Kinetic Theory Issues in Nonlocal Transport

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July, 2001

Introduction

- step size $\approx$ gradient scale length
- edge region : $\Delta r \sim L_n$
- ERS - mode : $\Delta r \sim L_n$
- parallel heat transport : $\lambda_{\parallel} \sim L_T$
- Δr : step size or orbit width

  L_n : $\frac{1}{n}\frac{dn}{dr}$, r : radius

  L_T : $\frac{1}{T}\frac{dT}{dl}$, l : distance along the field line.

  $\lambda_{\parallel}$: mean-free-path

Kinetic Issues

- When $\Delta r \sim L_n$ or $\lambda_{\parallel} \sim L_T$, equilibrium distribution is not a Maxwellian
- Can not be handled by the existing codes.

Simple Argument

- Radial transport:
  - Rate for relaxing to a Maxwellian distribution τ_r^{-1}:

$$\tau_r^{-1} \sim \nu \quad : \text{collision frequency}$$

  - Rate for the radial transport τ_D^{-1}:

$$\tau_D^{-1} \sim \nu \left(\frac{\Delta r}{L_n}\right)^2$$

- When $(\Delta r / L_n) \sim 1$, $\tau_r^{-1} \sim \tau_D^{-1}$,

$\Rightarrow$ equilibrium distribution is not a Maxwellian.

- $\langle C(f_0) \rangle$

$$= \frac{\nu_0}{2} \frac{1}{\sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial f_0}{\partial\theta}\right)$$

gives Maxwellian distribution

$$+ \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^3\left(\frac{\mu}{M}\nu_s f_0 + \frac{\nu_{\parallel} v}{2}\frac{\partial f_0}{\partial v}\right)\right]$$

$$+ \frac{\nu_0}{2}\left(\frac{v^2}{2\Omega^2}\right)^{\sim \rho^2}(1+\cos^2\theta)(\nabla_g^2) f_0$$

$$+ \frac{\nu_{\parallel}}{2}\left(\frac{v^2}{2\Omega^2}\right)^{\sim \rho^2}\sin^2\theta\,(\nabla_g^2) f_0$$

radial diffusion

- when $(\rho/L_W)^2 \sim 1$, f_0 is not a Maxwellian, because spatial diffusion is just as fast as energy relaxation!

- Parallel transport:
  - Rate for parallel transport $\tau_{\parallel}^{-1}$:

$$\tau_{\parallel}^{-1} \sim \nu \left(\frac{\lambda_{\parallel}}{L_T}\right)^2$$

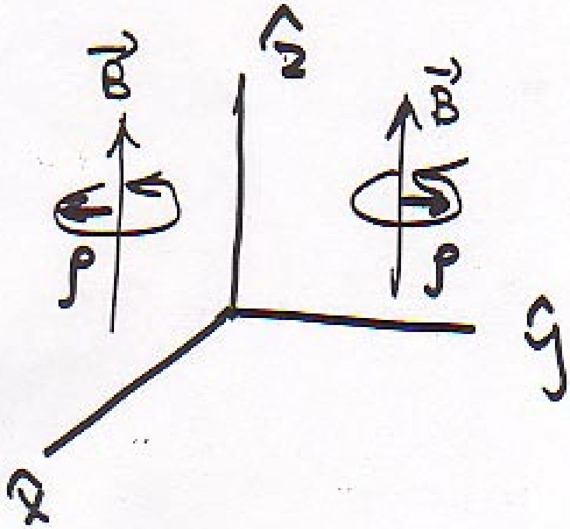
  - When $\lambda_{\parallel} \sim L_T$

$$\tau_r^{-1} \sim \tau_{\parallel}^{-1}$$

  - Again, equilibrium distribution is not a Maxwellian.

Two Models

- Classical Transport : (with Hirshman, in 1983)



- $\vec{B}$: uniform magnetic field.
- ρ : Gyro-radius.
- $\rho \sim L_n$

- Kinetic equation:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \Omega \frac{\partial f}{\partial \phi} = C(f) ,$$

  - $f = f(\vec{r}, \vec{v}, t)$
  - $\vec{v}$: velocity
  - Ω: Gyro-frequency
  - ϕ: Gyro-phase

- Steady state:

$$\underbrace{\vec{v} \cdot \vec{\nabla} f - \Omega \frac{\partial f}{\partial \phi}}_{\sim \Omega} = \underbrace{C(f)}_{\sim \nu}$$

- Collisionless: $\nu \ll \Omega$

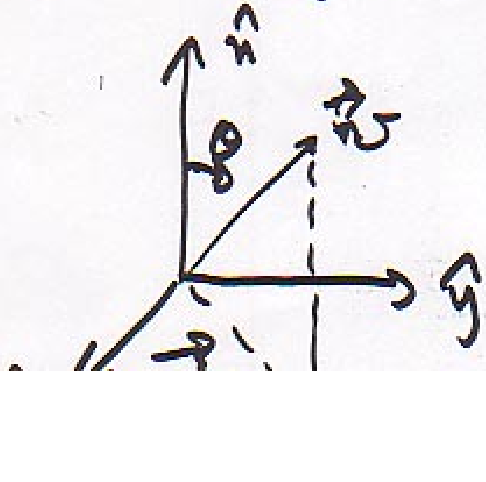
$\left(\frac{\nu}{\Omega}\right)^0$: $\quad \vec{v}_\perp \cdot \vec{\nabla} f_0 - \Omega \frac{\partial f_0}{\partial \phi} = 0$, (1)

$\left(\frac{\nu}{\Omega}\right)^1$: $\quad \vec{v}_\perp \cdot \vec{\nabla} f_1 - \Omega \frac{\partial f_1}{\partial \phi} = C(f_0)$. (2)

- To solve (1), we go to guiding center coordinates

  - $\vec{r}_g = \vec{r} + \vec{v}_\perp \times \hat{n} / \Omega$, $\hat{n} = \vec{B}/B$

- $(\vec{r}, \vec{v}) \Longrightarrow (\vec{r}_g, v, \theta, \phi)$



- $\frac{\partial}{\partial v_{\parallel}}\Big|_{\ell} \rightarrow \frac{\partial}{\partial v_{\parallel}}\Big|_{\ell_0} - \underbrace{t}\frac{\partial}{\partial \ell_0}$

mean-time between collision $\sim \frac{\lambda_{\parallel}}{v_{\parallel}}$

- Again, when $\lambda_{\parallel} \sim L_T$, spatial diffusion is coupled to the energy scattering.

- f_0 is not a Maxwellian.

• In guiding center coordinates:

(1): $\Omega \frac{\partial f_0}{\partial \phi} = 0 \Rightarrow f_0 = f_0(\vec{r}_g, v, \alpha)$

(2) $-\Omega \frac{\partial f_1}{\partial \phi} = C(f_0)$

• To determine f_0:

• Average (2):

$$\oint d\phi \, C(f_0) = 0$$

• Now, collision operator is in $(\vec{r}, \vec{v})$ space. When it is transformed to $(\vec{r}_g, \vec{v})$ space, it picks up

- Because f_0 is not a Maxwellian, $C(f_0)$ we used is not valid.

- Difficult to solve even numerically

Parallel Transport

- $\frac{\partial f}{\partial t} + v_{\parallel} \hat{n} \cdot \vec{\nabla} f = C(f)$

- ignore radial drift

- $f = f(v, v_{\parallel}, l, t)$

- Assume collisionless :

  - $\frac{\partial f_0}{\partial t} + v_{\parallel} \hat{n} \cdot \vec{\nabla} f_0 = 0$

  - $\frac{\partial f_1}{\partial t} + v_{\parallel} \hat{n} \cdot \vec{\nabla} f_1 = C(f_0)$

- parallel motion:

$$l = l_0 + v t$$

- Change variables from $(E, v_{\parallel}, l, t)$ to $(E, v_{\parallel}, l_0, t)$

- $$\frac{\partial f_0}{\partial t} + v_{\parallel} \hat{n} \cdot \vec{\nabla} f_0 = 0$$

$$\Rightarrow f = f(E, v_{\parallel}, l_0)$$

- Again, in $(E, v_{\parallel}, l_0)$ coordinates, $C(f_0)$ has "diffusion" term.

Conclusions

- When $\delta r \sim L_n$ or $\lambda_{\parallel} \sim L_T$, equilibrium distribution is not a Maxwellian.
- In non-local transport, $C(f)$ is not a differential operator.
- Difficult to calculate even numerically
- In the radial transport case, orbit squeezing is the solution to this difficult problem. bi-Max??
- In parallel transport case ???