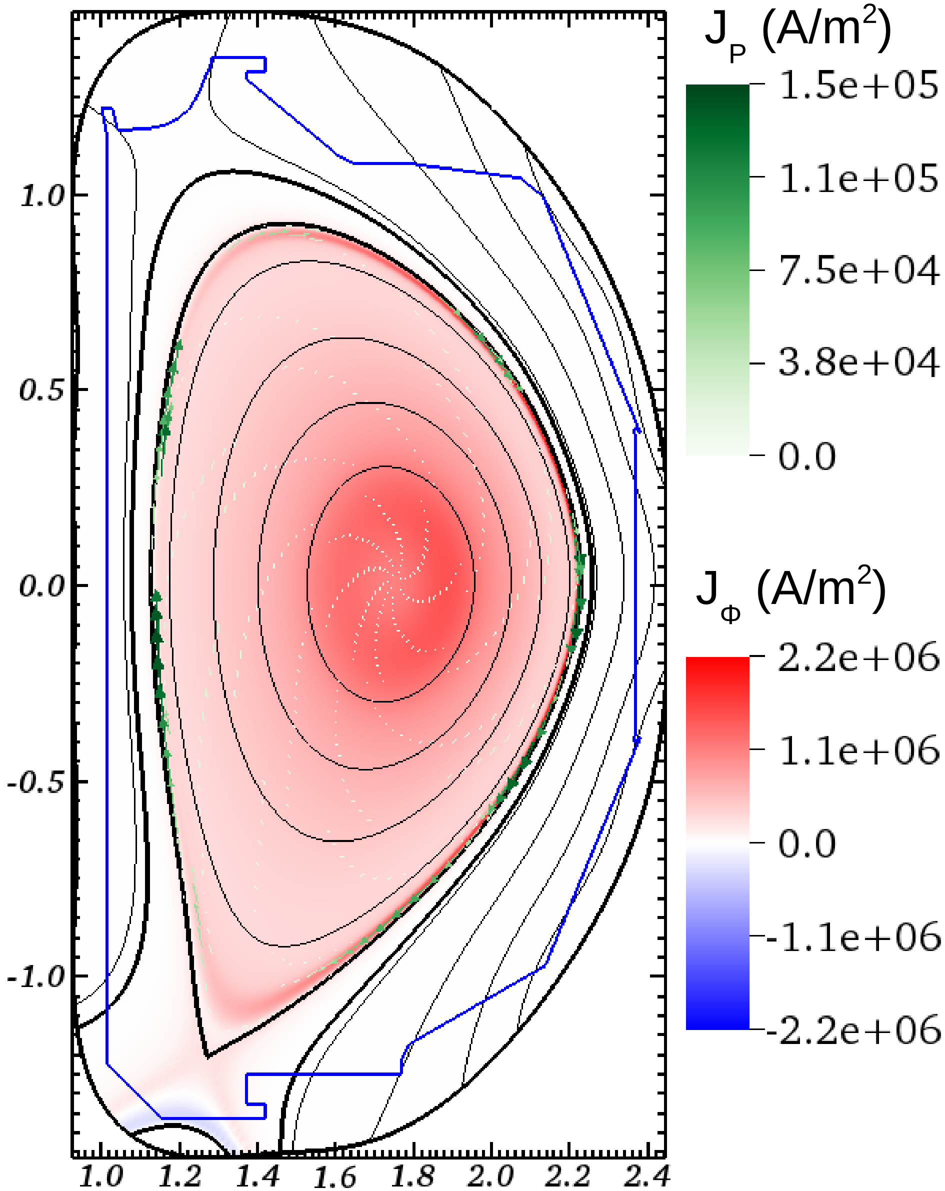
For the low edge-current case, inclusion of SOL produces similar current



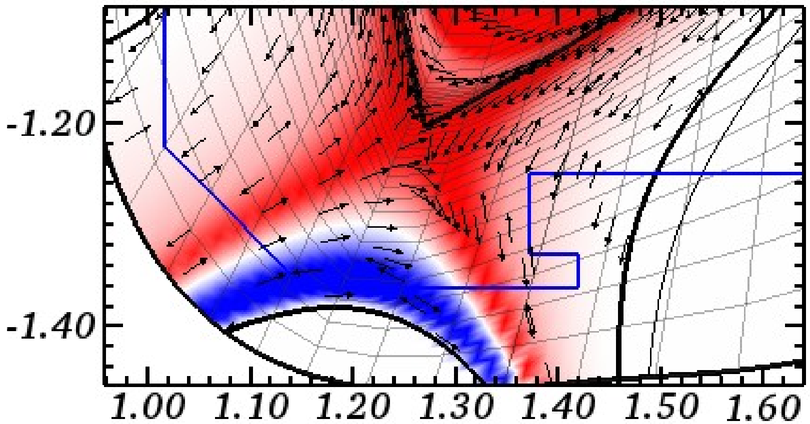
For high edge-current case, inclusion of SOL moves LCFS by ~2cm



Currents (and flows) extend into the divertor.



- Poloidal currents determined by RB_Φ profile
- Constraint: Divertor current limited to less than the ion saturation current [~2×10^4 A/m^2 for this case]
- Max values on plate are 500 A/m^2 for 145098 and 4000 A/m^2 for 160414



Comparison of measurements to NIMROD fields investigates reliability of final state

Thomson/CER/MSE/coils

p/n/T/B(R,Z)

EFIT

ψ(R,Z); p/n/T(ψ)

Comparison

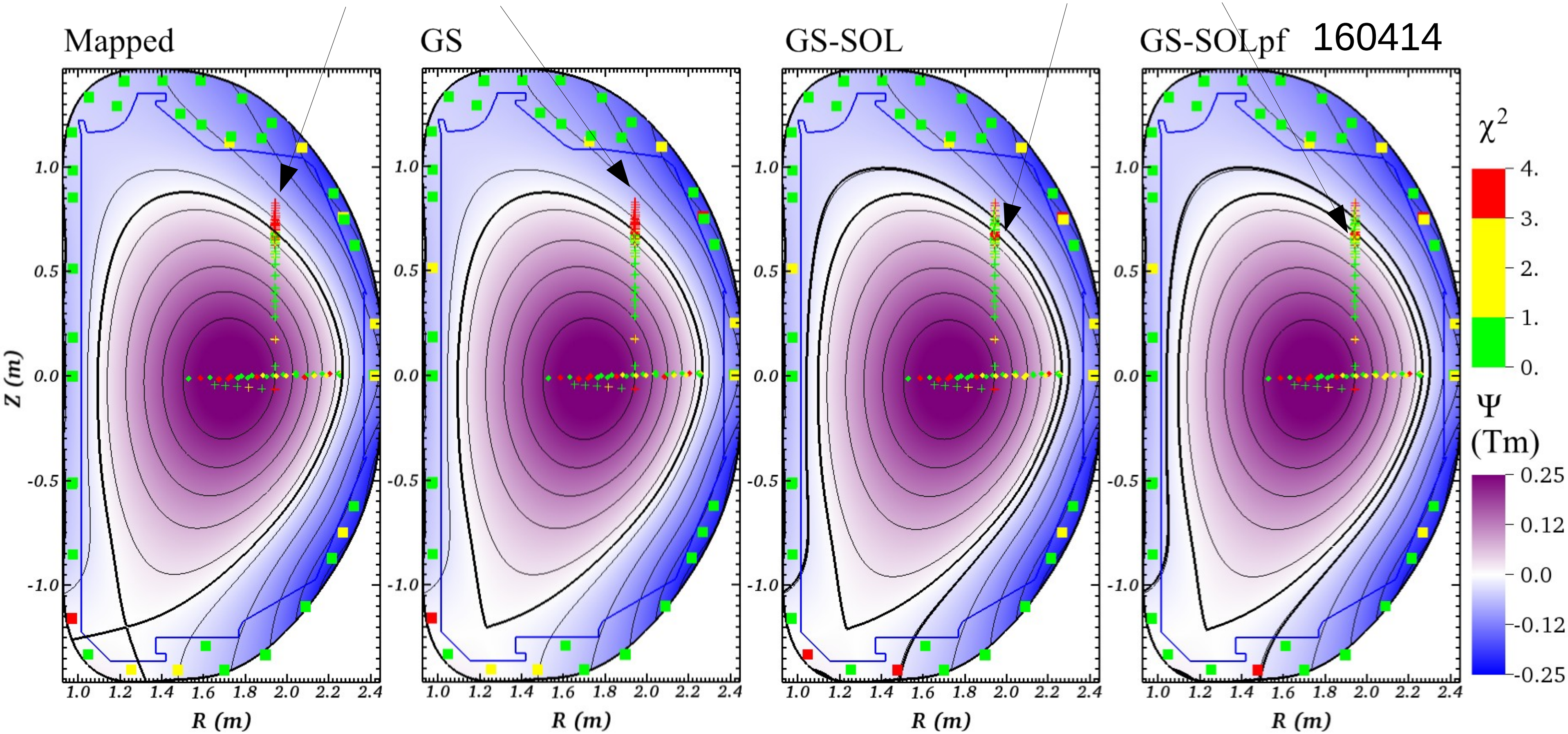
GS re-solve

p/n/T/B(R,Z)

For the low edge-current case, fit to data improves modestly

Poor agreement outside LCFS

Slight mismatch on LCFS location

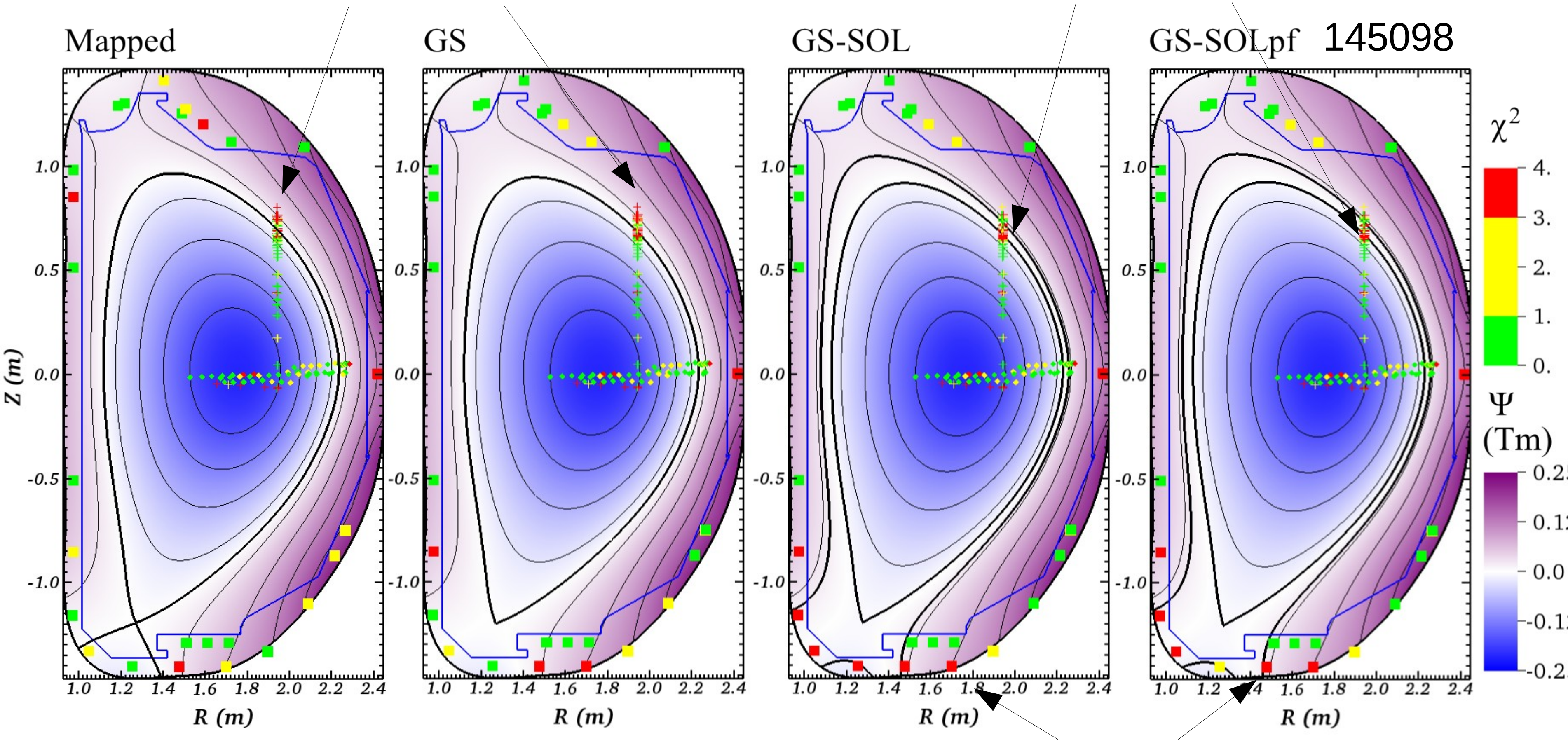


Crosses: Thomson n_e ; Diamonds: MSE; Squares: Coil B_p

For the high edge-current case, the benefits on including the SOL are mixed

Poor agreement outside LCFS

Slight mismatch on LCFS location



Coil comparison near footpoints poor

Crosses: Thomson n_e ; Diamonds: MSE; Squares: Coil B_p

Table summaries of modifications

Low current, 160414

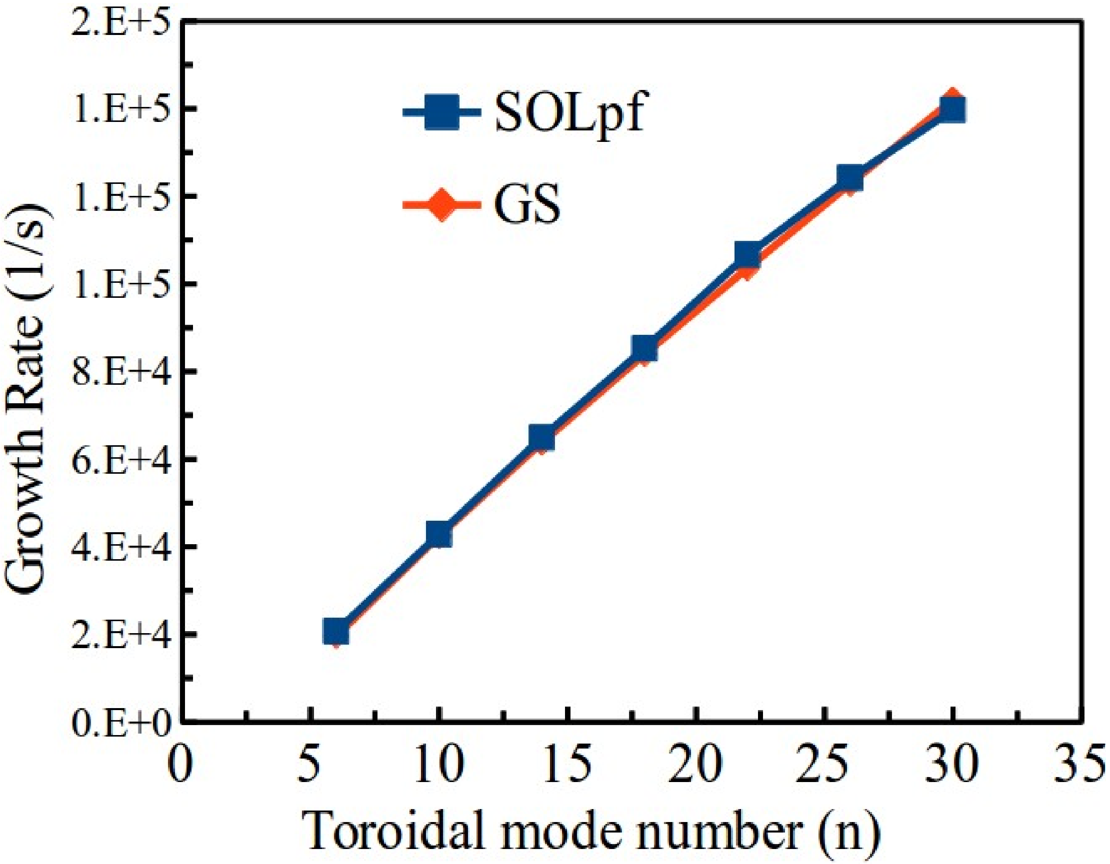
χ^2/N	mapped	GS	GS+SOL	GS+SOLpf
Thom. T_e	22.3	23.4	4.80	4.15
Thom. n_e	19.4	20.5	4.07	3.33
CER T_i	6.98	6.96	6.74	6.84
MSE	1.49	1.49	1.46	1.47
Mag. Coils	0.61	0.63	0.82	0.70
Δs	mapped	GS	GS+SOL	GS+SOLpf
$\Delta I/I_0$	6.95×10^{-5}	7.97×10^{-4}	3.22×10^{-7}	3.22×10^{-7}
$\Delta \mathbf{r}_{xpt}$ (cm)	N/A	ref.	0.72	1.07
$\Delta \mathbf{r}_{zmax}$ (cm)	N/A	ref.	0.35	0.19

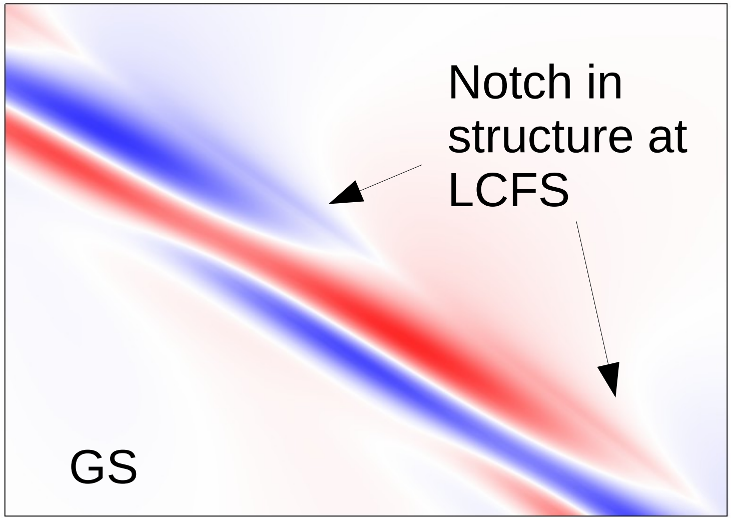
High current, 145098

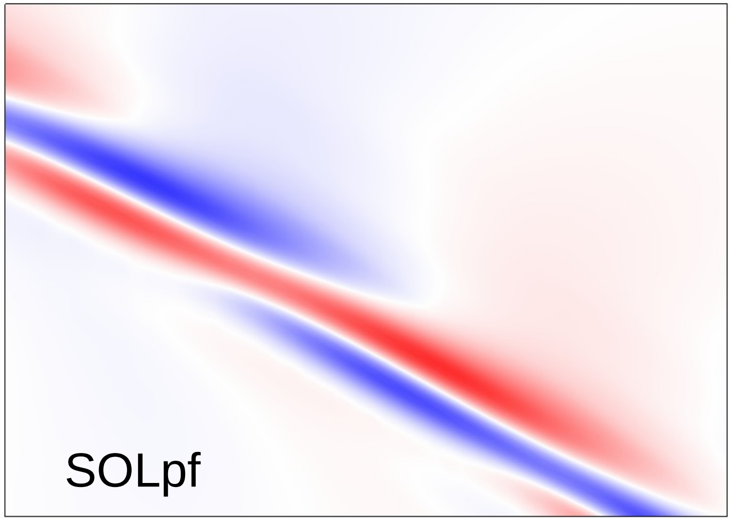
χ^2/N	mapped	GS	GS+SOL	GS+SOLpf
Thom. T_e	60.9	61.7	7.77	6.99
Thom. n_e	2.87	5.22	11.4	9.93
CER T_i	10.2	10.3	19.7	19.6
MSE	1.14	1.13	1.13	1.13
Mag. Coils	1.65	1.60	4.57	3.27
Δs	mapped	GS	GS+SOL	GS+SOLpf
$\Delta I/I_0$	9.86×10^{-5}	5.46×10^{-3}	-3.57×10^{-7}	-3.57×10^{-7}
$\Delta \mathbf{r}_{xpt}$ (cm)	N/A	ref.	0.86	0.55
$\Delta \mathbf{r}_{zmax}$ (cm)	N/A	ref.	2.82	2.12

- Low-current case: better match to experiment
- High-current case: LCFS motion and BC affect Thomson n_e + CER T_i and coil comparisons, respectively.
- How do modifications to equilibrium state affect the mode dynamics in the high-current case?

The linear growth rates are largely unaffected by the inclusion of the SOL current in the high-current case

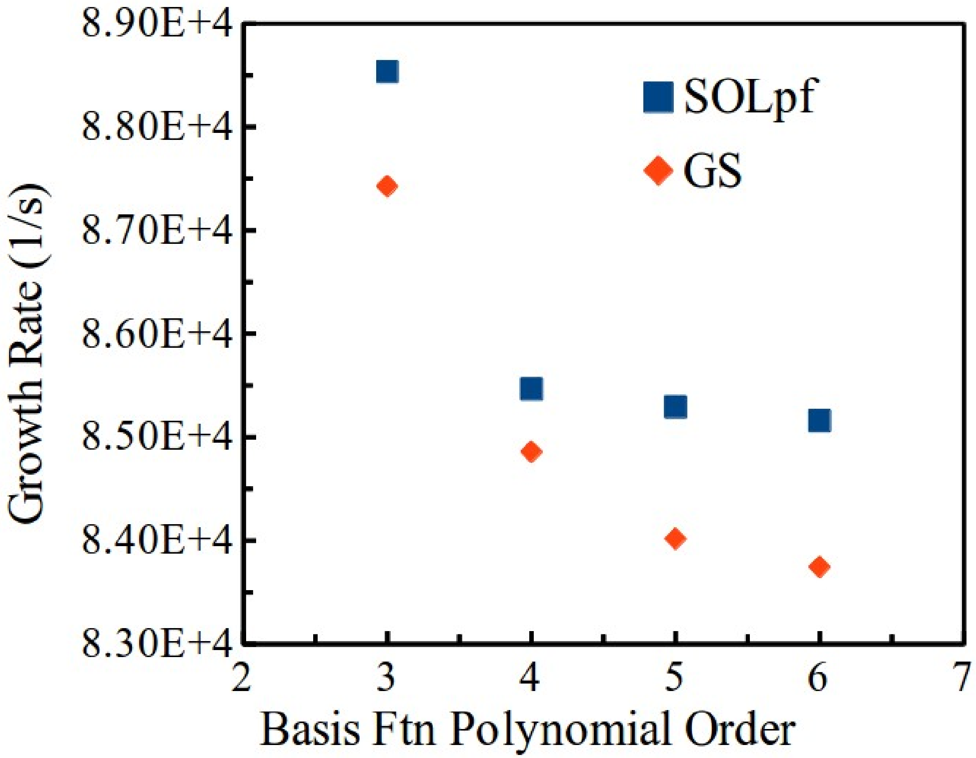


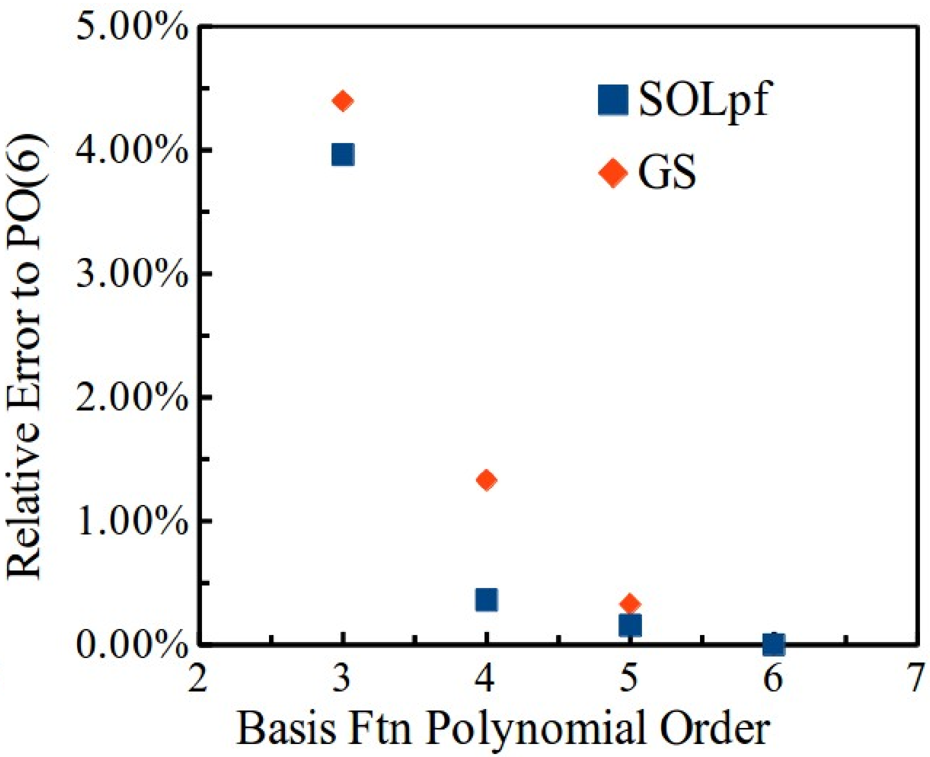




- Comparison with 72x512pd5 grid
- No flow

Inclusion of SOL currents improves rate of convergence





- Mode localized inside the LCFS
- Convergence effects likely more dramatic for nonlinear evolution of perturbations over the LCFS, but effect is difficult to quantify

Fixed boundary condition likely affects re-solved values at coils

- Currently we perform a fixed-boundary GS solve with the mapped ψ as the BC
- The boundary ψ is the superposition of the values resulting from the plasma current and external coils: $\psi = \psi_{plasma} + \psi_{coils}$
- A better solution may be possible with a free-boundary computation where ψ_{plasma} is allowed to change
- Free boundary solves have been implemented by C Sovinec

Nonlinear iteration: Converting to approximate-Newton starts with reorganization.

- The fixed-point iteration for nonlinear *F*(Λ) and *P*(Λ) had been organized for the linear Δ^* operator.

  For fixed-boundary computations, find $\Lambda_h^{k+1} \in L_{hp}$ such that

  $$\int_D R^2 \nabla\Omega \cdot \nabla\left(\Lambda_h^{k+1} + \Lambda_0\right) dVol = \int_D \Omega\left(FF' + \mu_0 R^2 P'\right)^k dVol$$

  for all $\Omega \in L_{hp}$ with the superscript *k* being the iterate label.

  - Note that $F' = \frac{d}{d\Psi} F\left(\Psi\right) = \frac{1}{R^2}\frac{d}{d\Lambda} F\left(\Lambda\right)$, for example.

- Equivalently, one may form a residual vector,

  $$H_{\Lambda_i}^k = \int_D \left[R^2 \nabla\alpha_i \cdot \nabla\left(\Lambda_h^k + \Lambda_0\right) - \alpha_i\left(FF' + \mu_0 R^2 P'\right)^k \right] dVol$$

  for all α_i and update with $\delta\underline{\Lambda} = -\underline{\underline{M}}^{-1}\underline{H}_\Lambda^k$, $M_{ij} = \int_D R^2 \nabla\alpha_i \cdot \nabla\alpha_j$.

With the residual-based formulation, full and approximate Newton result with different matrices.

- Formally, Newton's method uses the complete Jacobian matrix, and it changes with the iteration.

$$\underline{\underline{M}} \rightarrow \underline{\underline{M}}^k = \nabla_{\Lambda_h} \underline{H}_\Lambda^k$$

- With NIMEQ, as with other solvers, it is easier to use approximate Newton iteration.
  - Partial derivatives of the nonlinear terms are found via numerical difference approximation. For example,

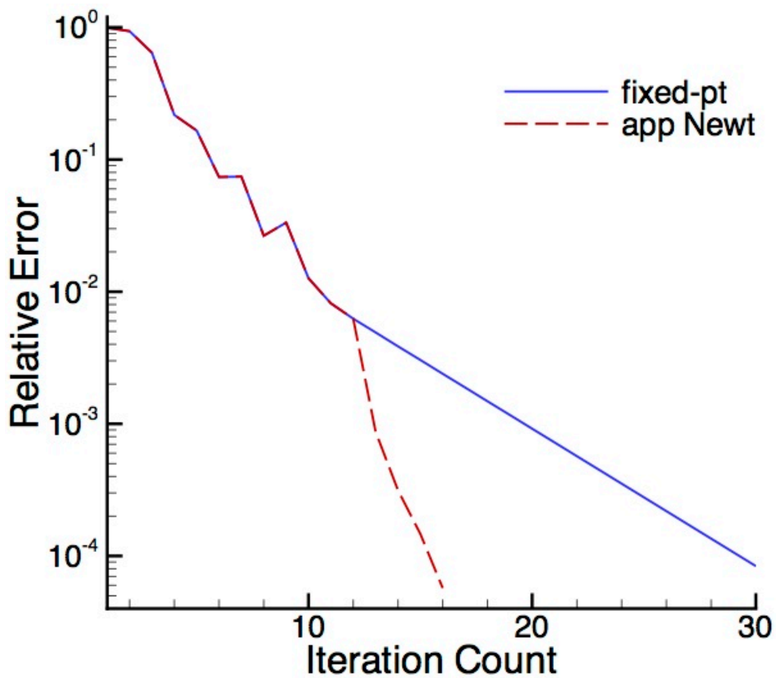
$$\frac{\partial}{\partial \Lambda} P' \cong \frac{\delta}{\delta \Lambda} P' \equiv \frac{P'(\Lambda + \delta\Lambda) - P'(\Lambda - \delta\Lambda)}{2\delta\Lambda}$$

    computed at nodes of the expansion and interpolated for the element computations. The *FF'* is treated similarly.

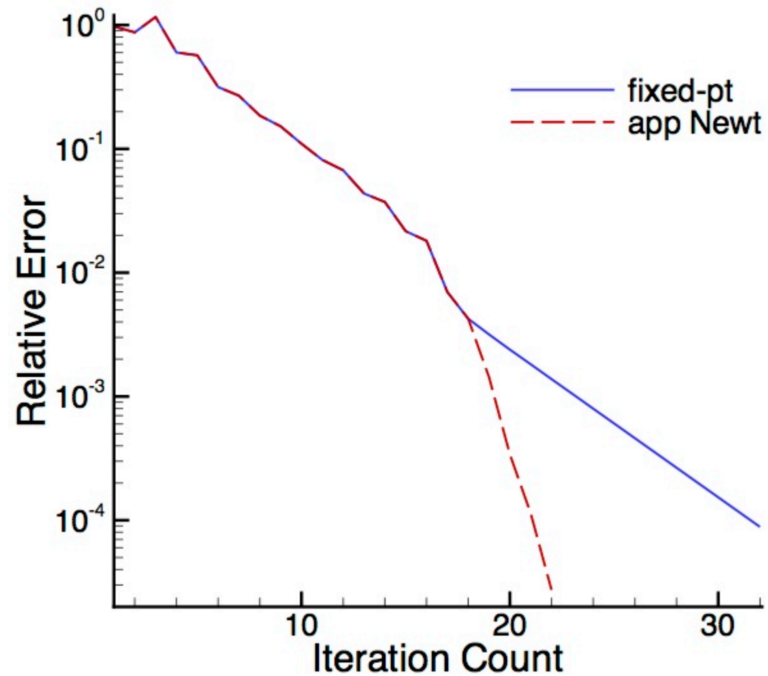
  - Also, the approximate differencing considers the separatrix shape to be held constant.

Initial results are encouraging in that approximate-Newton substantially reduces iteration.

- Approximate Newton reduces the iteration count in these fixed-boundary tests with or without **B**-tracing for distinguishing open and closed flux.



Fixed-boundary comparison without tracing (centering=0.75 for both).



Fixed-boundary comparison with B tracing (centering=0.75 for both).

Summary

- Modeling with SOL profiles eliminates edge-current (and flow) discontinuity
- Re-solved solution is consistent with GS equation
- Inclusion of SOL current impacts comparisons to experimental measurements:
  - Improves comparison with low SOL current
  - Mixed effect on comparison with higher SOL current
- Linear rate of convergence on edge modes are improved with SOL current
  - Effect likely more important for nonlinear evolution of perturbations through the LCFS
- Newly developed (C Sovinec) free-boundary and approximate Newton methods can impact experimental comparisons and performance through convergence rates