Nonlinear Extended MHD Simulation Using High-Order Finite Elements

C. Sovinec, C. Kim, H. Tian, *Univ. of WI-Madison*

D. Schnack, A. Pankin, *Science Apps. Intl. Corp.-San Diego*

D. Brennan, *MIT*

E. Held, *Utah State University*

D. Kaushik, *Argonne Nat. Lab.*

S. Kruger, *Tech-X Corp.*

X. Li, *Lawrence-Berkeley Lab.*

S. Jardin, *Princeton Pl. Phys. Lab.*

and the NIMROD Team

Scientific Discovery through Advanced Computing

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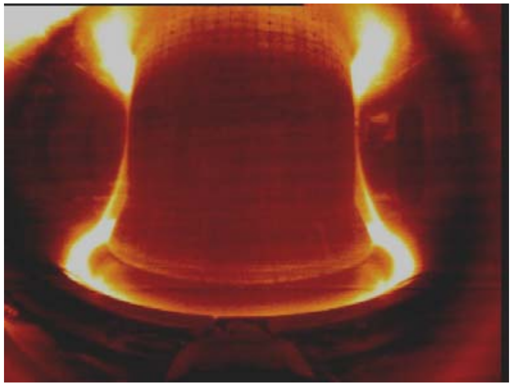
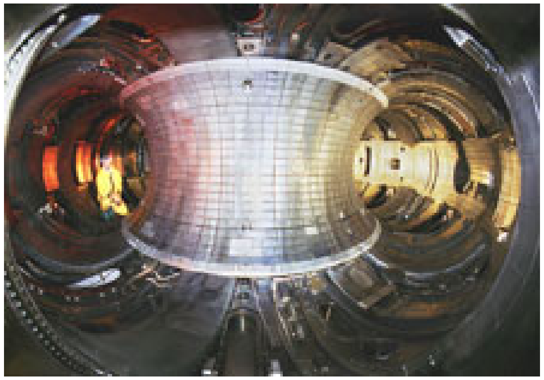
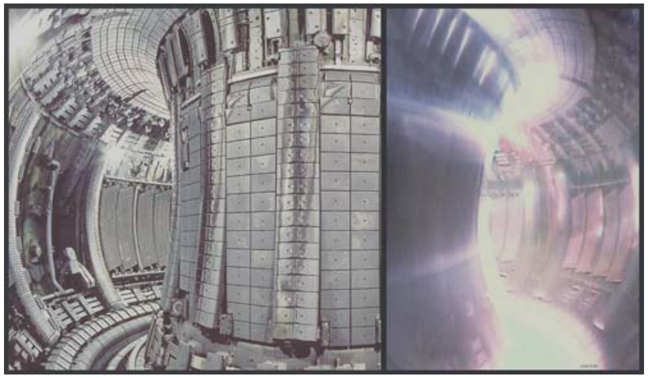
As the development of magnetic plasma confinement approaches conditions for ignition, ...



Alcator C-mod, Massachusetts Institute of Technology



TFTR, Princeton Plasma Physics Laboratory



Joint European Torus, European Fusion Development Agreement

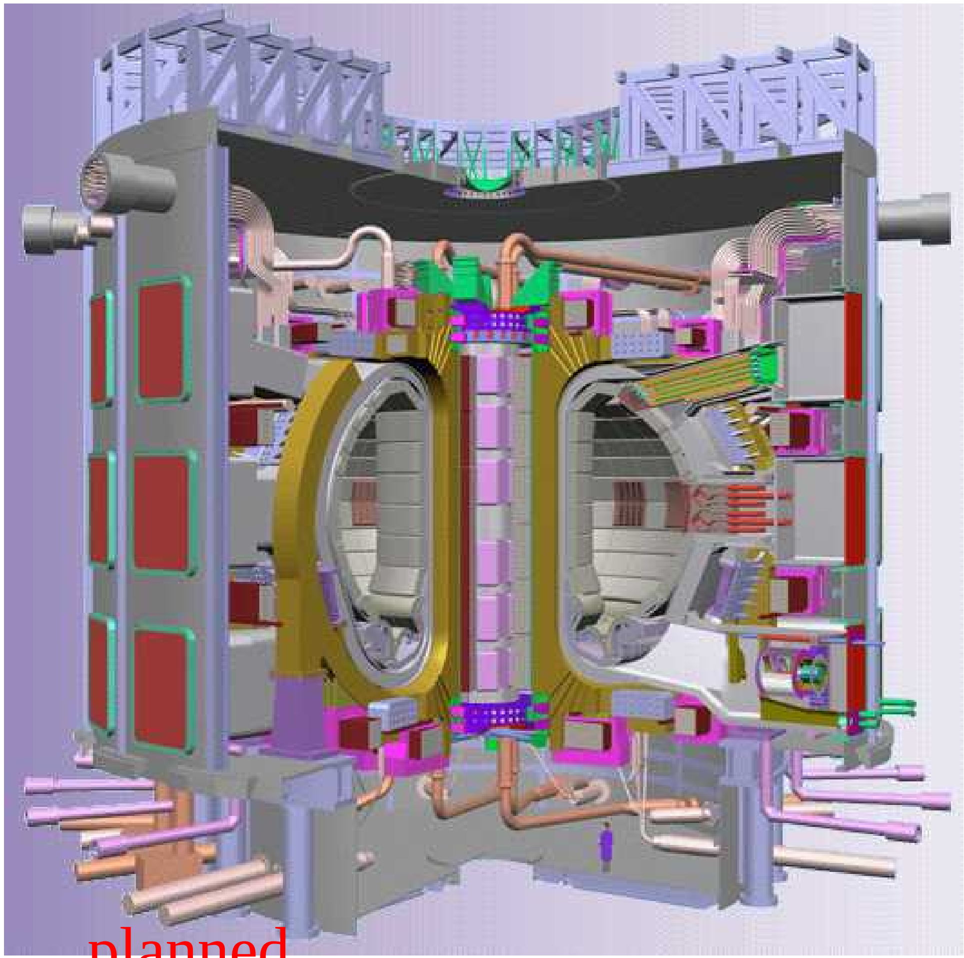
DIII-D, General Atomics Corporation

… the need for predictive simulation increases.

Critical ‘macroscopic’ topics include:

1. Internal kink stability
2. Neoclassical tearing excitation and control
3. Edge localized mode control
4. Wall-mode feedback

[2002 Snowmass Fusion Summer Study]



planned

~~proposed~~ International Thermonuclear Experimental Reactor (ITER)

- Fusion power: 500 MW
- Stored thermal energy: 10s of MJ

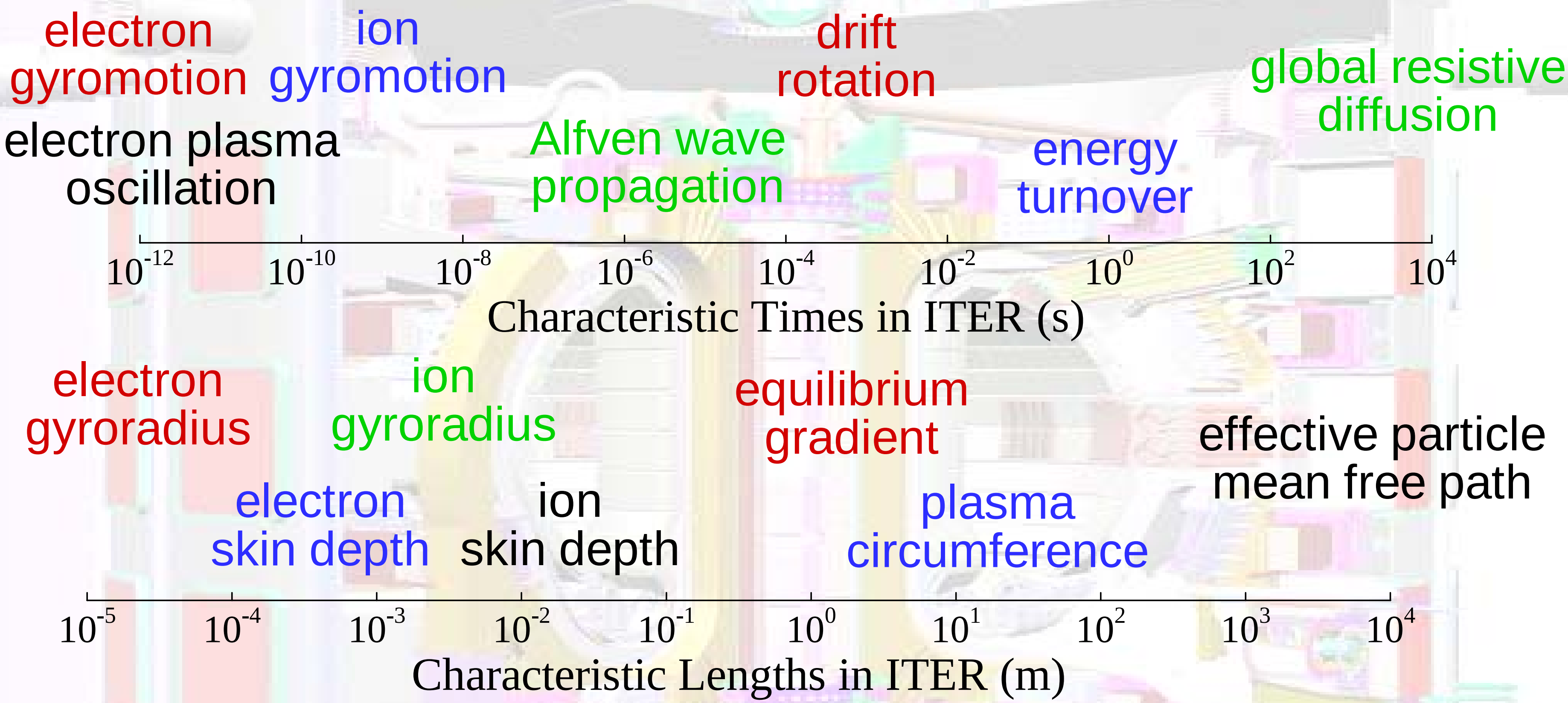
Outline

- Introduction
- Macroscopic plasma dynamics
  - Characteristics
  - Simulation examples
- Computational modeling
  - Numerical methods
    - High-order spatial representation
    - Time-advance for drift effects
  - Implementation
- Conclusions

Macroscopic Plasma Dynamics

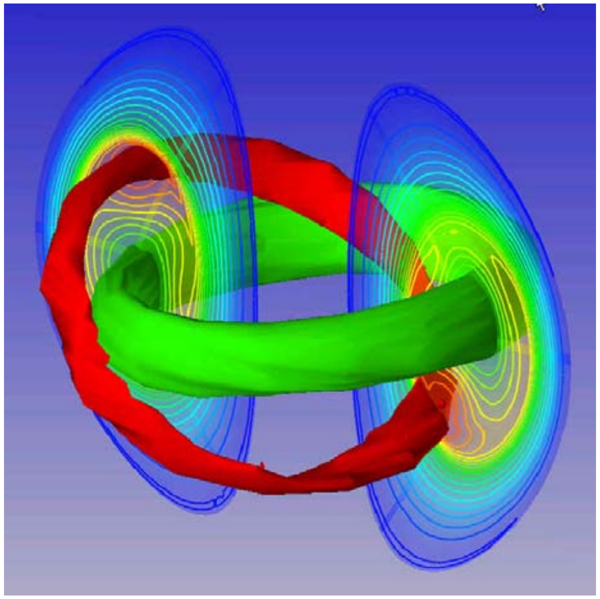
- **Magnetohydrodynamic (MHD) or MHD-like activity limits operation or affects performance in all magnetically confined configurations.**
- **Analytical theory has taught us which physical effects are important and how they can be described mathematically.**
- **Understanding consequences in experiments (and predicting future experiments) requires numerical simulation:**
  - **Sensitivity to equilibrium profiles and geometry**
  - **Strong nonlinear effects**
  - **Competition among physical effects**

Fusion plasmas exhibit enormous ranges of temporal and spatial scales.



- **Nonlinear MHD-like behavior couples many of the time- & length-scales.**
- **Even within the context of resistive MHD modeling, there is stiffness and anisotropy in the system of equations.**

Examples of Nonlinear Macroscopic Simulation

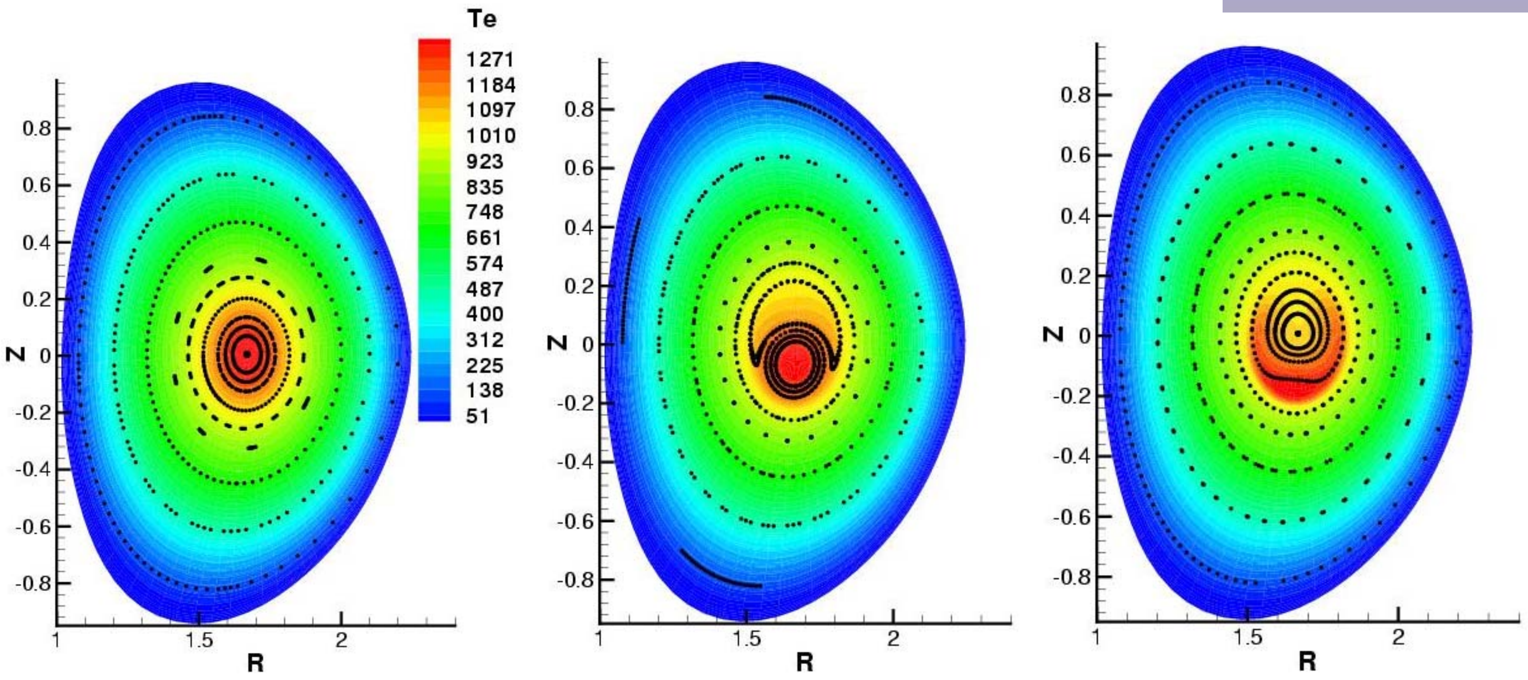


1) MHD evolution of the tokamak internal kink mode (m=1, n=1)

- Plasma core is exchanged with cooler surrounding plasma.

M3D simulation of NSTX [W. Park]

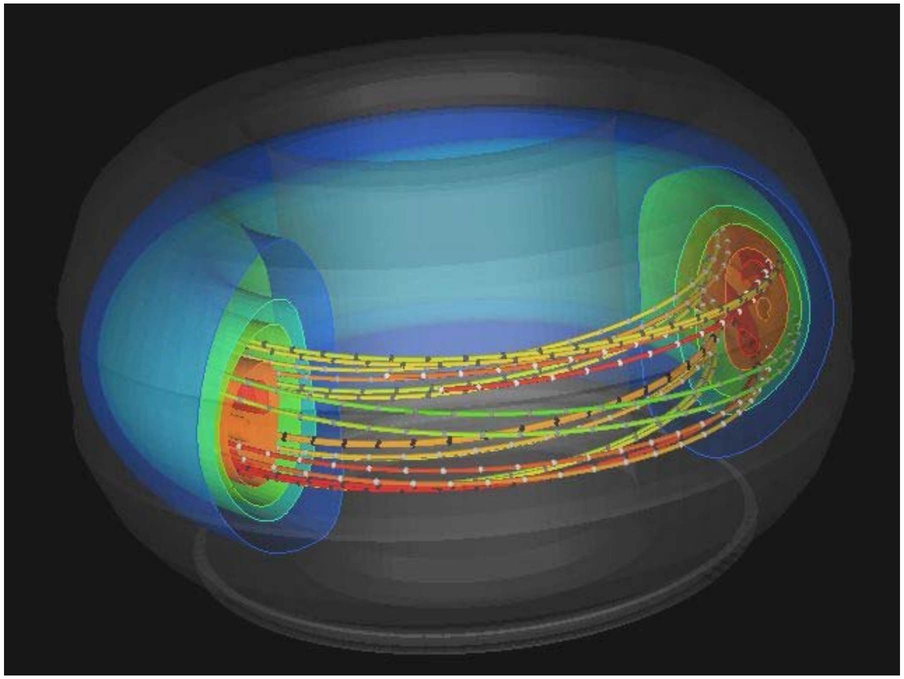
Evolution of pressure and magnetic topology from a NIMROD simulation of DIII-D



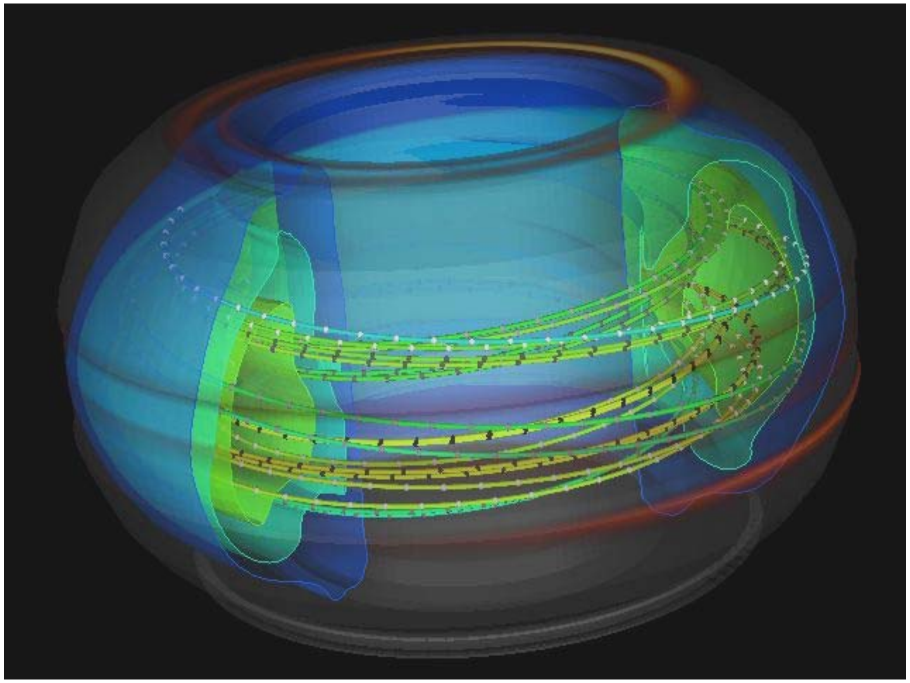
Examples of Nonlinear Macroscopic Simulation (continued)

2) Disruption (Loss of Confinement) events

• Understanding and mitigation are critical for a device the size of ITER.



Simulated event in DIII-D starts from an MHD equilibrium fitted to laboratory data.



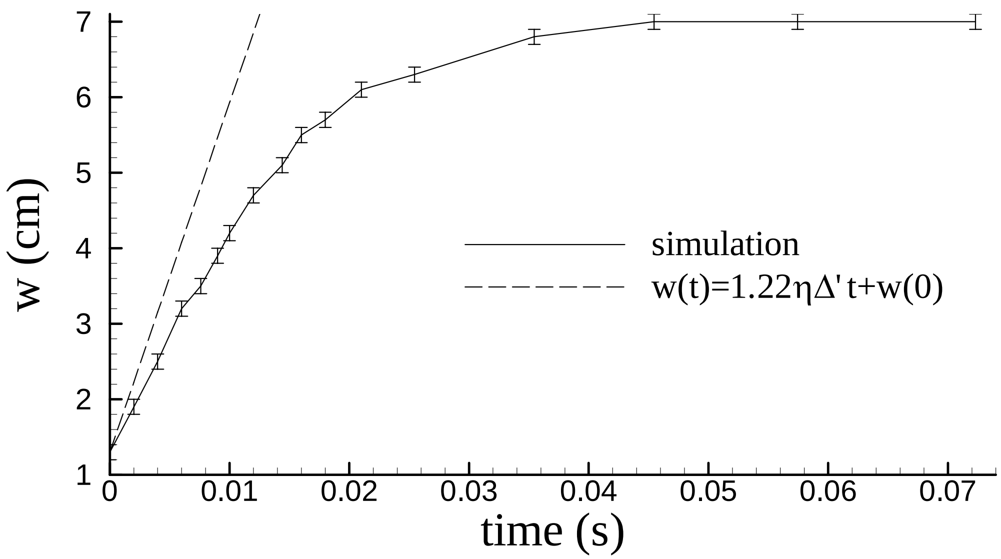
Resulting magnetic topology allows parallel heat flow to the wall.

simulation & graphics by S. Kruger and A. Sanderson

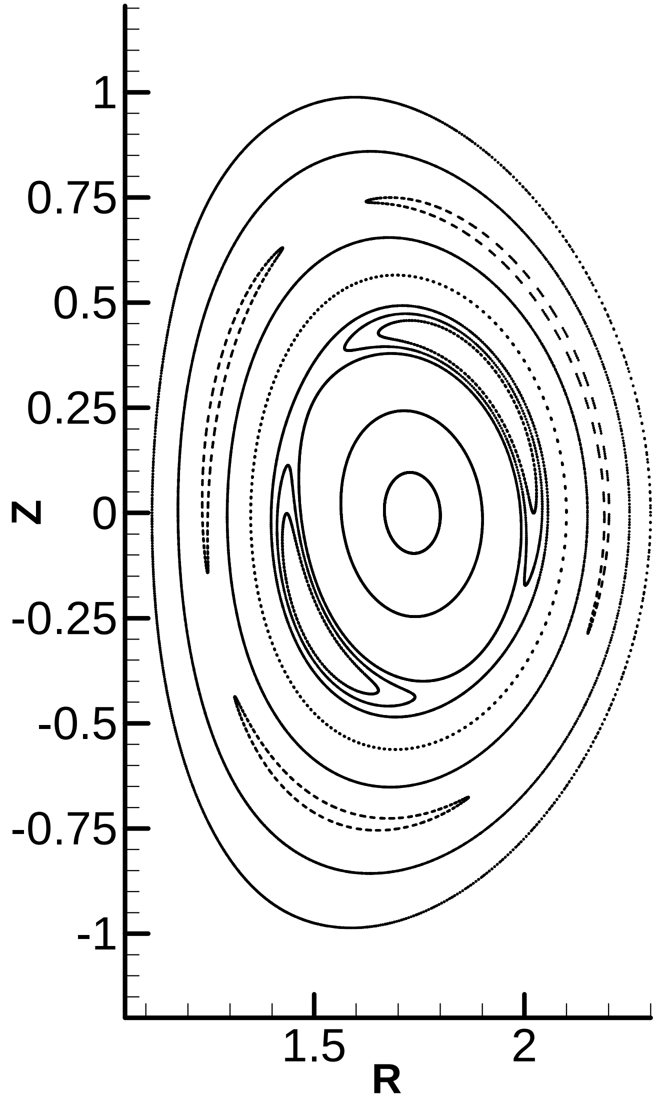
Examples of Nonlinear Macroscopic Simulation (continued)

3) Helical island formation from tearing modes

- Being weaker instabilities, tearing modes are heavily influenced by non-MHD effects.
- Tearing modes are usually non-disruptive but lead to significant performance loss.
- Slow evolution makes the system very stiff.



Magnetic Island Width vs. Time

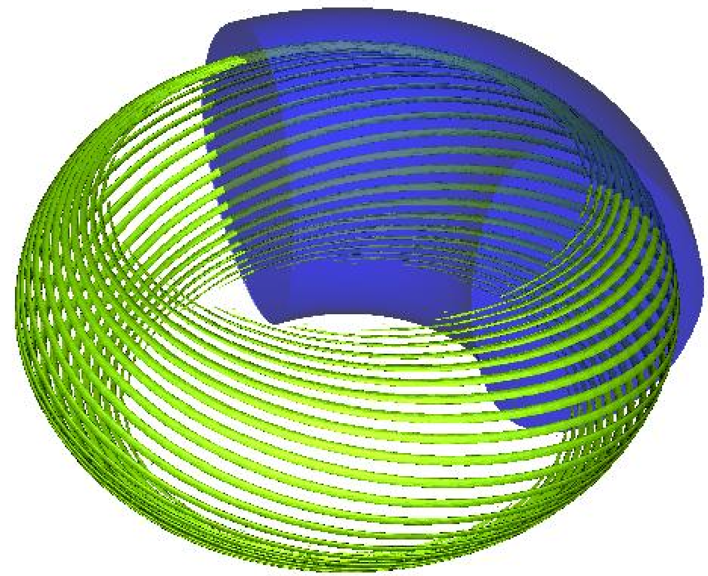


Resistive tearing evolution in toroidal geometry generating coupled island chains.

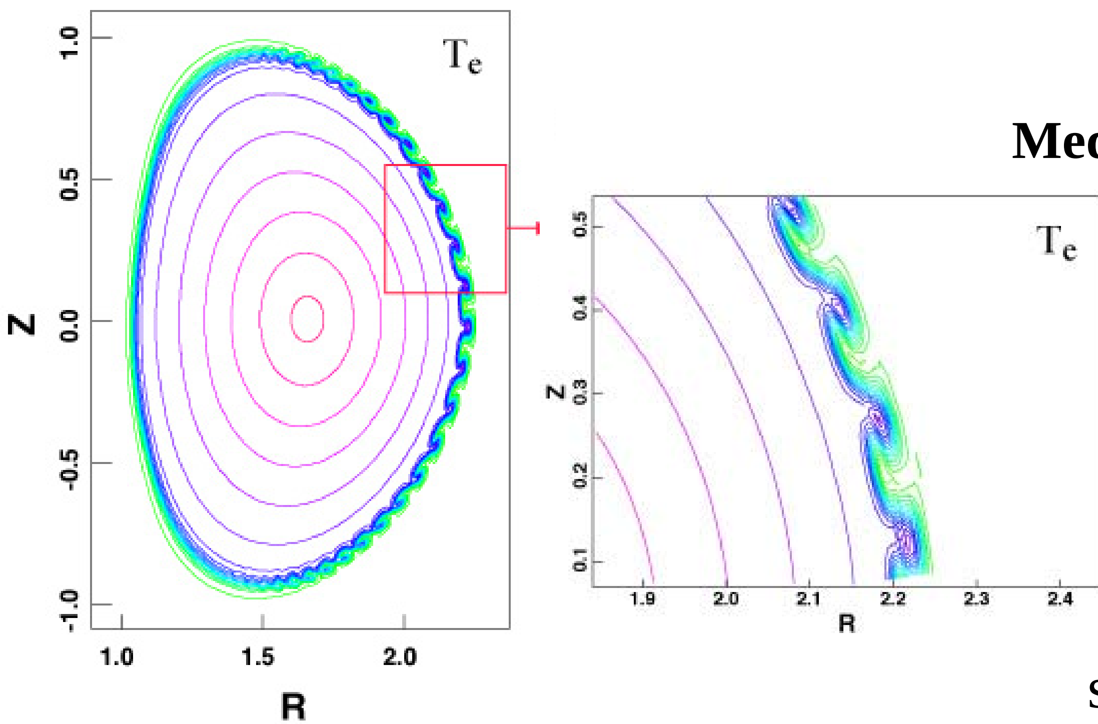
Examples of Nonlinear Macroscopic Simulation (continued)

4) Edge localized modes

- Strong gradients at the open/closed flux boundary drive localized modes.
- Heat transport to the wall occurs in periodic events—can be damaging if not controlled.



Medium-wavenumber modes are unstable.



Nonlinear coupling in MHD simulation leads to localized structures that are suggestive of bursty transport.

simulation by D. Brennan

MHD description is insufficient, however.

Computational Modeling

Important considerations:

- Stiffness arising from multiple time-scales
    - Fastest propagation is determined by linear behavior
- Anisotropy relative to the strong magnetic field
    - Distinct shear and compressive behavior
    - Extremely anisotropic heat flow
- Magnetic divergence constraint
- Weak resistive dissipation
- Typically free of shocks

Equations (for the mathematicians if not the physicists)

$$m_{\mathrm{i}} n\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=\mathbf{J}\times\mathbf{B}\;-\nabla p-\nabla\cdot\Pi_{\mathrm{i}}(\mathbf{V})$$

$$\frac{\partial n}{\partial t}+\nabla\cdot(\mathbf{V}n)=0$$

$$\frac{3n}{2}\left(\frac{\partial T_\alpha}{\partial t}+\mathbf{V}_\alpha\cdot\nabla T_\alpha\right)=-nT_\alpha\nabla\cdot\mathbf{V}_\alpha\;-\nabla\cdot\mathbf{q}_\alpha+Q_\alpha \qquad \alpha=i,e$$

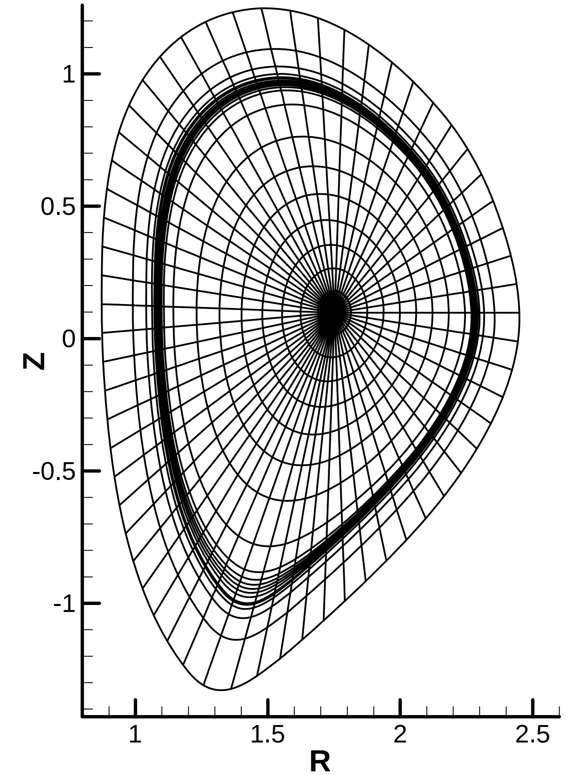
$$\frac{\partial \mathbf{B}}{\partial t}=-\nabla\times\left[\frac{1}{ne}(\mathbf{J}\times\mathbf{B}-\nabla p_{\mathrm{e}})-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}\right] \qquad \mu_0\mathbf{J}=\nabla\times\mathbf{B}$$

$$\Pi_{\mathrm{gv}}=\frac{m_{\mathrm{i}}p_{\mathrm{i}}}{4eB}\left[\hat{\mathbf{b}}\times\mathbf{W}\cdot\left(\mathbf{I}+3\hat{\mathbf{b}}\hat{\mathbf{b}}\right)-\left(\mathbf{I}+3\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\cdot\mathbf{W}\times\hat{\mathbf{b}}\right], \qquad \left(\mathbf{W}\equiv\nabla\mathbf{V}+\nabla\mathbf{V}^{\mathrm{T}}-\frac{2}{3}\mathbf{I}\nabla\cdot\mathbf{V}\right)$$

$$\mathbf{q}_i=-n\left\lfloor\chi_{\|_i}\hat{\mathbf{b}}\hat{\mathbf{b}}+\chi_{\perp i}\left(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\right\rfloor\cdot\nabla T_i\;+2.5p_{\mathrm{i}}(eB)^{-1}\hat{\mathbf{b}}\times\nabla T_{\mathrm{i}}$$

$$\mathbf{q}_e=-n\left\lfloor\chi_{\|_e}\hat{\mathbf{b}}\hat{\mathbf{b}}+\chi_{\perp e}\left(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\right\rfloor\cdot\nabla T_e\;-2.5p_{\mathrm{e}}(eB)^{-1}\hat{\mathbf{b}}\times\nabla T_{\mathrm{e}}$$

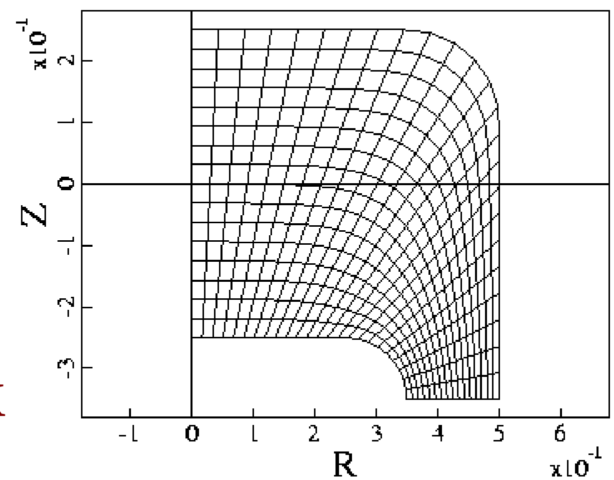
Modeling: Spatial Representation



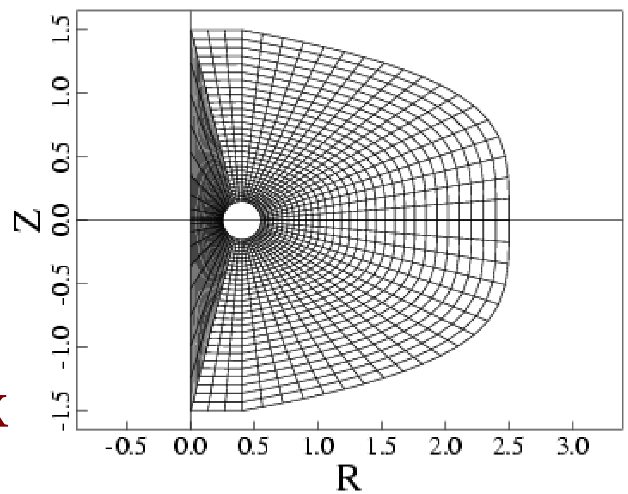
Packed mesh for DIII-D ELM study

- The NIMROD code (http://nimrodteam.org) uses finite elements to represent the poloidal plane and finite Fourier series for the periodic direction.
- Polynomial basis functions may be Lagrange or Gauss-Lobatto-Legendre. Degree>1 provides
  - **High-order convergence without uniform meshing**
  - **Curved isoparametric mappings**

SSPX

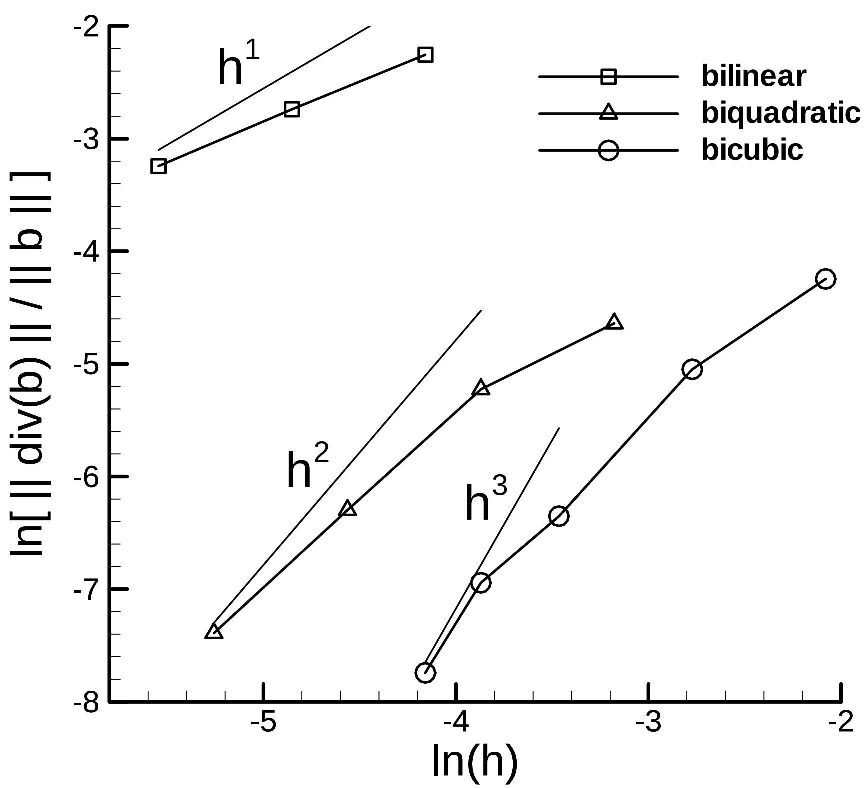


LDX



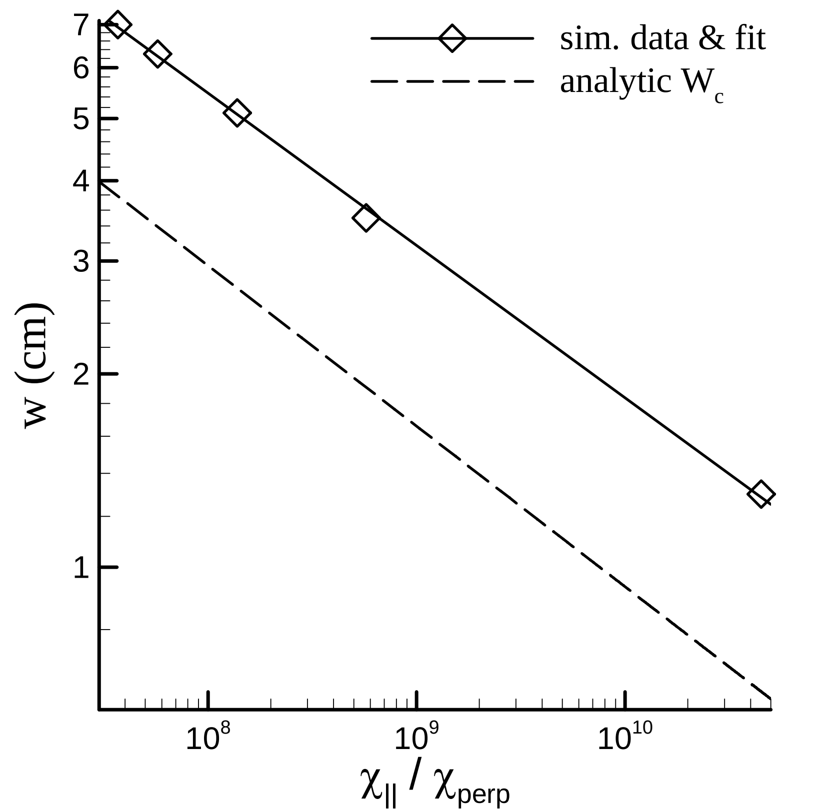
Modeling: Spatial Representation (continued)

- Polynomials of degree>1 also provide
  - **Control of magnetic divergence error**
  - **Resolution of extreme anisotropies (Lorentz force and diffusion)**



Magnetic divergence constraint

Scalings show the convergence rates expected for first derivatives.



Critical island width vs. $\chi_{||}/\chi_{perp}$

W_c shows where diffusion time-scales match [Fitzpatrick, PoP **2**, 825 (1995)].

See JCP **195**, 355 (2004).

Modeling: Time-advance algorithms

• Stiffness from fast parallel transport and wave propagation requires implicit methods.

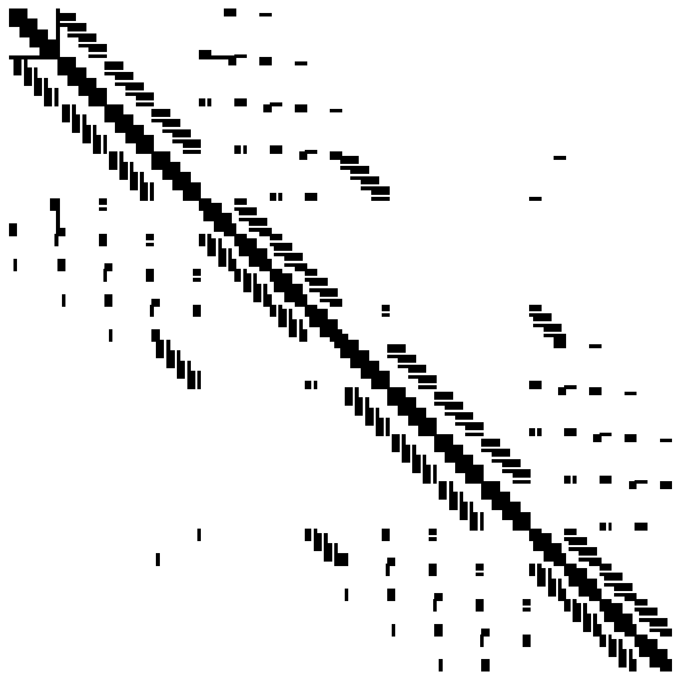
• Semi-implicit methods for MHD have been refined over the last two decades.

• Time-centered implicit methods are becoming more practical with matrix-free Newton-Krylov solves.

• A new implicit leapfrog advance is being developed for NIMROD modeling of two-fluid effects (drifts and dispersive waves).

• Matrices are sparse and ill-conditioned.

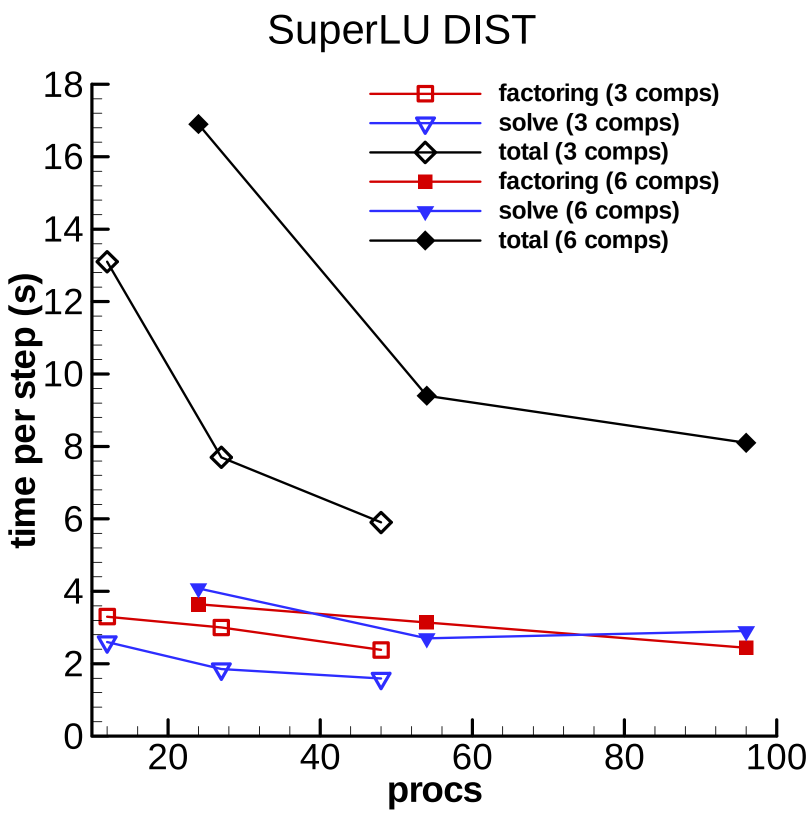
- They are solved during each time-step (~10,000s over a nonlinear simulation).
- Factoring is less frequent.

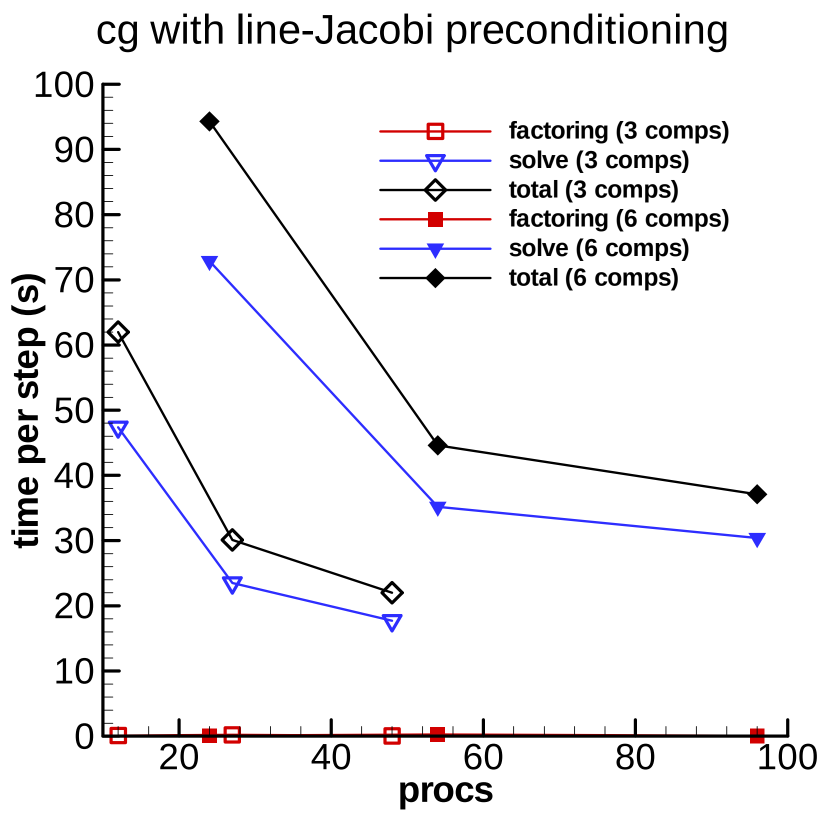


Example sparsity pattern for a small mesh of biquartic elements

Modeling: Implementation

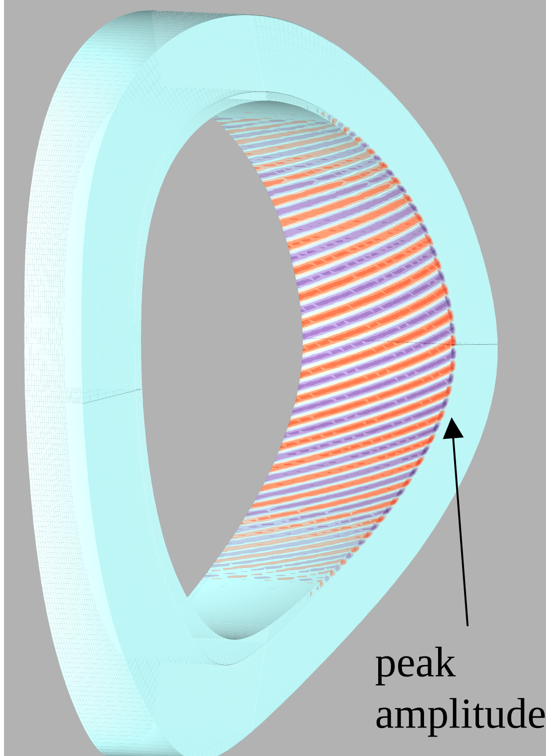
- Solving algebraic systems is the dominant performance issue.
- Iterative methods scale well but tend to perform poorly on ill-conditioned systems.
- Collaborations with **TOPS** Center researchers Kaushik and Li led us to parallel direct methods with reordering→SuperLU (http://crd.lbl.gov/~xiaoye/SuperLU/).



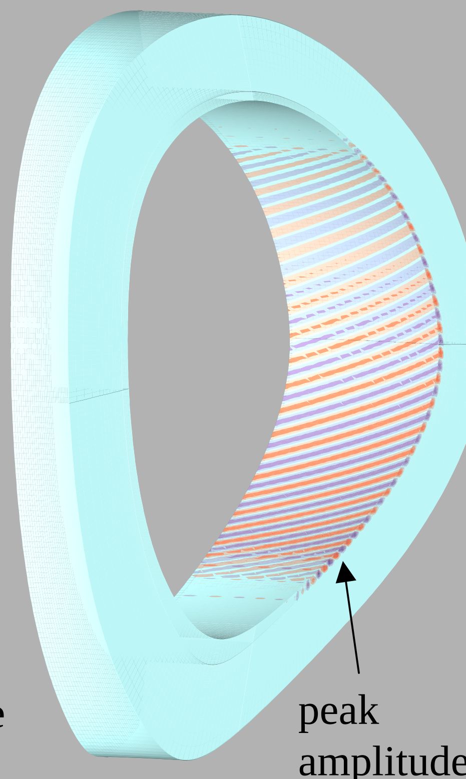


SuperLU improves NIMROD performance by a factor of **5** in nonlinear simulations.

Initial Application of the Two-Fluid Model to ELMs

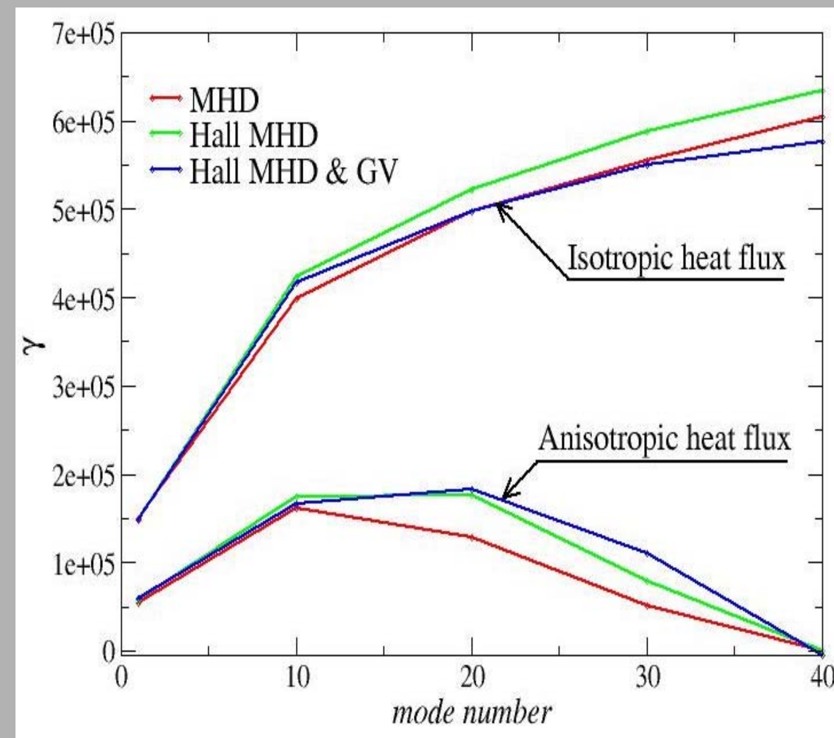


MHD



Two-fluid

The two-fluid model (including Hall and gyroviscous effects) shifts the mode downward and induces rotation.



Nonetheless, preliminary scans are indicating that anisotropic conduction is more important for stabilizing short wavelengths.

calculations by A. Pankin

Also see two-fluid modeling by Sugiyama in poster WED08.

Conclusions

The challenges of macroscopic modeling are being met by developments in numerical and computational techniques, as well as advances in hardware.

• High-order spatial representation controls magnetic divergence error and allows resolution of anisotropies that were previously considered beyond reach.

• SciDAC-fostered collaborations have resulted in huge performance gains through sparse parallel direct solves (with SuperLU).

Other Remarks

• SciDAC support for computing and collaborations is benefiting the fusion program at an opportune time.

• Integrated modeling (macro+turbulence+RF+edge) is the new horizon.

• The macroscopic modeling tools are also applicable to problems in space and astrophysical plasmas.