M3D Simulation Studies of ST's and Stellarators

W. Park, J. Breslau, J. Chen, G.Y. Fu, S.C. Jardin, S. Klasky,
J. Menard, A. Pletzer, D. Stutman (PPPL)
H.R. Strauss (NYU)
L.E. Sugiyama (MIT)

Outline

- M3D code

  MHD, two-fluids, hybrid models.

- NSTX studies including flow effects

  2D steady states.

  Evolutions of IRE's.

  BAE modes.

- Stellarator studies

  Two-fluid results compared to MHD.

  TAE mode study using hybrid model.

M3D Project

W. Park et al., Phys. Plasmas **6,** 1796 (1999)
http://w3.pppl.gov/~wpark/pop_99.pdf

Multilevel 3D Project for Plasma Simulation studies
Various physics levels are needed to understand the physics.
The best method depends on the problem at hand.

Physics	Geometry	State
MHD 2 Fluids Gyrokin. Hot P./MHD Gyrokin.Ion/Fluid Elect. ….	MPP Serial Unstructured FE Structured FD	Equilibrium Linear Nonlinear

MHD model

- Solves MHD equations.

$$
\begin{cases}
\rho\partial \mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v} \\
\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B} \\
\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0 \\
\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\nabla(p/\rho)
\end{cases}
$$

The fast parallel equilibration of T is modeled using wave equations;

$$
\begin{cases}
\partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u \\
\partial u/\partial t = s\,\mathbf{B}\cdot\nabla T + \upsilon\nabla^2 u
\end{cases}
$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscousity and neoclassical parallel viscousity terms in a torus.

- **Equations**

$$
\begin{cases}
\mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_{\parallel}/en, \\
\mathbf{v}_e^* \equiv -\mathbf{B}\mathrm{x}\nabla p_e\ /(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_{\perp}/en,
\end{cases}
$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_{\perp} = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_{\parallel}p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi e,$$
$$\mathbf{J} = \nabla\times\mathbf{B},$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$
\begin{aligned}
\partial p/\partial t + \mathbf{v}\cdot\nabla p = & -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p/\rho) \\
& - \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e \\
& - \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en)
\end{aligned}
$$

$$
\begin{aligned}
\partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = & -\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p_e/\rho) \\
& + (1/en)\mathbf{J}_{\parallel}\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_{\parallel}/en)
\end{aligned}
$$

GK Hot Particle /MHD Hybrid MH3D-K

- **Fluid equations**

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p - (\nabla\cdot\mathbf{P}_h)_\perp + \mathbf{J}\times\mathbf{B} \quad \text{(Pressure coupling)}$$

or

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + (\nabla\times\mathbf{B} - \mathbf{J}_h)\times\mathbf{B} + q_h\mathbf{V}\times\mathbf{B}$$

(Current coupling)

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = \mathbf{v}\times\mathbf{B} - \eta(\mathbf{J} - \mathbf{J}_h), \quad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\cdot\nabla\,(p/\rho)$$

- **Gyrokinetic equations for energetic particles**

$$d\mathbf{R}/dt = u[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,] + (1/\Omega)\mathbf{b}\times(\mu\nabla\mathbf{B} - q\mathbf{E}/m),$$

$$du/dt = -[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,]\cdot(\mu\nabla\mathbf{B} - q\mathbf{E}/m).$$

GK Particle Ion / Fluid Electron Hybrid

- **Pressure coupling**

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla\cdot\mathbf{P}_i - \nabla P_e + \mathbf{J}\times\mathbf{B}$$

$$= -\nabla\cdot\mathbf{P}_i^{CGL} - \nabla\cdot\Pi_i - \nabla P_e + \mathbf{J}\times\mathbf{B}$$

$\nabla\cdot P_i^{CGL}$: from particles following GK eqns.

$\nabla\cdot\Pi_i$: fluid picture as 2 fluid eqns,

or from particles.

- **Fluid electrons**

$$\mathbf{E} = -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla\cdot\mathbf{P}_e/ne$$

$$= -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla P_e/ne + \mathbf{bb}\cdot\nabla\cdot\Pi_e/ne$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{J} = \nabla\times\mathbf{B}$$

P_e eqn currently, but $P_{\parallel}$ and $P_\perp$ eqns are planned.

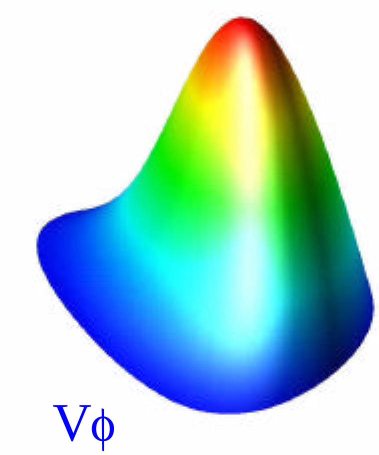
2D steady state with toroidal sheared flow

Quasi neutrality: $r\mathbf{V}\bullet\nabla\mathbf{V} + \nabla\bullet\overleftrightarrow{\mathbf{P}} - \mathbf{J}\times\mathbf{B} = 0$

$$\overleftrightarrow{\mathbf{P}} = \overleftrightarrow{\mathbf{P}}^{CGL} + \overleftrightarrow{\Pi}_g$$

$$= \mathrm{p}\overleftrightarrow{\mathrm{I}} \qquad + (P_{\|} - P_{\perp})\overleftrightarrow{\Pi}_{ii} \qquad + \overleftrightarrow{\Pi}_g$$

MHD Hot Particle/MHD 2-Fluids



MHD:

At the magnetic axis: $\mathbf{J}\times\mathbf{B} = 0$

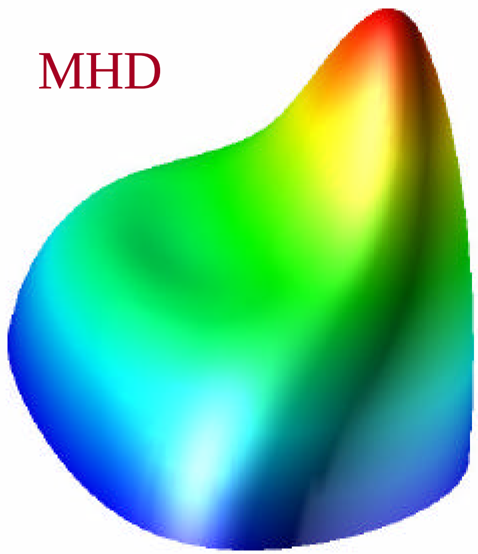
$$-\frac{r\mathrm{V}_f^{\ 2}}{R} + \frac{T\partial r}{\partial R} = 0$$

Relative shift of $r \equiv \frac{R\partial r}{r\partial R} = \frac{\mathrm{V}_f^{\ 2}}{T} = \frac{2M_A^{\ 2}}{b}$

Density profile dependence on Physics model

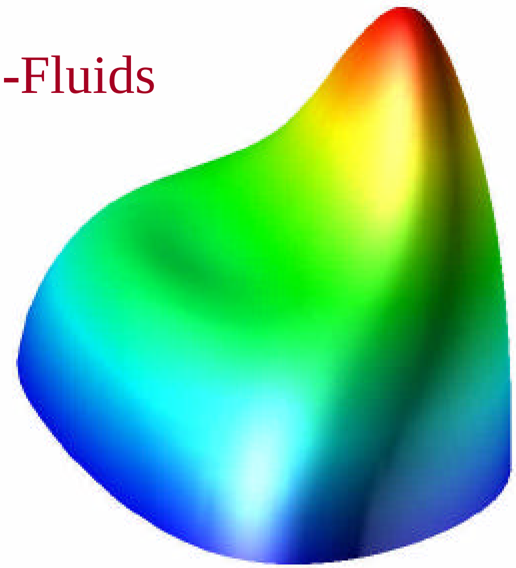
NSTX $\varepsilon=1.3$ $q_o=0.8$ $q_b=5$

MHD



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.1$
$\rho_{min}=0.5$
$RelSh=1$

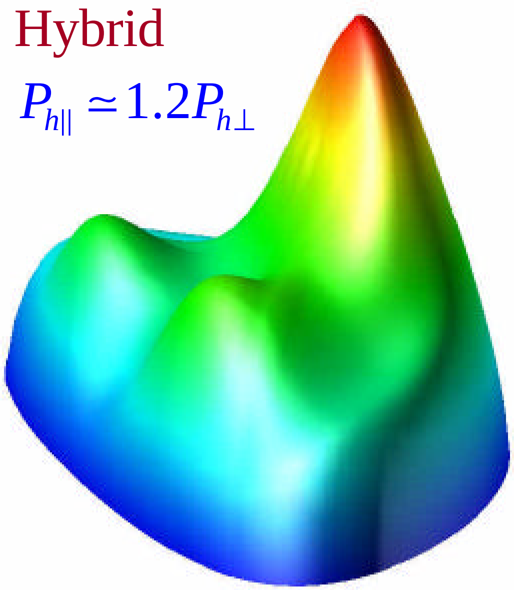
Two-Fluids



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.1$
$\rho_{min}=0.5$
$RelSh=1$

Hybrid

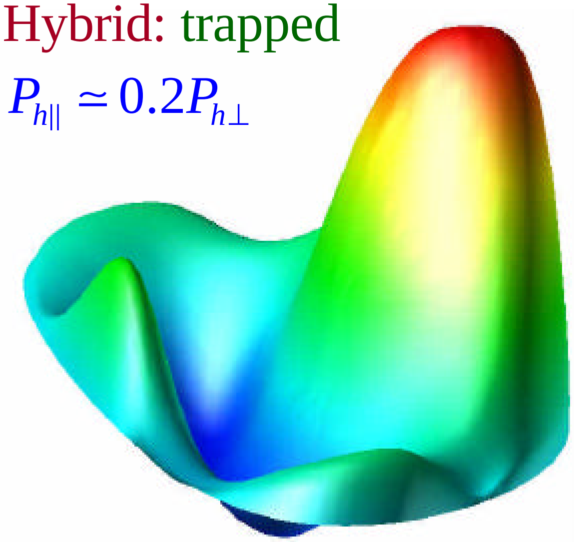
$P_{h\|} \simeq 1.2 P_{h\perp}$



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.2$
$\rho_{min}=0.5$
$RelSh=0.8$

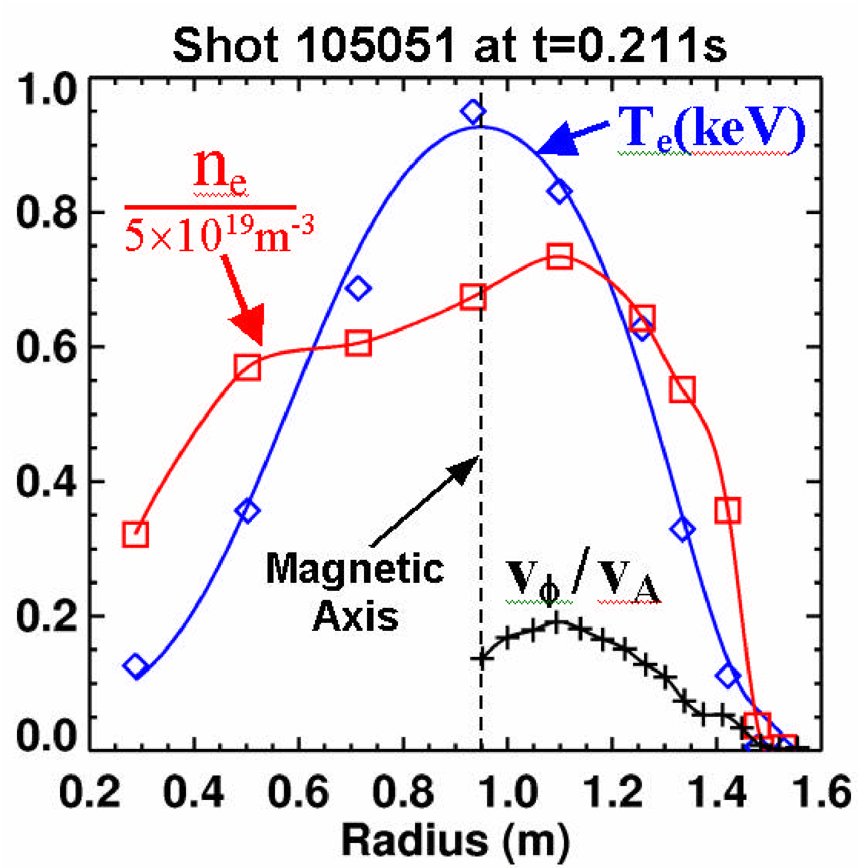
Hybrid: trapped

$P_{h\|} \simeq 0.2 P_{h\perp}$



$M_A=0.2$
$Sh=0.3$
$\rho_{max}=1.8$
$\rho_{min}=0.15$
$RelSh=1.9$

NSTX experimental data

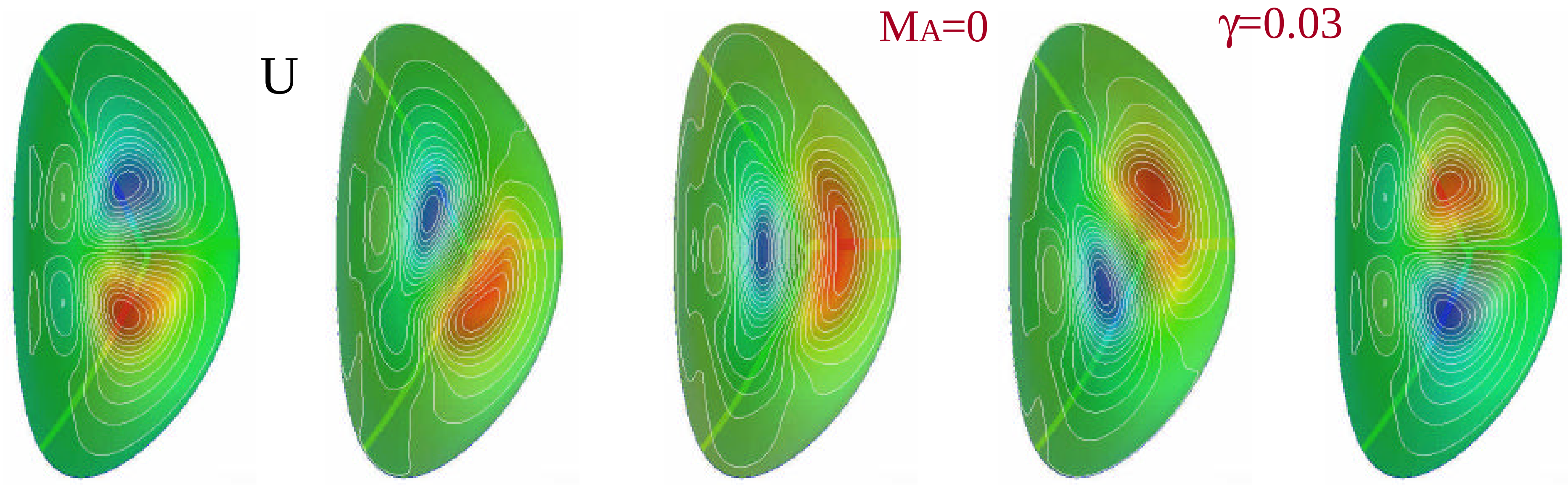


Relative shift of r

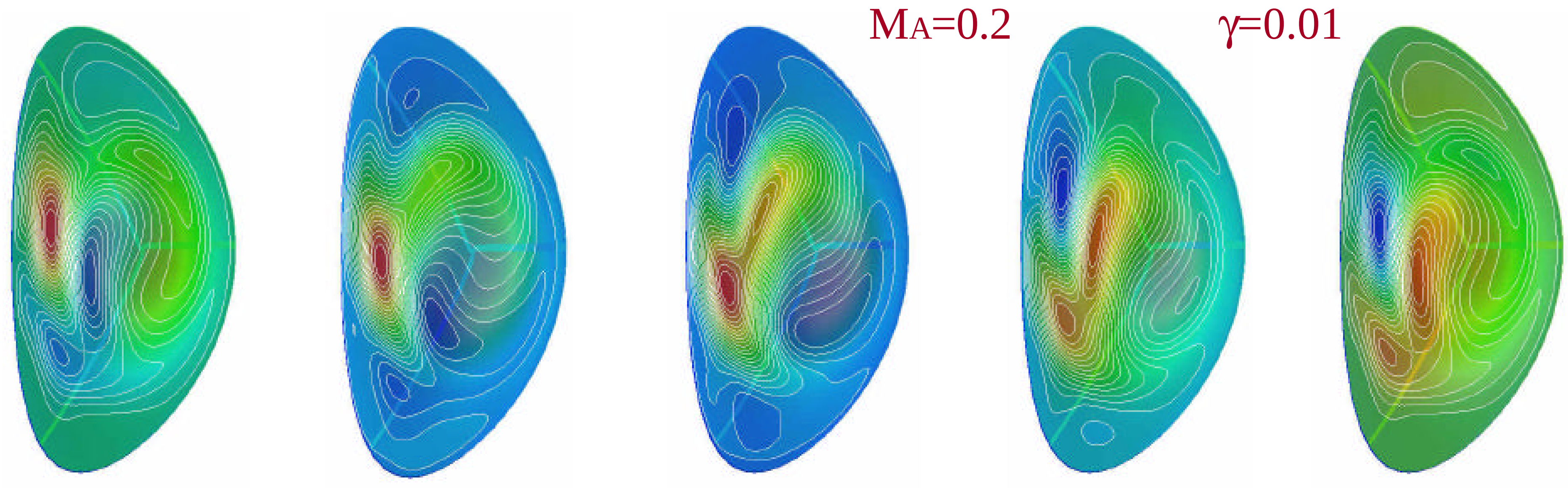
$$\frac{R\partial r}{r\partial R} = \frac{2M_A^{\,2}}{b}$$

Hot particle centrifugal force
~ Bulk plasma

Linear Eigenmodes: shear flow reduces growth rate



$f = 0$ $f = 0.25p$ $f = 0.5p$ $f = 0.75p$ $f = p$

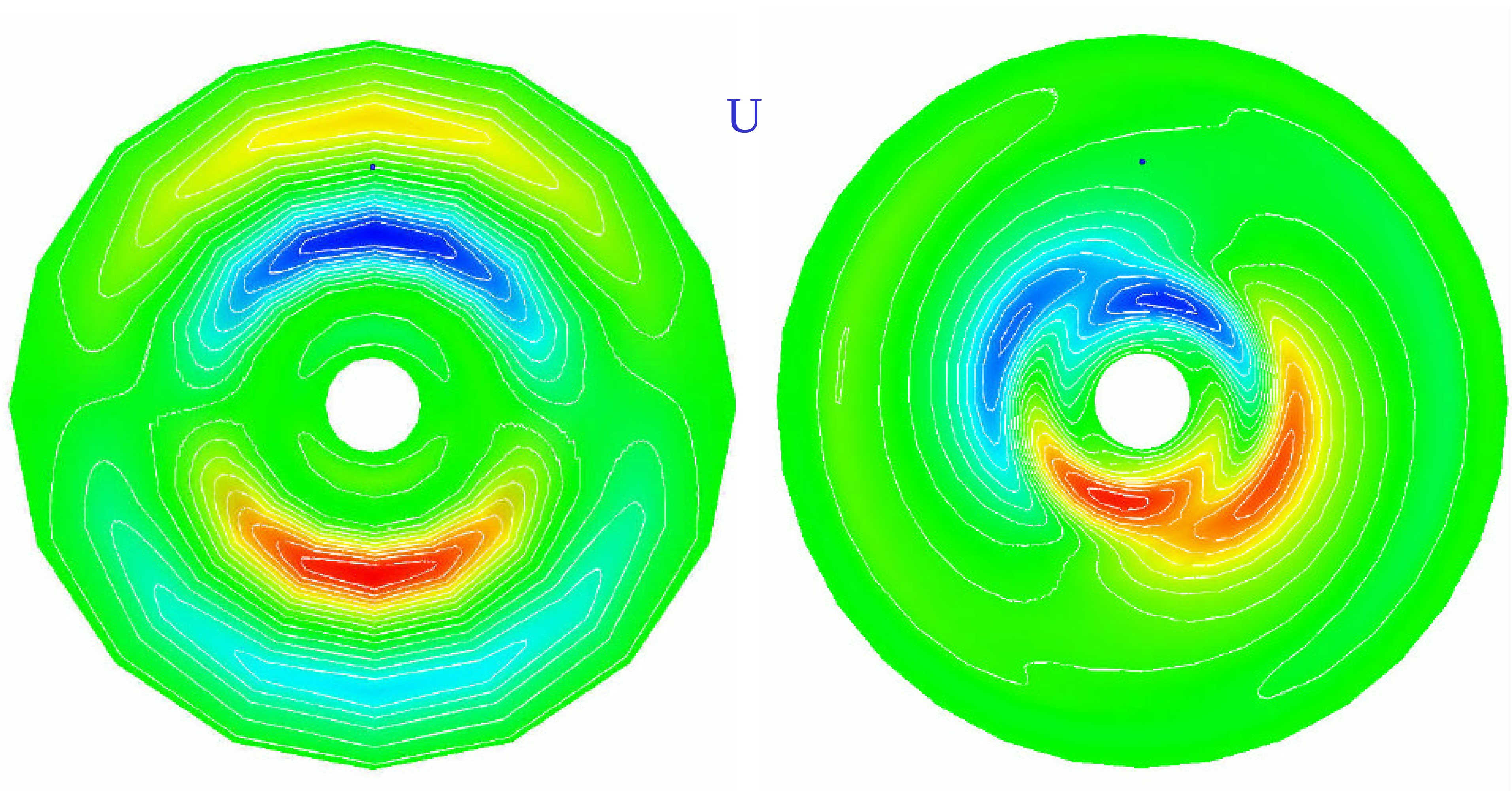


Linear Eigenmodes

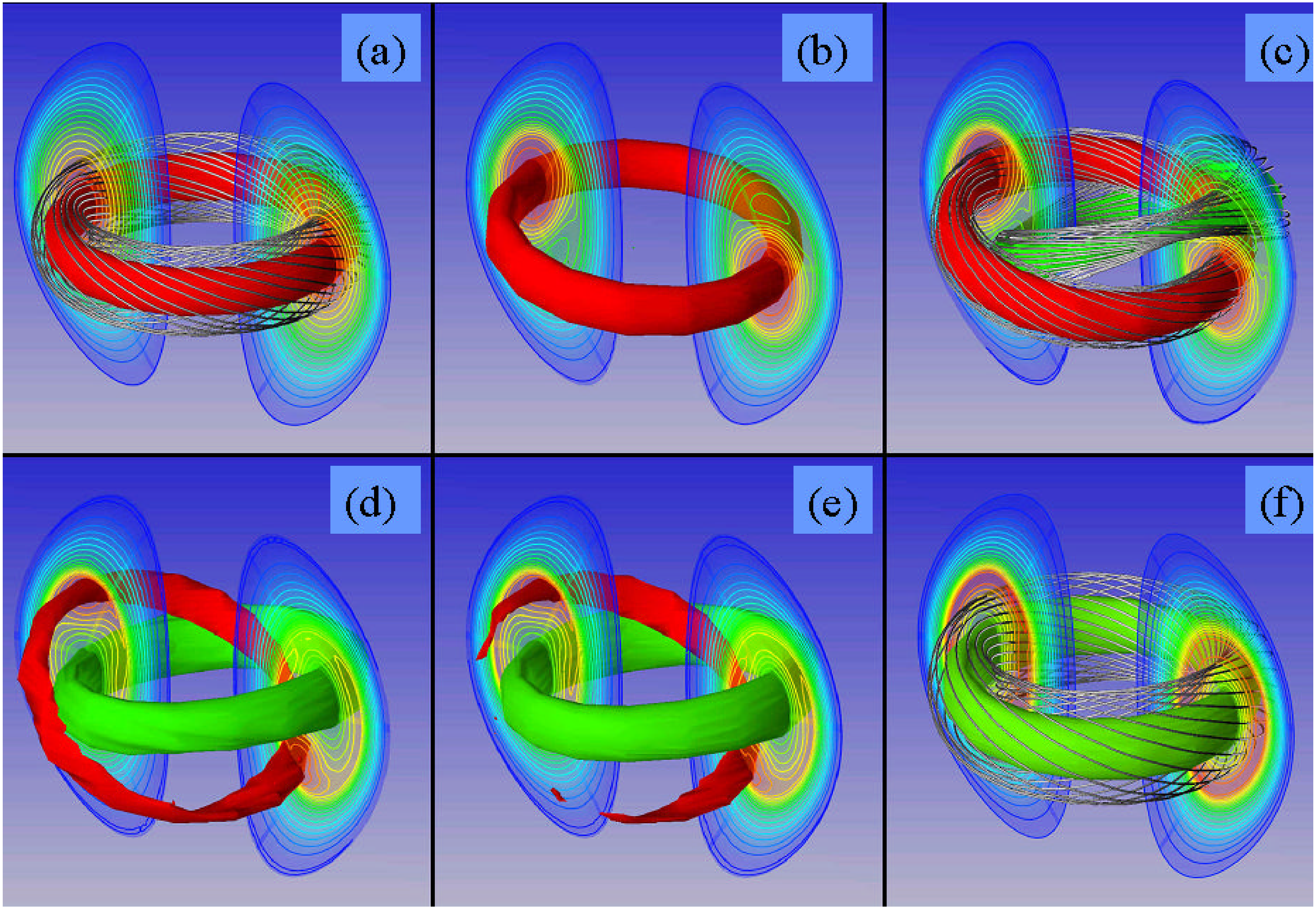
Top view on the mid-plane

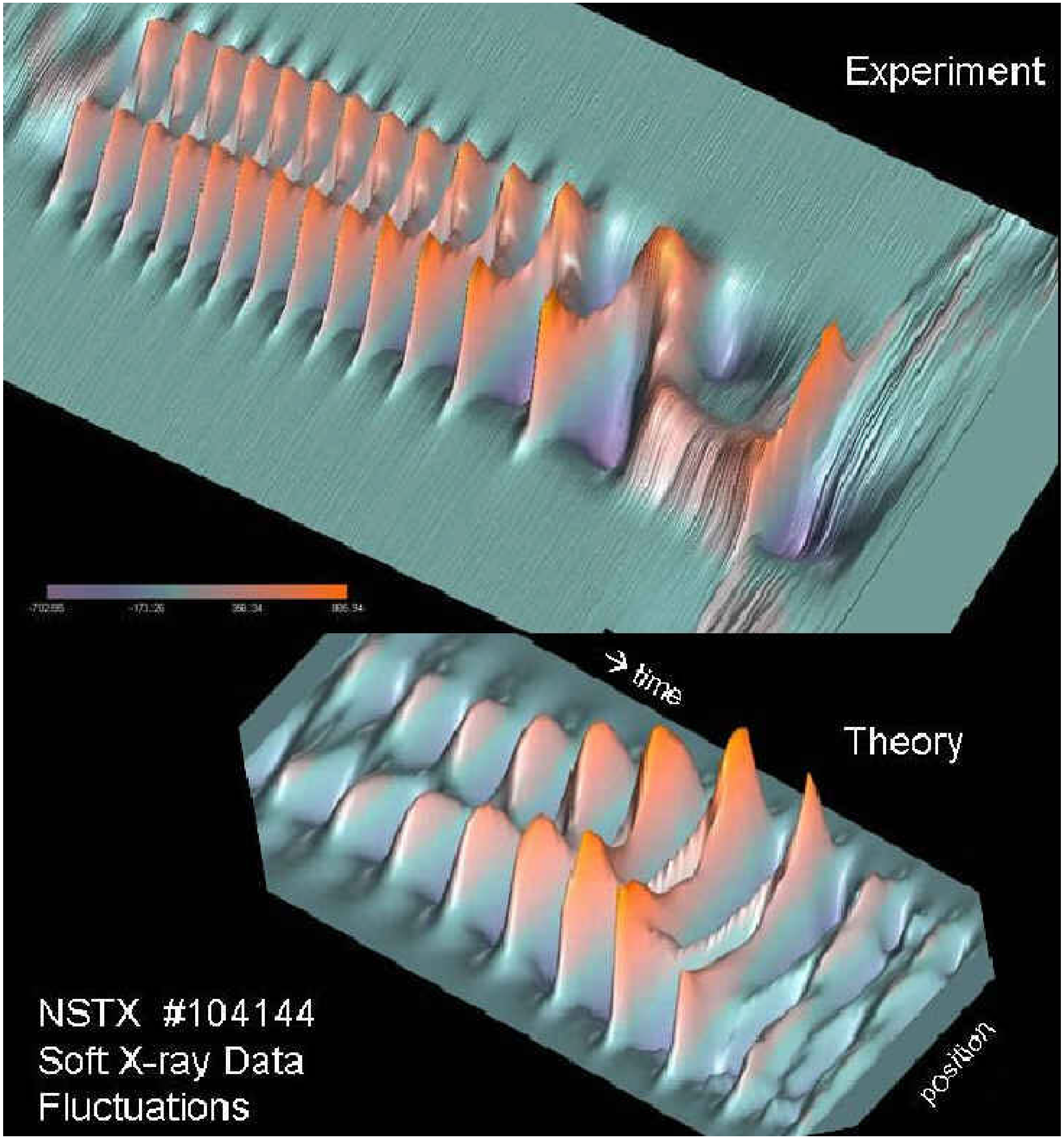
$M_A=0$
$\Omega m=0$

With shear flow: $M_A=0.2$
Rotating mode: $\Omega m=0.13$



Nonlinear Evolution without strong flow: similar to a sawtooth crash

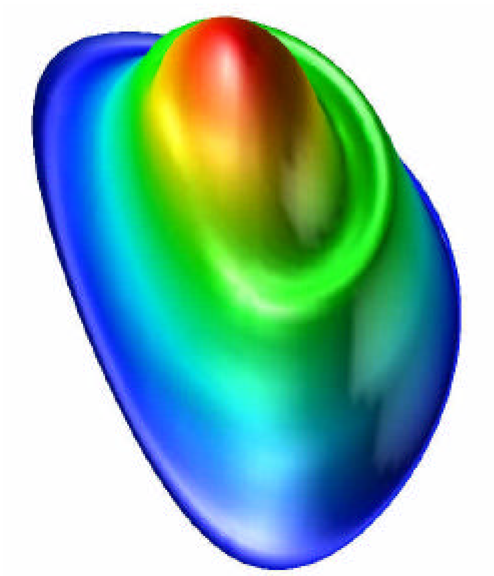




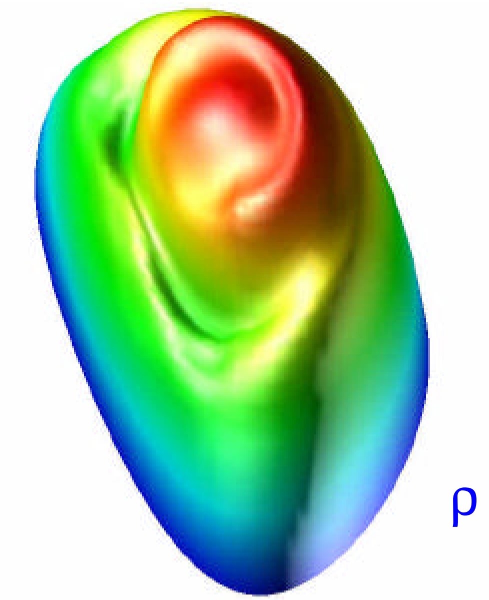
Soft X-ray signals compared:

Theory agrees with experiment on general characters, but does not have wall locking and a saturation phase.

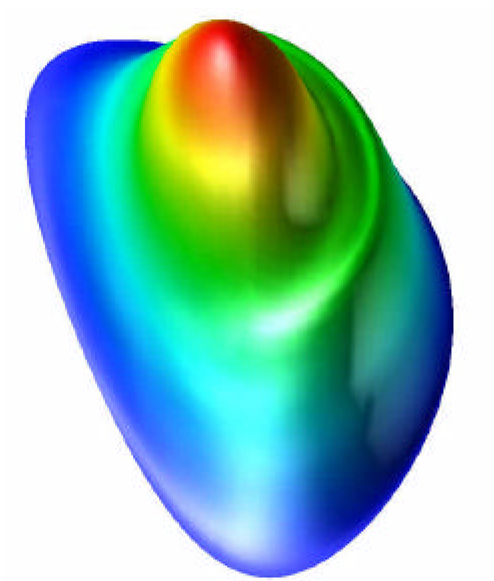
Nonlinear Evolution without strong flow



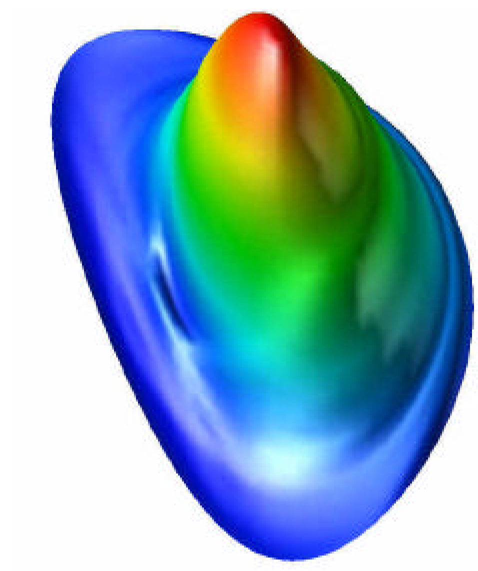
T



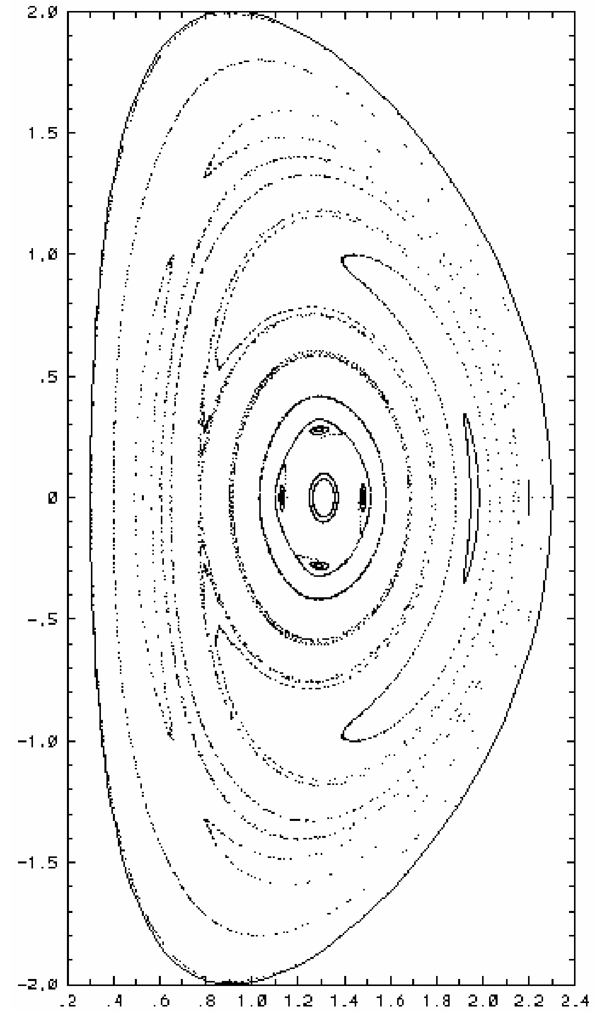
ρ

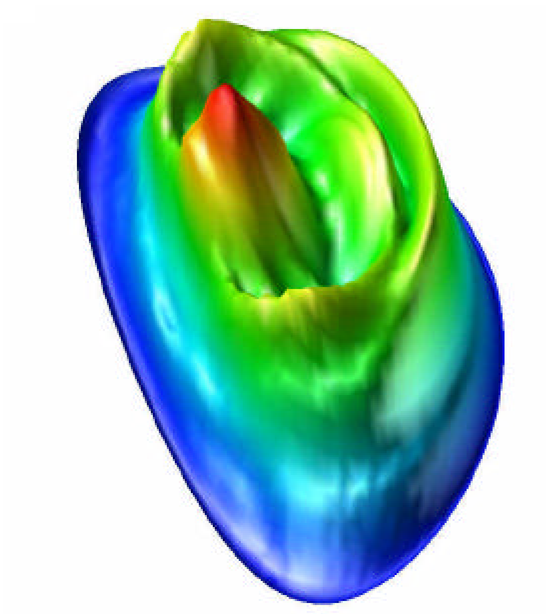


P



Jϕ





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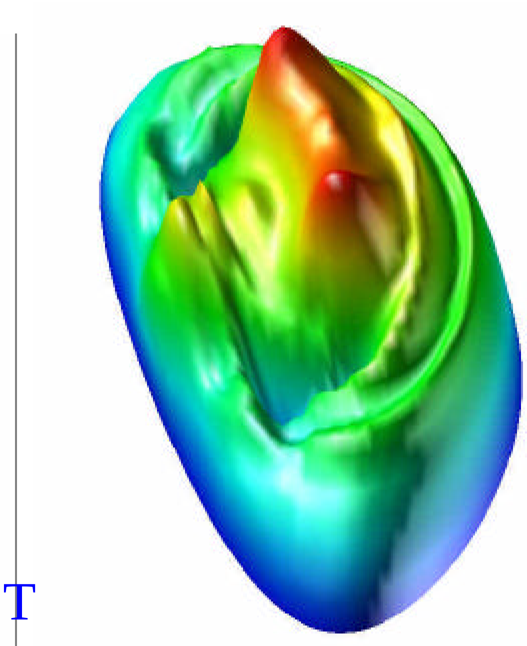
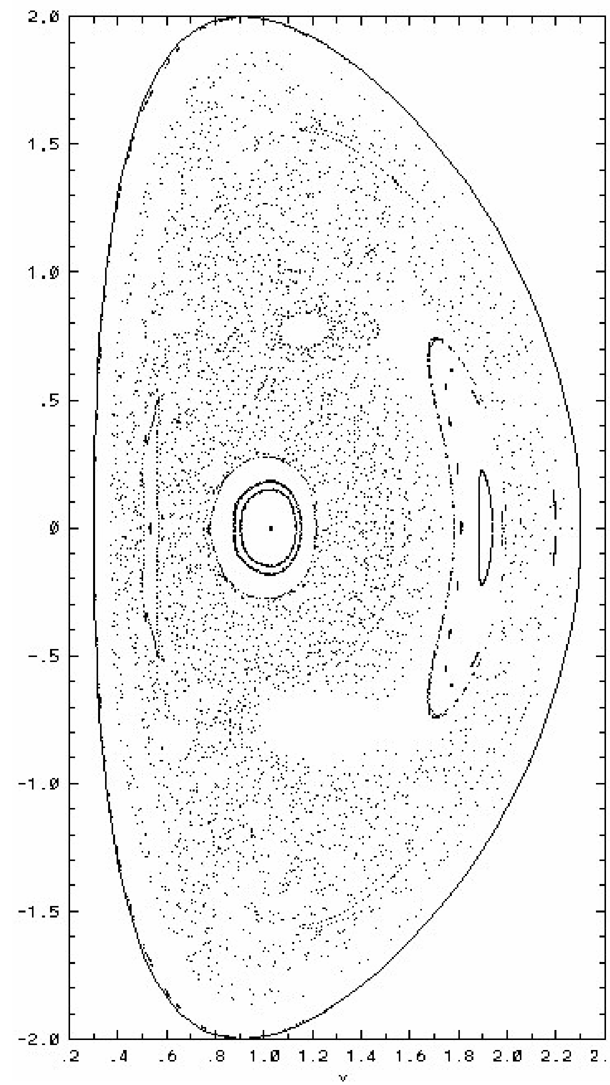
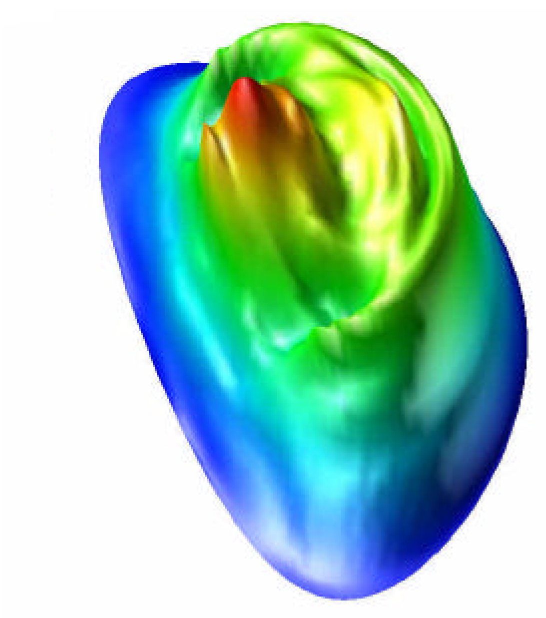


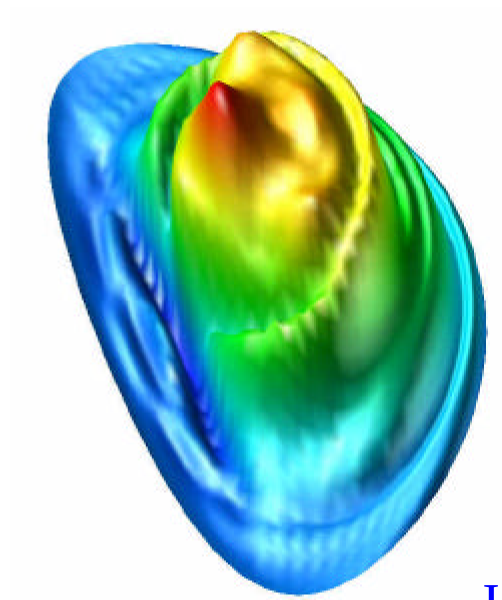
ρ

mostly stochastic



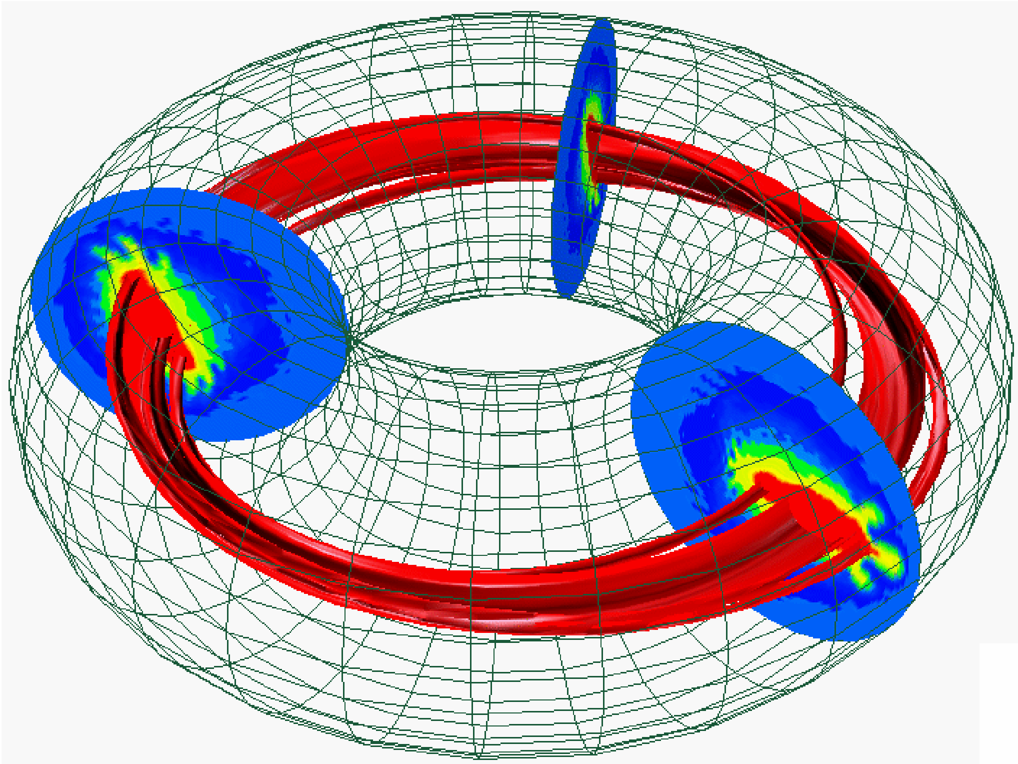


P



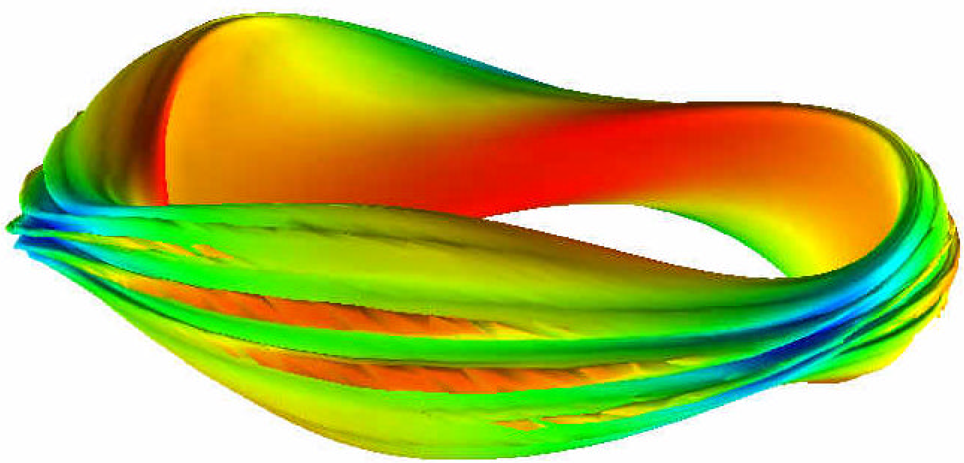
Jϕ

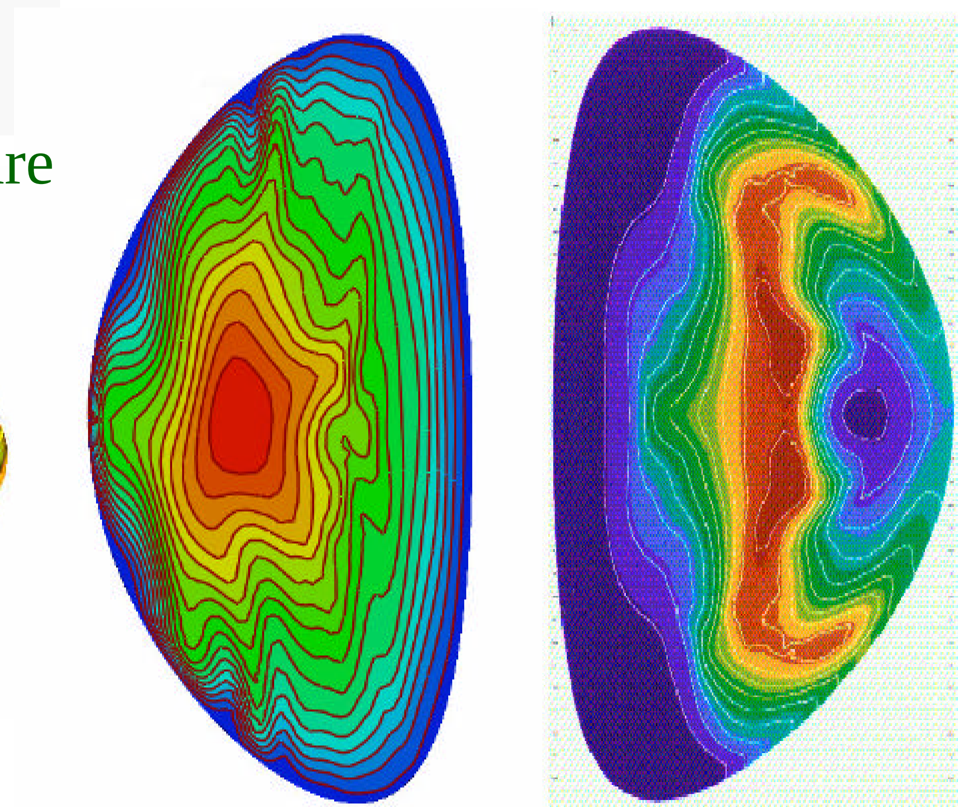
IRE : Disruption



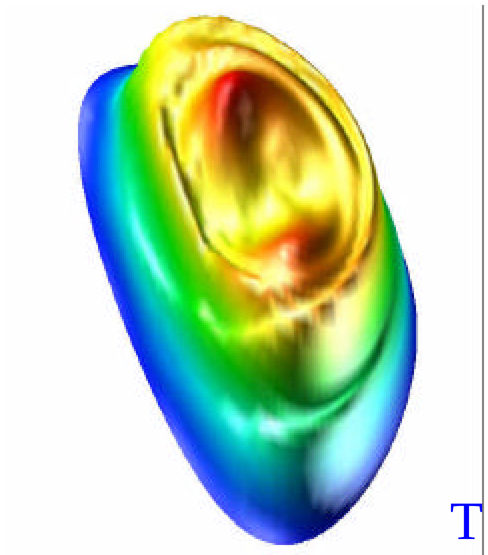
•Stochasticity as shown before.
•Localized steepening of pressure driven modes as shown here.

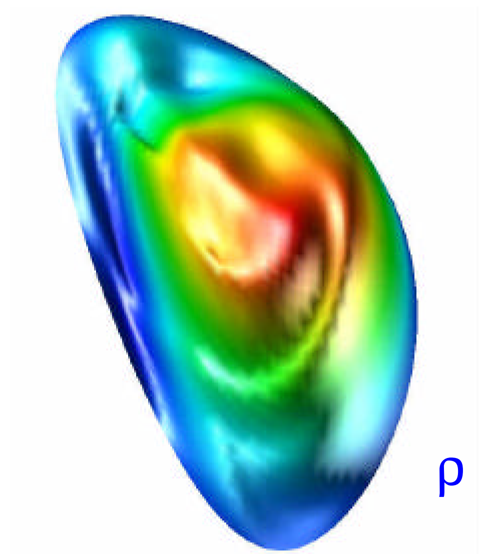
Pressure





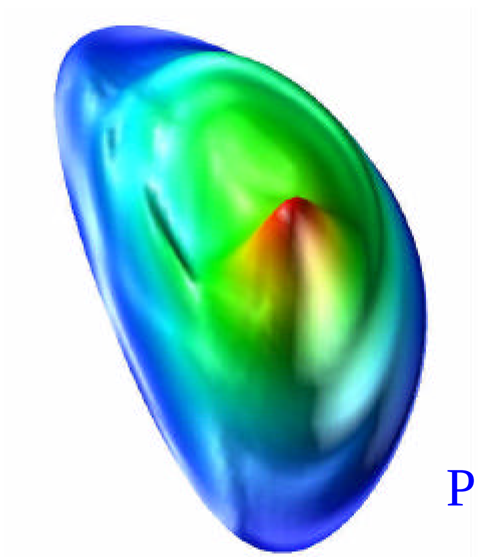
Nonlinear Evolution with peak rotation of M_A=0.2

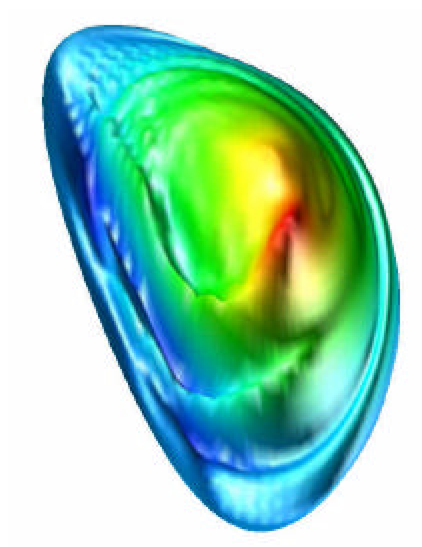


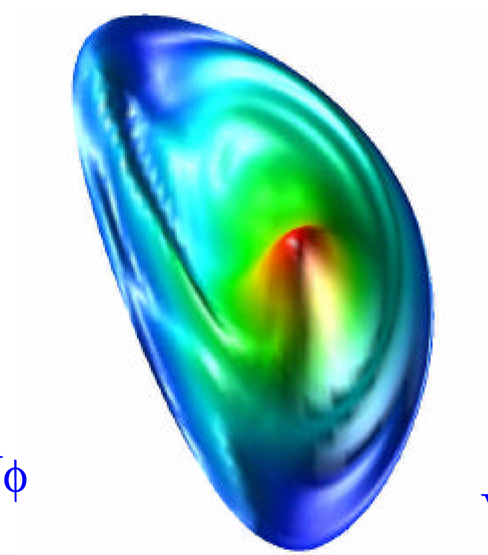


Vϕ profile evolves
with reconnection

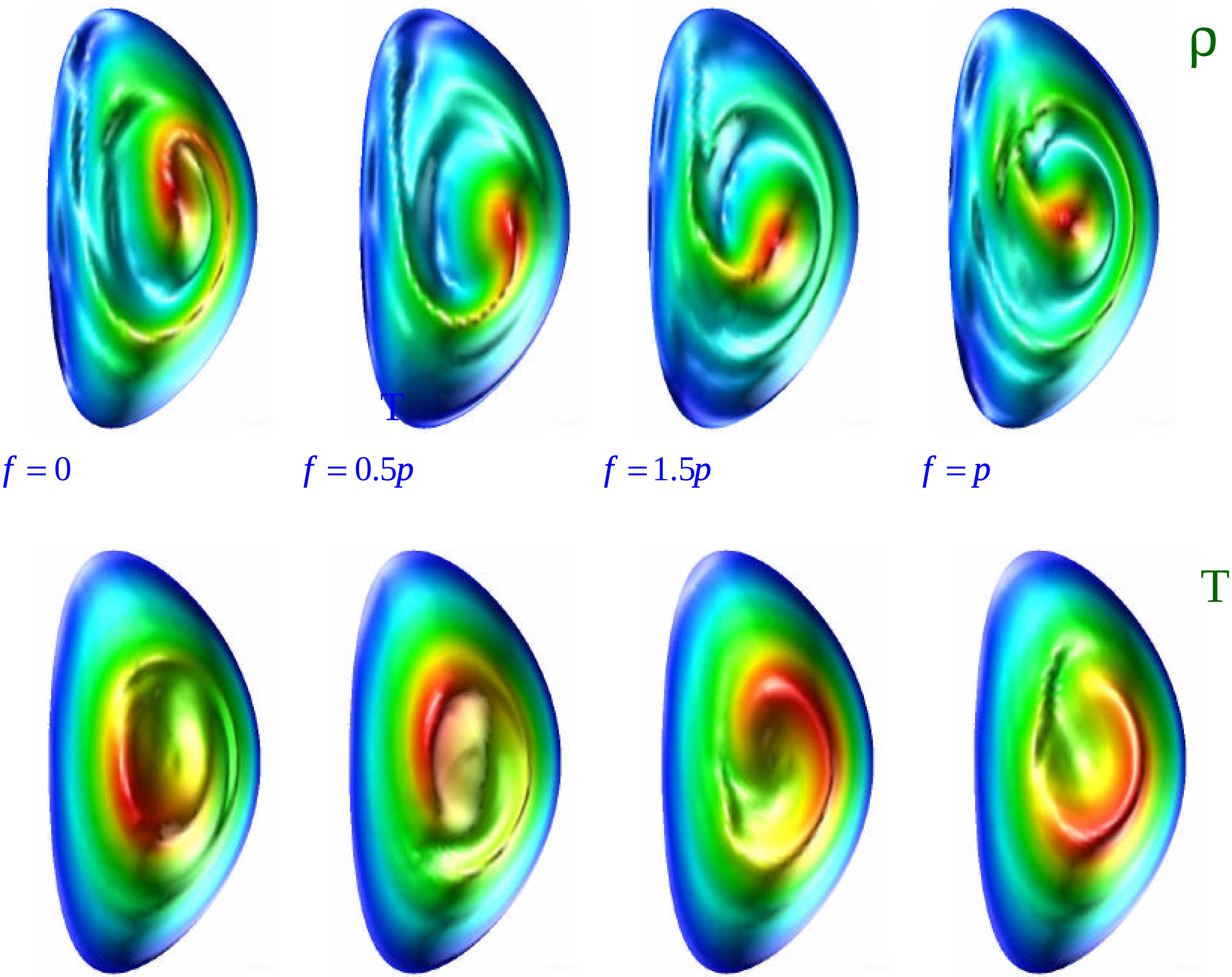
Momentum source rate
is an important factor



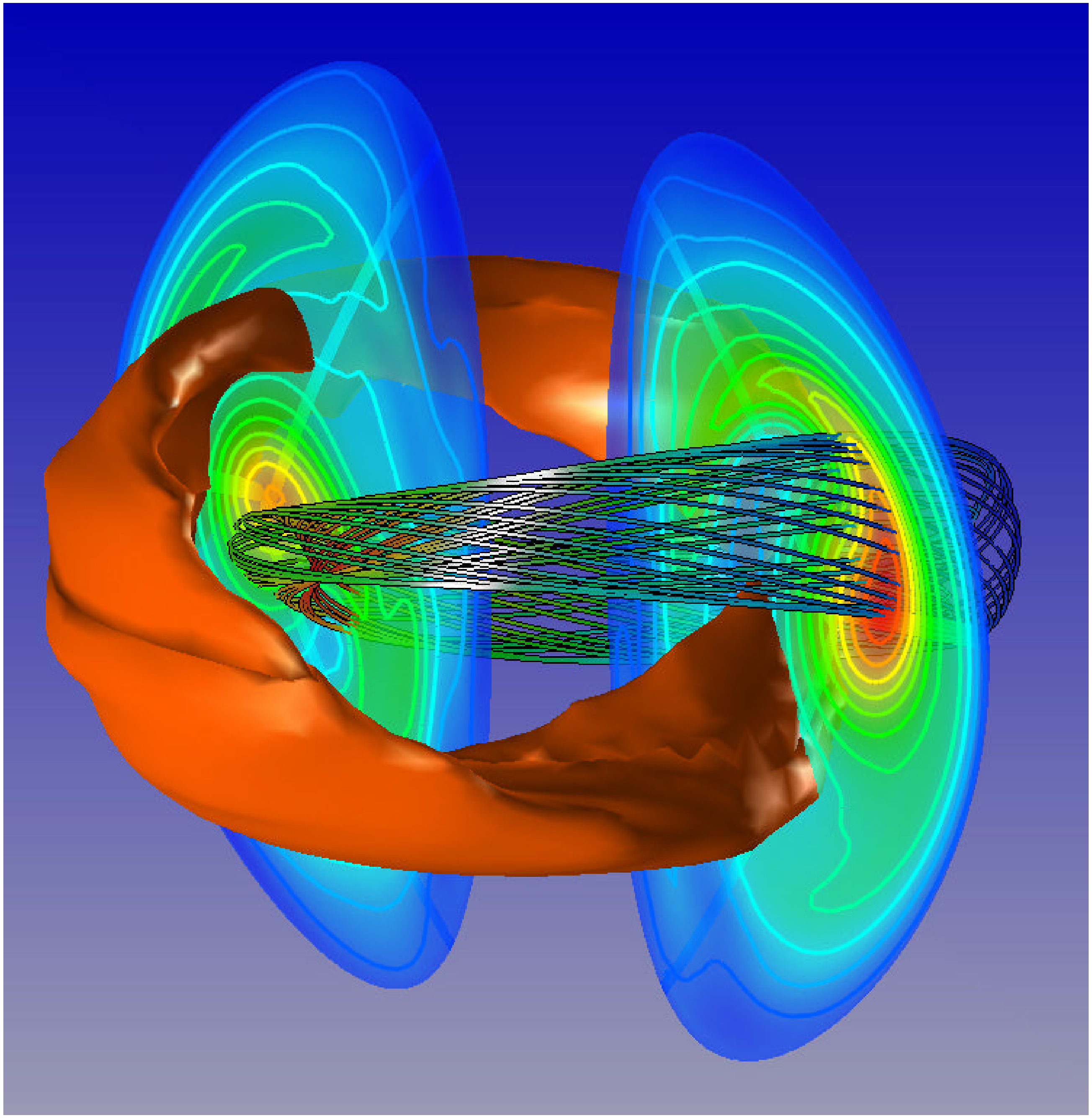




ρ (P) and T out of phase in a saturated case



Saturated steady state with strong sheared flow



B Field line
in the island
Density (Pressure)
contours
Temperature
isosurface

Pressure peak inside the island together with shear flow causes the mode saturation.

EPM (BAE) is excited at high beta in hybrid simulations

More coupling to sound wave due to stronger curvature and high beta.

May explain experimental data.

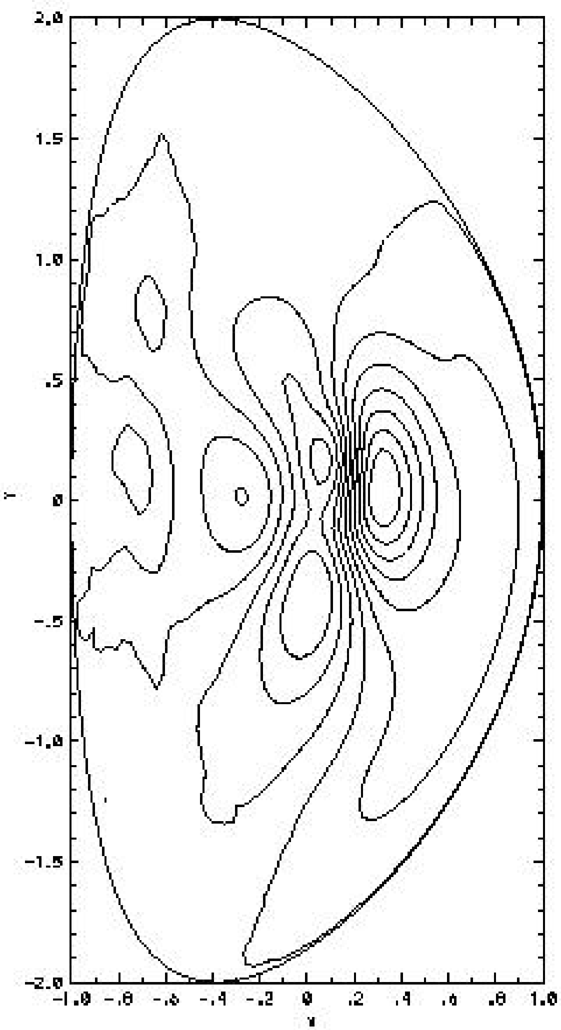
TAE

total=3.8%

h=3.8%

=0.327

=0.013



BAE

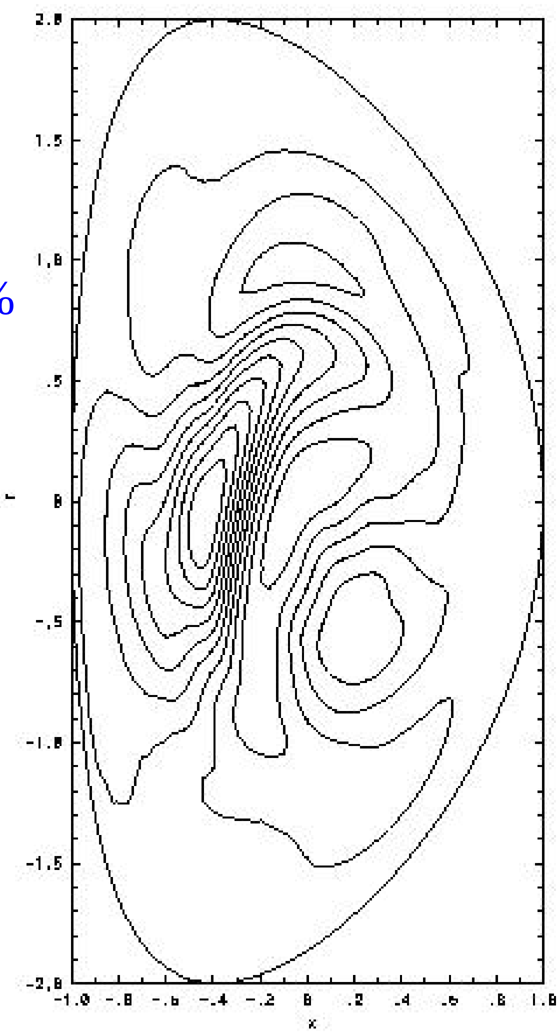
total=28%

h=3.8%

=0.067

=0.007

U



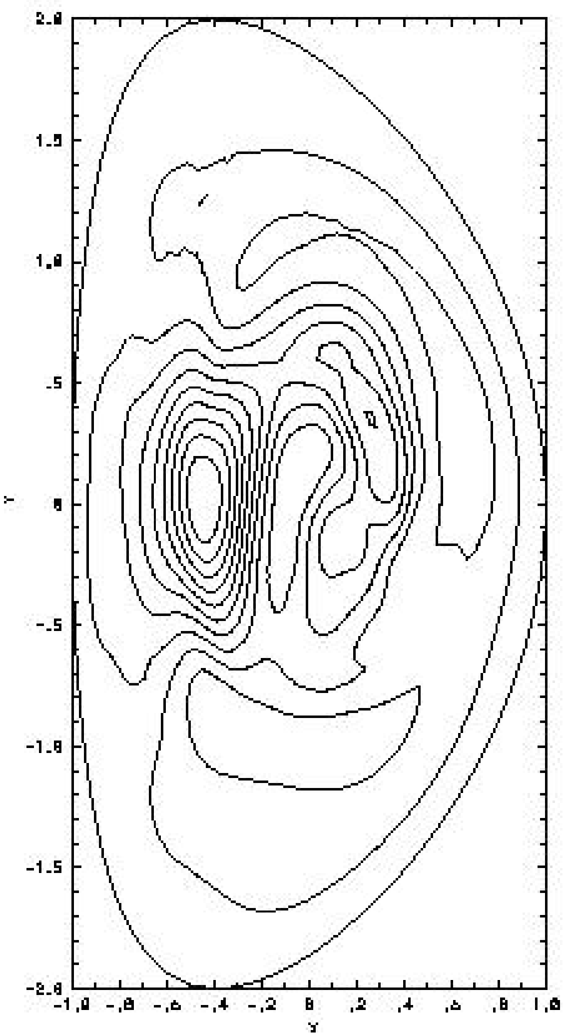
BAE changes to TAE when Γ is set to zero

BAE

Γ=1.666

=0.05

=0.003

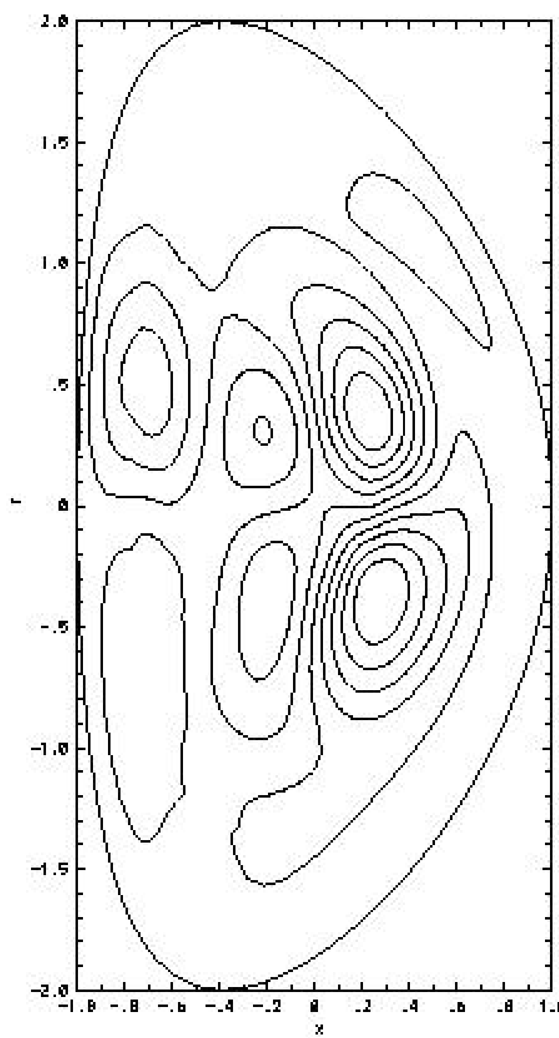


TAE

Γ=0

=0.24

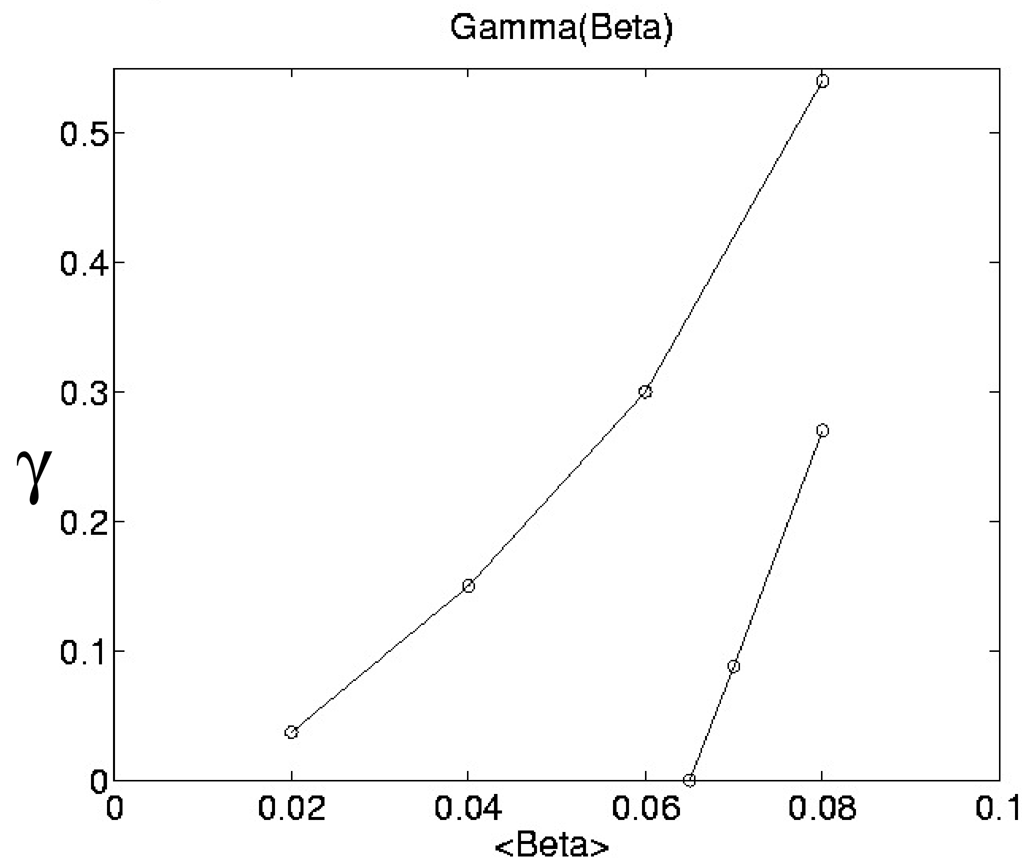
=0.02



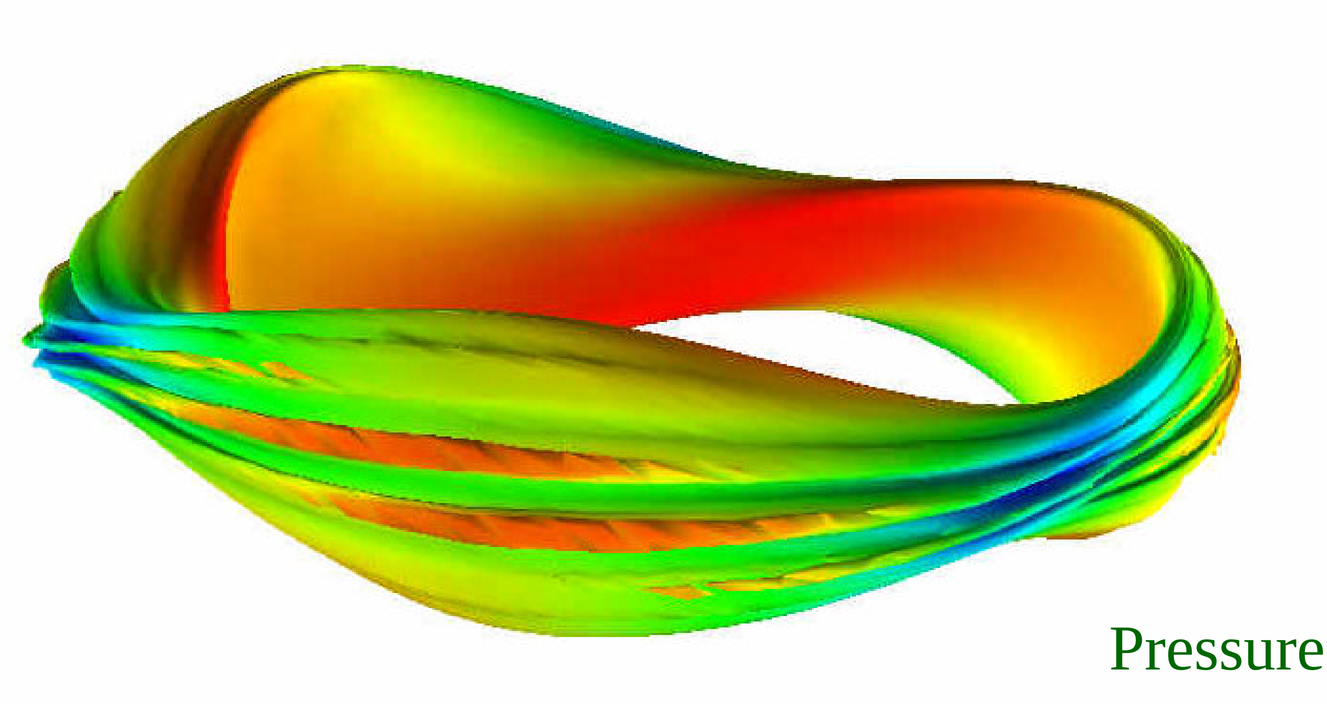
Stellarator Studies

NCSX li 383 with MHD model: Resistive ballooning and resistive interchange unstable below the design beta.

Growth rate γ of Ideal and Resistive Mode vs. β

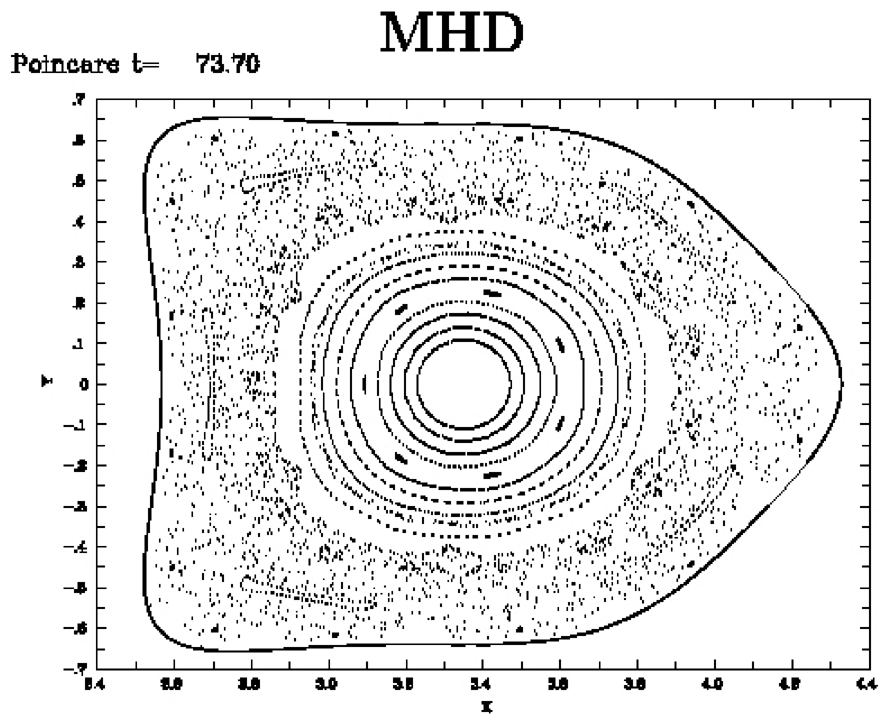


At β=8%, disruption can occur due to localized steepenings of pressure driven modes.

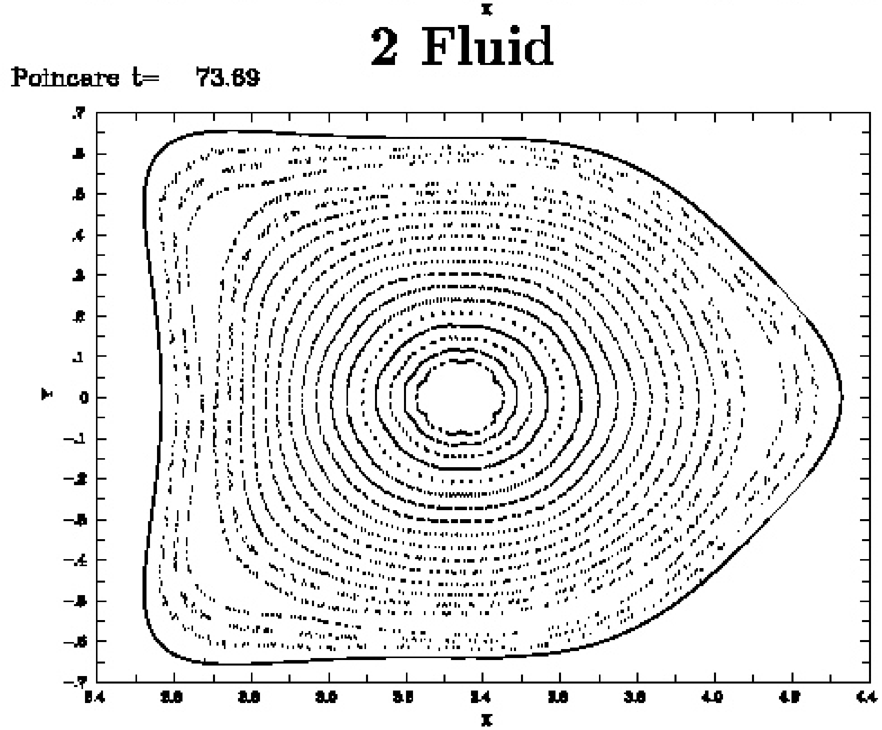


Pressure

Two-fluid effects seem to stabilize the resistive modes. May explain the absence of resistive modes in experiments.



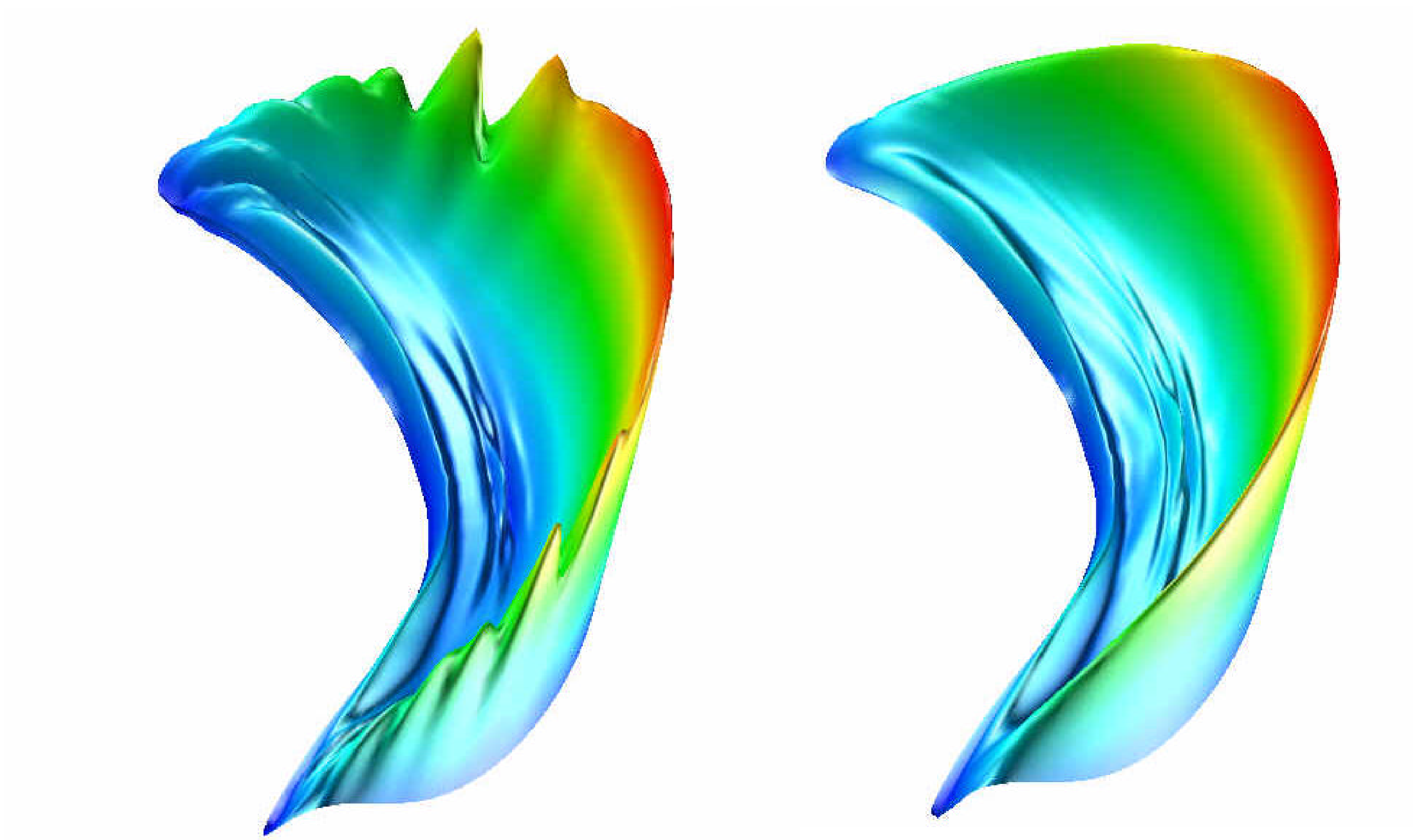
β=4%

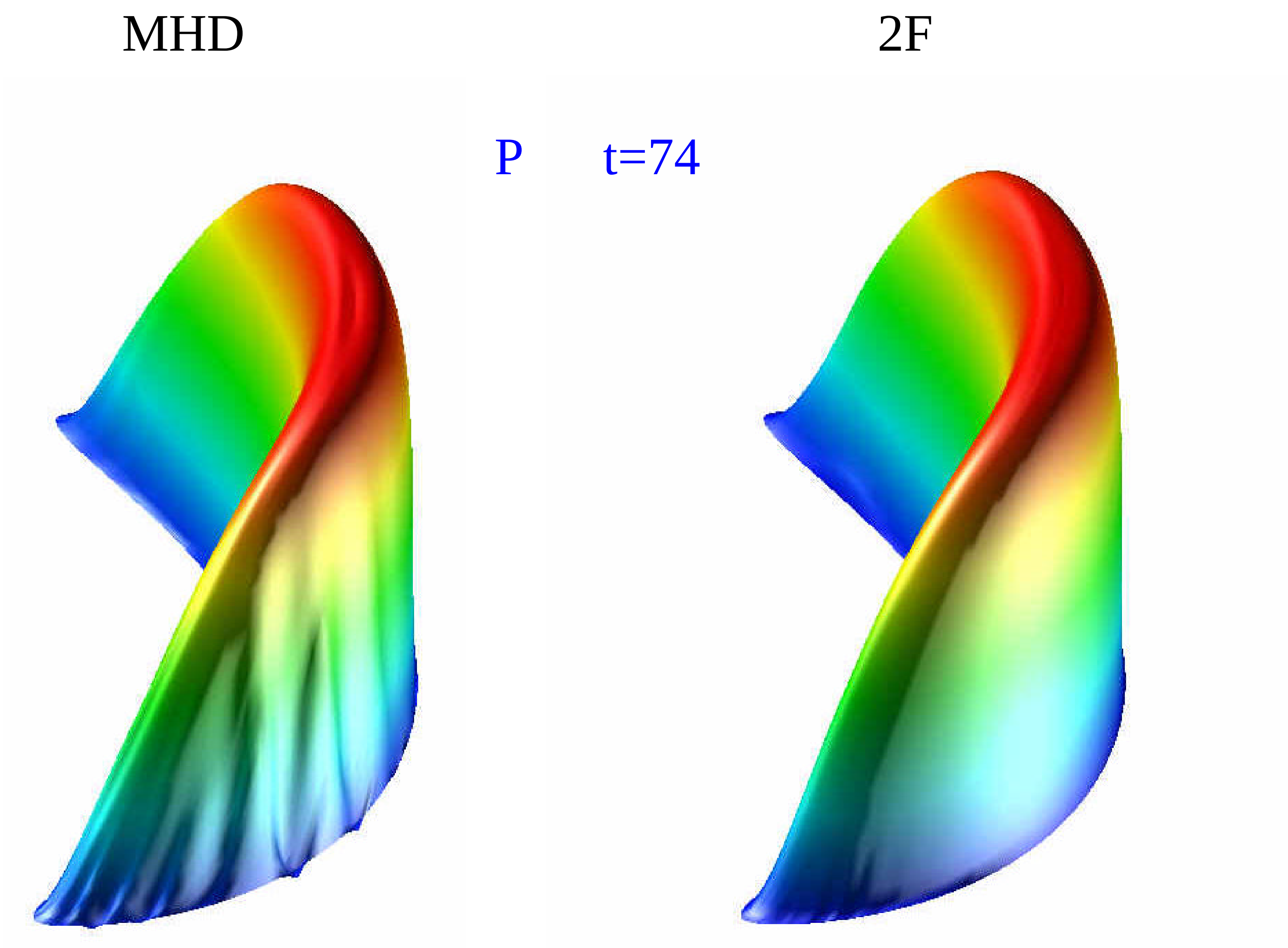


ω* stabilization is more effective for higher *n* modes which tend to be dominant in stellarators. This may also give substantially higher ideal beta limit.

MHD 2F

J_ϕ t=74

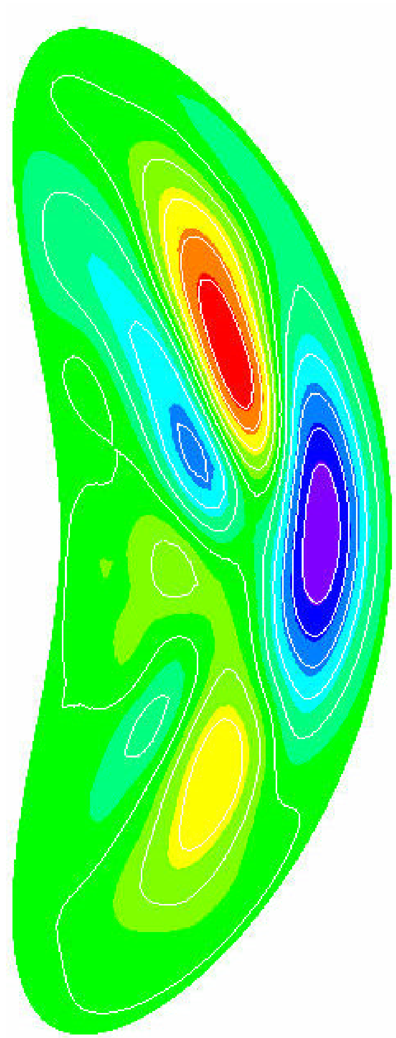



MHD
2F
P t=74

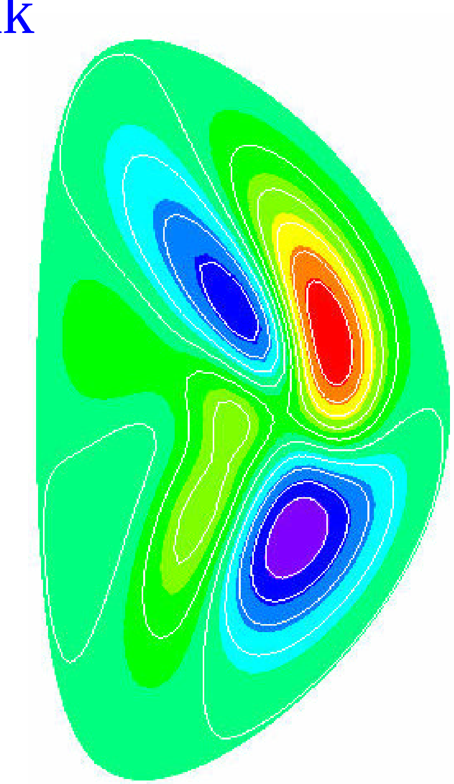
TAE Modes in Stellarators in Hybrid simulations

A 2-period QAS stellarator case is compared
to the case when the 3D shape is suppressed.

Stellarator



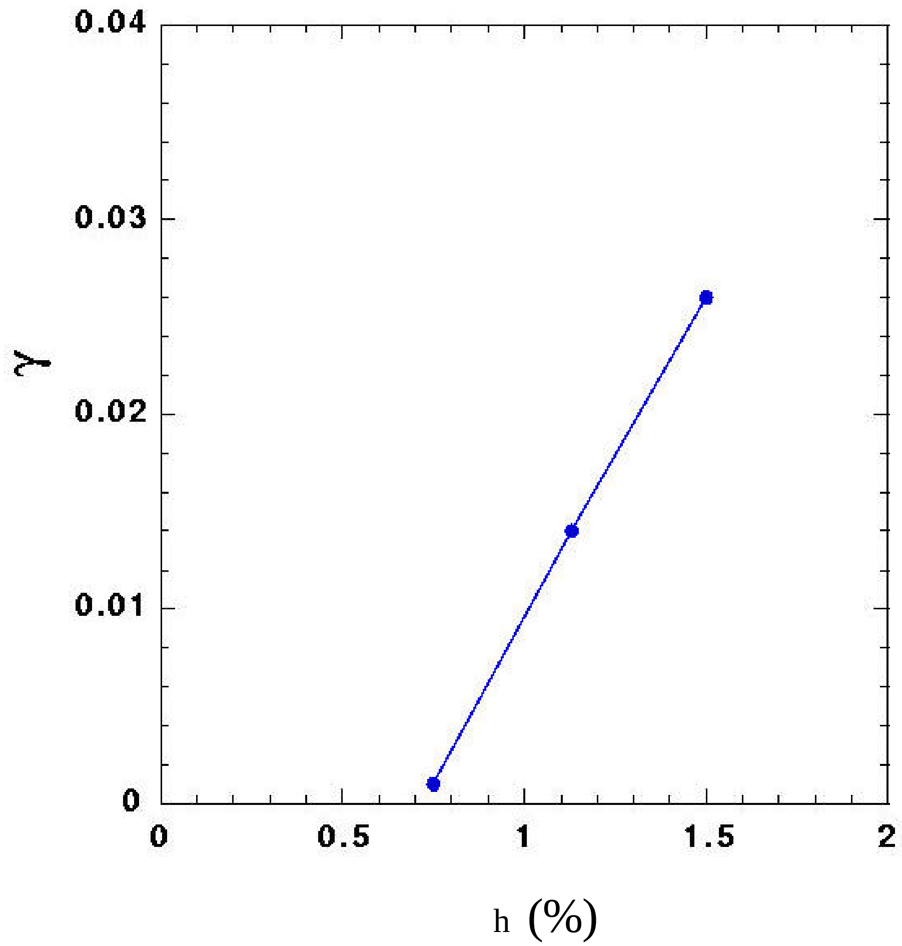
Tokamak

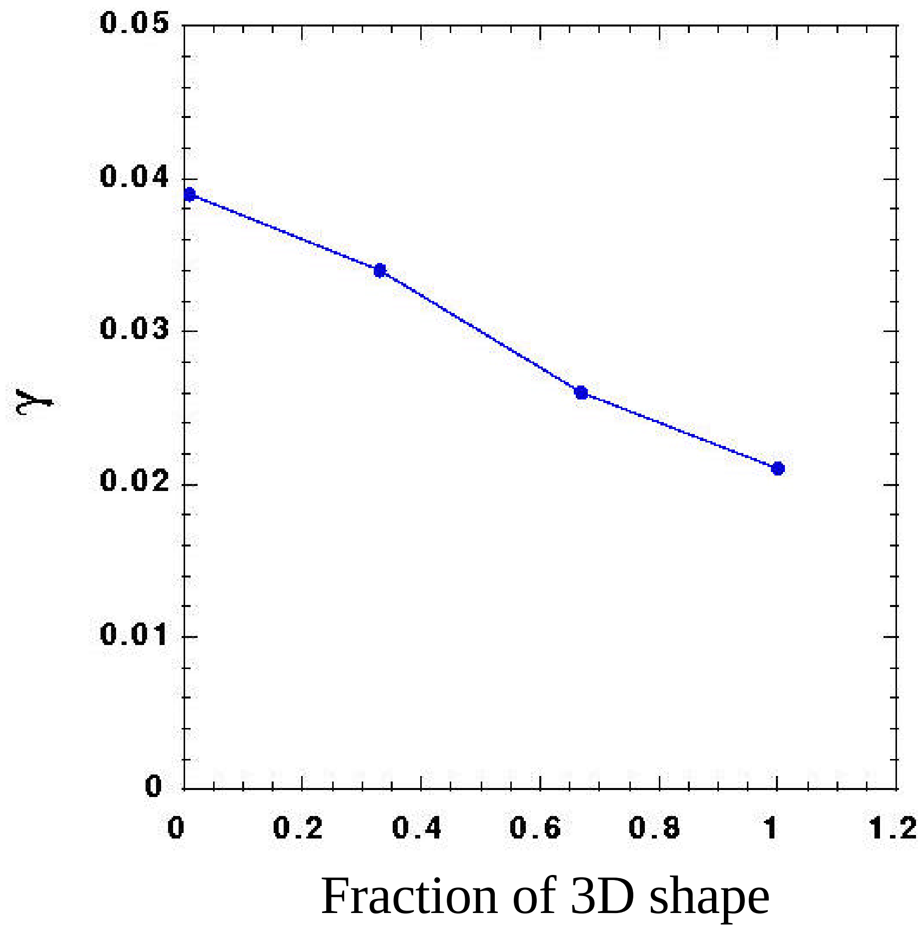


U

TAE growth versus hot ion beta: the growth rate is linear in hot ion beta.

TAE growth versus the fraction of 3D shape: 3D geometry is stabilizing.





Summary

- M3D code is used for simulation studies.
- NSTX studies including flow effects:

  The relative density shift relation holds in 2D steady states.

  Toroidal sheared rotation reduces linear growth and can saturate internal kink.

  IRE:Disruption can occur in at least two ways; due to stochasticity, and due to localized steepening of pressure driven modes.

  BAE mode is found which may explain experimental data.
- Stellarator studies:

  Two-fluid effects seem to stabilize resistive interchange, and may also give significantly higher ideal mode beta limit.

  3D shape is stabilizing to TAE.
- For more quantitative studies with more realistic parameters, better physics models, mesh schemes, numerical algorithms, and parallel processing structures are being pursued.