Braginskii's gyroviscous force

J.J. Ramos (March, 2005)

Braginskii's gyroviscous stress tensor is:

$$\Pi_{ij} \;=\; \frac{mp}{4eB^2}\left[\epsilon_{ikl}\Big(\delta_{jm}+3\frac{B_jB_m}{B^2}\Big)+\epsilon_{jkl}\Big(\delta_{im}+3\frac{B_iB_m}{B^2}\Big)\right]B_k\Big(\frac{\partial u_l}{\partial x_m}+\frac{\partial u_m}{\partial x_l}\Big). \tag{1}$$

Its divergence, in coordinate-free vector notation, is exactly:

$$\begin{aligned}\nabla\cdot\Pi \;=\;& \Big\{\Big[\nabla\times\Big(\frac{mp}{eB^2}\mathbf{B}\Big)\Big]\cdot\nabla\Big\}\mathbf{u} \;-\; \nabla\Big[\frac{mp}{2eB^2}\mathbf{B}\cdot(\nabla\times\mathbf{u})\Big] \;-\\ &-\nabla\times\left\{\frac{mp}{eB^2}\left[(\mathbf{B}\cdot\nabla)\mathbf{u} \;+\; \frac{1}{2}\left(\nabla\cdot\mathbf{u}-\frac{3}{B^2}\mathbf{B}\cdot\Big[(\mathbf{B}\cdot\nabla)\mathbf{u}\Big]\right)\mathbf{B}\right]\right\}+\\ &+(\mathbf{B}\cdot\nabla)\left\{\frac{mp}{eB^2}\left(\frac{3}{B^2}\mathbf{B}\times\Big[(\mathbf{B}\cdot\nabla)\mathbf{u}\Big] \;+\; \frac{3}{2B^2}\Big[\mathbf{B}\cdot(\nabla\times\mathbf{u})\Big]\mathbf{B} \;-\; \nabla\times\mathbf{u}\right)\right\}.\end{aligned} \tag{2}$$

In the momentum equation, the first term of $\nabla\cdot\Pi$ can be combined with the convective derivative term $mn(\mathbf{u}\cdot\nabla)\mathbf{u}$. There, it cancels partially (i.e. except for derivatives of the magnetic field) the part convected by the diamagnetic drift velocity, $mn(\mathbf{u}_d\cdot\nabla)\mathbf{u}$, where $\mathbf{u}_d=\mathbf{B}\times\nabla p/(enB^2)$. This is the famous gyroviscous cancellation which, given the numerous terms remaining both in $\nabla\cdot\Pi$ and in $mn(\mathbf{u}\cdot\nabla)\mathbf{u}$, I consider quite useless. As I like to put it, the gyroviscous cancellation is very important when you throw away all the terms that do not cancel. The second and fourth terms can be combined with the divergence of the Chew-Goldberger-Low pressure tensor:

$$\nabla\cdot\left[p\mathbf{I}+(p_\parallel-p_\perp)\Big(\frac{1}{B^2}\mathbf{B}\mathbf{B}-\frac{1}{3}\mathbf{I}\Big)\right] \;=\; \nabla\Big[p-\frac{1}{3}(p_\parallel-p_\perp)\Big] \;+\; (\mathbf{B}\cdot\nabla)\Big[\frac{(p_\parallel-p_\perp)}{B^2}\mathbf{B}\Big]. \tag{3}$$

Therefore, the terms proportional to the parallel vorticity $\mathbf{B}\cdot(\nabla\times\mathbf{u})$ in $\nabla\cdot\Pi$, can be regarded as an additional contribution to the pressure anisotropy or parallel viscosity $(p_\parallel-p_\perp)$, not to the scalar pressure p:

$$(p_\parallel-p_\perp) \;\rightarrow\; (p_\parallel-p_\perp) \;+\; \frac{3mp}{2eB^2}\mathbf{B}\cdot(\nabla\times\mathbf{u}) \tag{4}$$