Speculations on the Form of Kinetic-MHD Closures

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Continuum ("Vlasov") & Particle Gyrokinetics w/ Electromagnetics making impressive progress

⊛ Dorland - Kotschenreuther "GS2"
3D toroidal fluxtube nonlinear fully electromagnetic gyrokinetic continuum code, doing high-n ITG/driftwave turbulence with $\delta \underset{\sim}{B}$

→ fully implicit linear terms (Kotschenreuther, Rewoldt, Tang, CPC 1995 for neat trick)

→ Direct implicit tricks harder in Waltz-Candy Global code, but nonlinear δt small electron Courant condition
⇒ Brute force
or use Hybrid method.

⊛ Hybrid Kinetic / fluid approach using $V_{te} \gg \frac{\omega}{k_{\parallel}}$

Snyder & Hammett, Y. Chen & Parker, Z. Lin & L. Chen, B. Cohen
(Varenna 9?d?)

Phys. Plasmas 2000/2001

Particle codes no longer limited to $\beta < \frac{m_e}{m_i}$.

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But if you want to try Landau-fluid closures...

Suggested Refs:

(*) Hammett, Dorland, Perkins Phys. Fluids B4, 2052 (1992)

Hammett et. al. PPCF 35 973 (1993)

(Chang & Callen Refs. in above)

* "Landau fluid model of Kulsrud's Collisionless MHD"

Snyder, Hammett, Dorland Phys. Plasmas 4, 3974 (1997)

(ignores FLR, which is treated in ~~the~~ Snyder's Thesis 1999 & papers (2001)).

One of the major challenges for fluid-like closures:

Handling all 3+ time scales:

Fast (ITG, drift waves)

$\tau \ll \tau_B$

Polarization Density

$k_\perp^2 \rho_i^2 \; n_{i0}$

Medium

$\tau_B \ll \tau \ll \tau_{ii} \frac{r}{R}$:

$k_\perp^2 \rho_b^2 \sqrt{\frac{r}{R}} \; n_{i0}$

"Neoclassical enhancement of polarization current"

reduces, but leaves an undamped component of, poloidal rotation

Long (longer than collisions)

$\tau_{ii} \frac{r}{R} \ll \tau$

"neoclassical" closures.

MHD

Prefer evolution of separate $\frac{\partial p_\parallel}{\partial t}$ & $\frac{\partial p_\perp}{\partial t}$ Eqs.

(1) Proper dependence on $\parallel$ & $\perp$ compression

$$\nabla_\parallel u_\parallel \quad \& \quad \nabla_\perp \cdot \underset{\sim}{u}_\perp$$

(2) Proper constraints on $\frac{\partial p_\perp}{\partial t}$ from μ conserv.

(3) Simplifies gyroviscous cancellations.

But have to include parallel conduction of $\parallel$ & $\perp$ heat:

$$q_\parallel = \int d^3v\, f m (v_\parallel - u_\parallel)^3$$

$$q_\perp = \int d^3v\, f m (v_\parallel - u_\parallel) \frac{v_\perp^2}{2}$$

Because simple CGL ignoring heat flows can be worse than MHD...

out problems with the CGL simpler systems with fewer re work could try to extend etic gyrokinetic equation or Chang and Callen for the

who have tried some forms equations. Bondeson and e-damped models of Landau ilization of external MHD gns. An important feature of gian variables so that the $|k_\|}|$ iid closures would (at least ng perturbed magnetic field showed was important to do. 'ard model was a relatively und was not entirely consis- ty in the derivation of the collisionality elsewhere. A Diamond[19] has incorporated into a set of two fluid equa- mplitude shear Alfvén and erplanetary plasmas. The ions assume isotropic pres- imited parameter regime (β l presented here should pro- is work, useful for the study well as for general problems ration in both laboratory and

er is as follows. In Section II onless MHD formulation. In based on Kulsrud's kinetic d. In Sections IV and V clo- models are derived, follow- Dorland.[12] In Section VI we icluding the reduction of the f the Braginskii equations. In nonlinear implementation of III, the Landau MHD formu- nirror instability, and in Sec- narks.

for the zeroth-order distribution function of each species $f_{0_s}(v_\|,\mu,\mathbf{r},t)$:

$$\frac{\partial f_{0_s}}{\partial t}+(v_\|\hat{\mathbf{b}}+\mathbf{v}_E)\cdot\nabla f_{0_s}+\left(-\hat{\mathbf{b}}\cdot\frac{D\mathbf{v}_E}{Dt}-\mu\hat{\mathbf{b}}\cdot\nabla B+\frac{e_s}{m_s}E_\|\right)\times\frac{\partial f_{0_s}}{\partial v_\|}=0, \tag{1}$$

where e_s is the charge on species s, $\hat{\mathbf{b}}$ is a unit vector in the magnetic field direction $\hat{\mathbf{b}}=\mathbf{B}/B$, $\mathbf{v}_E\doteq c(\mathbf{E}\times\mathbf{B})/B^2$, $\mu\doteq v_\perp^2/2B$, and $D/Dt\doteq\partial/\partial t+(v_\|\hat{\mathbf{b}}+\mathbf{v}_E)\cdot\nabla$.

Combining moments of this kinetic equation with Maxwell's equations and taking the usual low Alfvén speed limit $v_A^2\ll c^2$ yields Kulsrud's set of collisionless MHD equations:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{U})=0, \tag{2}$$

$$\rho\left(\frac{\partial\mathbf{U}}{\partial t}+\mathbf{U}\cdot\nabla\mathbf{U}\right)=\frac{(\nabla\times\mathbf{B})\times\mathbf{B}}{4\pi}-\nabla\cdot\mathbf{P}, \tag{3}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{U}\times\mathbf{B}), \tag{4}$$

$$\mathbf{P}=p_\perp\mathbf{I}+(p_\perp-p_\|)\hat{\mathbf{b}}\hat{\mathbf{b}}, \tag{5}$$

$$p_\perp=\sum_s\frac{m_s}{2}\int f_{0_s}v_\perp^2d^3v, \tag{6}$$

$$p_\|=\sum_s m_s\int f_{0_s}(v_\|-\mathbf{U}\cdot\hat{\mathbf{b}})^2d^3v, \tag{7}$$

$$\sum_s e_s\int f_{0_s}\,d^3v=0, \tag{8}$$

where ρ is the total mass density, $\mathbf{U}=\mathbf{v}_E+u_\|\hat{\mathbf{b}}$ is the fluid velocity, and $\mathbf{P}$ is the pressure tensor.

The above set of equations is exact to zeroth order in the expansion parameter, but the kinetic equation itself, Eq. (1), must be used to evaluate $p_\|$ and $p_\perp$ to close the system. Because Eq. (1) is difficult to solve directly, this system is rarely employed without further simplification.

One such simplification is the introduction of the double adiabatic law (also known as CGL theory[3,1]). In the CGL

GL model, while increasing
cluding models of kinetic
ccomplished by first taking
next section, closing the
analogous to those devel-
Hammett and Perkins.[10]
l adding Eq. (4) multiplied
in the phase space conserv-

$$\left[f_s B\left(-\hat{\mathbf{b}}\cdot \frac{D\mathbf{v}_E}{Dt} \right.\right.$$

$$). \qquad (11)$$

a suppressed. All calcula-
er distribution function in
ugh a subsidiary ordering
ndau closures.
operator to the right hand
low for generalization to
important role. Here a
employed:

$$(12)$$

n rate of species j with
f_j to relax to a shifted
perature of species j and

$$\left.\frac{(v_\parallel - u_{\parallel k})^2}{2T_j} - \frac{m_j \mu B}{T_j}\right], \qquad (13)$$

K collision operator in
m, and energy.
nents as follows:

... of Eq. (11) then leads to the following set of exact moment equations:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{U}) = 0, \qquad (14)$$

$$\frac{\partial u_\parallel}{\partial t} + \mathbf{U}\cdot\nabla u_\parallel + \hat{\mathbf{b}}\cdot\left(\frac{\partial \mathbf{v}_E}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{v}_E\right) + \frac{1}{nm_s}\nabla\cdot(\hat{\mathbf{b}}p_{\parallel_s}) - \frac{p_{\perp_s}}{nm_s}\nabla\cdot\hat{\mathbf{b}} - \frac{e_s}{m_s}E_\parallel = 0, \qquad (15)$$

$$\boxed{\frac{\partial p_{\parallel_s}}{\partial t}} + \nabla\cdot(\mathbf{U}p_{\parallel_s}) + \nabla\cdot(\hat{\mathbf{b}}q_{\parallel_s}) + 2p_{\parallel_s}\hat{\mathbf{b}}\cdot\nabla\mathbf{U}\cdot\hat{\mathbf{b}} - 2q_{\perp_s}\nabla\cdot\hat{\mathbf{b}} = -\tfrac{2}{3}\nu_s(p_{\parallel_s} - p_{\perp_s}), \qquad (16)$$

$$\boxed{\frac{\partial p_{\perp_s}}{\partial t}} + \nabla\cdot(\mathbf{U}p_{\perp_s}) + \nabla\cdot(\hat{\mathbf{b}}q_{\perp_s}) + p_{\perp_s}\nabla\cdot\mathbf{U} - p_{\perp_s}\hat{\mathbf{b}}\cdot\nabla\mathbf{U}\cdot\hat{\mathbf{b}} + q_{\perp_s}\nabla\cdot\hat{\mathbf{b}} = -\tfrac{1}{3}\nu_s(p_{\perp_s} - p_{\parallel_s}), \qquad (17)$$

$$\frac{\partial q_{\parallel_s}}{\partial t} + \nabla\cdot(\mathbf{U}q_{\parallel_s}) + \nabla\cdot(\hat{\mathbf{b}}r_{\parallel,\parallel_s}) + 3q_{\parallel_s}\hat{\mathbf{b}}\cdot\nabla\mathbf{U}\cdot\hat{\mathbf{b}} - \frac{3p_{\parallel_s}}{nm_s}\hat{\mathbf{b}}\cdot\nabla p_{\parallel_s} + 3\left(\frac{p_{\perp_s}p_{\parallel_s}}{nm_s} - \frac{p_{\parallel_s}^2}{nm_s} - r_{\parallel,\perp_s}\right)\nabla\cdot\hat{\mathbf{b}} = -\nu_s q_{\parallel_s}, \qquad (18)$$

$$\frac{\partial q_{\perp_s}}{\partial t} + \nabla\cdot(\mathbf{U}q_{\perp_s}) + \nabla\cdot(\hat{\mathbf{b}}r_{\parallel,\perp_s}) + q_{\perp_s}\nabla\cdot(u_\parallel\hat{\mathbf{b}}) - \frac{p_{\perp_s}}{nm_s}\hat{\mathbf{b}}\cdot\nabla p_{\parallel_s} + \left(\frac{p_{\perp_s}^2}{nm_s} - \frac{p_{\perp_s}p_{\parallel_s}}{nm_s} - r_{\perp,\perp_s} + r_{\parallel,\perp_s}\right)\nabla\cdot\hat{\mathbf{b}} = -\nu_s q_{\perp_s}, \qquad (19)$$

where $\rho = n(m_e + m_i)$, $\mathbf{U} = \mathbf{v}_E + u_\parallel\hat{\mathbf{b}}$, and $\nu_i = \nu_{ii} + \nu_{ie}$ and $\nu_e = \nu_{ee} + \nu_{ei}$.

Using the condition $u_{\parallel_i} = u_{\parallel_e}$ to solve for $E_\parallel$ [as given in Kulsrud's Eq. (49)], it is straightforward to show that the

Kind of like Grad 13-moment or 20-moment approach, uses $\nu, \omega << \Omega_c$ to reduce off-diagonal moments.

and $q_\perp$. We refer to such a model as a "3+1 model" because it evolves 3 parallel moments (n, $u_\parallel$, $p_\parallel$) and 1 perpendicular moment ($p_\perp$). Note that the CGL model is a 3 +1 model which invokes the simple closure $q_\parallel = q_\perp = 0$.

The 3+1 closures can be derived following the procedure laid out in the previous section, by writing $q_\parallel$ and $q_\perp$ as a sum of the lower moments and B_1, and solving for coef-

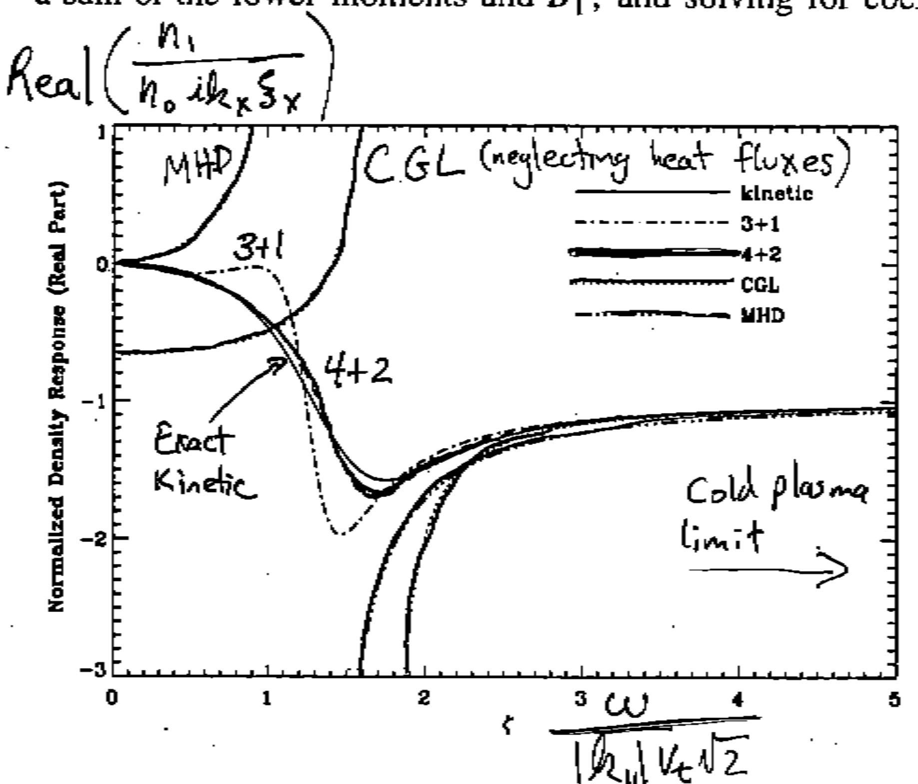


FIG. 1. The real part of the normalized linear density response $(n_1/ik_x\xi_x n_0)$, versus real normalized frequency ($\zeta_i = \omega/\sqrt{2}|k_\parallel| v_{T_{\parallel i}}$). The 3 +1 and 4+2 moment Landau MHD models are compared with linear kinetic theory. Predictions of CGL theory and ideal MHD theory are also shown. Parameters chosen are $Z=1$, $T_{\perp 0}/T_{\parallel 0}=1$, $T_{\perp 0i}=T_{\perp 0e}$, $T_{\parallel 0i}=T_{\parallel 0e}$, and $\sqrt{m_i/m_e}=40$.

$$n_{1s} = -\frac{in_0}{k_z T_{\parallel 0s}} e_s E_\parallel \mathscr{R}_3(\zeta_s) + \frac{B_1 n_0}{B_0}\left[1 - \frac{T_{\perp 0s}}{T_{\parallel 0s}} \mathscr{R}_3(\zeta_s)\right] \quad (4$$

and the perpendicular pressure response:

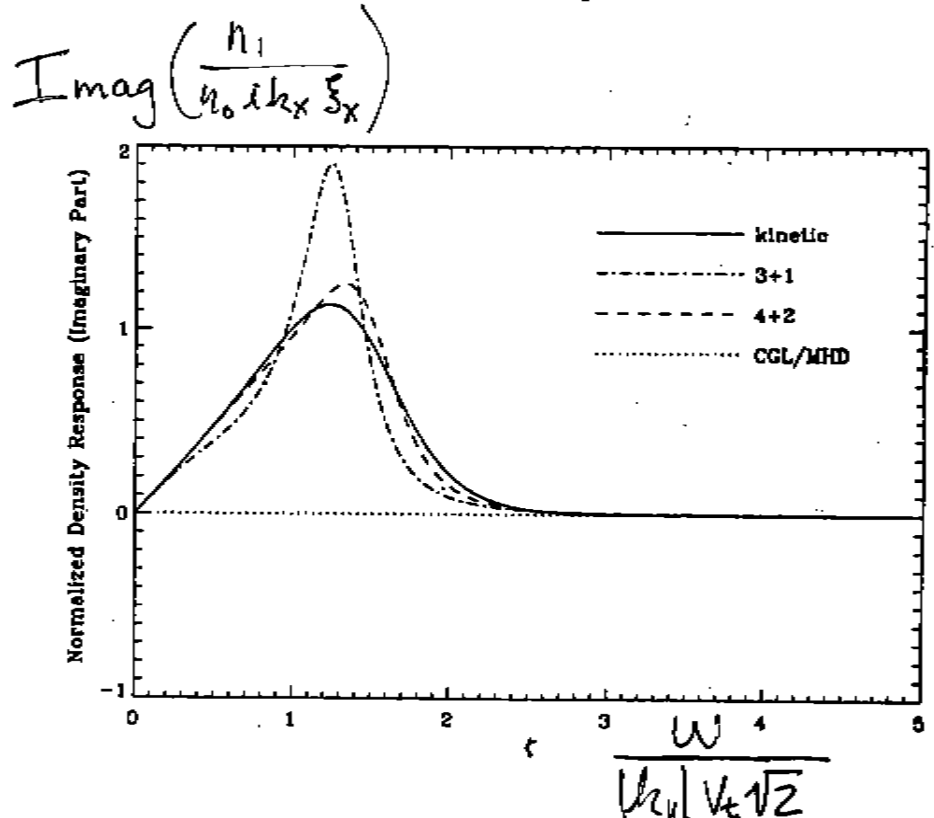


FIG. 2. The imaginary part of the normalized linear density respon $(n_1/ik_x\xi_x n_0)$, versus real normalized frequency ($\zeta_i = \omega/\sqrt{2}|k_\parallel| v_{T_{\parallel i}}$). The +1 and 4+2 moment Landau MHD models are compared with linear k netic theory. Both CGL theory and Ideal MHD theory predict no imagina density response. Parameters are identical to those in Fig. 1.

Mirror Growth rate

R INSTABILITY

; of our model, and the effects in simple colli- investigate the magnetic ; example to demonstrate theory and to expose the such as CGL.[1] We will iodels recover the exact iode, and provide a good rate above the threshold. l, homogeneous plasma charged ions. Take the direction, $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. The) be an anisotropic bi- and perpendicular tem- ectron and ion tempera- $T_{\parallel 0i} = T_{\parallel 0e} = T_{\parallel 0}$ and $T_{\perp 0i}$ n by writing the wave 'plasma displacement''

ming Eqs. (2) through ns of motion:

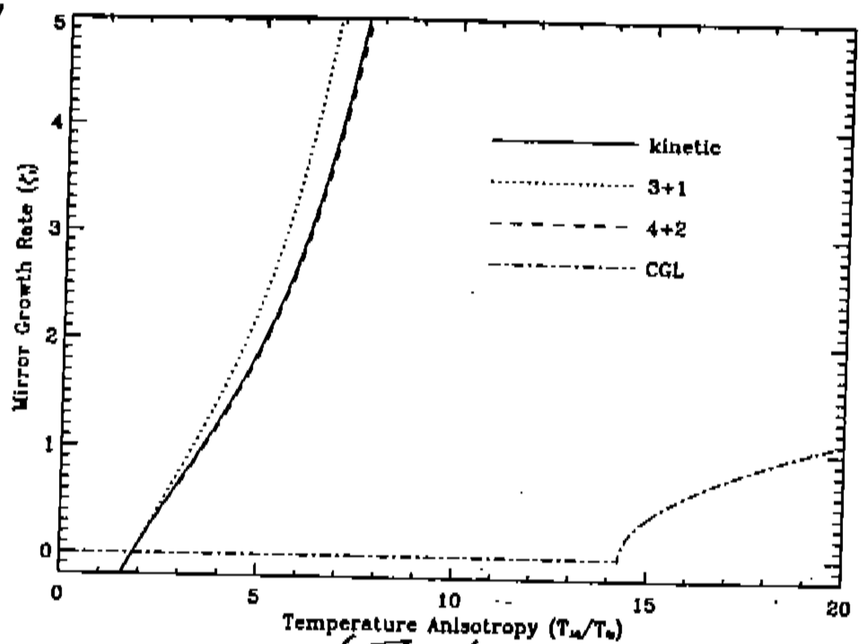
$${}_{0})\xi_x - (k_x^2 + k_z^2) \qquad (53)$$

$${}_{\perp 0})\xi_x, \qquad (54)$$

pressures is again sup- d pressures $p_\parallel$ and $p_\perp$ solve for the instability in four different ways: sing CGL theory, then and finally with the 4 compare the instability ermined by each.

proceed exactly as in itrality to solve for $E_\parallel$, yields

γ



$(T_{\perp 0}/T_{\parallel 0})$

FIG. 5. The linear growth rate of the mirror instability ($k_\perp^2 \gg k_\parallel^2$) as predicted by kinetic theory, 3+1 and 4+2 Landau MHD models, and CGL theory (ideal MHD cannot predict the mirror growth rate as it posits an isotropic pressure). The normalized growth rate $[\zeta_i = \mathrm{Im}(\omega)/\sqrt{2}|k_\parallel|v_{T_{0i}}]$ is plotted versus the temperature anisotropy $(T_{\perp 0}/T_{\parallel 0})$ at constant $\beta = \{(2/3)p_{\perp 0} + (1/3)p_{\parallel 0}\}/(B_0^2/8\pi)$. The parameters chosen are $Z=1$, $T_{\perp 0i} = T_{\perp 0e}$, $T_{\parallel 0i} = T_{\parallel 0e}$, $\beta = 1$, and $\sqrt{m_i/m_e} = 40$.

$$\zeta_i^2 + \zeta_e^2 = 2\frac{k_x^2}{k_z^2}\left(-\frac{T_{\perp 0}^2}{T_{\parallel 0}^2}\mathscr{A}_k(\zeta) + \frac{T_{\perp 0}}{T_{\parallel 0}} + \frac{B_0^2}{8\pi p_{\parallel 0}}\right) + \left(\frac{T_{\perp 0}}{T_{\parallel 0}} - 1 + \frac{B_0^2}{4\pi p_{\parallel 0}}\right), \qquad (58)$$

where the function $\mathscr{A}_k(\zeta)$ is defined by $\mathscr{A}_k(\zeta) = \{\mathscr{R}(\zeta_i)^2 + 6\mathscr{R}(\zeta_i)\mathscr{R}(\zeta_e) + \mathscr{R}(\zeta_e)^2\}/\{4(\mathscr{R}(\zeta_i) + \mathscr{R}(\zeta_e))\}$. For parallel propagation ($|k_z| \gg |k_x|$), the above reduces to the dispersion relation for the "firehose" instability, and the kinetic effects drop out within our ordering (note that a different ordering can be used to analyze these much smaller kinetic effects for limited parameter regimes—see Medvedev and Diamond[19]). All of the models considered will reproduce the firehose linear growth rate exactly. In the opposite limit ($|k_x| \gg |k_z|$), the dispersion relation becomes

Neoclassical Improvements to Gyrofluid Closures

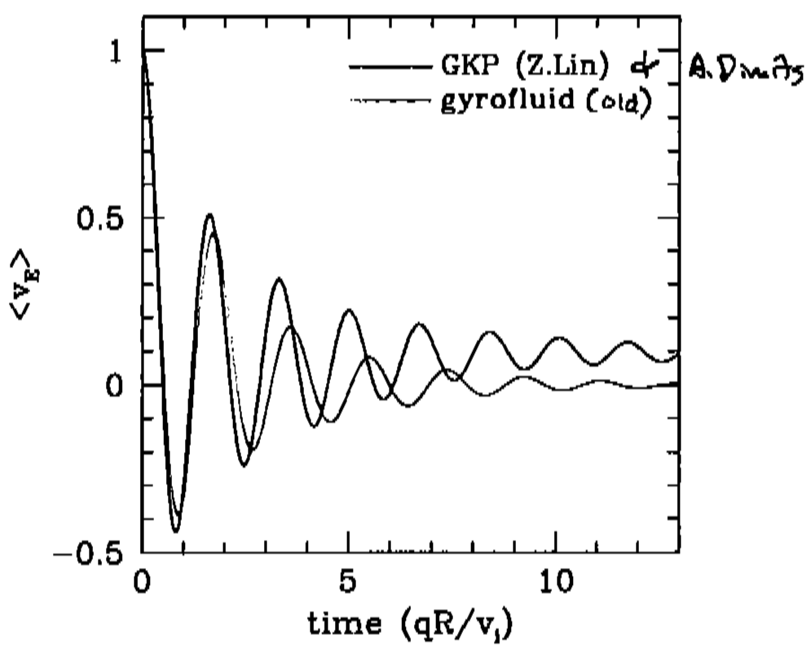
Status Summary:

* First cut at neoclassical improvement accounts for $\sim \frac{1}{2}$ of Rosenbluth-Hinton zonal flows & $\sim \frac{1}{2}$ of Gyrofluid / Gyrokinetic χ_i differences.

* 6 ideas to further improve closures.

Importance of Linear Zonal Flow Damping

- Two phases: fast collisionless damping & slow collisional damping. Depends on initial flow conditions
- In [Beer, Ph.D. Thesis (1995)] showed that our gyrofluid equations accurately model the fast linear collisionless damping for $t < qR/v_{ti}\sqrt{\epsilon}$. Argued that long time linear flow dynamics are not important, nonlinear effects will dominate long term nonlinear flow evolution.



- [Rosenbluth & Hinton, PRL (1998)] emphasized a linearly undamped flow component. This "residual" flow damped by collisional effects. Argued that *nonlinearly*, residual component should grow in time $\sim \sqrt{t}$ in collisionless limit. Modeled nonlinear drive term as a white noise source.
- Since our original gyrofluid eqns underestimate residual component, if residual component is important nonlinearly, gyrofluid simulations would underestimate $\mathbf{E} \times \mathbf{B}$ flow levels and overpredict χ_i.

Orignal Gyrofluid Closure:

$$\frac{\partial T_\parallel}{\partial t} = \quad \cdots \quad - \nabla_\parallel q_\parallel$$

$$\frac{\partial q_\parallel}{\partial t} = \quad \cdots \quad - 3 \nabla_\parallel T_\parallel \underbrace{- |k_\parallel| v_t q_\parallel}$$

causes $q_\parallel$ ($+ T_\parallel$) to relax to be constant on a flux surface.

But actual equilibrium is <u>not</u> constant on flux surface.

$$v_\parallel \nabla_\parallel f_0 + \overbrace{\underbrace{\vec{v}_d \cdot \nabla f_0}}^{\mathcal{O}(\frac{\rho_b}{a}) \text{ smaller}} = 0$$

radial gradients must be balanced by parallel gradients

In equilibrium:

$$\nabla_{\parallel} q_{\parallel 0} \propto \frac{\partial T_0}{\partial r}$$

Like "Pfirsch-Schluter" current:

$$\nabla \cdot \left(\vec{j}_{TOT} \right) = 0 = \nabla \cdot \left(j_{\parallel} \hat{b} + \vec{j}_{\perp} \right)$$

$$\vec{j}_{\perp} = \frac{c \vec{B} \times \nabla p}{B^2}$$

$$\boxed{\nabla_{\parallel} j_{\parallel, p.s.} \propto \frac{\partial p}{\partial r}}$$

So modify

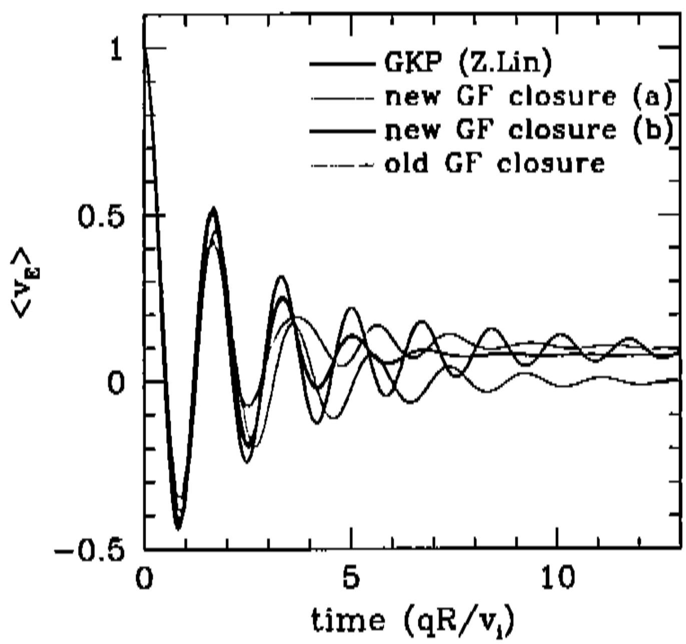
$$|k_{\parallel}| V_t \, q_{\parallel} \longrightarrow |k_{\parallel}| V_t \underbrace{\left(q_{\parallel} - q_{\parallel \, p.s.} \right)}_{\substack{\text{Turns off} \\ \text{dissipation when} \\ q_{\parallel} = q_{\parallel \, p.s} = q_{\parallel 0}}}$$

(Also set $|\omega_d| = 0$ for $m = n = 0$ zonal flows.)

This simple change accounted for $\sim \frac{1}{2}$ of Gyrofluid / Gyrokinetic differences

Comparison of Gyrokinetic and Gyrofluid Flow Damping With New Closures

New closures agree reasonably well with gyrokinetic results on amplitude of residual component for $k_r \rho_i = 0.2$:



This is for DIII-D 81499 parameters, $\epsilon = .18$, $q = 1.4$.

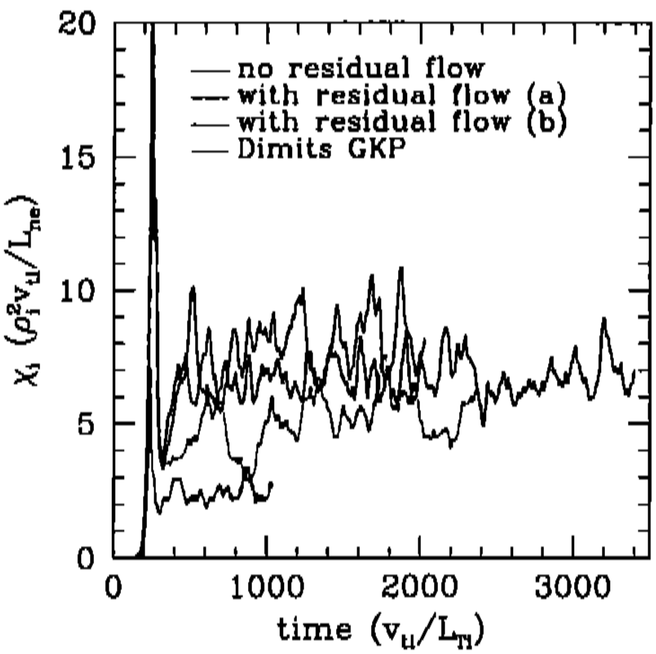
Reasonable agreement with Rosenbluth-Hinton formula:

$$\frac{v_{Ef}}{v_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

where $c = 0.625$, which predicts $v_{Ef}/v_{Ei} = 0.12$.

Nonlinear Tests of Importance of Residual Flow

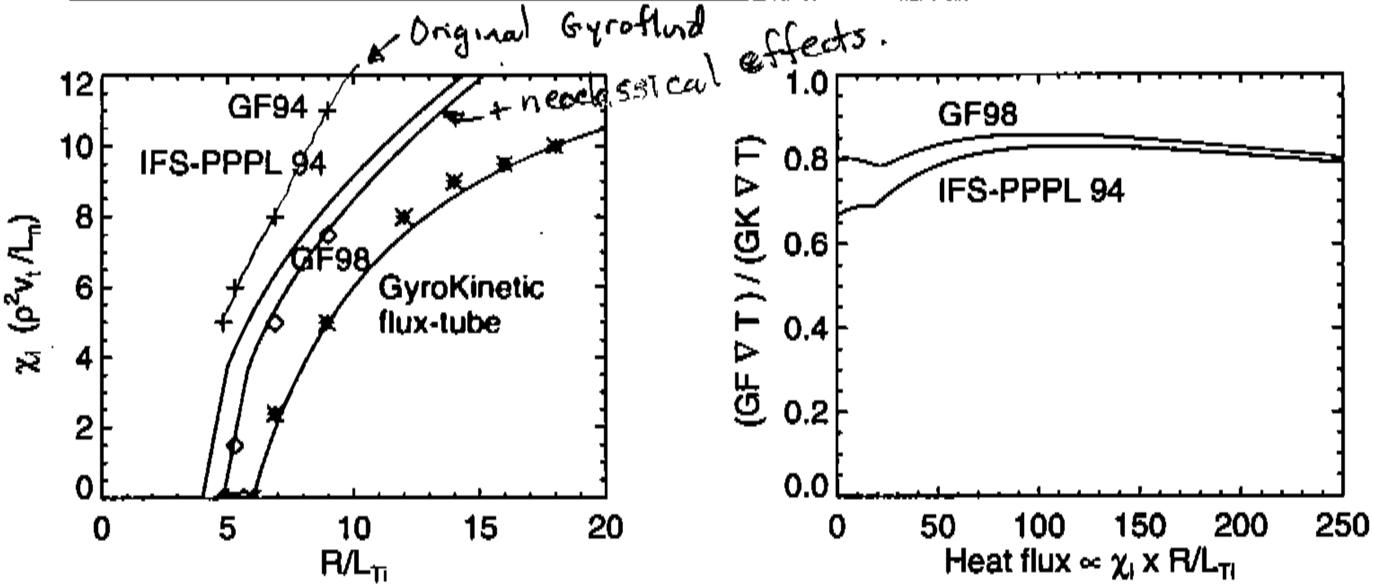
For parameters from DIII-D shot 81499 (the Cyclone base case, with $R/L_{Ti} = 6.9$), we repeat nonlinear runs with the new closures (a) and (b), both including undamped components of the zonal flow.



With residual flows, flux drops by up to about 35%, for this case

Nonlinear effects (*e.g.* turbulent viscosity) keep linearly undamped residual components from growing indefinitely

Gyrofluid/gyrokinetic (GF/GK) simulation differences correspond to 20-30% differences in the predicted temperature gradient.



•Dimits (LLNL): good convergence in his gyrokinetic particle simulations

•New neoclassical gyrofluid closure significantly improves GF/GK comparison

•Turning this plot around, for a fixed amount of heat flux, $\propto \chi \nabla T$, the temperature gradient predicted by the original gyrofluid-based IFS-PPPL model is 20-33% low. But $P_{\rm fusion} \propto T^2$, and so may increase by $\times 2$ or more.

•Nonlinear upshift in critical gradient may depend on: Rosenbluth-Hinton undamped zonal flows ↑ with elongation (W. Dorland), ↓ with weak collisions, ↓ with non-adiabatic electrons [may limit inverse cascade that drives zonal flows (Diamond, Liang, Terry-Horton, Waltz, ...) and ↑ turbulent viscosity].

Further Improvements to try soon:

(1) Also replace $|\omega_d|\, T_\parallel \rightarrow |\omega_d|\,(T_\parallel - T_{\parallel 0})$

$\propto \nabla_\parallel q_\parallel$

instead of $|\omega_d| = 0$ always for zonal flow.

(2) Instead of using flux surface averaged ~~[illegible]~~

$$\nabla_\parallel q_{\parallel 0} \propto \frac{\partial \langle T \rangle}{\partial r}$$

Use local moments of $v_\parallel \nabla_\parallel f_0 + v_d \cdot \nabla f_0 = 0$

(3) Finite banana width corrections
(Should get $k_r \rho_b$ equivalent of $J_0(k_\perp \rho_i)$ Bessel functions).

(4) Test on exact CGL equilibria in Mirror limit

$$|k_\parallel|\, q_\parallel \rightarrow B\, |k_\parallel| \left(\frac{q_\parallel}{B} \right)$$

(5) Frequency dependent closures á lá Mattor + Callen?
(improve $\sqrt{\frac{\omega}{\omega_d}}$ branch cut in toroidal resonance response).

(6) Revisit $|\omega_d|$ closure coefficients.
(complicated fit of 10 coefficients, local minima?).

Frequency - Dependent Closures

* I've use ~~ξ~~ ω-independent closures
  ($\omega = 0$ limit of Chang-Callen $\Rightarrow$ gives n-pole approx. to $Z\left(\frac{\omega}{|k_\parallel| v_t}\right)$
  $\omega \gg k_\parallel v_t$ is cold plasma limit where closure irrelevant.)

* Chang-Callen approach: exact linear closure in terms of $Z\left(\frac{\omega}{|k_\parallel| v_t \sqrt{2}}\right)$

* Mattor suggested an instantaneous WKB approx.

$$\omega \approx i \frac{\partial \Phi}{\partial t} / \Phi \qquad \left(\text{or } \omega = i \frac{\partial \tilde{T}}{\partial t} / \tilde{T}\right.$$

or WKB analog for real fields)

Rationale:

1) Near Marginal Stability, $\gamma \ll Re(\omega)$
   this is an accurate measure of ω

2) far from Marginal Stability, this instantaneous approx. for ω will vary in time & sample $Z(\omega)$ over a relevant range of ω.

[But is this WKB approx well-behaved? ...]