General parallel closures for plasma fluid equations [1]

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Closures Workshop

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Overview

- **<u>Objective</u>: Numerically simulate plasmas confined by slowly evolving magnetic fields.**

- **Requires closed system of equations with <u>general</u> treatment of parallel dynamics.**

- **"General" implies as much physics as possible.**

$$\frac{\partial}{\partial t} \sim \nu \sim k_{\|} v_{th}$$

Collisional or collisionless transport?

- **Nature of parallel transport in vicinity of magnetic island varies.**

  Collisional near X-points: $k_\| v_{th}/\nu \to 0$.

  Moderately collisional inside island: $k_\| v_{th}/\nu \geq 1$

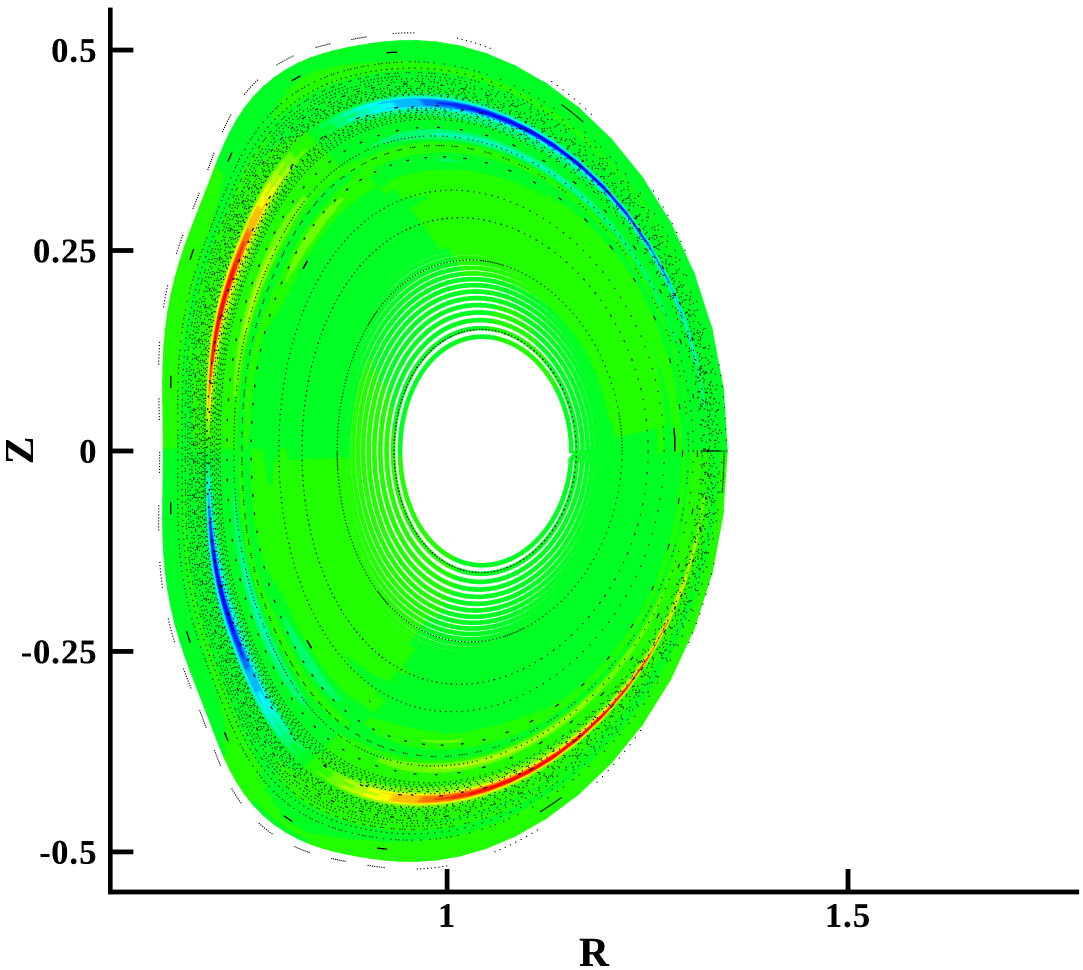


Figure 1: Perturbed heat flow contours due to a **2/1** magnetic island.

- **Closures should allow for arbitrary collisionality.**

General $\Longrightarrow$ Intractable?

- **Model requirements for quantitative parallel closures.**
  1. **Relevant electron and ion kinetic equations**

     Drift kinetic equations (DKE)

  2. **Good collision operator**
     Lorentz scattering with nonlocal restoring terms

  3. **Free-streaming and time-dependent physics**
     Integration involving characteristics.

- **Closures must also be numerically tractable.**
  1. **Time spent calculating closures $\sim$ time for nonlinear step.**

  2. **Closures must be robust, numerically stable.**

Take Chapman-Enskog-like (CEL) approach.

- **Write f as the sum of Maxwellian, f_M, and kinetic distortion, F:**

$$f = f_M + F = n(\vec{x},t)\left[\frac{m}{2\pi T(\vec{x},t)}\right]^{\frac{3}{2}} exp\left[-\frac{m(\vec{v}-\vec{V})^2}{2T(\vec{x},t)}\right] + F,$$

- **Why 5-moment CEL approach?**
  1. **Keeps fluid and kinetic physics separate.**
  2. **$\vec{V}$ and T (or p) readily available in most plasma fluid codes.**
  3. **Density does not appear directly as drive.**
  4. **Fewer fluid equations.**

WARNING: 5-moment approach is less messy but requires better solution of kinetic equation.

- **Write but <u>do not</u> solve full CEL kinetic equation:**

$$\frac{\partial F}{\partial t} + \vec{v}\cdot\vec{\nabla}F + \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\vec{\nabla}_{\vec{v}'}F - C(f) =$$

$$m\left(\vec{v}'\vec{v}' - \frac{v'^2}{3}\mathrm{I}\right) : \vec{\nabla}\vec{V}\frac{f_M}{T} + \vec{v}'\cdot\left(\vec{\nabla}\cdot\Pi - \vec{R}\right)\frac{f_M}{p} -$$

$$\frac{2}{3}L_1^{(\frac{1}{2})}\left(\Pi : \vec{\nabla}\vec{V} + \vec{\nabla}\cdot\vec{q} - Q\right)\frac{f_M}{p} + L_1^{(\frac{3}{2})}\vec{v}'\cdot\vec{\nabla}T\frac{f_M}{T}.$$

Gyroaverage kinetic equations and define parallel closure moments.

- Gyroaverage full kinetic equations, order out drift effects and ignore remaining annoying terms (nonlinearities, parallel drift terms, etc...).

$$\left(\frac{\partial}{\partial t} + \vec{v}_{\|} \cdot \vec{\nabla}\right) \bar{F} - \left\langle C(\bar{F} + f_M) \right\rangle =$$

$$\begin{aligned}
\frac{f_{M0}}{p_0}(- \quad & P_0(\xi) \quad && (2/3) L_1^{(\frac{1}{2})} \nabla q_{\|} \\
+ \quad & P_1(\xi) \quad && v \left(\hat{b} \cdot \vec{\nabla} \cdot \Pi_{\|} - \mathrm{R}_{\|} + L_1^{(\frac{3}{2})} n_0 \nabla_{\|} T \right) \\
- \quad & P_2(\xi) \quad && n_0 m v^2 \left(\hat{b}\hat{b} - \frac{\mathrm{I}}{3} \right) : \vec{\nabla}_{\|} \vec{V}_{\|}),
\end{aligned}$$

where $\xi = v_{\|}/v$, P_0, P_1 and P_2 are Legendre polynomials, and $\hat{b} \cdot \vec{\nabla} \cdot \Pi_{\|} = \hat{b} \cdot \vec{\nabla} \cdot \pi_{\|} \left(\hat{b}\hat{b} - (\mathrm{I}/3) \right) = (2/3) \nabla_{\|} \pi_{\|}$.

- Define collision operator.

$$\left\langle C(\bar{F}_s + f_{Ms}) \right\rangle = \underbrace{\sum_{s'} \nu_{ss'}(v) \frac{\partial}{\partial \xi}(1 - \xi^2) \frac{\partial \bar{F}_s}{\partial \xi}}_{\text{Lorentz operator}} + \underbrace{\sum_{s'} \sum_{k} \nu_{s'k}^{\mathrm{res}}(v) \psi_k(\bar{F}_{s'}) f_{Ms}}_{\text{restoring terms}}.$$

- Define parallel closures.

$$\pi_{\|} \equiv m \int d^3 v v^2 P_2(\xi) \bar{F},$$

and

$$q_{\|} \equiv -T \int d^3 v v P_1(\xi) L_1^{(\frac{3}{2})} \bar{F},$$

Homogeneous solution written in terms of characteristics.

- **Write homogeneous solution in terms of Legendre polynomials and characteristics, $\tau_{\pm} = (L/v) \mp t$.**

$$\bar{F}_h = \sum_{n=0}^{N} P_n(\xi) \sum_{i=0}^{N} \left[a_{ni}^{+} e^{k_i^{+} \nu_s \tau_{+}'} + a_{ni}^{-} e^{k_i^{-} \nu_s \tau_{-}'} \right],$$

where $a_{ni}^{\pm}$ and $k_i^{\pm}$ are constants and $\nu_s(v) = \sum_{s'} \nu_{ss'}(v)$.

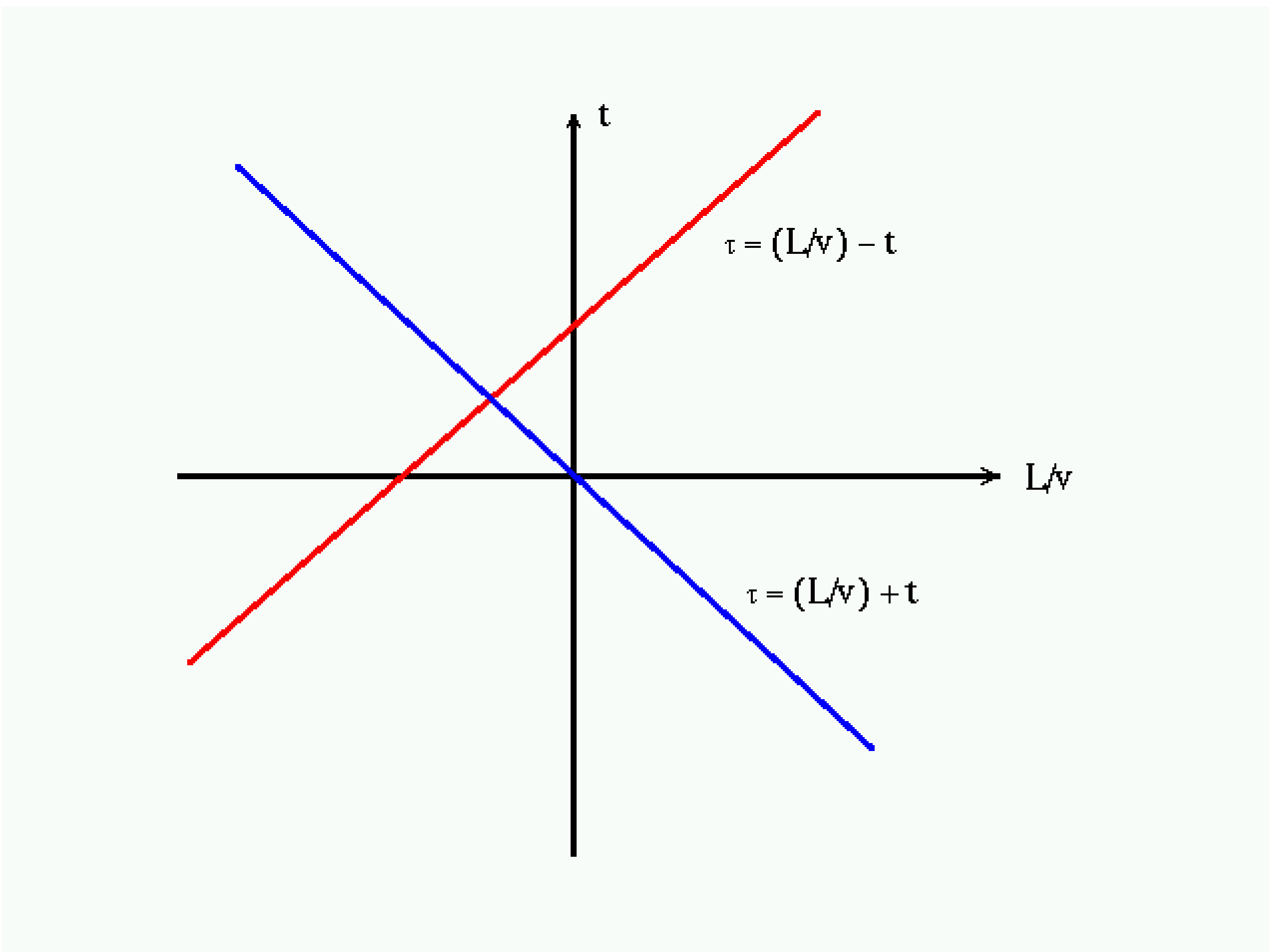


Figure 2: Sample characteristics along which homogeneous solution is constant.

Integration along characteristics captures nonlocal effects.

- **Inhomogeneous solution shows nonlocal dependence on fluid and closure moments.**

$$\bar{F} = \sum_{n=0}^{N} P_n(\xi) \sum_{i=0}^{N} \sum_{\pm} \bar{a}_{ni}^{\pm} \int^{\tau'} d\tau g(v,t,L) \frac{f_{M0}}{p_0} e^{k_i^{\pm} \nu_s (\tau'_{\pm} - \tau_{\pm})},$$

where $g = L_1^{(\frac{1}{2})} \underline{\nabla_{\|} q_{\|}} + v \underline{\left(\hat{b} \cdot \vec{\nabla} \cdot \Pi_{\|} - R_{\|}\right)} + n_0 v L_1^{(\frac{3}{2})} \nabla_{\|} T$
$- n_0 m v^2 \left(\hat{b}\hat{b} - \frac{I}{3}\right) : \vec{\nabla}_{\|} \vec{V}_{\|}$.

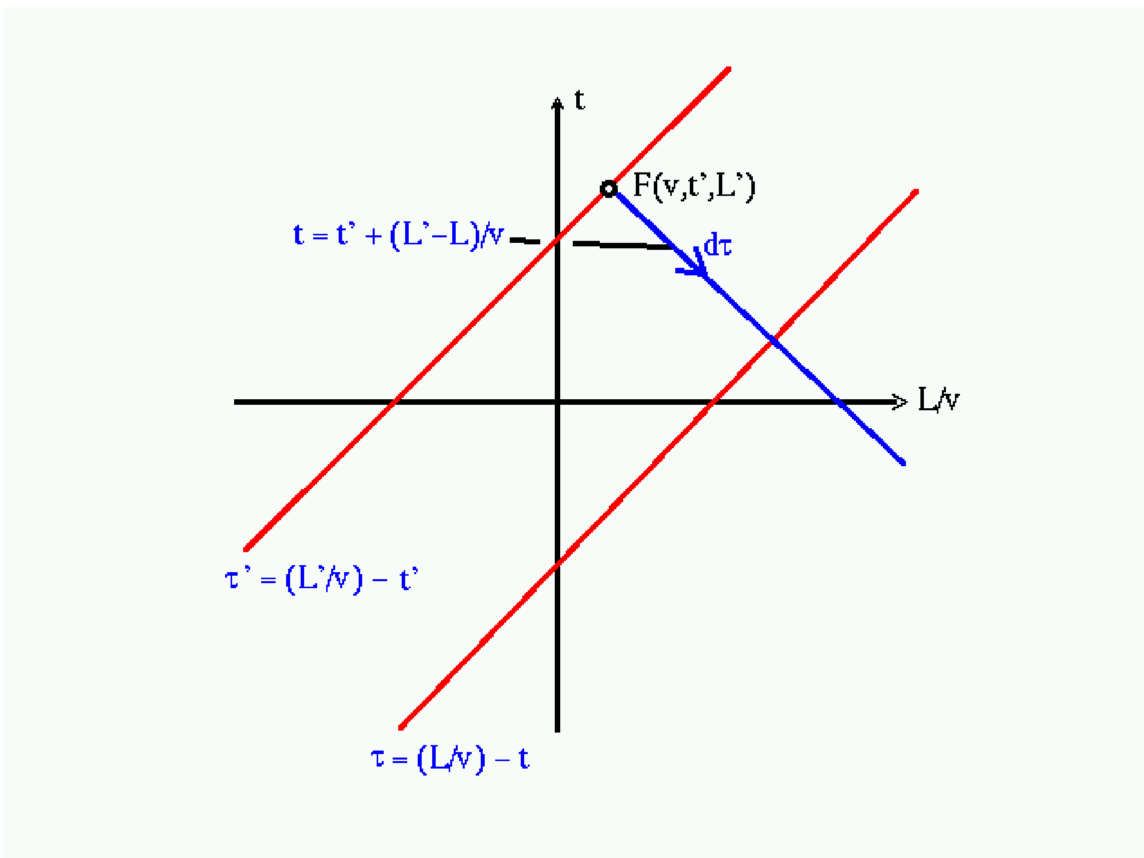


Figure 3: Integrals along $t = t' + (L' - L)/v$ connect characteristics when evaluating $\bar{F}$ at (v, t', L').

Closures take form of singular Fredholm integral equations.

- **Taking moments yields coupled system of integral equations.**

$$\begin{pmatrix} q_\parallel + Q(q_\parallel) & Q(\pi_\parallel) \\ P(q_\parallel) & \pi_\parallel + P(\pi_\parallel) \end{pmatrix} = \begin{pmatrix} Q(T) & Q(V_\parallel) \\ P(T) & P(V_\parallel) \end{pmatrix}$$

where, for example,

$$\begin{aligned} Q(T) &= -\frac{4\pi}{3}\int_0^\infty dv v^4 \left(L_1^{(\frac{3}{2})}\right)^2 f_{M0} \\ &\sum_{i=0}^{N}\left[\bar{a}_{1i}^{+}\int_{\tau'}^{\infty} d\tau(\nabla_\parallel T)e^{-k_i^{+}\nu_s(\tau-\tau')} + \bar{a}_{1i}^{-}\int_{-\infty}^{\tau'} d\tau(\nabla_\parallel T)e^{k_i^{-}\nu_s(\tau-\tau')}\right], \end{aligned}$$

and

$$\begin{aligned} Q(q_\parallel) &= -\frac{4\pi}{3}\int_0^\infty dv v^3 L_1^{(\frac{1}{2})} L_1^{(\frac{3}{2})} \frac{f_{M0}}{n_0} \\ &\sum_{i=0}^{N}\left[\bar{a}_{1i}^{+}\int_{\tau'}^{\infty} d\tau(\nabla_\parallel q_\parallel)e^{-k_i^{+}\nu_s(\tau-\tau')} + \bar{a}_{1i}^{-}\int_{-\infty}^{\tau'} d\tau(\nabla_\parallel q_\parallel)e^{k_i^{-}\nu_s(\tau-\tau')}\right], \end{aligned}$$

$$\begin{aligned} P(\pi_\parallel) &= \frac{8\pi}{15} m \int_0^\infty dv v^6 \frac{f_{M0}}{p_0} \\ &\sum_{i=0}^{N}\left[\bar{a}_{2i}^{+}\int_{\tau'}^{\infty} d\tau(\nabla_\parallel \pi_\parallel)e^{-k_i^{+}\nu_s(\tau-\tau')} + \bar{a}_{2i}^{-}\int_{-\infty}^{\tau'} d\tau(\nabla_\parallel \pi_\parallel)e^{k_i^{-}\nu_s(\tau-\tau')}\right], \end{aligned}$$

- **Solution exists in limit of zero mode frequency and sinusoidal perturbations in T and $V_\parallel$.**

Heat flux closure approximate for arbitrary collisionality.

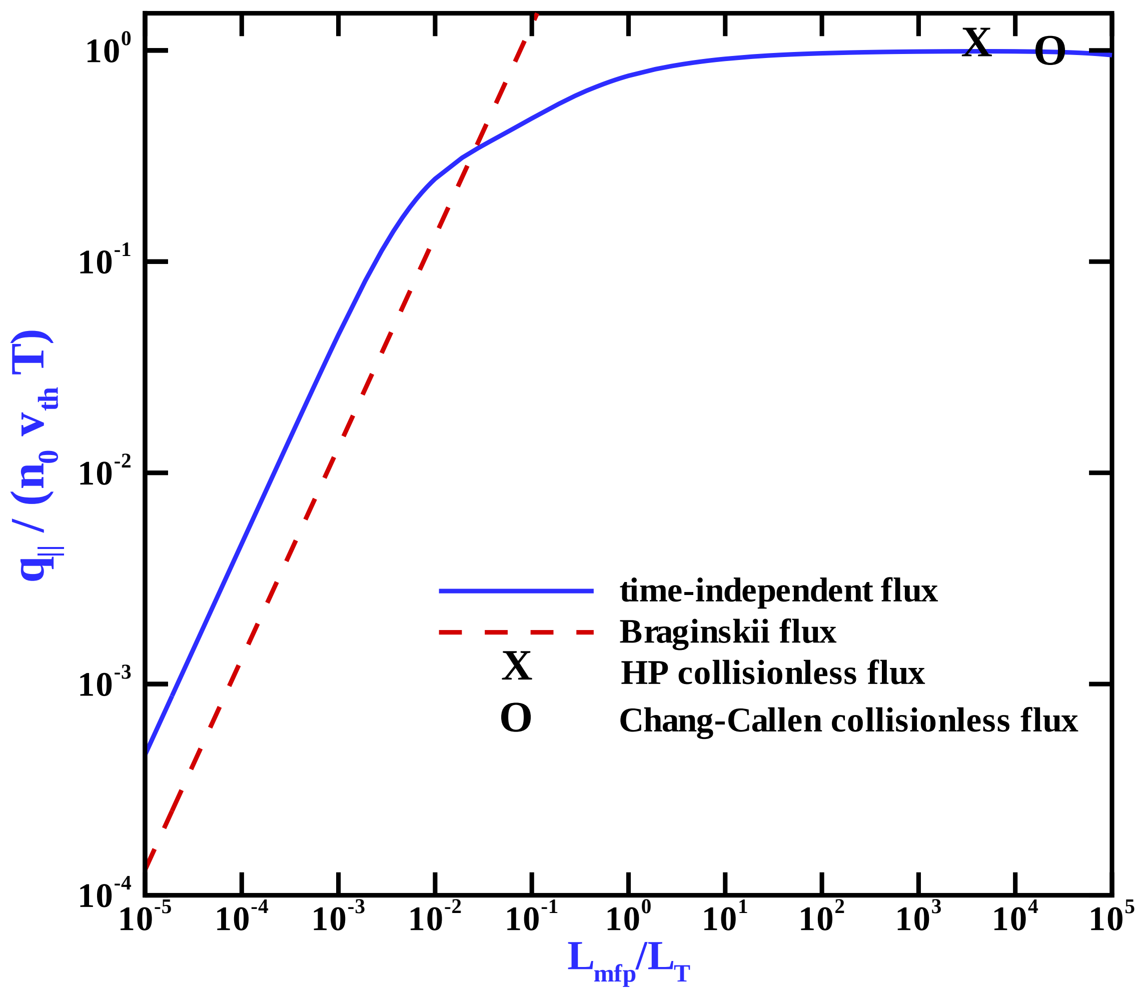


Figure 4: Temperature-driven heat flux for homogeneous magnetic field and sinusoidal temperature perturbations.

Parallel viscous stress approximate for arbitrary collisionality.

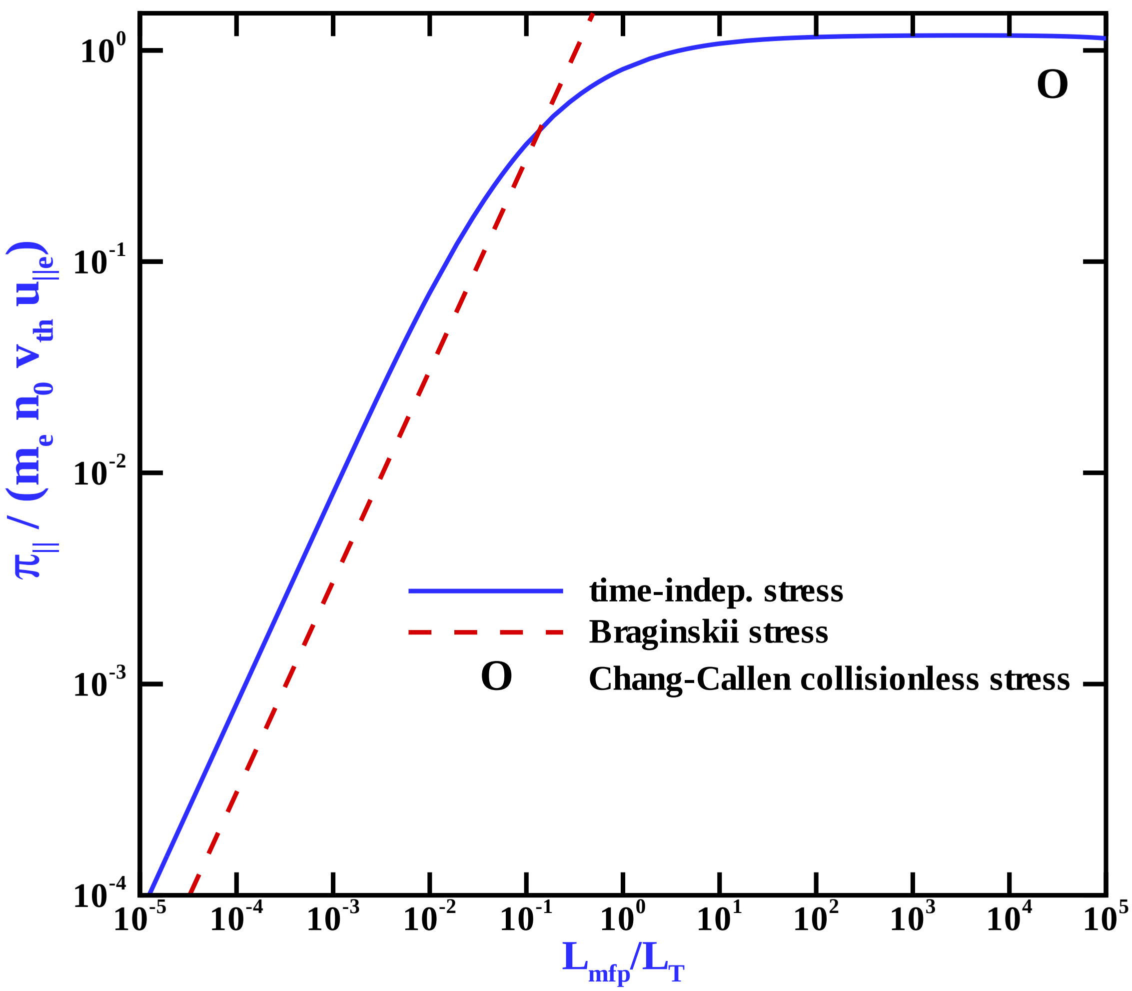
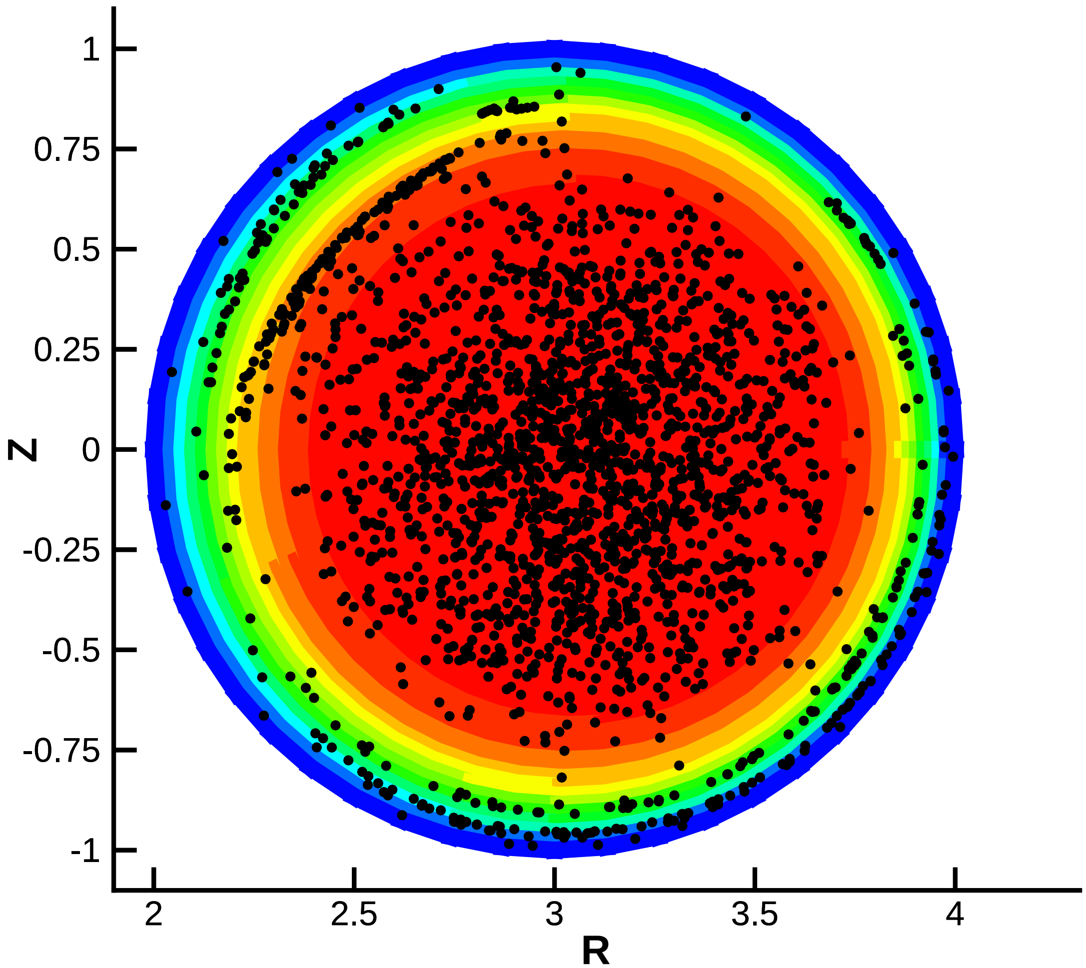


Figure 5: Flow-driven parallel viscous stress for homogeneous magnetic field and sinusoidal parallel flow perturbations.

Understanding electron heat confinement in RFP's requires nonlocal heat flux closure.

- **Field line diffusivity causes χ_ψ to greatly exceed predictions of classical transport.**



Nonlocal heat flux closure predicts heat flow against local gradients.

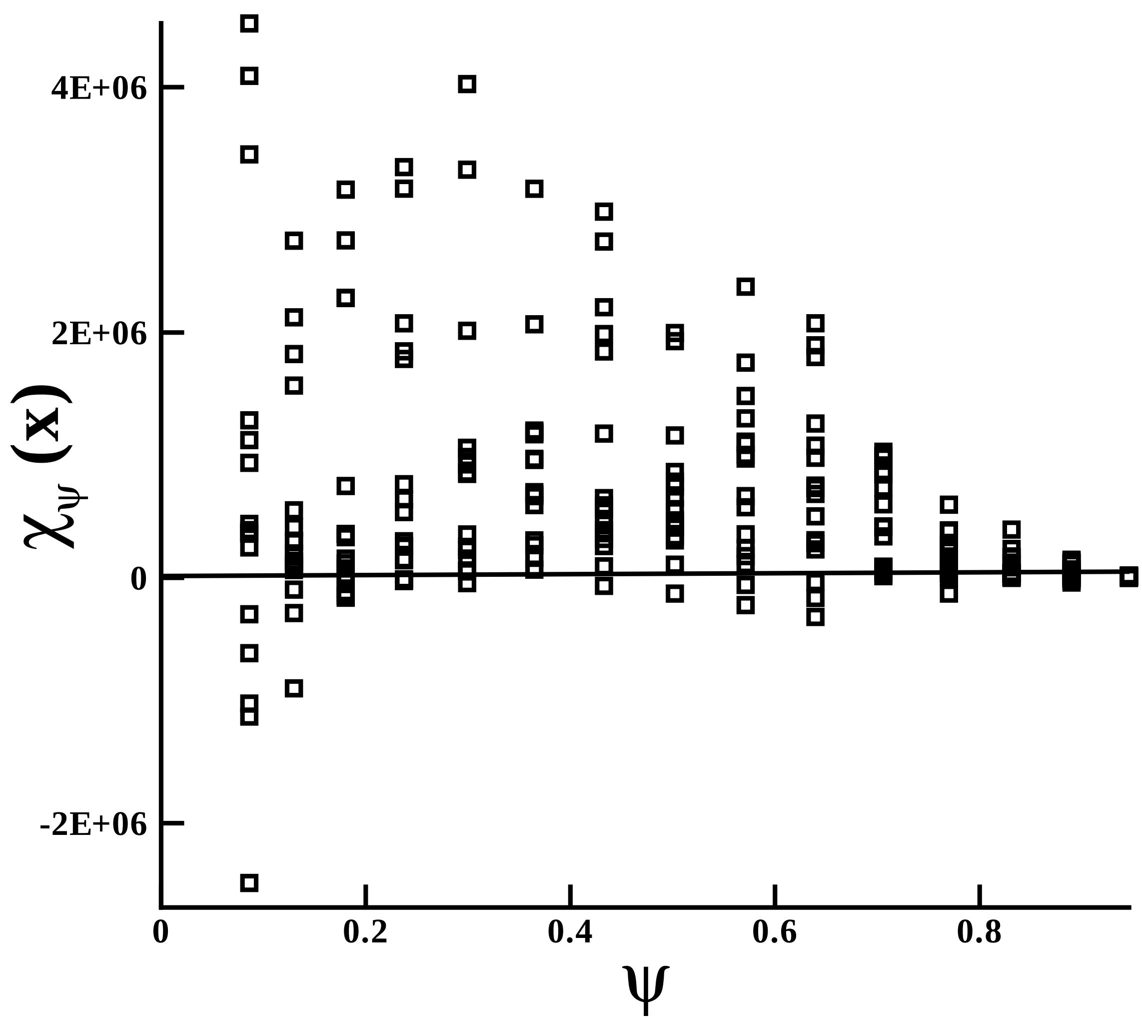


Figure 6: In a small subset of cases heat flows against local gradients for the superimposed axisymmetric temperature profile.

Sales Pitch and Shortcomings

- **General parallel closures promise novel physics from simulations of plasmas confined by slowly evolving magnetic fields.**

- **Outstanding issues exist.**

  1. **Incorporation of drift effects for electromagnetic, neoclassical closures.**

  2. **Inclusion of trapped particle effects in tori of arbitrary aspect ratio and shaping.**

  3. **Implementation of fully time-dependent solution desirable but difficult.**

  4. **Numerical performance still unknown.**