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Enhancement of Residual Zonal Flows in Helical Systems with Radial Electric Fields

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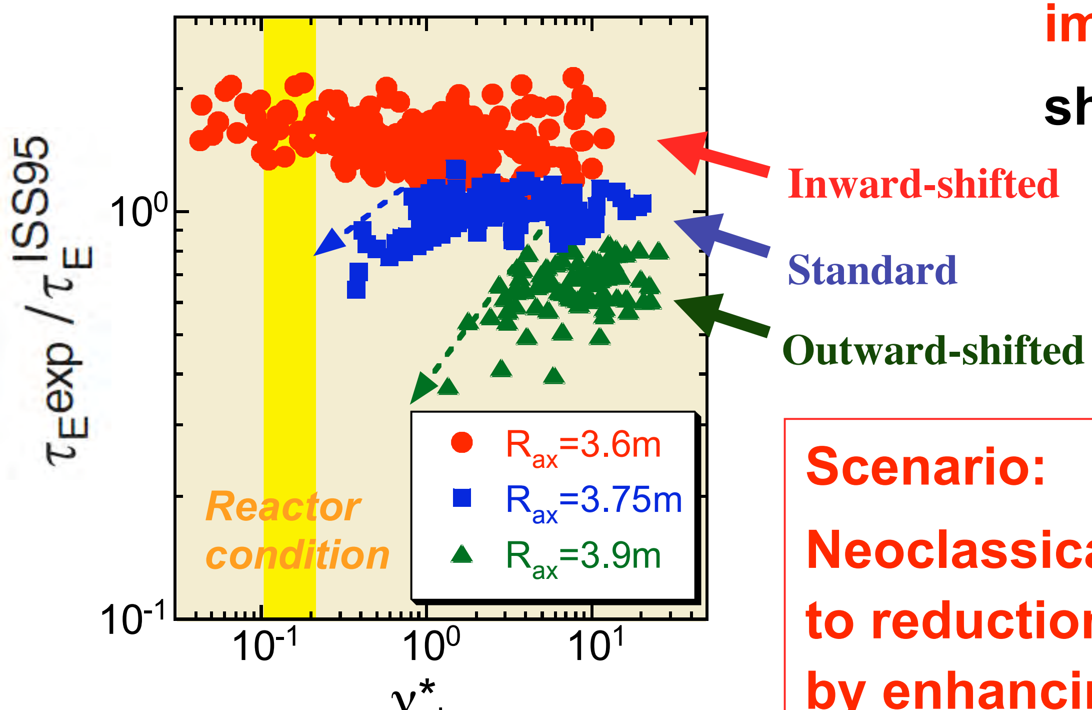
OUTLINE

- **Introduction**
- **Zonal Flows and ITG Turbulence in Helical Systems with $E_r = 0$**
- **Effects of Equilibrium Electric Field E_r on Zonal Flows in Helical Systems**
- **Summary**

Introduction : Results from LHD experiments

For low collisionality, better confinement is observed in the inward-shifted magnetic configurations, where lower neoclassical ripple transport but more unfavorable magnetic curvature driving pressure-gradient instabilities are anticipated.

Anomalous transport is also improved in the inward shifted configuration.



H. Yamada *et al.* (PPCF2001)

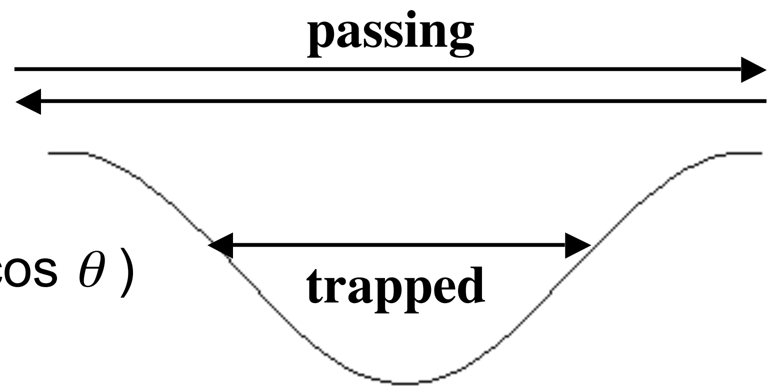
Scenario:

Neoclassical optimization contributes to reduction of anomalous transport by enhancing the zonal-flow level.

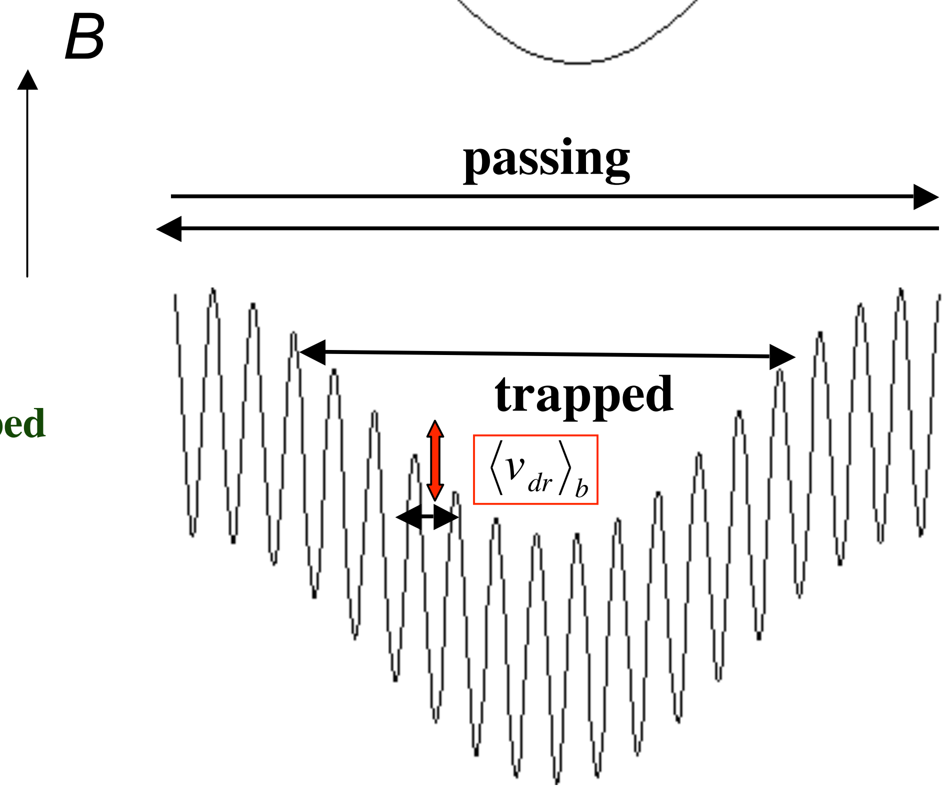
Classification of particle orbits

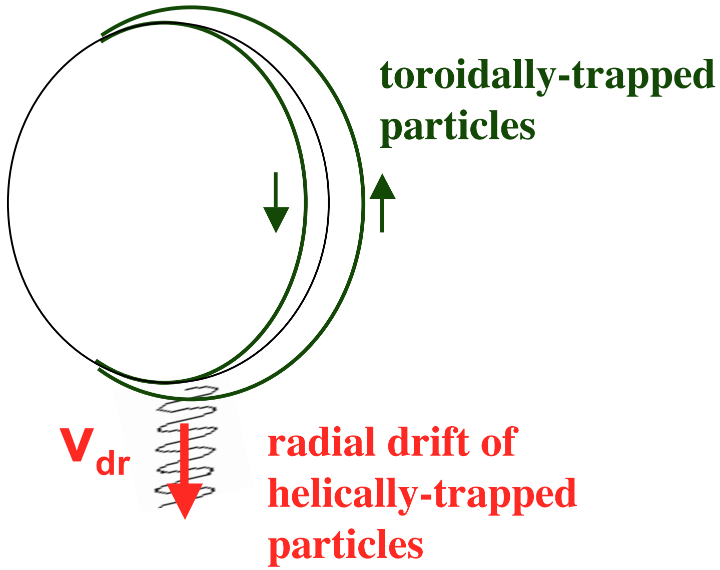
Tokamak

$$B = B_0\,(1 - \varepsilon_t \cos\theta)$$



Helical System





$$B = B_0\,[\,1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\zeta)\,]$$

Collisionless Time Evolution of Zonal Flows in Helical Systems

[Sugama & Watanabe, PRL (2005), Phys.Plasmas (2006)]

Response of the zonal-flow potential to a given initial potential $k\,\rho_i < 1$

$$\langle \phi_k(t) \rangle = K(t)\,\langle \phi_k(0) \rangle$$

Response function = **GAM component** + **Residual component**

$$K(t) = K_{GAM}(t)[1 - K_L(0)] + K_L(t)$$

$K(t=0) = 1 \qquad K(t) \to K_L(t),\ K_{GAM}(t) \to 0 \text{ as } t \to +\infty$

$B = B_0\,[\,1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\zeta)\,]$

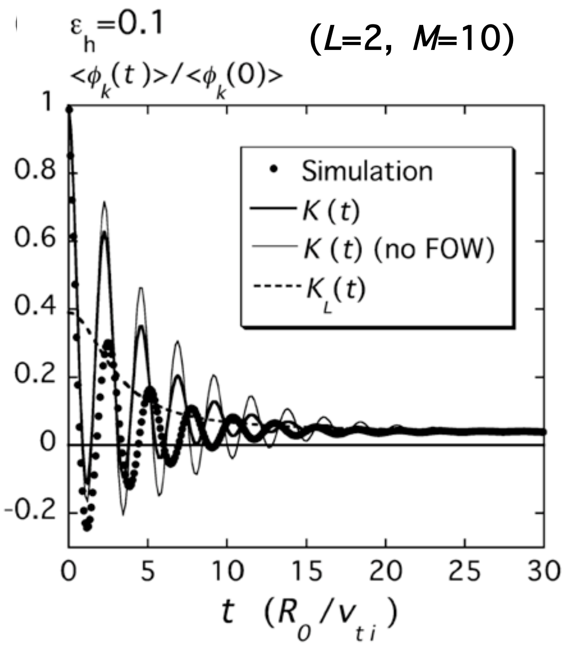
GAM response function $K_{GAM}(t) = \cos(\omega_G)\exp(-|\gamma|t)$

Long-time response function

$$K_L(t) = \frac{1 - (2/\pi)^{1/2}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle}{1 + G + E(t)\Big/\left(n_0\left\langle k_\perp^2 \rho_{ti}^2\right\rangle\right)}$$

***E*(*t*) represents effects of shielding of potential due to helical-ripple-trapped particles.**

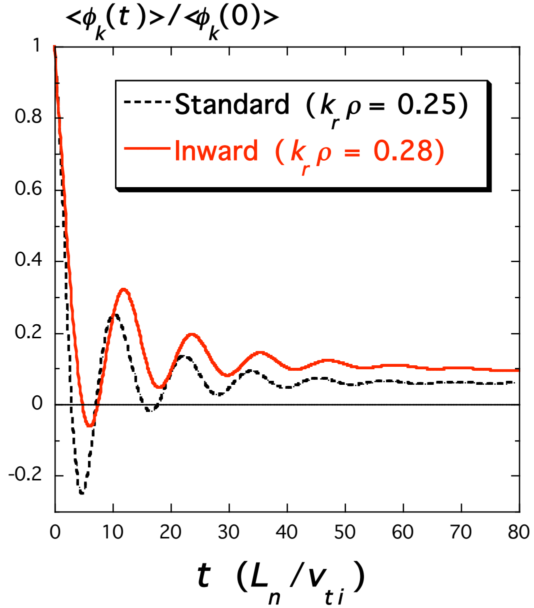
$$E(t) = \frac{2}{\pi} n_0 \Bigg[\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle - \frac{3}{2}\left\langle k_\perp^2 \rho_{ti}^2\right\rangle \left\langle (2\varepsilon_H)^{1/2}\{1 - g_{i1}(t,\theta)\}\right\rangle + \frac{T_i}{T_e}\left\langle (2\varepsilon_H)^{1/2}\{1 - g_{e1}(t,\theta)\}\right\rangle\Bigg]$$



Results from GKV simulation (flux tube, $E_r = 0$)

Smaller χ_i and larger zonal flows are found in the saturated turbulent state for the inward-shifted configuration than for the standard one !

Linear time evolution of zonal-flow potential



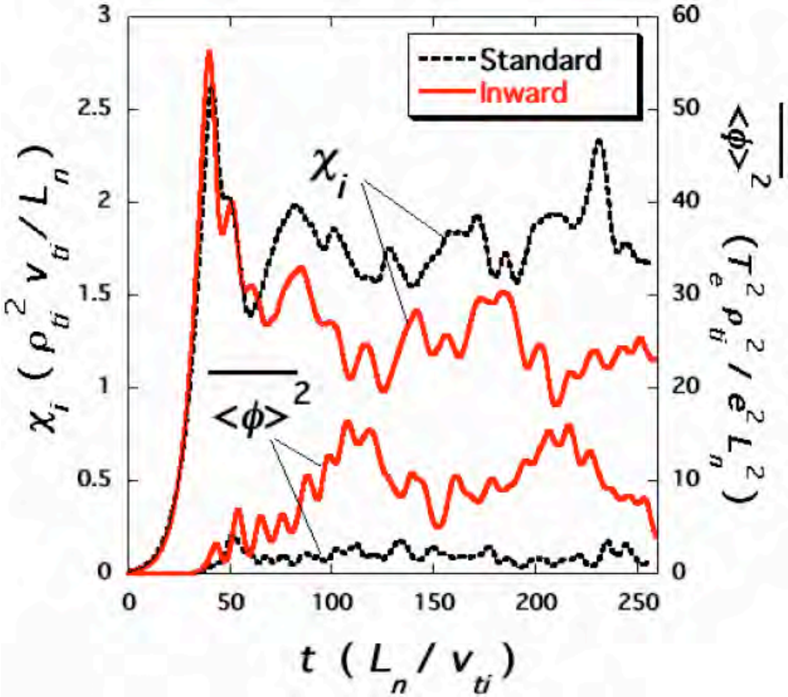
Larger residual zonal flow is found for the inward-shifted case.

Watanabe, Sugama & Ferrando, PRL(2008)
Sugama, Watanabe & Ferrando, PFR(2009)

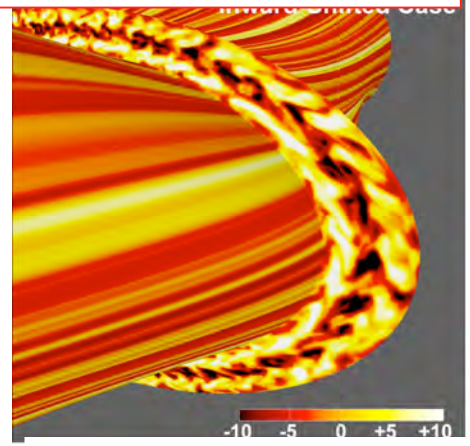
The GKV turbulence simulations were carried out by the Earth Simulator (JAMSTEC).

ITG turbulence

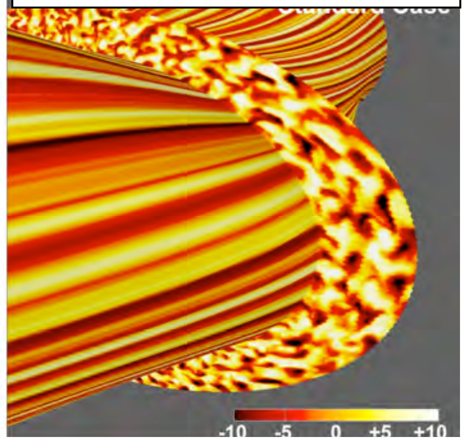
Turbulent thermal diffusivity and squared zonal-flow potential



inward-shifted configuration



standard configuration



Potential contours obtained from six copies of flux tube

Effects of Equilibrium Electric Field E_r on Zonal Flows in Helical Systems

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In helical systems
E_r is given from ambipolar condition of radial particle fluxes.
E_r reduces neoclassical ripple transport.

How does E_r influence zonal flows and anomalous transport?

Effects of E_r on gyrokinetic equation and zonal flows

Gyrokinetic equation for $\mathbf{k}_\perp = k_r \nabla r$

gyrophase average of zonal-flow potential

$$\left[\frac{\partial}{\partial t} + v_\parallel \hat{\mathbf{b}}\cdot\nabla + i\mathbf{k}_r\cdot\mathbf{v}_d - \mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)\frac{\partial}{\partial v_\parallel} + \omega_E\frac{\partial}{\partial\alpha}\right]\delta f = -i\mathbf{k}_r\cdot\mathbf{v}_d\frac{e\langle\phi(\mathbf{x}+\rho)\rangle}{T_i}F_M$$

angular velocity due to ExB drift $\omega_E = -\dfrac{c\,E_r}{r_0 B_0}$

field line label $\alpha = \theta - \zeta/q$

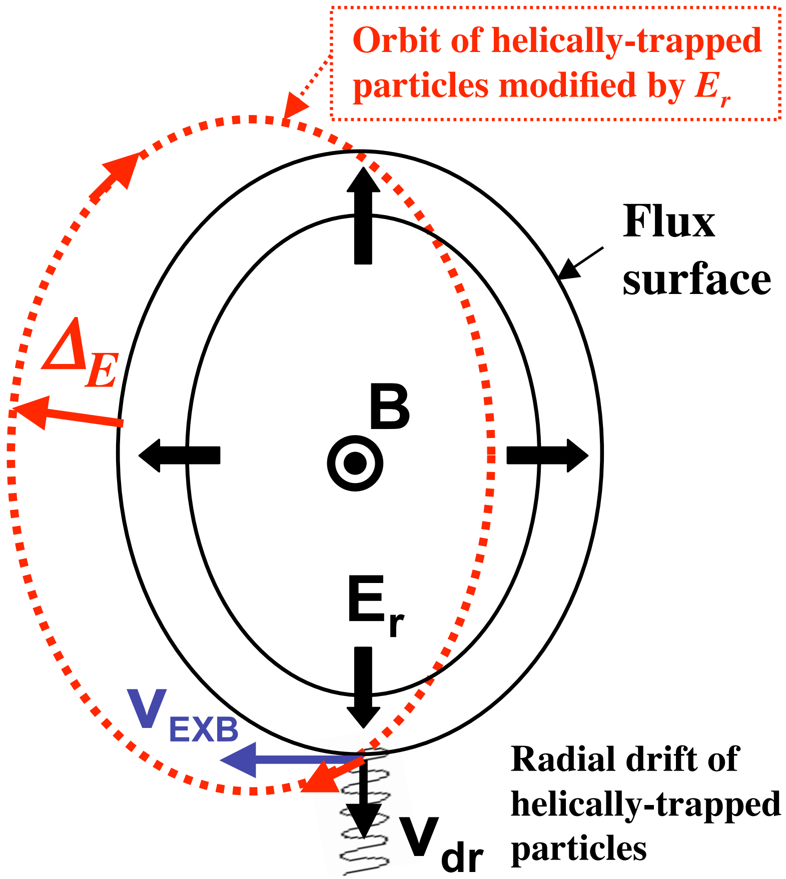
In helical systems, α **-dependence appears in** $\mathbf{k}_r\cdot\mathbf{v}_d$ **and** $\mu\left(\hat{\mathbf{b}}\cdot\nabla\Omega\right)$ **.**

Therefore, even if the zonal-flow potential ϕ **is independent of** α **,** δf **comes to depend on** α**.**

Thus, ω_E **influences** δf **and accordingly** ϕ **through quasineutrality condition.**

Effects of Equilibrium E_r on Zonal-Flow Response

Equilibrium E_r field generates a ExB component to the velocity.

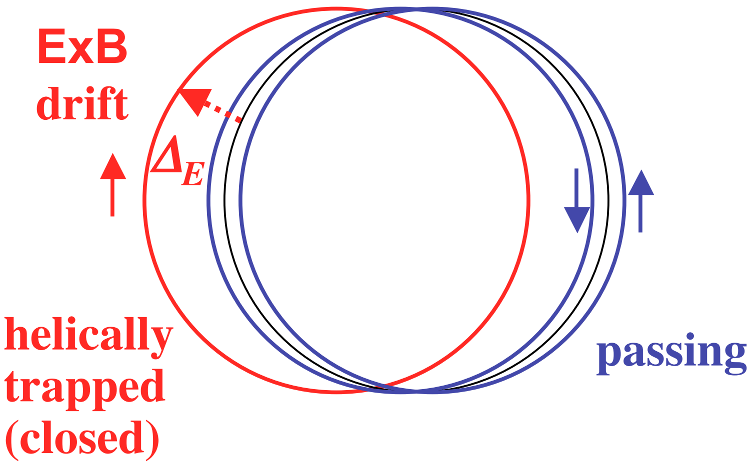


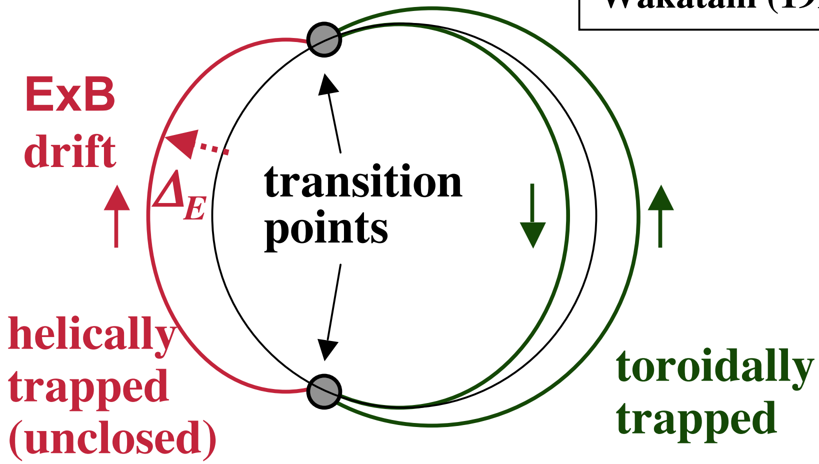
Poloidal ExB rotation of helically-trapped particles with reduced radial displacements Δ_E will decrease the shielding of zonal-flow potential and increase its response.

Mynick & Boozer, PoP(2007)
Action-Angle Formulation

Classification of particle orbits in the presence of E_r

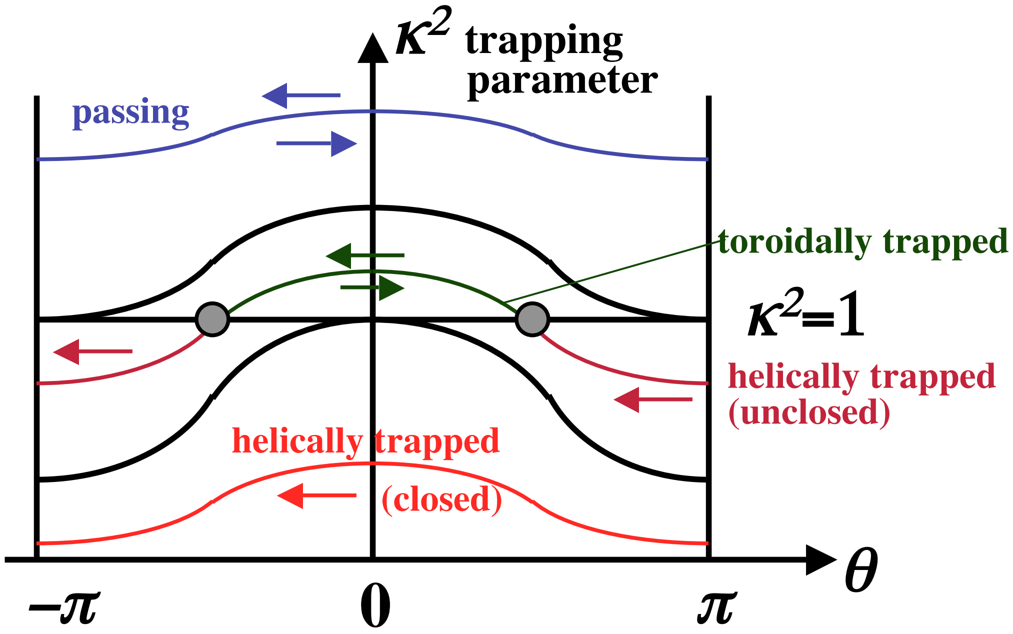
Cary *et al.*, PF (1988)
Wakatani (1998)





$$\Delta_E \sim r_0 \frac{v_{dr}}{v_{E\times B}}$$

radial displacement of
helically-trapped particle



trapping parameter

$$\kappa^2 = \frac{1-\lambda B_0[1-\varepsilon_T(\theta)-\varepsilon_H(\theta)]}{2\lambda B_0 \varepsilon_H(\theta)}$$

$$\lambda = \frac{1}{2}mv^2 \Big/ \mu$$

Solution of gyrokinetic equation to describe long-time evolution of zonal flows [Sugama & Watanabe, PoP(2009)]

Perturbed particle distribution function

$\rho_r \cdots$ gyro motion

$\Delta_r \cdots$ drift motion

$$\delta f_k(t) = -\frac{e}{T}\phi_k(t) F_M \left[1 - e^{-ik_r\rho_r} e^{-ik_r\Delta_r} \left\langle e^{ik_r\Delta_r} J_0(k_r\rho_r)\right\rangle_{\text{orbit}}\right]$$

→ **Polarization (classical & neoclassical)**

$$+ e^{-ik_r\rho_r} e^{-ik_r\Delta_r} \left\langle e^{ik_r\Delta_r}\left[\delta f_k^{(g)}(0) + F_M \int_0^t S_k(t)\,dt\right]\right\rangle_{\text{orbit}}$$

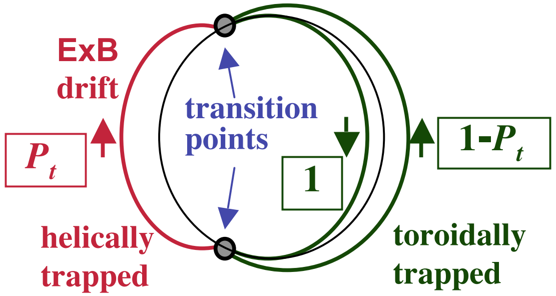
→ **Initial condition & Turbulence source**

Average along the orbit

$$\langle \cdots \rangle_{\text{orbit}} = \oint \cdots \frac{dl}{(dl/dt)} \Big/ \oint \frac{dl}{(dl/dt)}$$

$$P_t \approx \frac{4\sqrt{2}}{\pi}\left(\frac{c\,E_r}{v\,B_0}\right)\left(\frac{R_0 q}{r_0}\right)\left[(\varepsilon_H)^{-1/2} \frac{\partial\varepsilon_H/\partial\theta}{\partial(\varepsilon_H - \varepsilon_T)/\partial\theta}\right]_{P-}$$

For particles which show transitions



P_t transition probability

$$\oint \frac{dl}{(dl/dt)} = (1 - P_t)\int_{\substack{\kappa^2>1\\ v_\parallel>0}} \frac{dl}{|v_\parallel|} + \int_{\substack{\kappa^2>1\\ v_\parallel<0}} \frac{dl}{|v_\parallel|} + P_t \int_{\kappa^2<1} \frac{d\theta}{\omega_E}$$

Annotations on the equation: an arrow labelled 1-P_t and an arrow labelled P_t (transition probability) point between the integrals; "toroidally trapped" points to the two $\kappa^2>1$ integrals; "helically trapped" points to the $\kappa^2<1$ integral.

Long-time zonal-flow response to the initial condition and turbulence source

For radial wavenumbers $k_r\rho_{ti} < 1$ (ITG turbulence) and $k_r\Delta_E < 1$, the zonal-flow potential is derived from the quasineutrality condition as [Sugama & Watanabe, PoP(2009)]

$$\frac{e}{T_i}\langle\phi_k(t)\rangle = \frac{\left\langle n_0^{-1}\int d^3v\left[1 + i k_r\left(\Delta_{ir} - \langle\Delta_{ir}\rangle_{\text{orbit}}\right)\right]\left[\delta f_{ik}^{(g)}(0) + F_M\int_0^t S_{ik}(t)\,dt\right]\right\rangle}{(k_r\rho_{ti})^2\left[1 + G_p + G_t + M_p^{-2}(G_{ht} + G_h)(1 + T_e/T_i)\right]}$$

Geometrical factors G's represents shielding effects of neoclassical polarization due to particles motions in different orbits.

$G \propto$ (population) $\times(\Delta_r/\rho)^2$

G_p : passing

G_t : toroidally-trapped

G_{ht} : helicallly-trapped (unclosed orbit)

G_h : toroidally-trapped (closed orbit)

Zonal-flow generation can be enhanced when

G_{ht} and G_h decreases with neoclassical optimization (which reduces radial drift velocity v_{dr})

and when poloidal Mach number $M_p \equiv \left|(cE_r/B_0)/(r v_{ti}/Rq)\right|$ **increases**

with increasing E_r and using heavier ions.

Response to the initial condition

Assume the initial distribution to have Maxwellian dependence $\delta f_{ik}^{(g)}(0) = [\delta n_k^{(g)}(0)/n_0] F_M$

Then, we obtain $\delta n_k^{(g)}(0)/n_0 = (k_r \rho_{ti})^2 e\phi(0)/T_i$

$$\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + M_p^{-2}(G_{ht} + G_h)(1 + T_e/T_i)}$$

(no turbulence source)

For the single-helicity configuration

$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\varsigma)] \quad (\varepsilon_h : \text{independent of } \theta)$$

No transitions occur. $G_{ht} = 0, \quad G_h = (15\pi/4) q^2 (2\varepsilon_h)^{1/2}$

$$\langle \phi_k(t) \rangle = \frac{\langle \phi_k(0) \rangle}{1 + G_p + G_t + (15\pi/4) M_p^{-2} q^2 (2\varepsilon_h)^{1/2} (1 + T_e/T_i)}$$

This corresponds to the case considered by previous works.

Mynick & Boozer, PoP(2007)

Sugama, Watanabe & Ferrando, PFR(2008)

Extention of GKV code to poloidally global model

GKV code is extended from the flux tube to the poloidally global model for studying effects of E_r on zonal flows in helical systems [Watanabe, IAEA FEC 2008].

$\alpha \equiv \theta - \zeta / q$ **: field-line label** ζ **: toroidal angle**

- **Linear simulations for time evolution of zonal flows are done using**
  129 Fourier modes in the α direction ,
  1, 536 grid points in the ζ direction, and
  (512, 48) grid points in the ($v_{\|}$, μ) space
  for a fixed radial wavenumber k_r .

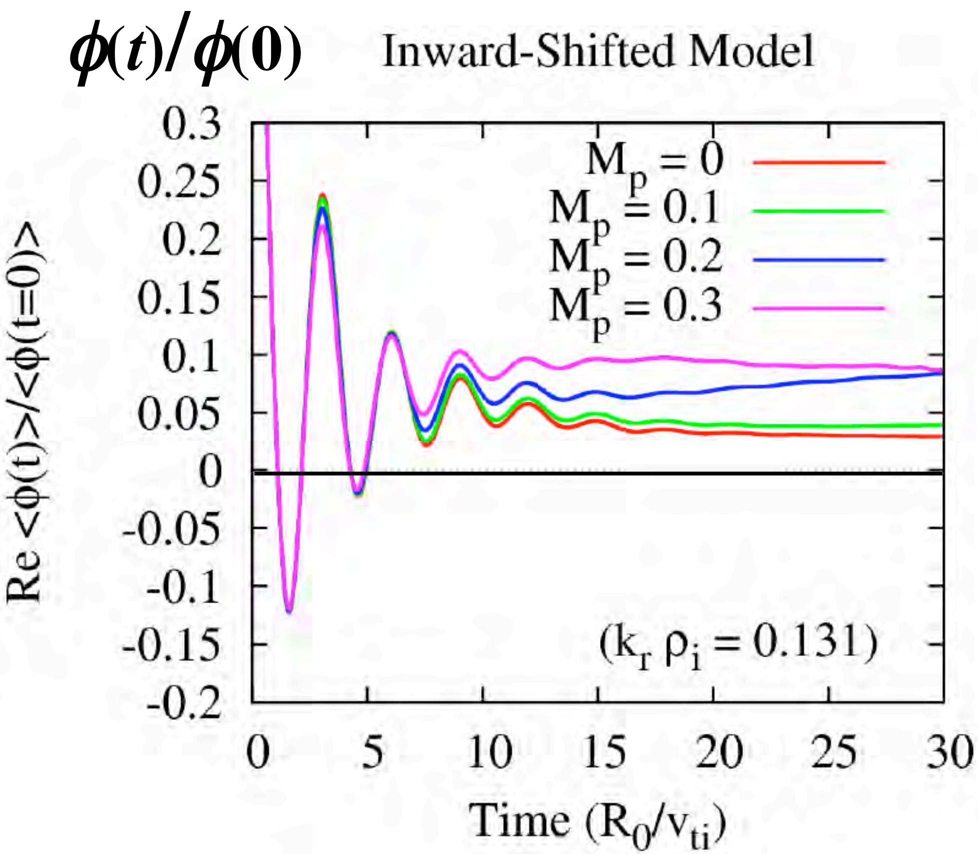
- **Standard configuration model (single helicity) :**

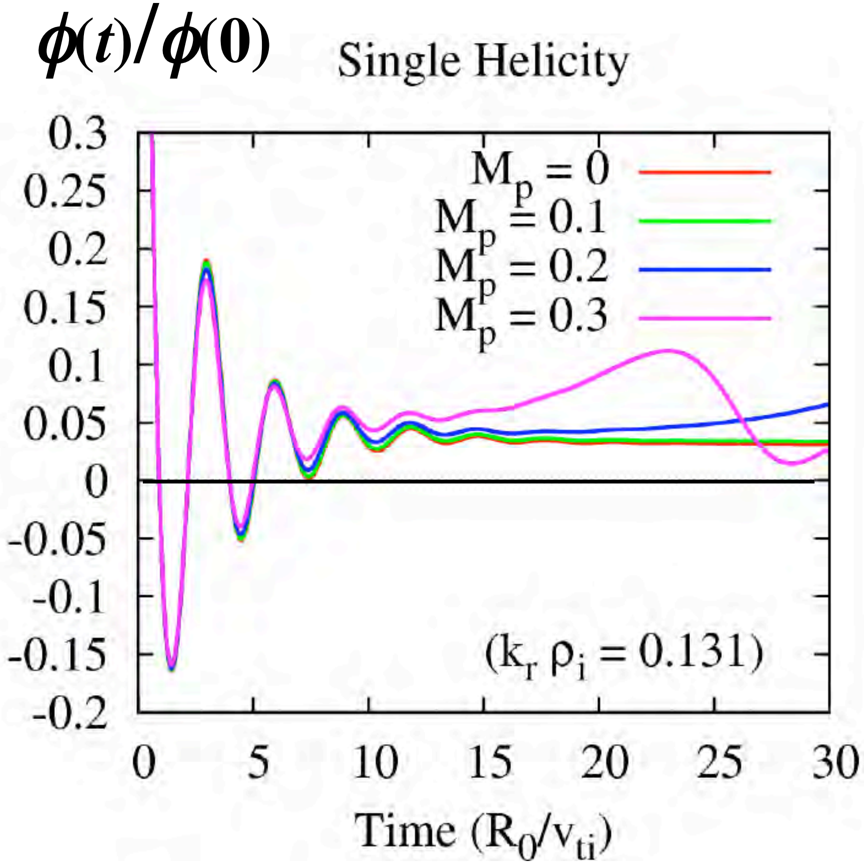
$$B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(L\theta - M\varsigma)], \quad \varepsilon_t = 0.1, \quad \varepsilon_h = 0.1, \quad q = 1.5, \quad L = 2, \quad M = 10$$

- **Inward-shifted configuration model :**
  Sideband helicity components $\left(\varepsilon_{L+1} = -0.02, \quad \varepsilon_{L-1} = -0.08 \right)$
  are included.

Collisionless time evolution of zonal flows in helical configurations with E_r

It is clearly shown for the inward-shifted model configuration that the residual zonal-flow potential amplitude (observed after Landau damping of GAM) is enhaced by increasing E_r.



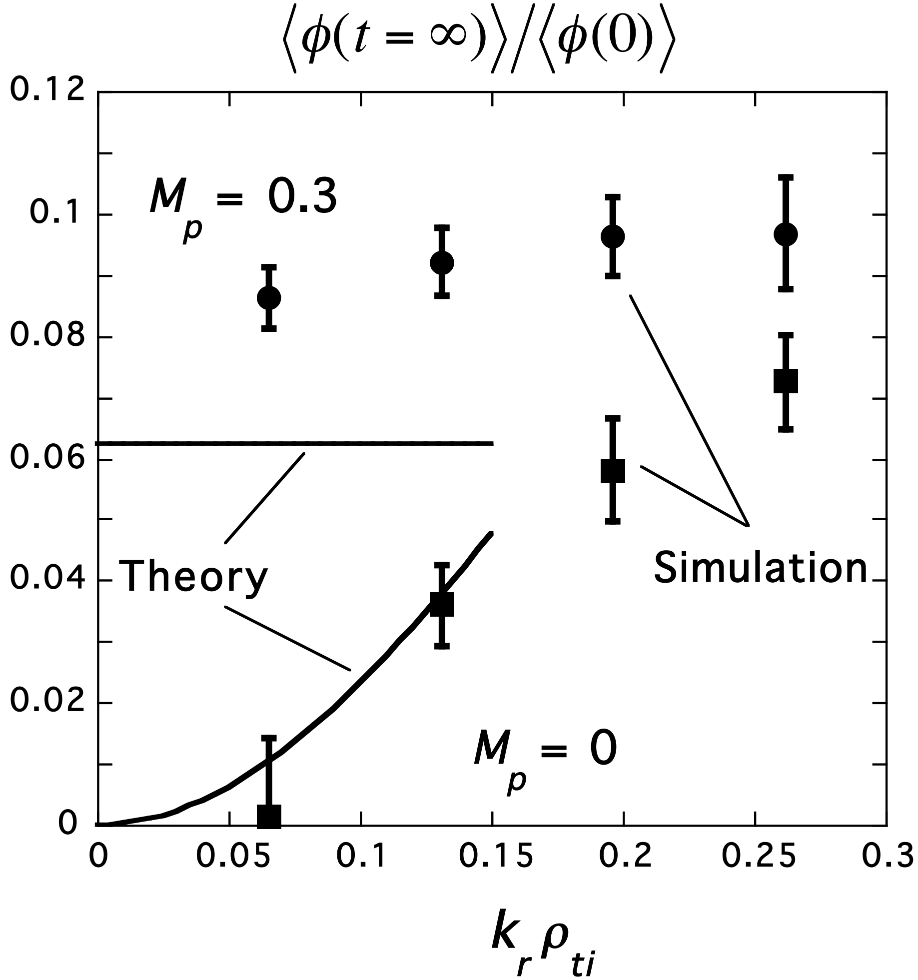


The residual zonal-flow potential as a function of $k_r\,\rho_{ti}$ for $M_p = 0$ and $M_p = 0.3$

Different $k_r\,\rho_{ti}$ dependences for $M_p = 0$ and $M_p = 0.3$ are theoretically predicted and confirmed by simulation.

Theoretical results are derived by assuming $k_r\,\rho_{ti} << 1$.

Inward-shited configuration



Summary

- It is verified from GKV simulation (for $E_r = 0$) that, in the inward-shifted (or neoclassically optimized) LHD configuration, zonal-flow generation is enhanced and the ITG turbulent thermal diffusivity is reduced.
  [Similar gyrokinetic simulation results for W7X will be reported by Dr.Xanthopoulos (I25) on Thursday in this Workshop.]

- Zonal-flow response theory is presented, in which effects of the equilibrium radial electric field E_r and transitions between toroidally- and helically-trapped particles are taken into account.

- The predicted enhancement of the zonal-flow response due to E_r is confirmed by the linear poloidally-global gyrokinetic simulation.

- The E_r effects appear through the poloidal Mach number M_p.
  For the same magnitude of E_r, higher zonal-flow response is obtained by using ions with heavier mass (favorable deviation from gyro-Bohm scaling).

- Nonlinear poloidally-global GKV simulation to study E_r effects remains as a future task.