Ideal Perturbed Equilibria in Tokamaks and Control of External Magnetic Perturbations

J.-K. Park[1],
A. H. Boozer[2], J. E. Menard[1],
S. P. Gerhardt[1], R. J. Hawryluk[1], M. S. Chance[1], S. R. Hudson[1],
H. Reimerdes[2], M. Lanctot[2], M. J. Schaffer[3], A. M. Garofalo[3],
T. Evans[3], J. Canik[4], C. Nuehrenberg[5]

1 Princeton Plasma Physics Laboratory
2 Columbia University
3 General Atomics
4 Oak Ridge National University
5 IPP-Euratom Association

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Overview

- Ideal Perturbed Equilibrium Code (IPEC) solves ideal 3D tokamak equilibria with free-boundary

- IPEC applications to tokamaks show importance and validity of ideal perturbed equilibria

  - Ideal plasma response and Resonant Field Amplifications (RFAs)
  - Resonant field driving islands and Locked Modes (LMs)
  - Variation in the field strength and Neoclassical Toroidal Viscosity (NTV)
  - Resonant Magnetic Perturbations (RMPs) and Edge Localized Modes (ELMs)

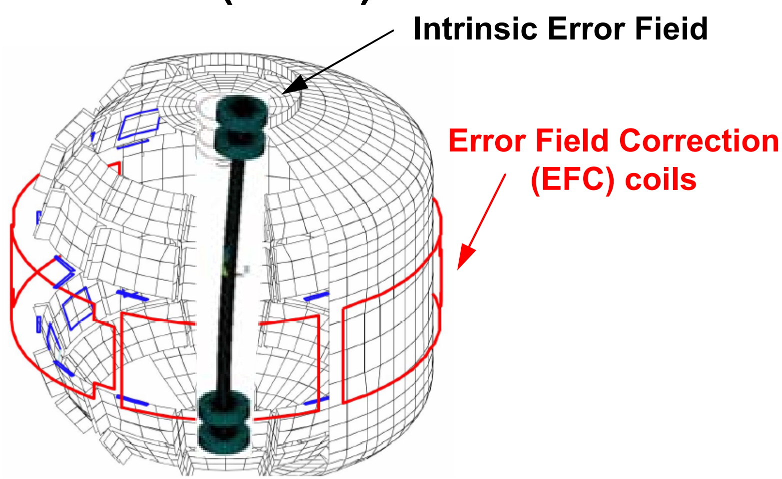
- Coupling between external magnetic field and physical parameters such as total resonant field can be an effective tool for control

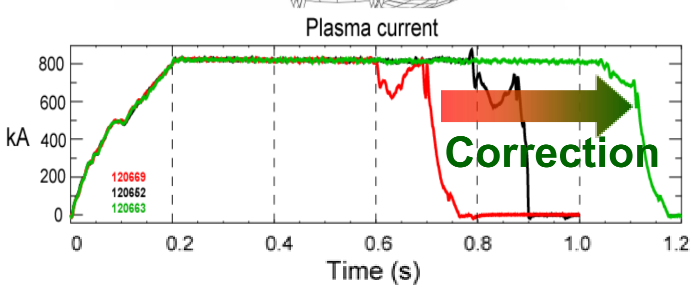
- Summary and Future Work

Tokamaks are sensitive to a small 3D field

- Tokamaks are almost axisymmetric, but a small non-axisymmetric magnetic field $\delta B/B_0 \sim 10^{-4}$ can
  - **Degrade plasma performance by locking or non-ambipolar transport**
  - **Improve plasma performance by change of local transport**

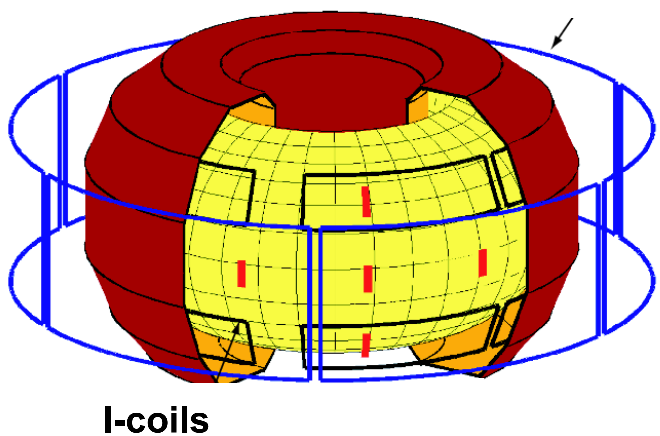
NSTX (PPPL)

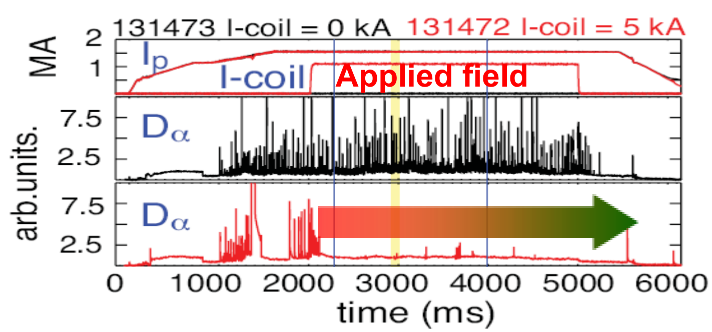




DIII-D (GA)

C-coils





Ideal Perturbed Equilibrium Code (IPEC) solves ideal 3D tokamak equilibria with a small perturbation

- Given an axisymmetric equilibrium, $\vec{\nabla} p_0 = \vec{j}_0 \times \vec{B}_0$, and given an non-axisymmetric field, $\delta \vec{B}^x(\vec{x})$ **[*Park et al, Phys. Plasmas* *14*, *052110 (2007)*]**

$$\vec{F}[\vec{\xi}] = \vec{0} = \delta\vec{j} \times \vec{B}_0 + \vec{j}_0 \times \delta\vec{B} - \vec{\nabla}\delta p$$

$$\delta\vec{B} = \vec{\nabla} \times (\vec{\xi} \times \vec{B}_0) \text{ and } \delta\vec{j} = (\vec{\nabla} \times \delta\vec{B}) / \mu_0$$

$$\delta p = -\vec{\xi} \cdot \vec{\nabla} p_0 - \gamma p_0 (\vec{\nabla} \cdot \vec{\xi})$$

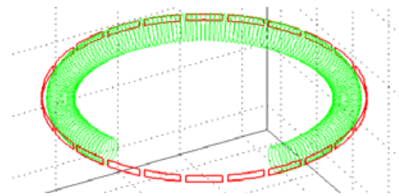
$p_0(\psi)$ and $q_0(\psi)$ profiles are preserved (ideal constraints)

- Ideal constraints (internal boundary condition) :
  - Imply no islands, no resonant field, and shielding currents
  - Plasma rotation halts reconnection and maintains the shielding currents that prevent islands from opening, and give nearly ideal plasma response

- Non-axisymmetric field $\delta \vec{B}^x(\vec{x})$ (external boundary condition) :
  - Is represented by equivalent surface currents at the boundary

IPEC uses equivalent surface currents to solve free-boundary perturbed equilibria

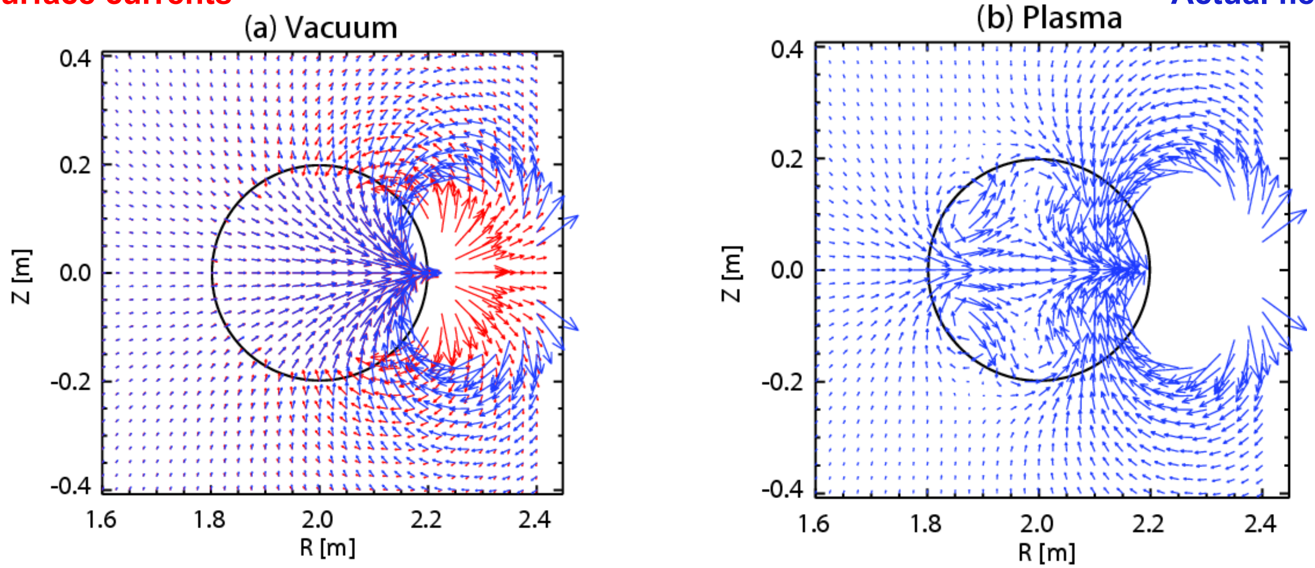
- IPEC uses equivalent surface currents (based on DCON and VACUUM stability codes) to solve free-boundary perturbed equilibria
- Total field including ideal plasma response is provided
  - Total Perturbed field (δB) = Plasma Field from perturbed plasma currents (δB^P) + External field from external currents (δB^X)



Actual field in vacuum

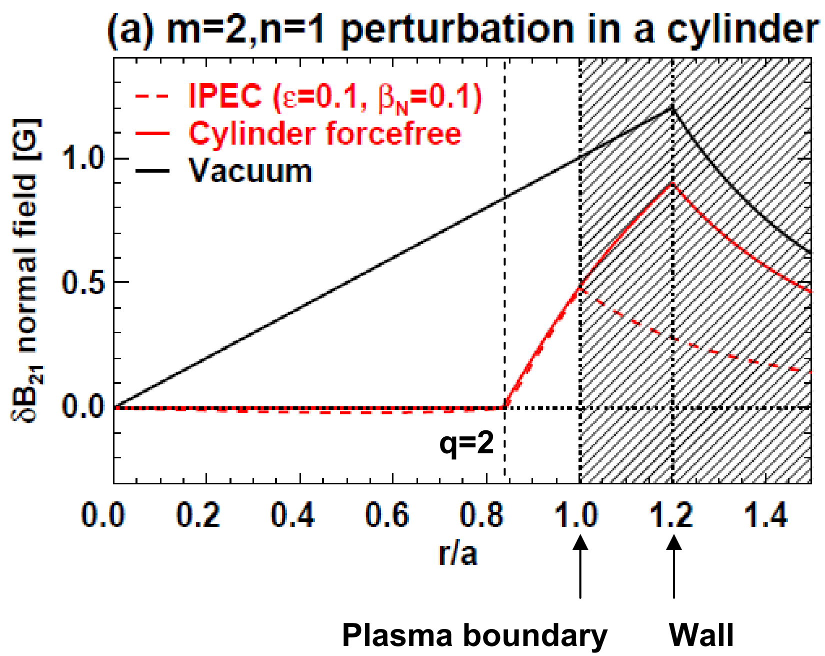
Field from surface currents

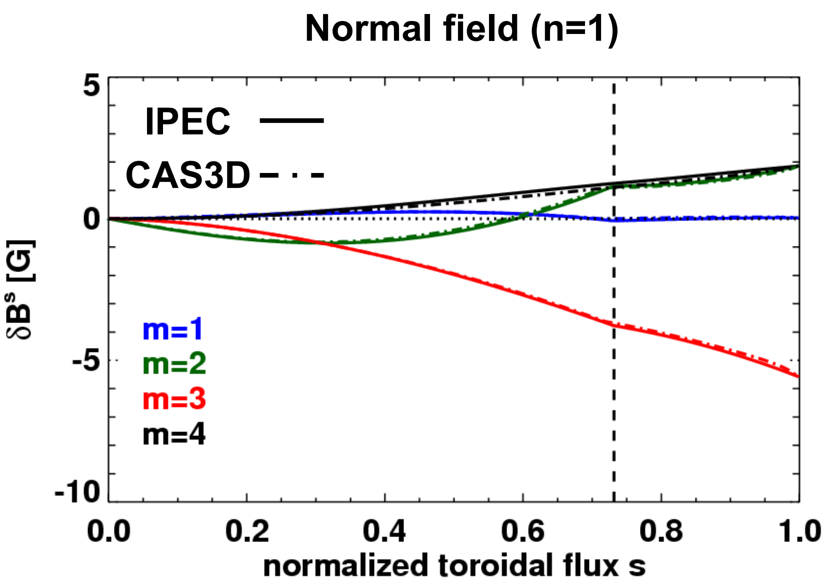
Actual field with plasma



Ideal plasma response by IPEC has been benchmarked in cylindrical limits and with CAS3D

- IPEC vs. Cylinder forcefree
  - Cylindrical forcefree examples can be solved by simple numerical routines
- IPEC vs. CAS3D
  - CAS3D calculates ideally perturbed equilibria in stellarators

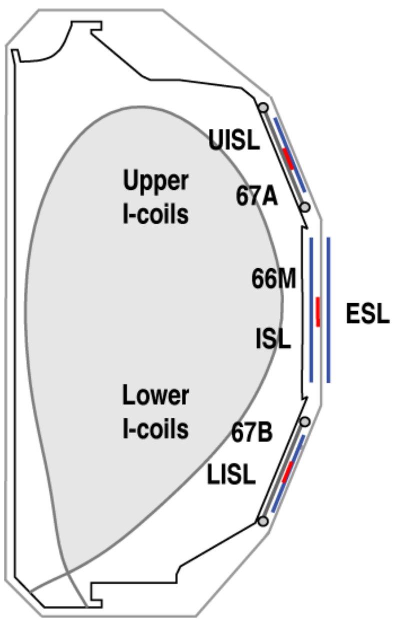


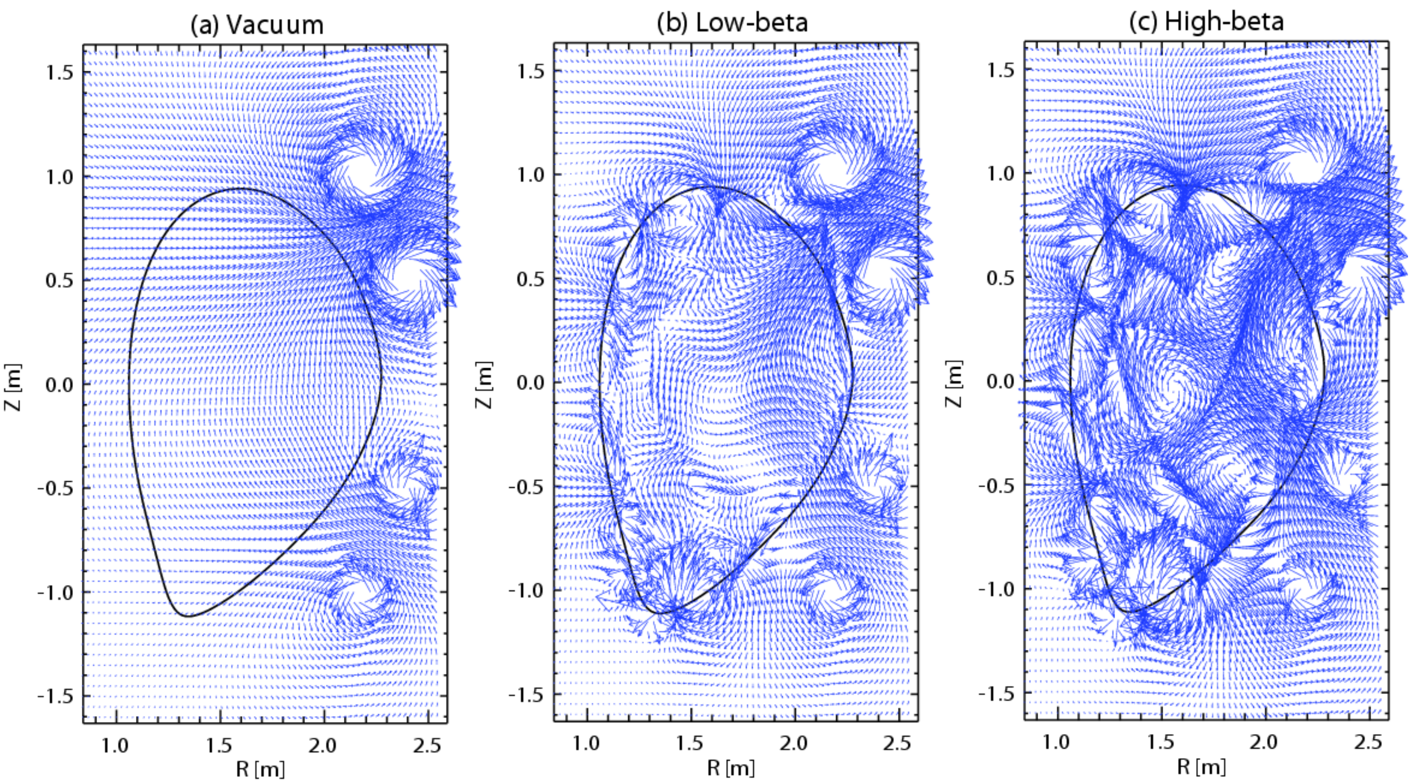


Ideal plasma response by IPEC can be directly compared with Resonant Field Amplification (RFA) measurements

- IPEC can calculate the field (δB^P) by ideally perturbed plasma currents
- Plasma response at sensors can be estimated by integrating IPEC field and can be directly compared with RFA measurements

DIII-D





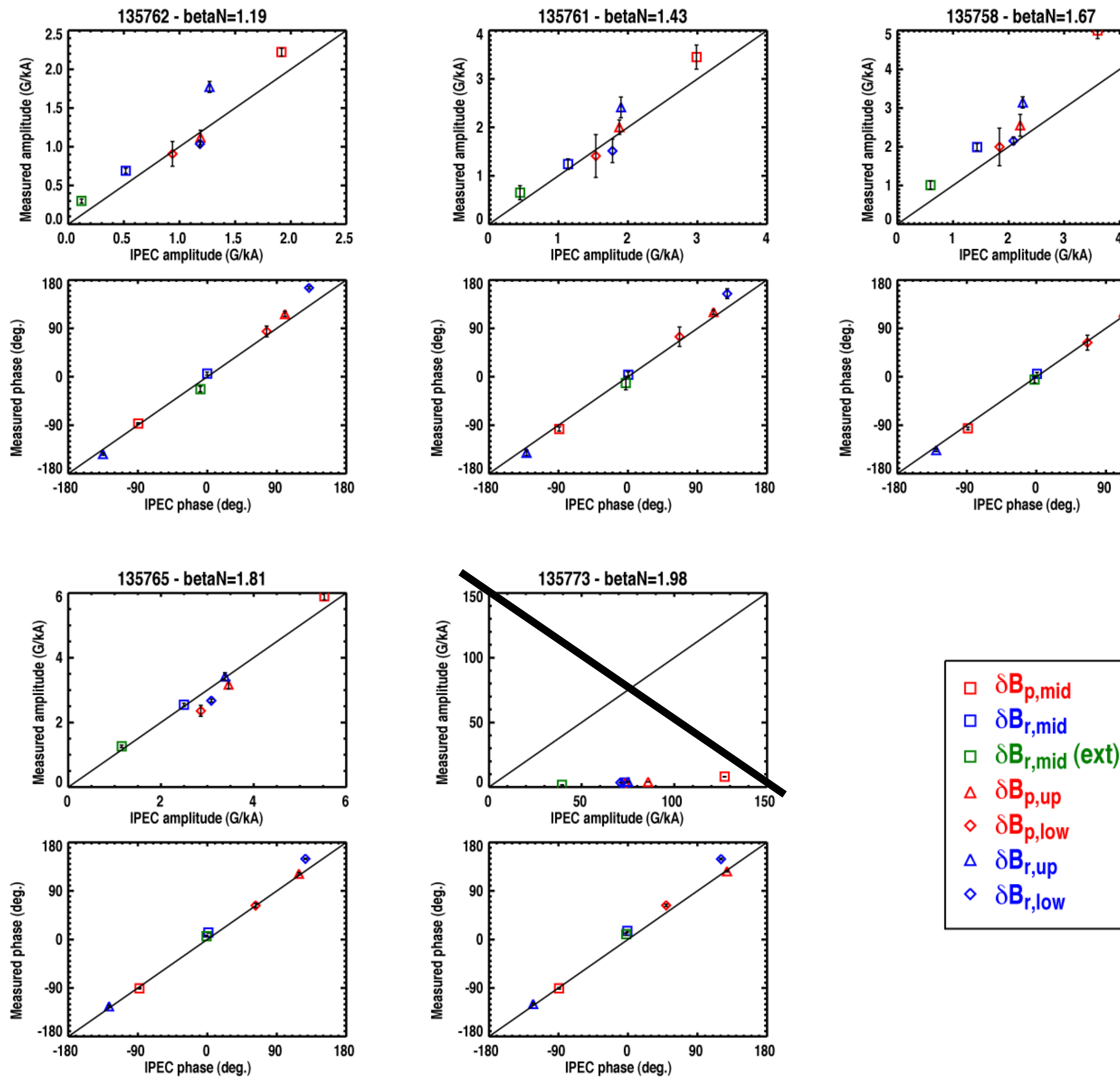
#135762, β_N=1.19

#135765, β_N=1.81

Comparison between IPEC and n=1 RFA DIII-D measurements showed good agreements below marginal stability

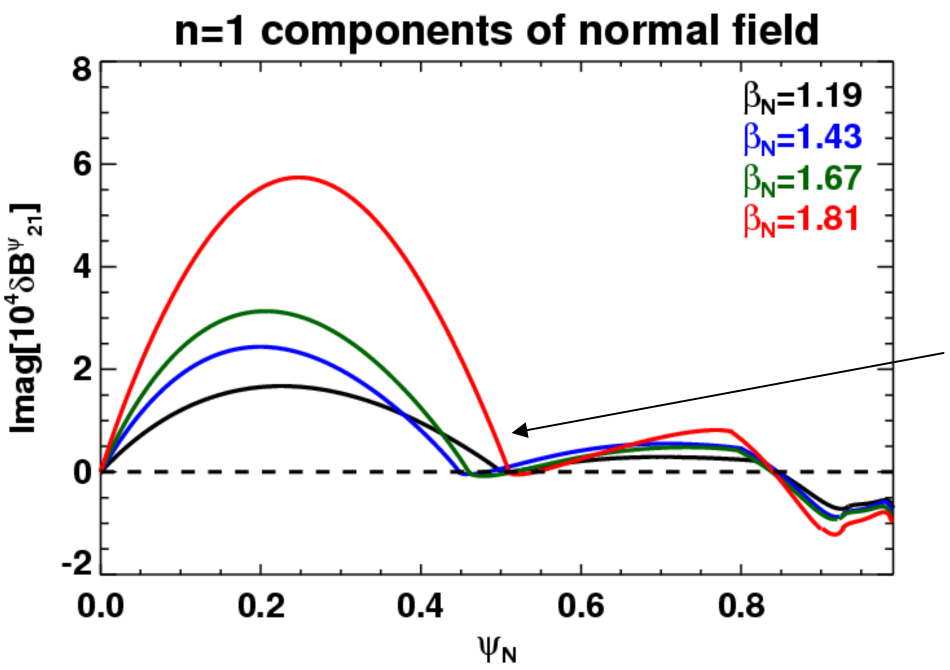
- IPEC and RFA measurements show good agreements for amplitudes and toroidal phases when plasma is below marginal stability

- IPEC becomes singular at the marginal limit and indicates that non-ideal effects become important for high-β plasmas



IPEC gives resonant field driving islands through shielded perturbed equilibria

- IPEC calculates shielded perturbed equilibria without islands, which are valid before the onset of islands, Locked Modes (LMs) or Neoclassical Tearing Modes (NTMs)



- External resonant field : δB_{mn}^{x}
  Directly calculated, but without plasma response, and often paradoxical

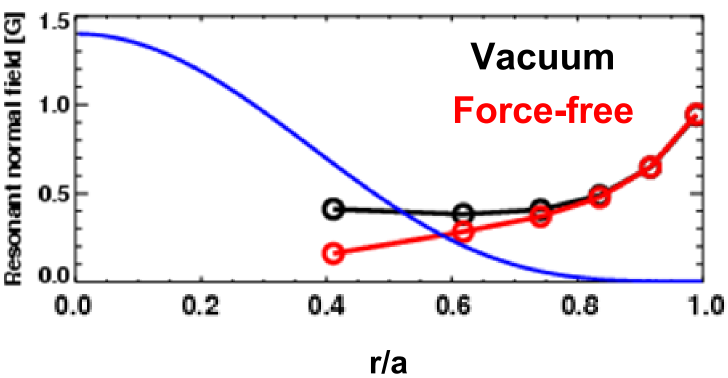
- Total resonant field : δB_{mn}
  Calculated from the shielding currents, including plasma response, and correct before the onset of locking

Ideal Plasma Response includes important shielding, amplification, and poloidal coupling

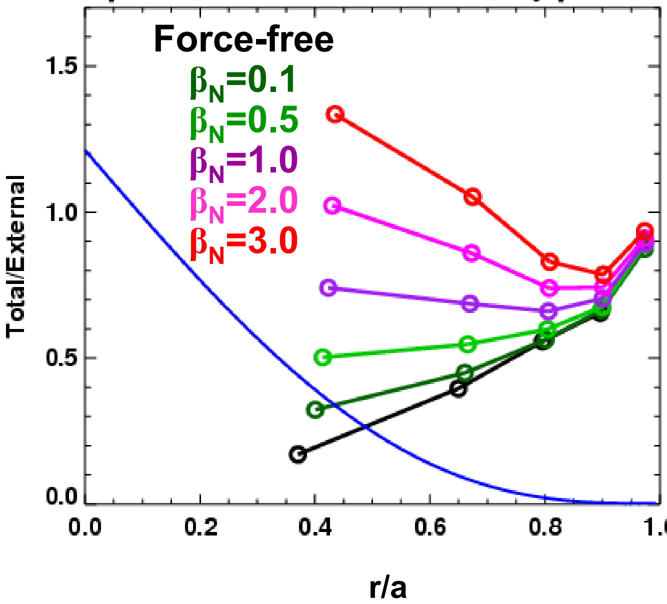
- Plasma currents tend to shield perturbation
- Plasma pressures tend to amplify perturbation
- Toroidicity gives strong poloidal coupling

Cylindrical (Large aspect ratio) example

Shielding of resonant field by current



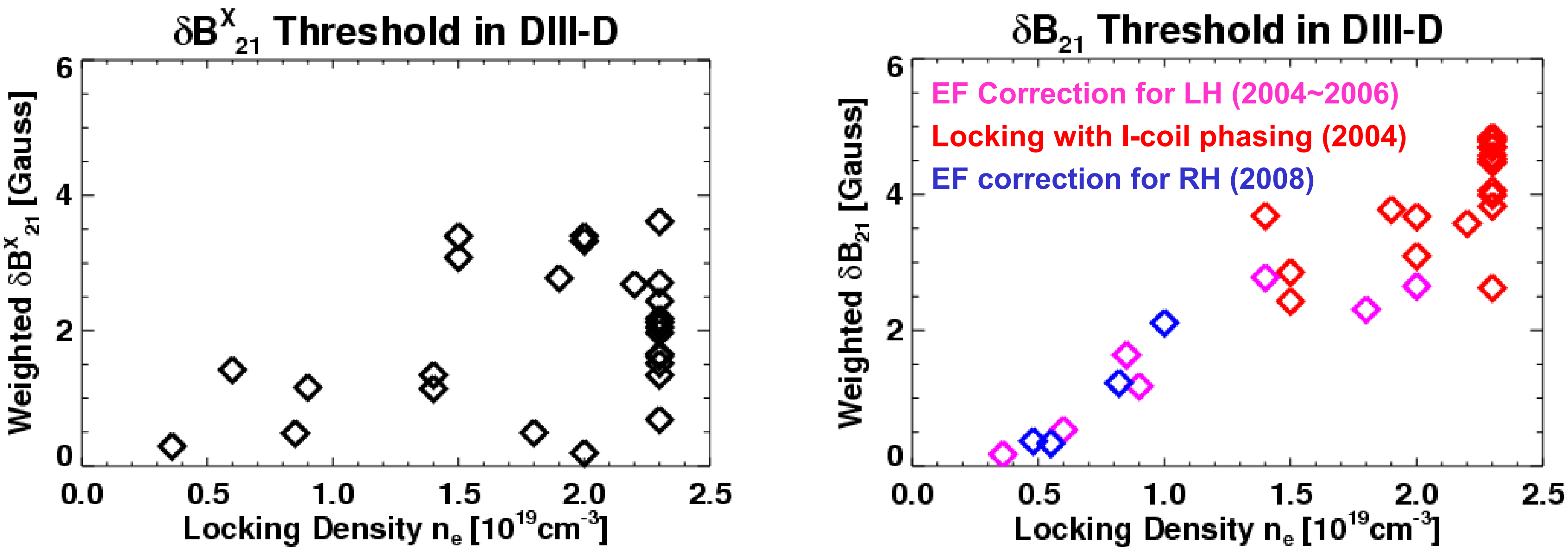
Amplification of resonant field by pressure



IPEC resonant field resolved paradoxical error field correction results in NSTX and DIII-D

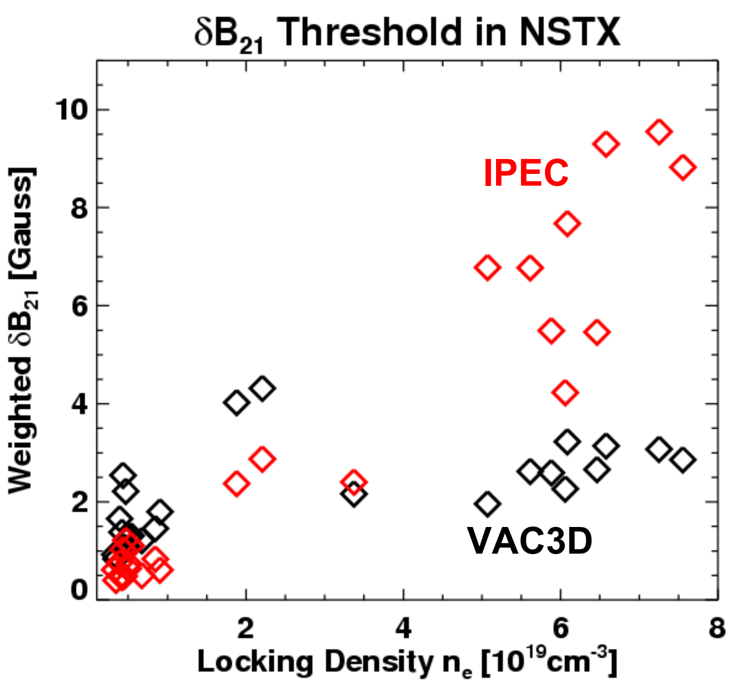
- Vacuum resonant field (based on standard vacuum superposition, $\delta B^P=0$) showed often paradoxical results
- IPEC resonant field restored good parametric correlation

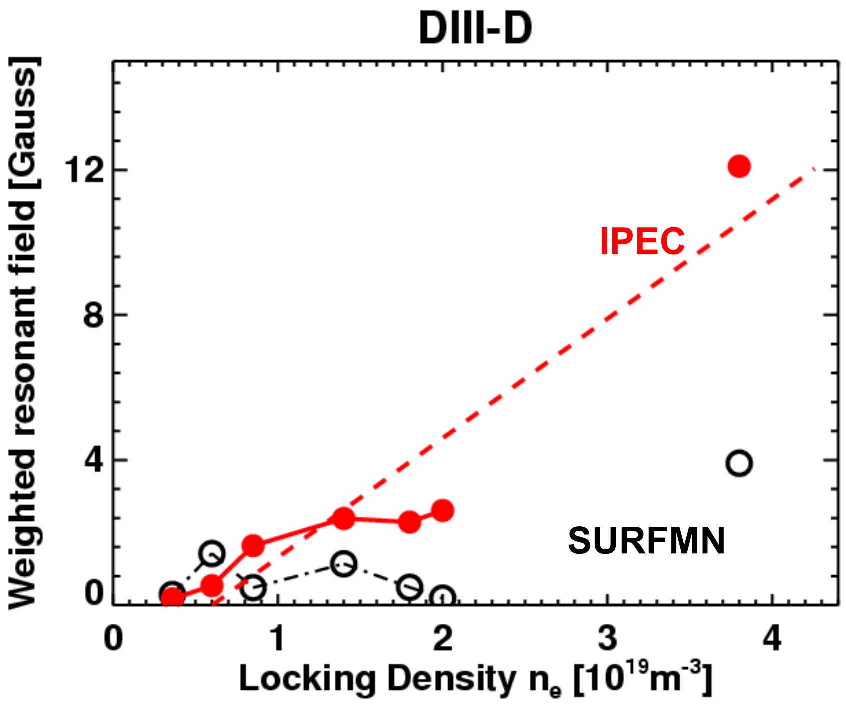
[Park et al, Phys. Rev. Lett. 99, 0195003 (2007)]



IPEC restored linear density scaling even for high-β cases

- NSTX and DIII-D both showed linear density correlation of total resonant field across low-β and high-β plasmas
- Systematic study on high-β locking has been started





Non-axisymmetric variation in the field strength produces non-ambipolar transport

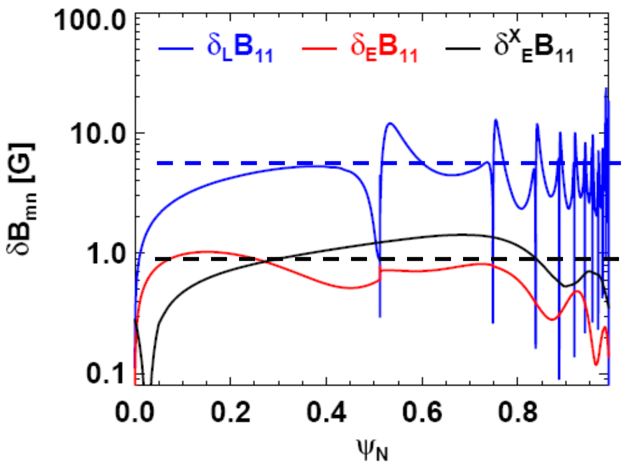
- Action is dependent on toroidal location in the presence of the nonaxisymmetric variation in the field strength

$$J = \oint v_{\|} dl \propto \oint \sqrt{H - \mu|B|} dl.$$

- Action must be conserved, so a particle must have an additional radial drift. It depends on species and give Neoclassical Toroidal Viscosity (NTV) torque

$\delta \vec{j} \cdot \vec{\nabla} \psi$ produces a toroidal torque $\delta \vec{j} \times \delta \vec{B}$ and rotational damping $\nu_{\text{damp}}[1/s]$

- Important variation occurs by the variation in the field strength along the perturbed field lines, not along unperturbed field lines
  - Lagrangian : $\delta_L B \equiv \delta_E B + \vec{\xi} \cdot \vec{\nabla} B_0$
  - Eulerian : $\delta_E B \equiv \delta \vec{B} \cdot \hat{b}_0$
  - Vacuum Eulerian : $\delta_E B^x \equiv \delta \vec{B}^x \cdot \hat{b}_0$



NSTX
n=1 EFC 1kA

$\delta_L B/B_0 \sim 10^{-3}$

$\delta_E B/B_0 \sim 10^{-4}$

NTV theory has been generalized with bounce-harmonic resonances

- NTV formula has been derived including resonaces between bounce motions, electric, and magnetic precessions
- With effective collisional operator and large aspect ratio approximations, generalized formula shows a small fraction of resonating particles can make a strong $1/\nu$ transport **[*Park et al, Phys. Rev. Lett. 102, 065002 (2009)*]**

$$\tau_\varphi \cong C\left[\delta B\right]^2 \frac{\nu(\omega_E - \omega_0)}{(\ell\omega_\ell - n\omega_E - n\omega_B)^2 + \nu^2}$$

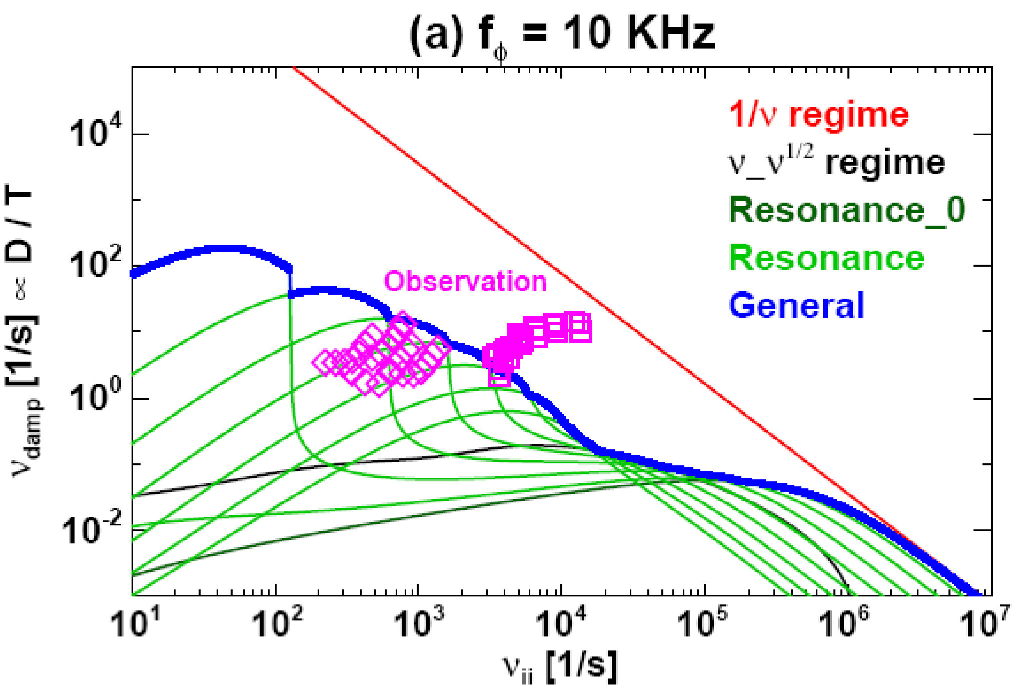
$$\langle \hat{\phi} \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}_a \rangle_\ell = \frac{\epsilon^{-1/2} p_a}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{w,\ell}^2 \int_0^\infty dx \mathcal{R}_{a1\ell} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT_a}{d\chi} \right| \right]$$

$$\delta_{w,\ell}^2 = \sum_{nmm'} \delta_{nmm'}^2 \frac{F_{nm\ell}^{-1/2} F_{nm'\ell}^{-1/2}}{4K(\kappa)}, \quad F_{nm\ell}^y \equiv \int_{-\vartheta_t}^{\vartheta_t} d\vartheta (\kappa^2 - \sin^2(\vartheta/2))^y \cos(m - nq - \sigma\ell)\vartheta$$

$$\mathcal{R}_{ay\ell} = \frac{1}{2} \frac{n^2(1 + (\frac{\ell}{2})^2)\frac{\nu_a}{2\epsilon} x (x - \frac{5}{2})^y e^{-x}}{\left[\ell \frac{\pi\sqrt{\epsilon}}{4\sqrt{2}} \omega_{ta}\sqrt{x} - n\omega_E - n\sigma\frac{q^3}{4\epsilon}(\omega_{ta}^2/\omega_{ga})x\right]^2 + \left[(1 + (\frac{\ell}{2})^2)\frac{\nu_a}{2\epsilon}\right]^2 x^{-3}}.$$

IPEC variation field strength + generalized NTV restored right order of magnitudes as observations

- General formula has been derived to combine the regimes and to include bounce-harmonic resonances $\ell\omega_b - n\omega_p \sim 0$
- Small fraction of resonant particles can significantly enhance the transport, up the level of observations [*Park et al, Phys. Rev. Lett.* 102*, 065002 (2009)*]



The most important external field gives convenient and reliable method for error field corrections

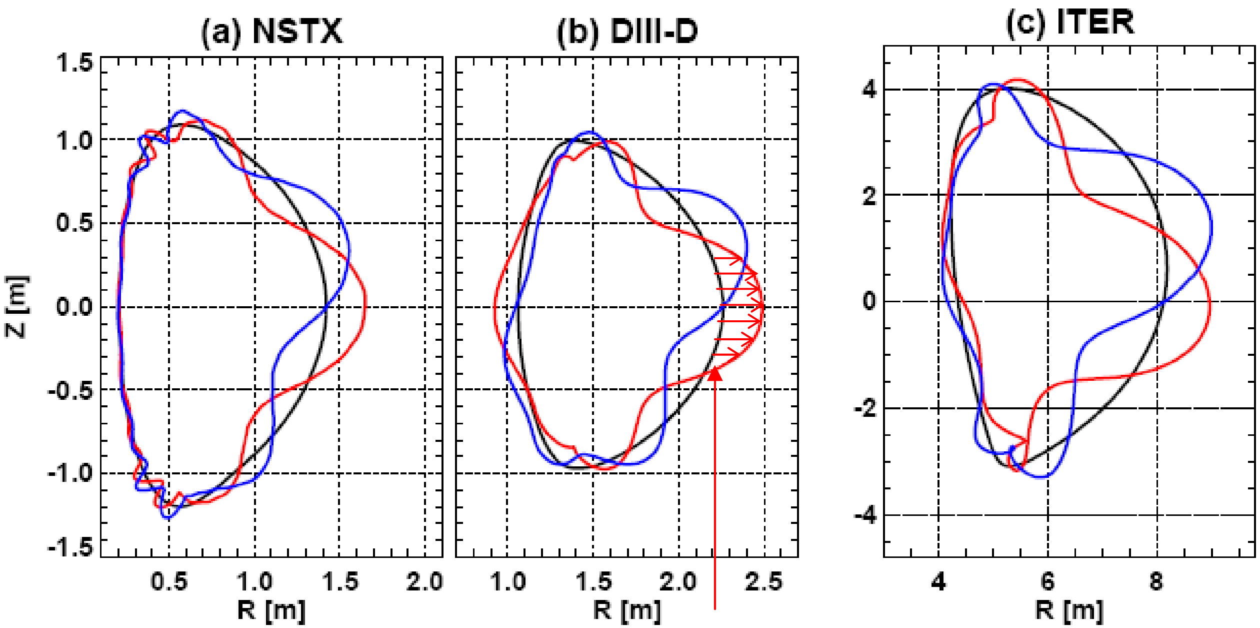
- Dominant external field for core : External normal field on the boundary maximizing the sum of the total resonant field for the core

[*Park et al, Nucl. Fusion 48 045006 (2008)*]

Shape of the dominant external field

<Cosine part (red) and Sine part (blue)> on the plasma boundary

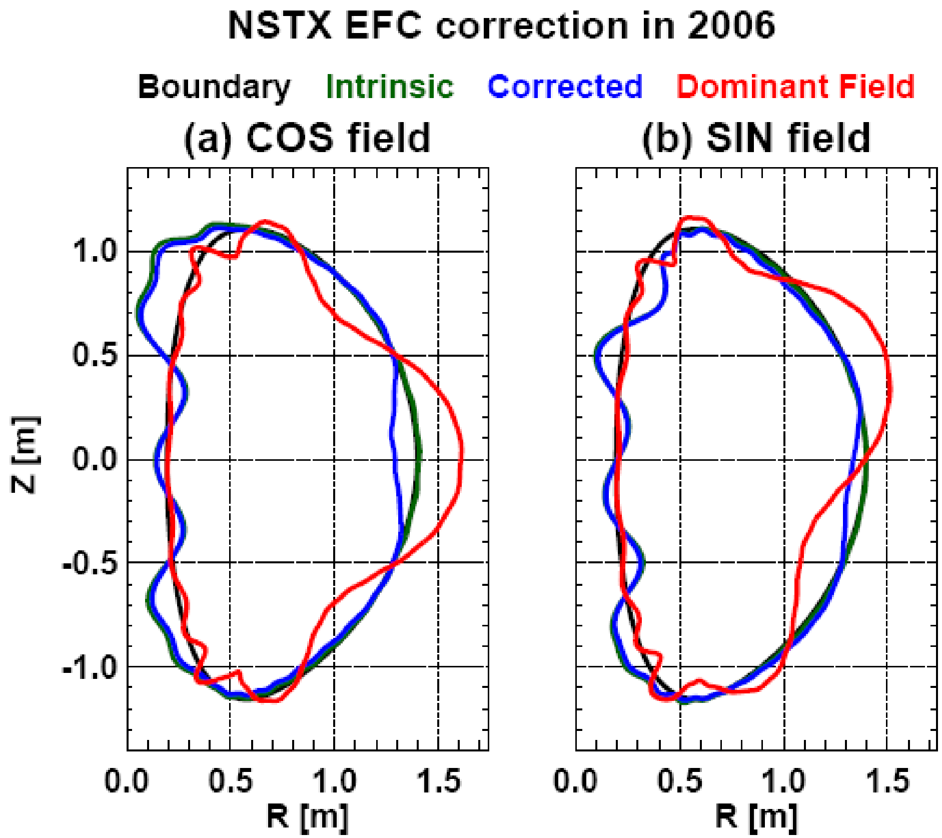
$$\delta \vec{B}^{x} \cdot \hat{n}_{b} = A(\theta)\cos(n\phi) + B(\theta)\sin(n\phi)$$



This is the shape of the external field to be minimized (can be quantified by overlap integral), and other distributions of the external field are less important roughly by an order of magnitude

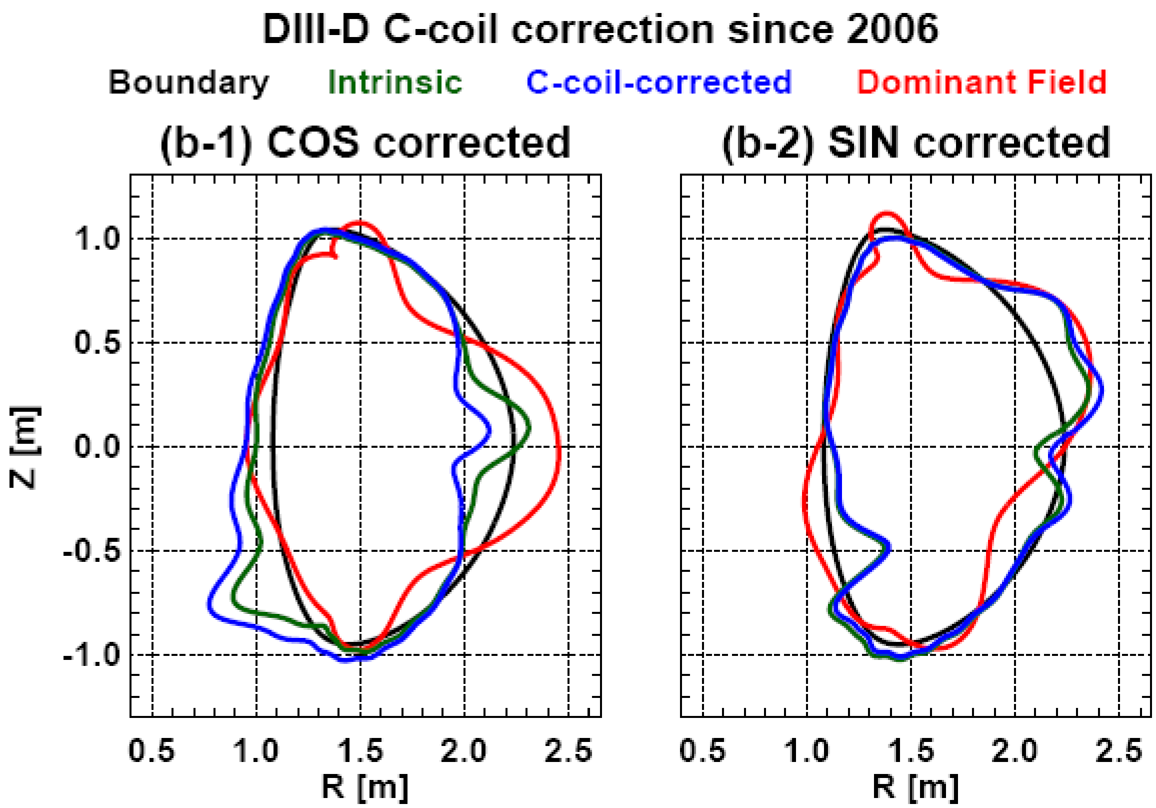
NSTX error field corrections can be understood in the view point of dominant external field

- OH-TF intrinsic error field in NSTX is mostly located in the inboard side
- ~60 Gauss intrinsic error field can be corrected by ~3 Gauss correction field, which can produce the dominant part of the external field



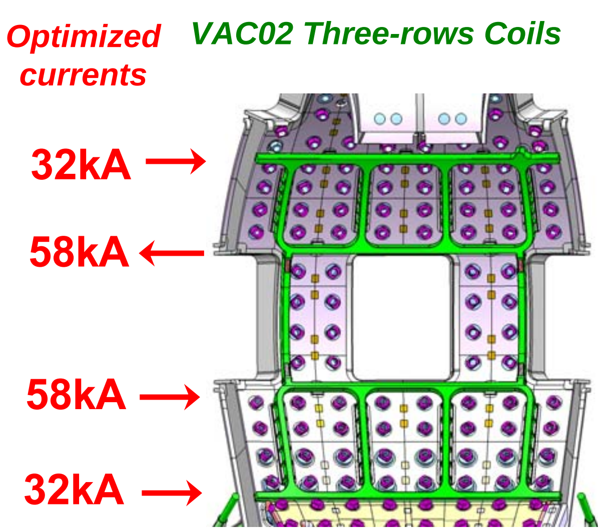
DIII-D C-coil error field correction can be understood in the view point of dominant external field

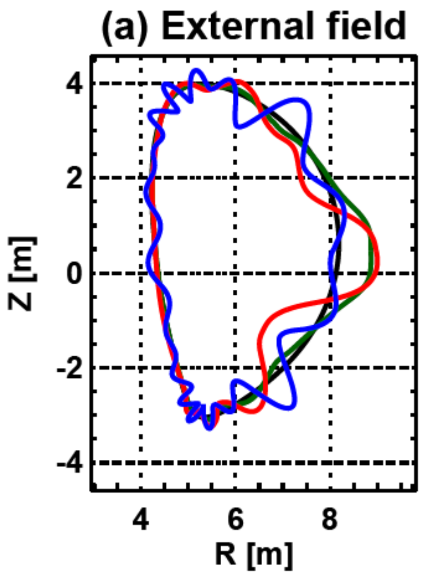
- DIII-D intrinsic error field produces dominant Sine part of the external field, but C-coil can produce only dominant Cosine part
- The correction is being made by increasing (-Cosine part) to cancel (+Sine part)

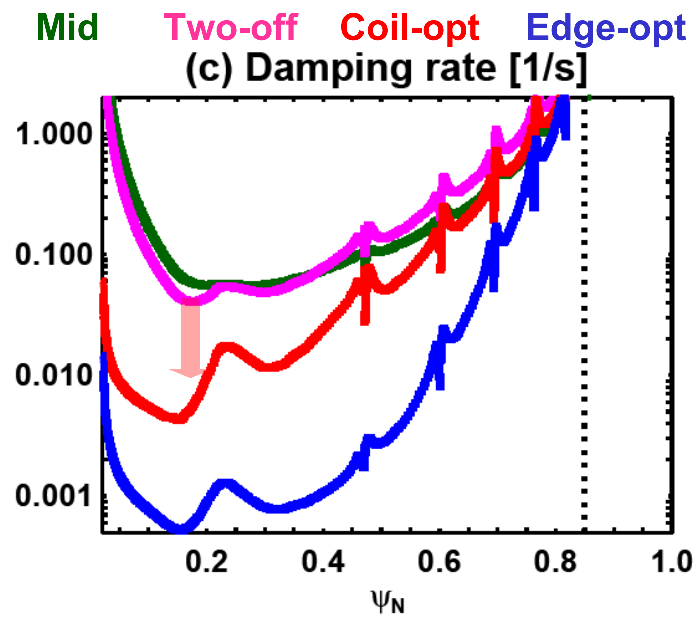


IPEC + NTV can help RMP coil design work in KSTAR, JET, and ITER

- Three rows of RMP coils can eliminate the irrelevant part of the midplane or the off-midplane field *[Todd, Schaffer]*
  - Mid : n=4 field using midplane coils
  - Two-off : n=4 field using two off-midplane coils
  - Coil-opt : Optimized n=4 field using three rows of coils
  - Edge-opt : Theoretical best n=4 field maximizing perturbations in the edge, but minimizing perturbations in the core
- Optimized field can reduce core NTV damping by an order of magnitude







Summary and Future Work

- IPEC solves free-boundary perturbed equilibria with shielded islands
- RFA n=1 results are successfully compared with IPEC and shows that plasma responds ideally to perturbations below marginal pressure
- IPEC total resonant field gives far better explanation for Locked Modes and provides promising scheme for the control of error field
- Generalized NTV has been developed and can improve consistency between theory and experiment when combined with IPEC
- IPEC + NTV can be used for RMP characterization
- Coupling between external magnetic field and physical parameters such as total resonant field can be an effective tool for control

Back up

Importance of non-ideal effects can be estimated by perturbed energy and toroidal torque

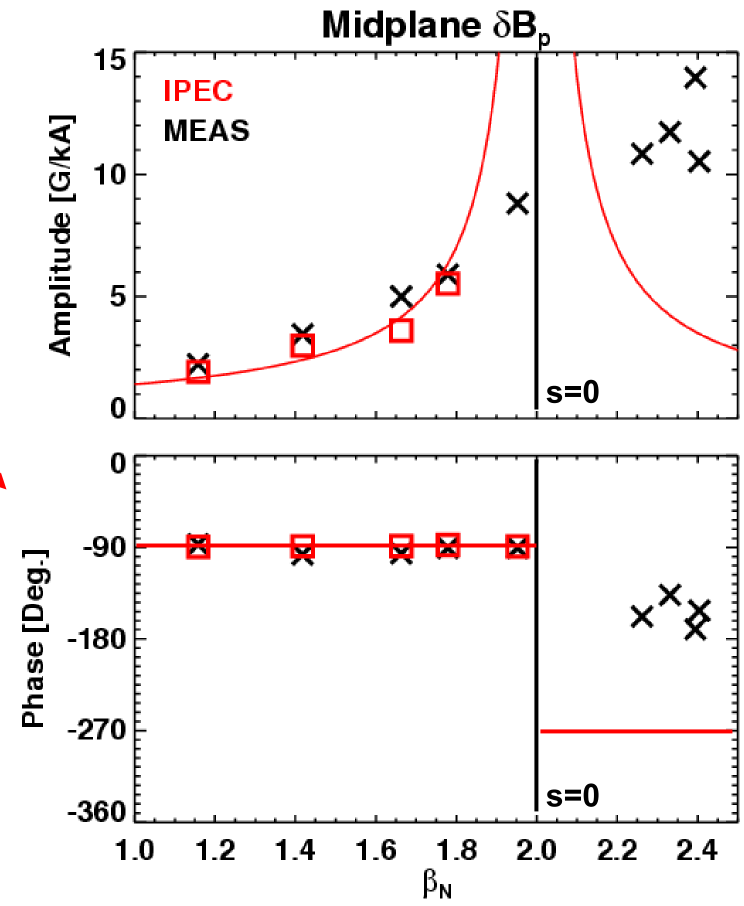
- The simplest model is to use perturbed energy (s) and torque (α)
  - Ideal response :

$$\text{RFA} \propto \frac{1}{s(\beta_N)}$$

  - Non-ideal response :

$$\text{RFA} \propto \frac{1}{s(\beta_N) + i\alpha(\beta_N)}$$

- The jump of the toroidal phases above the marginal point indicates the importance of toroidal torque

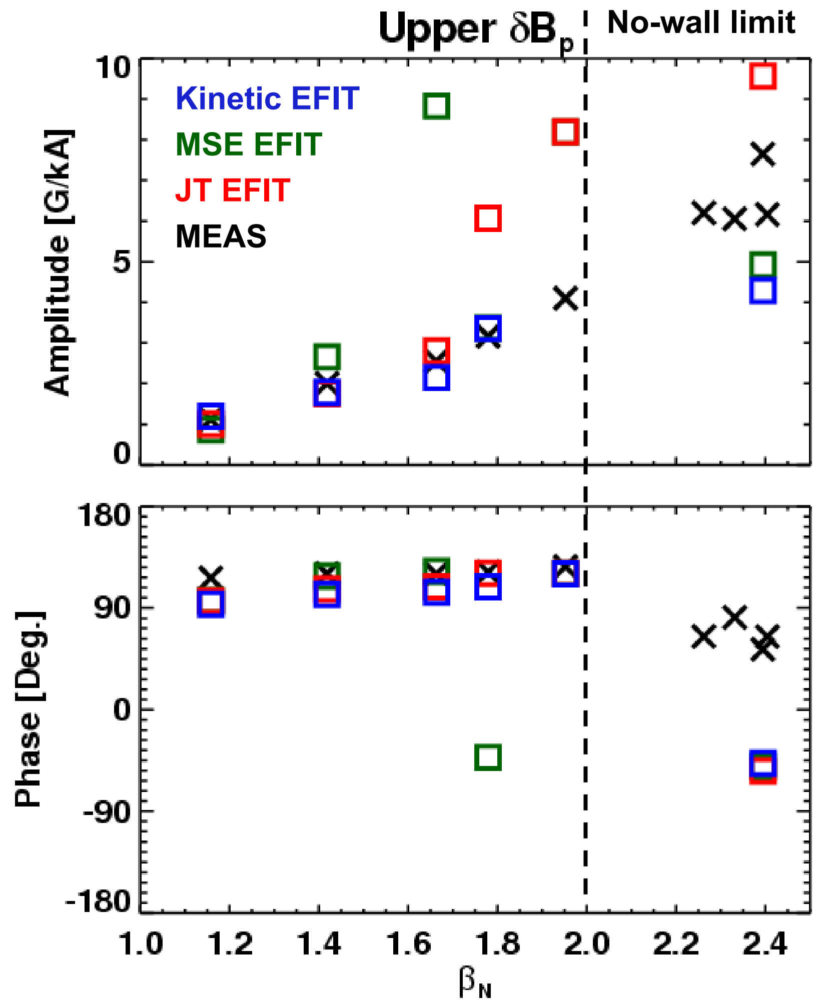


Good equilibrium reconstructions are important in high-β applications

- Typical EFITs work fine for 3D problem when plasma is reasonably far from marginal point

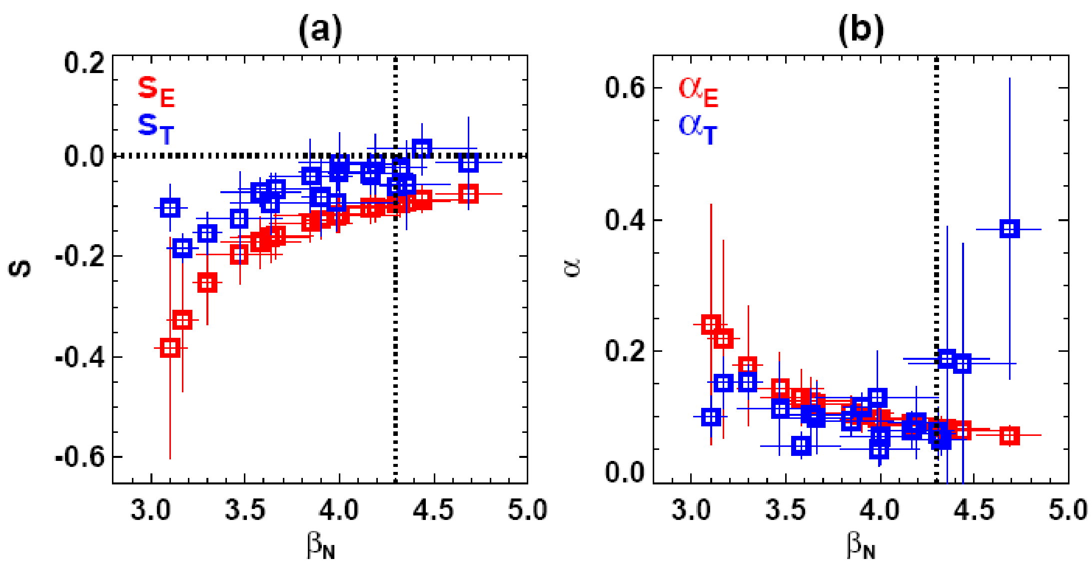
- Good equilibrium reconstructions are important to have sufficient precision in high-β applications

  [*Lanctot, Reimerdes*]



NSTX n=1 RFA results also shows that ideal plasma response is valid only below the marginally stable point

- Plasma response is almost ideal below the marginally stable point
- Non-ideal effects become important for high-β plasmas
- RFA results above the marginally stable case can give effective energy (s) and toroidal torque (α) ***[Park et al, Phys. Plasmas 16, 082512 (2009)]***
  - NSTX n=1 RFA experiments also imply torque becomes important above the marginal point, and gives stabilizing effects for s and shielding effects for α

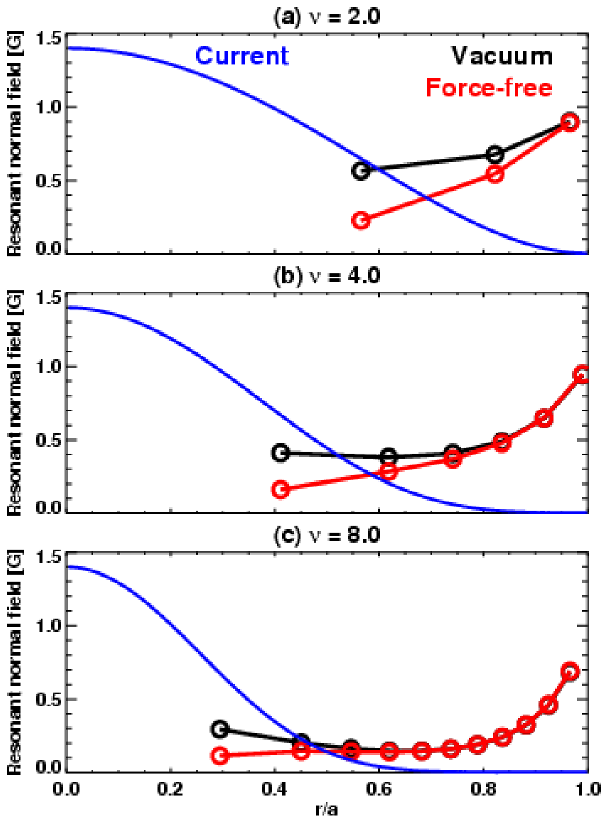


- This will be important for RWM applications (torque for VALEN3D)

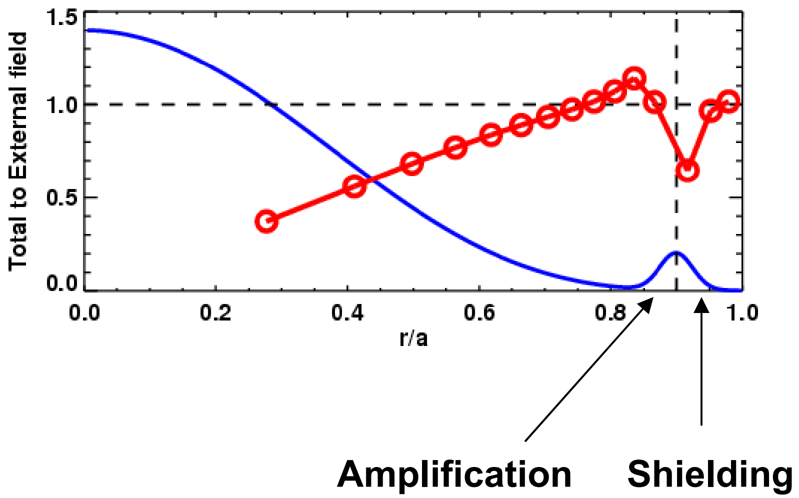
Total resonant field can show plasma response effects such as shielding

- Plasma currents tend to shield external perturbations

Force-free cylindrical example



Force-free cylindrical example with bootstrap currents



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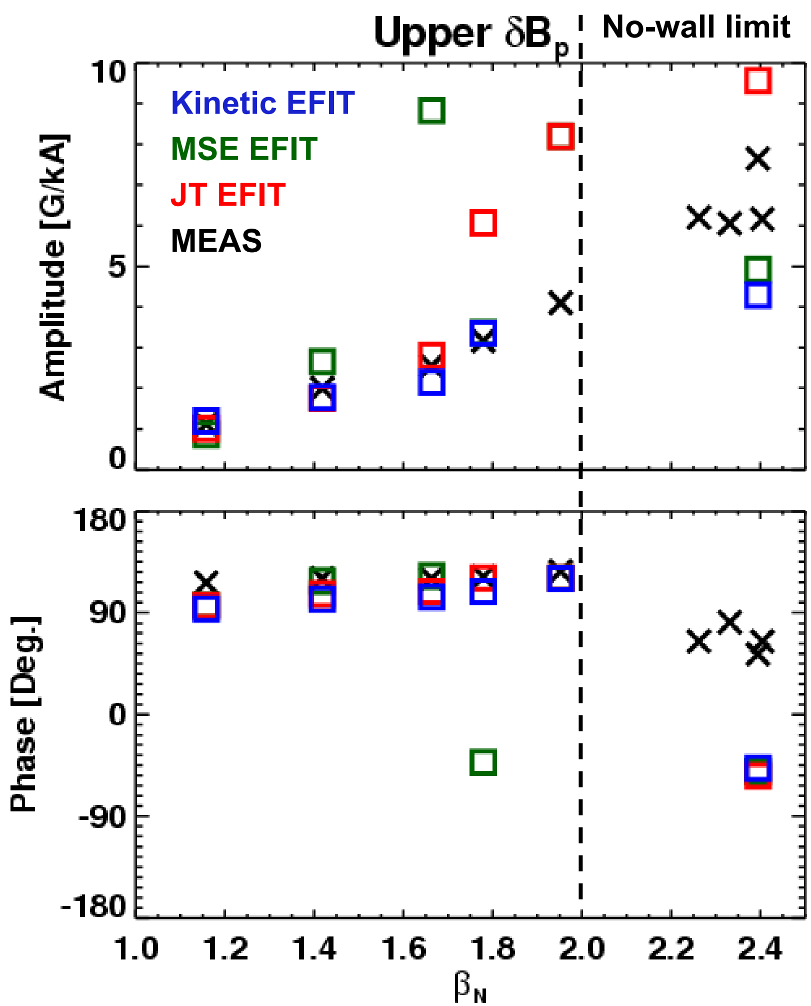
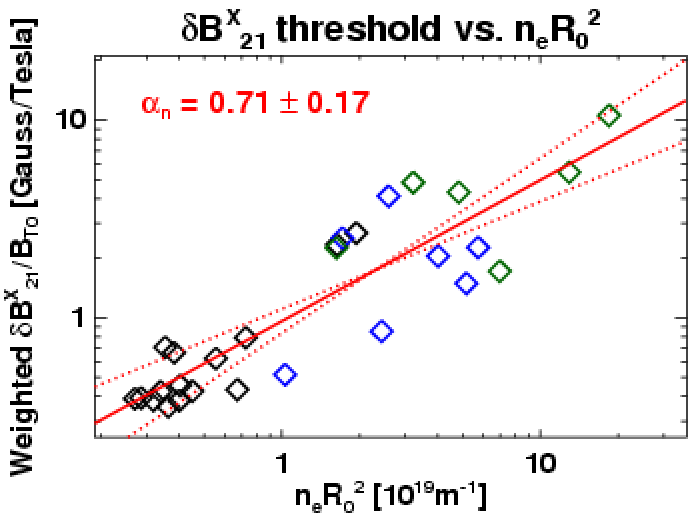


Locking scaling are being constructed using NSTX, DIII-D and CMOD data and will be used for KSTAR and ITER

- External (Vacuum) resonant field at q=2, Total resonant field at q=2, and the overlapped field with the dominant field for core will be used for locking scaling

Vacuum (External) field

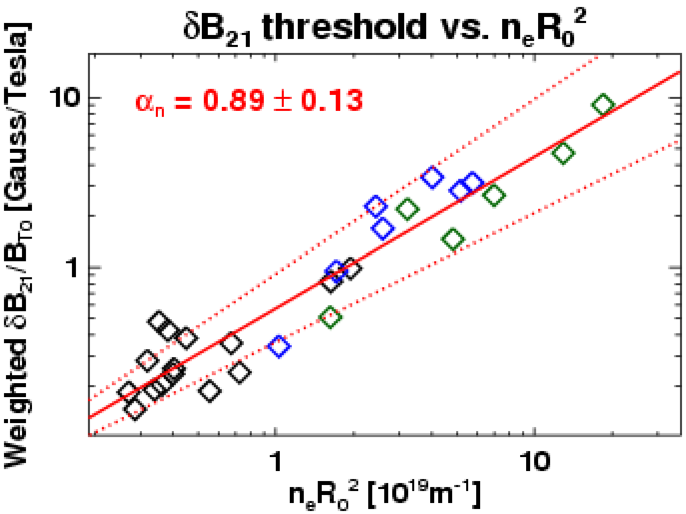


$$\frac{\delta B_{21}^{x}}{B_{T0}} \le 0.94\times10^{-4}\left(n[10^{19}m^{-3}]\right)^{0.71}\left(B_{T0}[T]\right)^{-1.1}\beta_N^{-0.48}$$

For $n=10^{19}m^{-3}$, $B_{T0}=3.5T$, and $\beta_N=0.9$,

$$\delta B_{21}^{x} \le 0.9\ Gauss$$

IPEC (Total) field



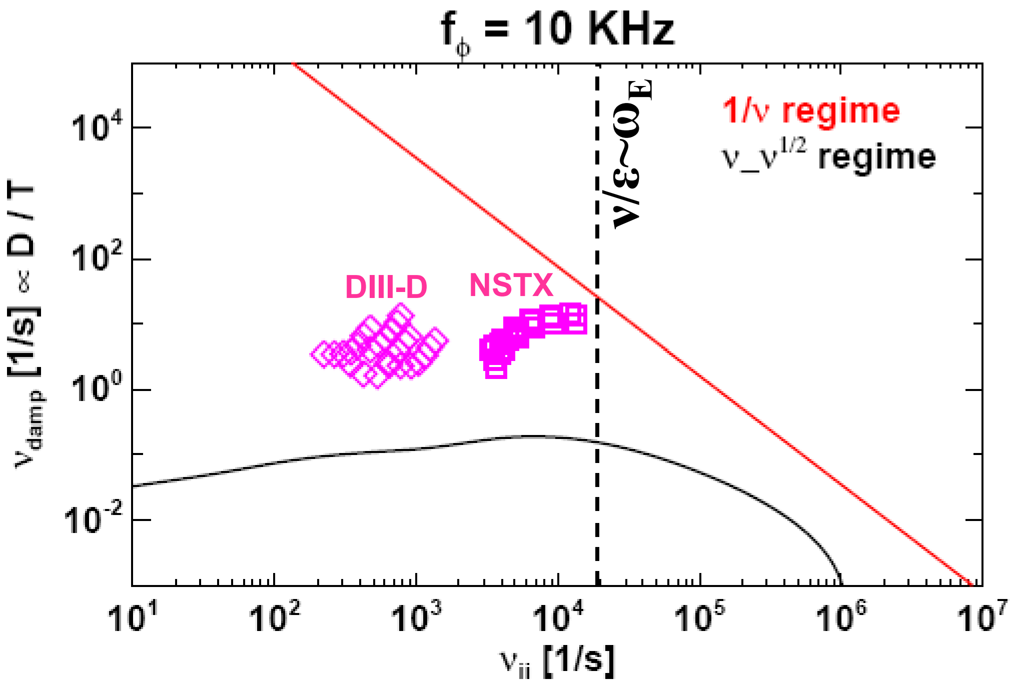
$$\frac{\delta B_{21}^{x}}{B_{T0}} \le 2.3\times10^{-4}\left(n[10^{19}m^{-3}]\right)^{0.89}\left(B_{T0}[T]\right)^{-1.3}$$

For $n=10^{19}m^{-3}$, and $B_{T0}=3.5T$,

$$\delta B_{21} \le 1.6\ Gauss$$

Standard presumption is not consistent with theory

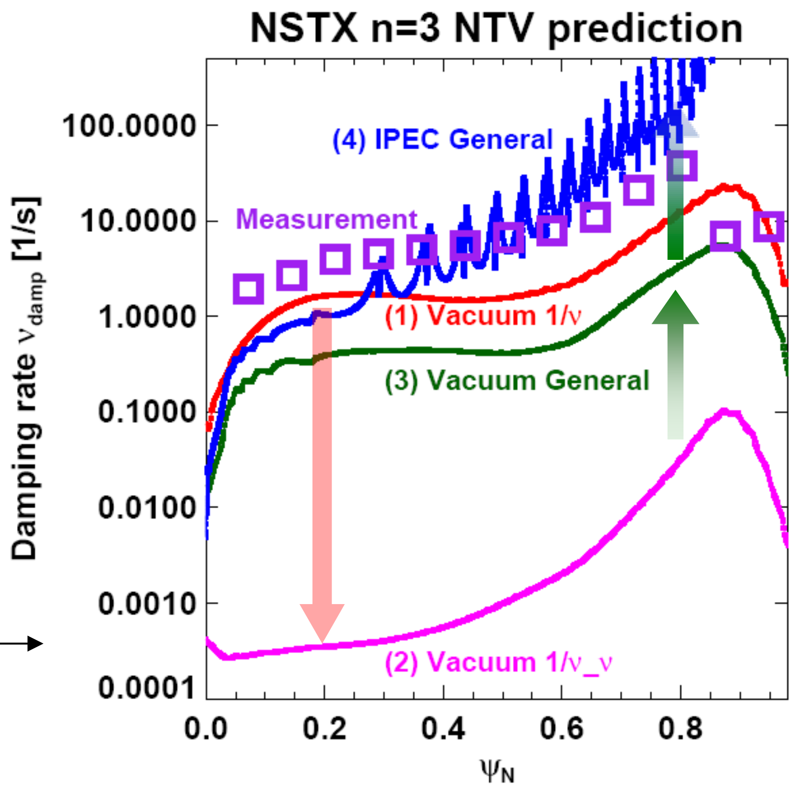
- IPEC gives the Lagrangian variation in the field strength, which can be coupled with analytic NTV evaluation (IDL routines)
- The previous $1/\nu$ and $\nu\_\nu^{1/2}$ evaluations differ by several orders of magnitude when they switch one to the other, and are inconsistent with observations
- Strong precession gives too small transport <u>even with 10^{-3} variation in the field strength</u>



Detailed NTV profiles are being compared with ideal perturbed equilibria

- Important new physics in NTV theory :
  - a) Toroidal precession rates (ω_E), which are often faster than the collisional rates (ν)
  - b) Trapped particle bounce rates (ω_b), which can resonate with the precession ($\omega_E+\omega_B$)
  - c) Variation of field strength along the perturbed magnetic field lines, which includes plasma response

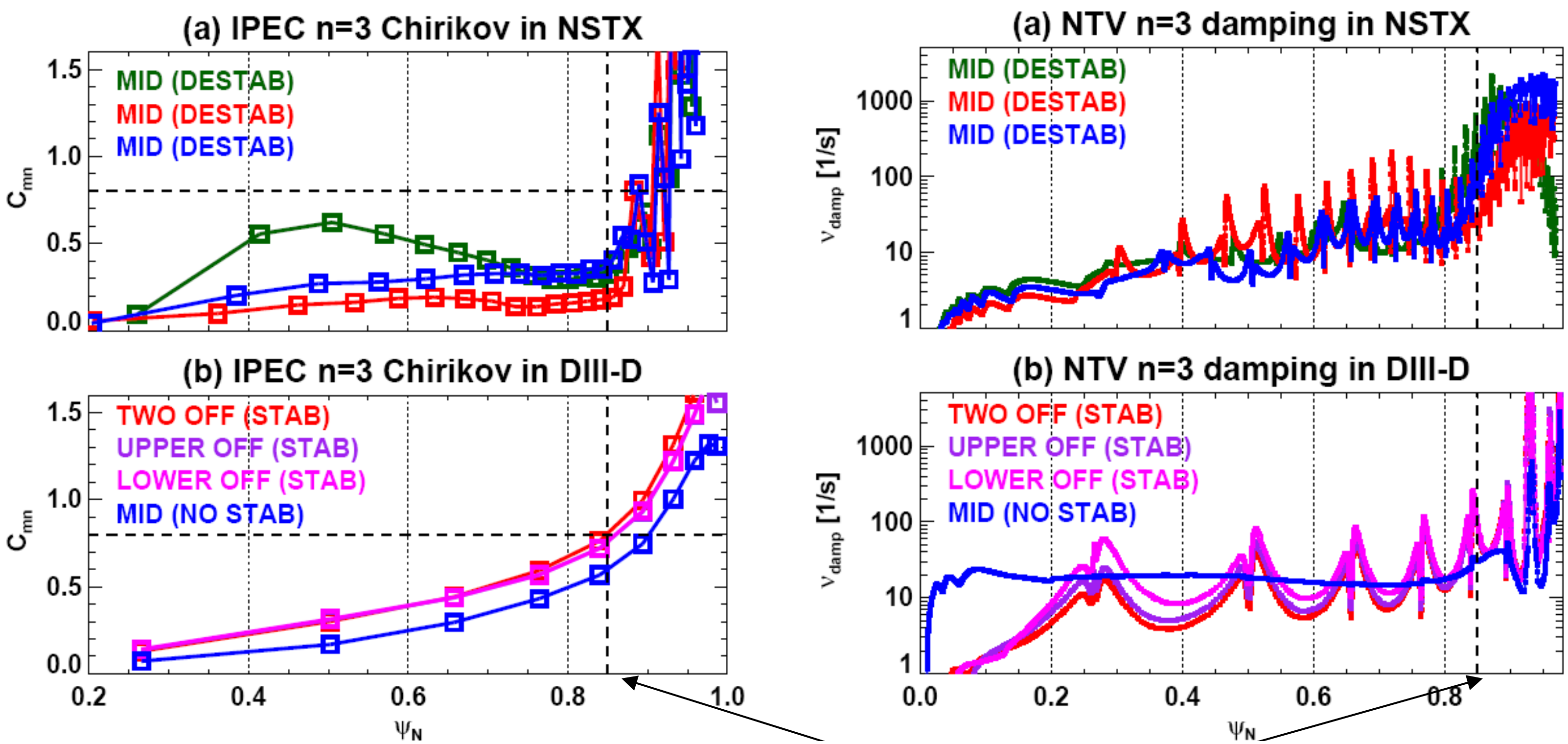
(1) (a), (b) and (c) are all ignored
(2) (a) is included
(3) (a) and (b) are included
(4) (a), (b) and (c) are all included



[*Park et al, Phys. Plasmas* *16*, *056115 (2009)*]

IPEC +NTV can be used RMP characterizations with perpendicular transport

- NSTX midplane n=3 applications destabilized ELMs (DESTAB)
- DIII-D off-midplane n=3 applications stabilized ELMs (STAB)
- DIII-D midplane n=3 applications did not influence ELMs (NO STAB)



The location where sufficient transport is required

- It is important to maximize the perturbation in the edge ψ_N~0.85

IPEC can be used RMP characterizations with parallel transport

- Field line tracing with IPEC field show similar structure of lobes indicating that plasma response does not greatly change parallel transport