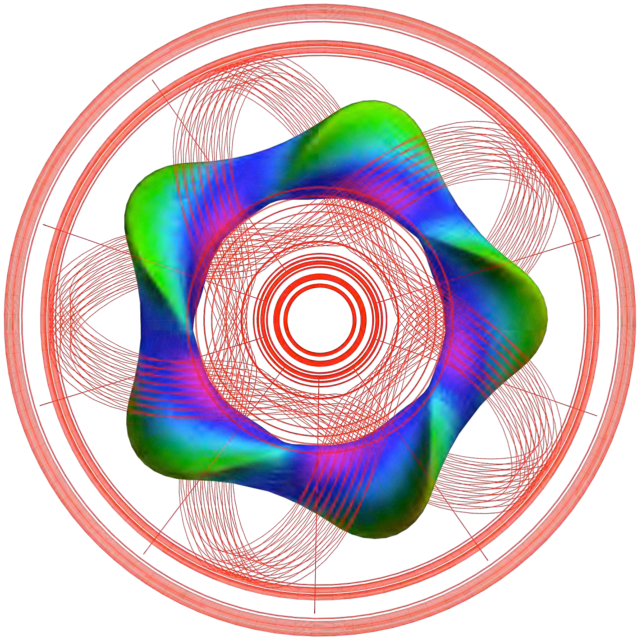
Three-Dimensional Equilibrium Reconstruction:
The V3FIT Code

James D. Hanson, Stephen F. Knowlton
Auburn University
Steven P. Hirshman, Edward A. Lazarus
Oak Ridge National Laboratory
Lang Lao
General Atomics

Equilibrium Reconstruction



CTH – Auburn U.

- Axisymmetric – EFIT
  - Observe magnetic diagnostic signals
  - Infer FF' and P' for Grad-Shafranov equation
- Non-axisymmetric – V3FIT
  - There is *no* Grad-Shafranov equation
  - There *are* MHD equilibrium solvers
  - Need to know pressure and current profiles
- A classic Inverse Problem
  - Forward problem: given parameters, determine signals.
    Known Function $\mathbf{S}^{\mathbf{m}}(\mathbf{p})$ - Model signals.
  - We know (observe) the signals $\mathbf{S}^{\mathbf{o}}$. What are the parameters?
    Determine Inverse Function $\mathbf{p}(\mathbf{S}^{\mathbf{m}}, \mathbf{S}^{\mathbf{o}})$
  - Use Maximum Likelihood - Least Squares.

V3FIT Code Design Goals

- Fast
  - Want reconstructions between shots
  - Design Choice: one reconstruction uses one CPU

    => Multiple reconstructions need multiple processors
- Flexible
  - Easy to understand, maintain, and modify
  - Written in Fortran 95
  - Clear and consistent data flow – modular coding
- Extensible
  - Initial equilibrium solver - VMEC
  - Localize VMEC code assumptions, so that could use a different equilibrium solver in the future
  - Initial signals – magnetic diagnostics
  - Other diagnostics can be added

VMEC

- Three-dimensional MHD equilibrium, *assumes* closed, nested flux surfaces
  - Can *not* resolve islands and chaotic regions
  - Uses inverse-coordinate representation
  - Spectral representation for angle coordinates
  - Grid representation for radial coordinate
  - Variational principle – minimizes radial forces on flux surfaces
  - Both free-boundary and fixed-boundary equilibria
- Fast, robust, widely used throughout the world.
- Parameters to use for reconstruction:

– Current and pressure profile parameters	am(i), ac(i)
– Pressure scale factor	pres_scale
– Total toroidal current	curtor
– External currents	extcur(i)
– Total toroidal flux within last closed flux surface	phiedge

General Algorithm

- Minimize deviation between observed and model signals

$$\chi^2(\mathbf{p}) \equiv \sum_i \left(\frac{S_i^o(\mathbf{d},\mathbf{p}) - S_i^m(\mathbf{p})}{\sigma_i} \right)^2$$

  - Minimize $\chi^2(\mathbf{p})$. Parameters $\mathbf{p}$, Observed signals $S_i^o(\mathbf{d},\mathbf{p})$.
  - Model-computed signals $S_i^m(\mathbf{p})$, uncertainties in signals σ_i.
- Definitions
  - Normalized parameters $a_j = p_j / \pi_j$
  - Error vector $e_i = \left(S_i^o(\mathbf{d},\mathbf{p}) - S_i^m(\mathbf{p}) \right) / \sigma_i$ $\quad \chi^2(\mathbf{p}) = \mathbf{e} \cdot \mathbf{e}$
  - Jacobian (unnormalized) $J_{ij} = \dfrac{\partial S_i^m}{\partial p_j}$

Minimization Algorithm

- Jacobian (normalized) $A_{ij} = \frac{\pi_j}{\sigma_i}\left(\frac{\partial S_i^o}{\partial p_j} - \frac{\partial S_i^m}{\partial p_j}\right)$ $\mathbf{A} = \nabla \mathbf{e}$

- V3FIT uses Quasi-Newton algorithm for new parameters

$$\mathbf{A}^T \cdot \mathbf{A} \cdot \delta \mathbf{a} = -\mathbf{A}^T \cdot \mathbf{e}$$

- Finite differences to compute Jacobian
  - Small steps in parameter space – VMEC converges rapidly
  - Need moderate accuracy in S_i^m
  - Needs well-converged VMEC
  - Does *not* need high radial resolution

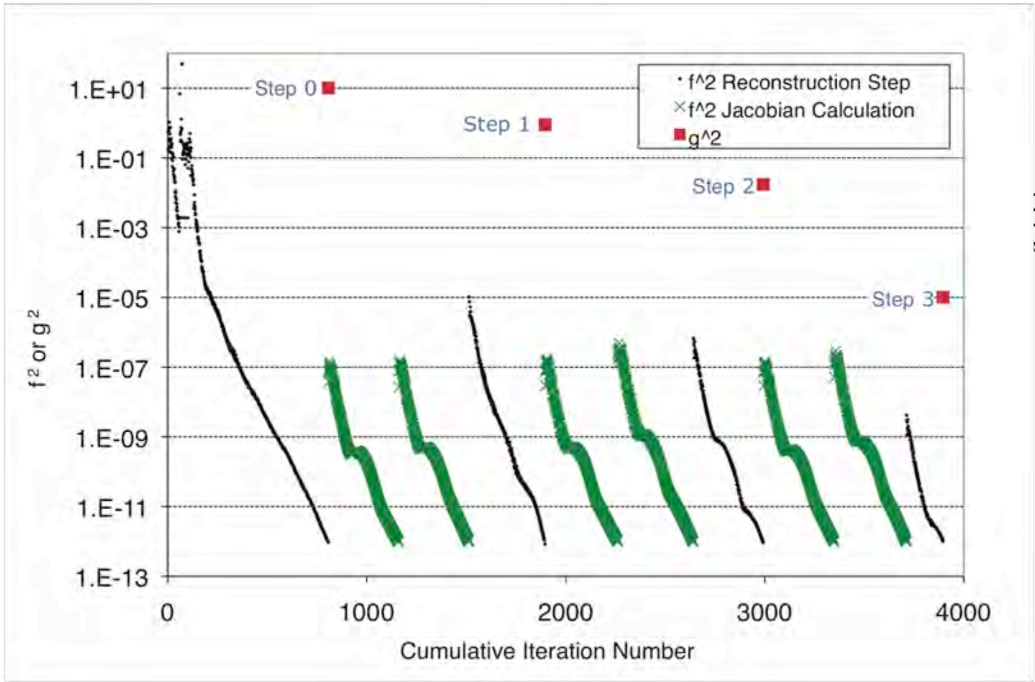
- Use Singular Value Decomposition (SVD) on Jacobian
  - Helps avoid large steps in parameter space

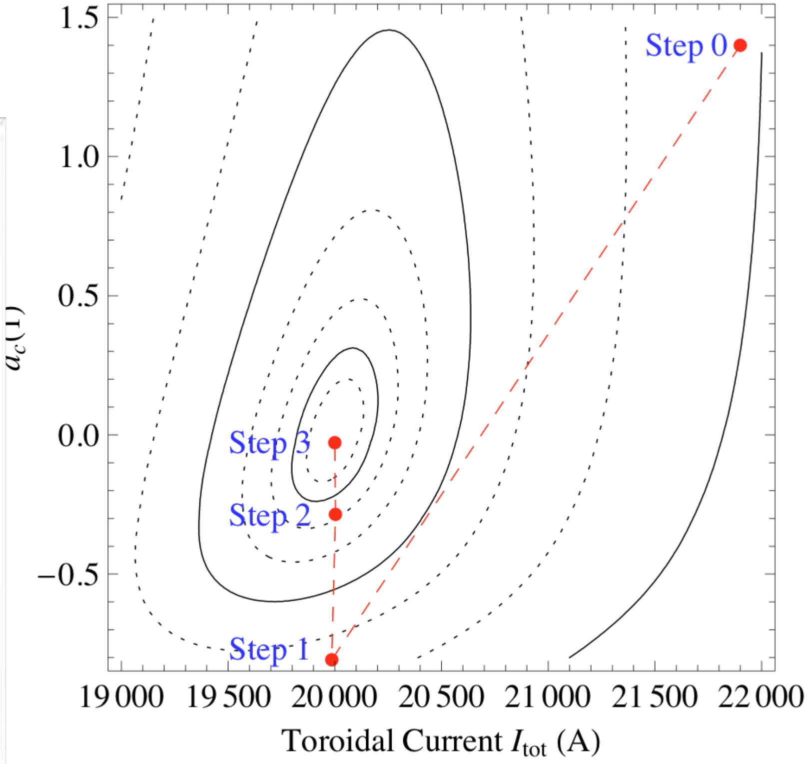
Posterior Sigmas: Confidence Limits on Parameters

- Assume uncorrelated Gaussian distribution of Signals
  - Signal covariance matrix – assume diagonal $C_{ij} = \sigma_i^2 \delta_{ij}$
- Expect nearly Gaussian distribution in parameter space
  - Parameter covariance matrix $\mathbf{C}_p = (\mathbf{J}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{J})^{-1}$
  - Also called posterior covariance
  - Confidence limit on parameter value $\sigma_{p\,j} = \sqrt{\left(\mathbf{C}_p\right)_{jj}}$
  - $\sigma_{p\,j}$ – Measures how accurately these signals determine the *j*th reconstruction parameter.

Reconstruction Illustration

- CTH Equilibrium
- 2 Parameters
  - Total toroidal plasma current
  - Toroidal current profile shape
- 12 Magnetic Diagnostic signals
  - Rogowski, 8-part Rogowski
  - Two flux loops, one magnetic probe





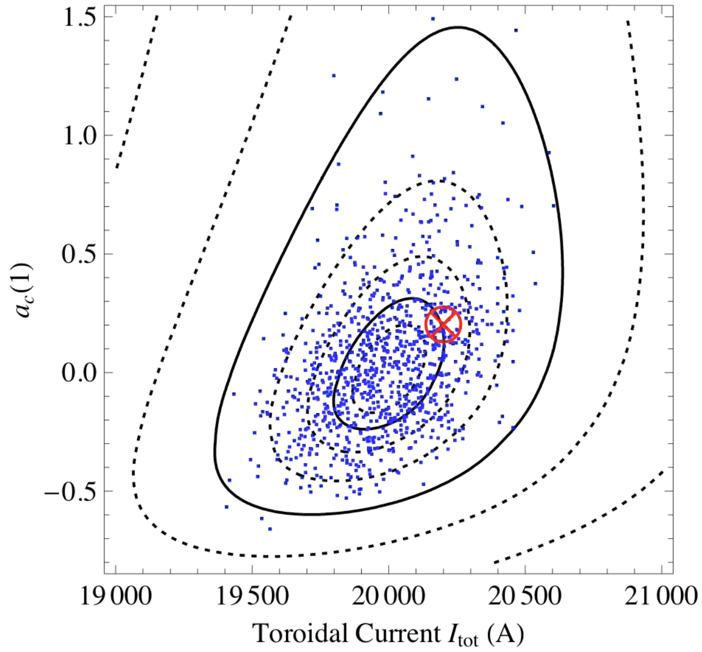
Reconstruction with Noise

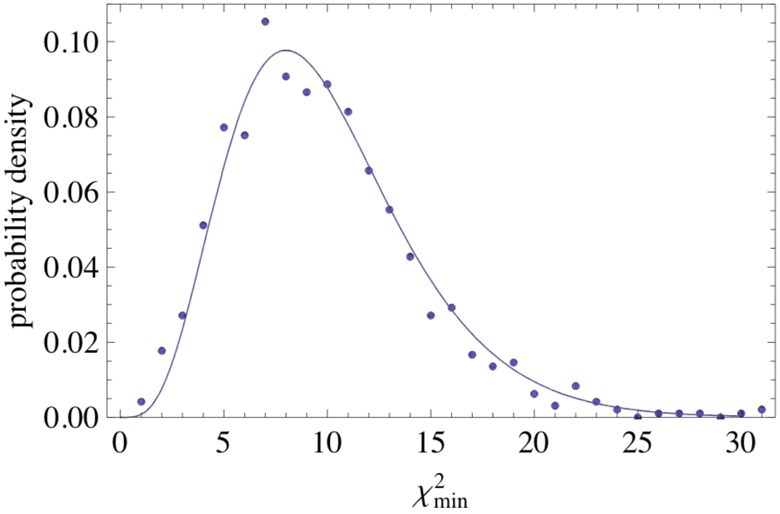
- Simulated signals

$$S_i^{"observed"} = S_i^{\text{model}}(p_0) + \delta S_i$$

δS_i - Gaussian distributed noise

- If noise is not too large, then:
  - Gaussian distribution of reconstructed parameters
  - χ^2 distribution of χ^2_{min} values





EFIT and VMEC / V3FIT Agree on Forward Problem

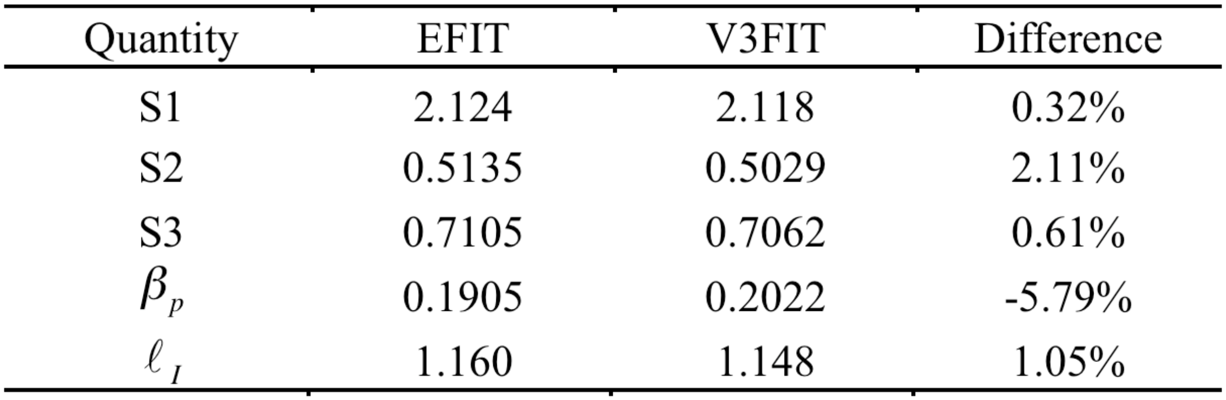
- Test equilibrium – DIII-D shot 118162.03030
- Use EFIT profiles for VMEC input

- Mutual inductances between magnetic diagnostics and external coils agree to 5 significant figures.
- 103 Magnetic Diagnostics agree to RMS 0.63%
- Integrated equilibrium quantities agree well:

Quantity	EFIT	VMEC	Difference
S1	2.124	2.120	0.21%
S2	0.5135	0.5085	0.98%
S3	0.7105	0.7119	-0.20%
β_p	0.1905	0.1898	0.37%
ℓ_i	1.160	1.157	0.22%

Use V3FIT to Reconstruct DIII-D Equilibrium

- *Experimental* observations of 31 partial Rogowskis and 36 flux loops
- Used 21 reconstruction parameters:
  - 18 F-coil currents
  - PRES_SCALE – overall pressure profile scaling factor
  - CURTOR – net toroidal plasma current
  - AC(1) – parameter that changes shape of current profile
- Comparison:

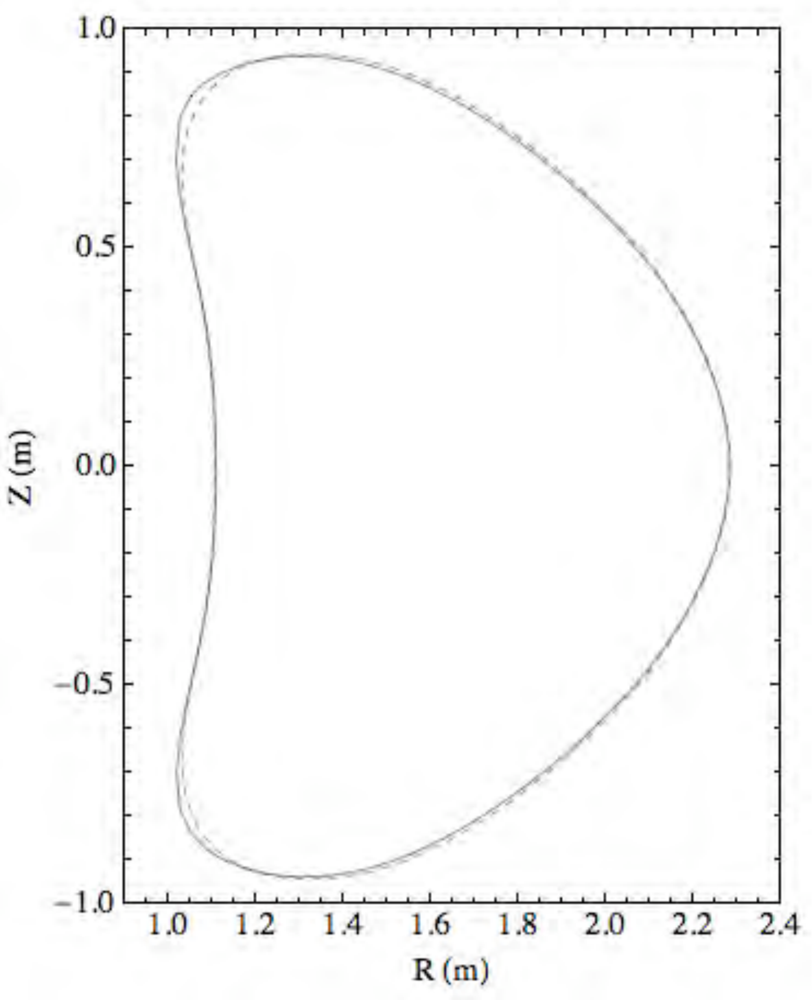


Quantity	EFIT	V3FIT	Difference
S1	2.124	2.118	0.32%
S2	0.5135	0.5029	2.11%
S3	0.7105	0.7062	0.61%
β_p	0.1905	0.2022	-5.79%
ℓ_I	1.160	1.148	1.05%

Use V3FIT to Reconstruct DIII-D Equilibrium

- Good agreement with EFIT reconstruction on parameter values, integrated quantities, and outermost flux surface shape.
- CONCLUDE: V3FIT can use real data to reconstruct equilibria. Reconstructed equilibrium is comparable to EFIT's.

Nucl. Fusion **49** (2009) 075031.



Equilibrium Reconstruction on CTH

- Goal: Use V3FIT for routine reconstructions on the Compact Toroidal Hybrid (CTH) at Auburn
- Poster P01-01, Stevenson et al., this workshop
- 25 Signals
  - Three 8-part Rogowski's at toroidal angles: σ ~3%

18°	1/4 field period
108°	1 1/2 field period
144°	2 field period

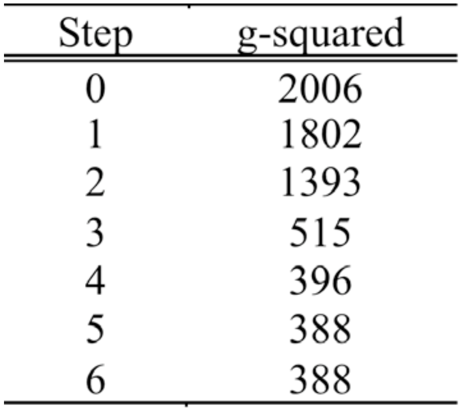
  - Limiter (circular) σ =1.0 mm
- 3 reconstruction parameters
  - CURTOR total toroidal current
  - AC(1) current profile parameter (changes breadth of profile)
  - PHIEDGE total toroidal flux (changes size of plasma)

Equilibrium Reconstruction on CTH

- VMEC Parameters (Stellarator Symmetric)

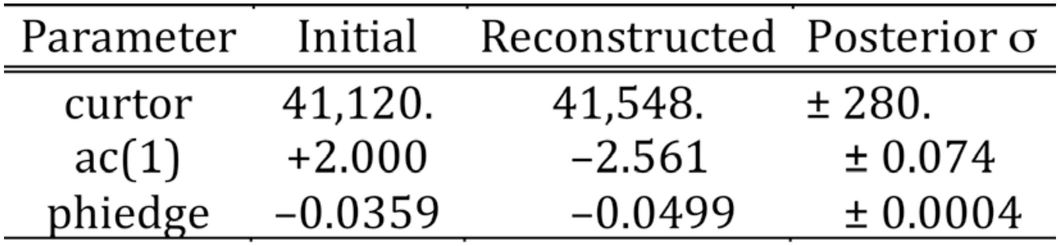
– MPOL=6	Number of poloidal modes
– NTOR=8	Number of toroidal modes
– NS=15	Number of radial grid points
– FTOL=5.E-14	Convergence Parameter

- Six reconstruction iterations:
  - Run time: 206 seconds



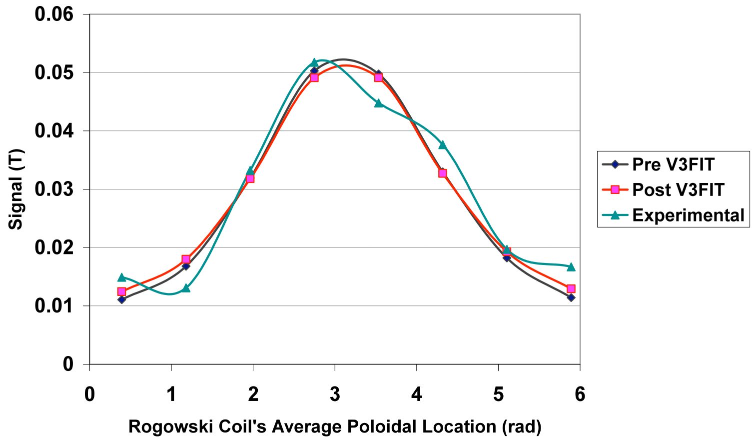
Step	g-squared
0	2006
1	1802
2	1393
3	515
4	396
5	388
6	388

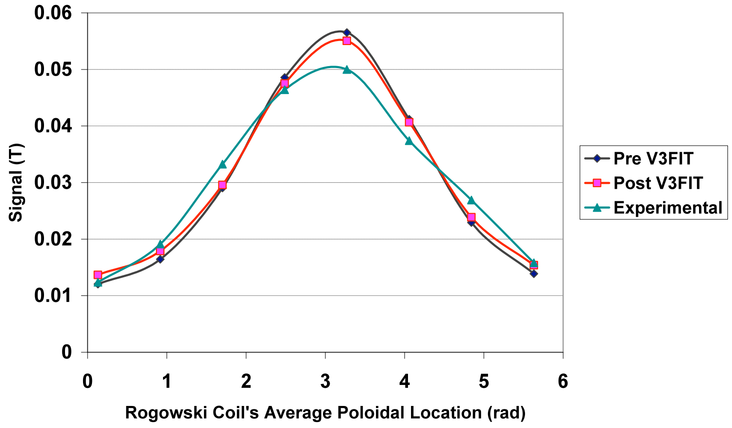
- Reconstructed Parameters:

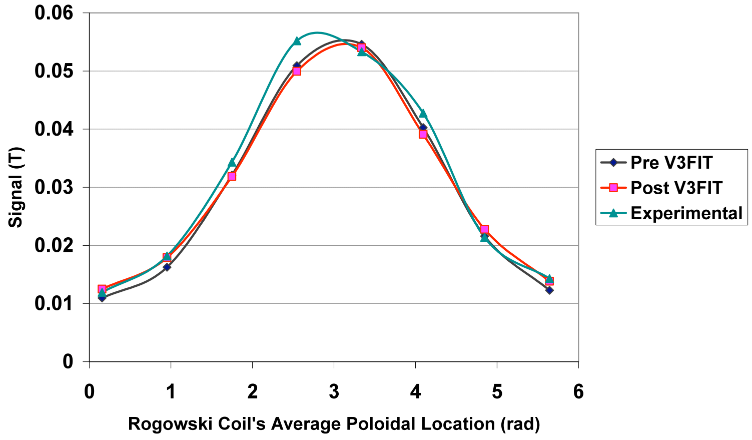


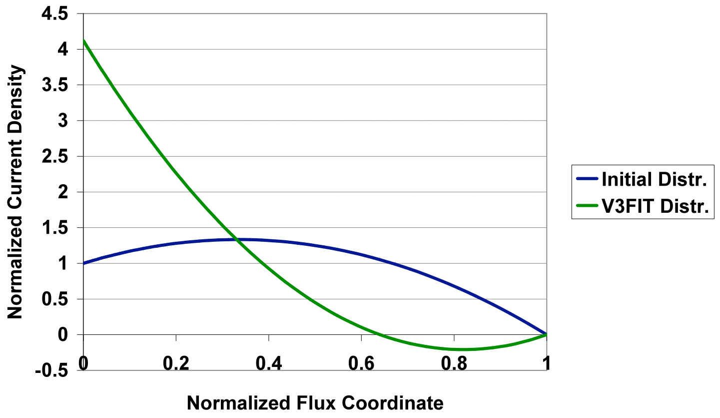
Parameter	Initial	Reconstructed	Posterior σ
curtor	41,120.	41,548.	± 280.
ac(1)	+2.000	−2.561	± 0.074
phiedge	−0.0359	−0.0499	± 0.0004

Equilibrium Reconstruction on CTH


Plasma Response Comparison for Rogowski Coil Set 8PO018


Plasma Response Comparison for Rogowski Coil Set 8PI108


Plasma Response Comparison for Rogowski Coil Set 8PI144


V3FIT Current Profile Modification at 1.66801 s

Equilibrium Reconstruction on CTH

- Final χ^2 is larger than expected
  - 25 signal – 3 parameters, expect χ^2 near 22
  - Indicative of possible systematic error
- Signal behavior also indicates systematic errors
- Possible corrections for systematic errors:
  - More accurate measurement of mutual inductances
  - Improve model of vacuum vessel currents
  - Allow for broken stellarator symmetry
  - Allow for broken field-period symmetry
- Run time could be improved with better initial choice of reconstruction parameters
- Need to automate the reconstruction process

Signal Effectiveness

- Motivation
  - Which magnetic diagnostics are most useful?
  - I wish to improve the measurement of the current profile. What magnetic diagnostics should I add?
  - I only have money for one more diagnostic. Where should I put it?
  - Magnetic diagnostics break. For which magnetic diagnostics do I need a spare, ready and waiting to put on the machine?
  - I'm building a new stellarator. What magnetic diagnostics should I build?

Pomphrey, Lazarus et al., *Phys Plasmas* **14** 056103 (2007).

- Design for NCSX
- Database of 2500 free-boundary VMEC equilibria
- Initial 600 trial flux loops, pruned to 225.

Signal Effectiveness

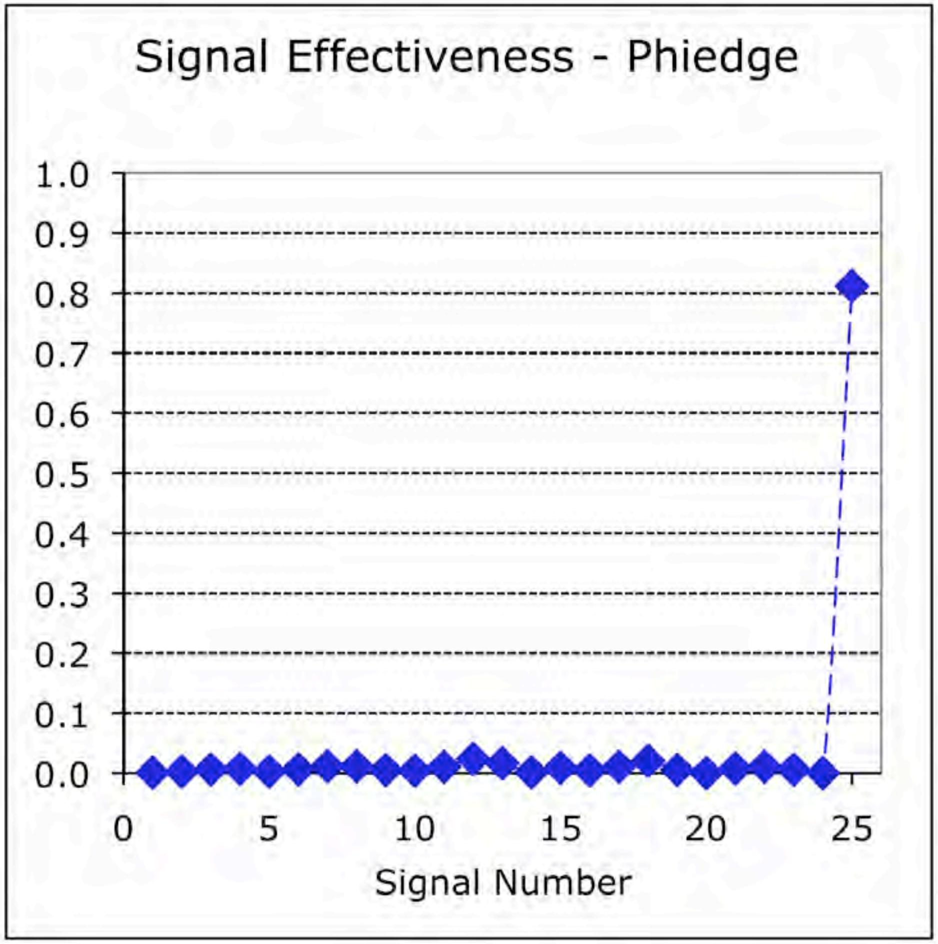
- Proposed measure of the effectiveness of a signal:

$$R_{ji} = \frac{d \ln \sigma_{p\,j}}{d \ln \sigma_i} = \frac{\sigma_i}{\sigma_{p\,j}} \frac{d\sigma_{p\,j}}{d\sigma_i}$$

  - Logarithmic derivative of the *j*th posterior parameter σ_p with respect to the *i*th signal σ
  - How much will the *j*th posterior σ_p improve if the noise level on the *i*th signal is reduced?
- With $\mathbf{C}_p = (\mathbf{J}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{J})^{-1}$, R is readily computable from the Jacobian $\partial S_i^m / \partial p_j$.
- Note:
  - R is dimensionless and non-negative.
  - $\sum_{i\,(signals)} R_{ji} = 1$
  - It is *local*. It only contains information about what happens near a particular point in parameter space.

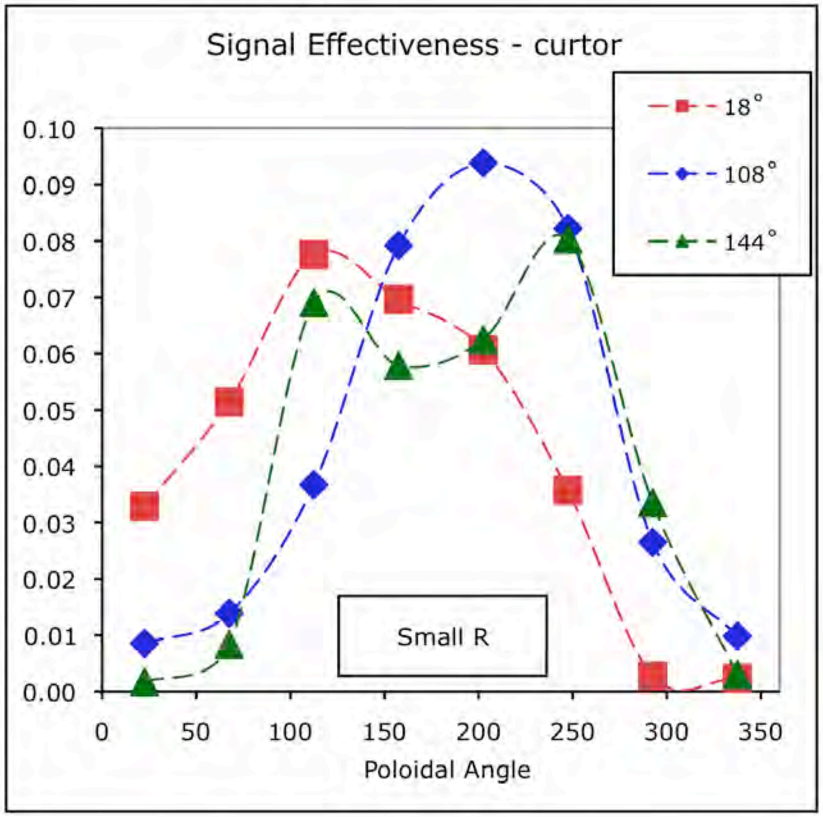
Signal Effectiveness

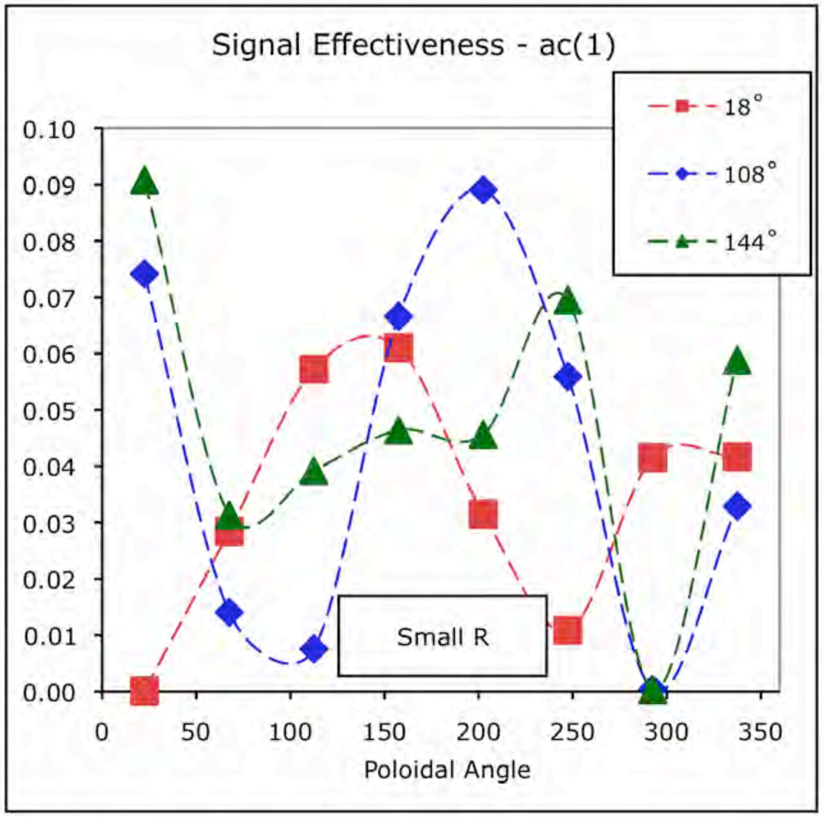
- Preliminary Results:
  - CTH Reconstruction
  - Signals 1-24 are partial Rogowski's
  - Signal 25 is the limiter
  - As expected, the limiter is by far the most effective diagnostic for determining phiedge



Signal Effectiveness

- Preliminary Results:
  - CTH Reconstruction
  - 24 partial Rogowski's in 3 different toroidal planes



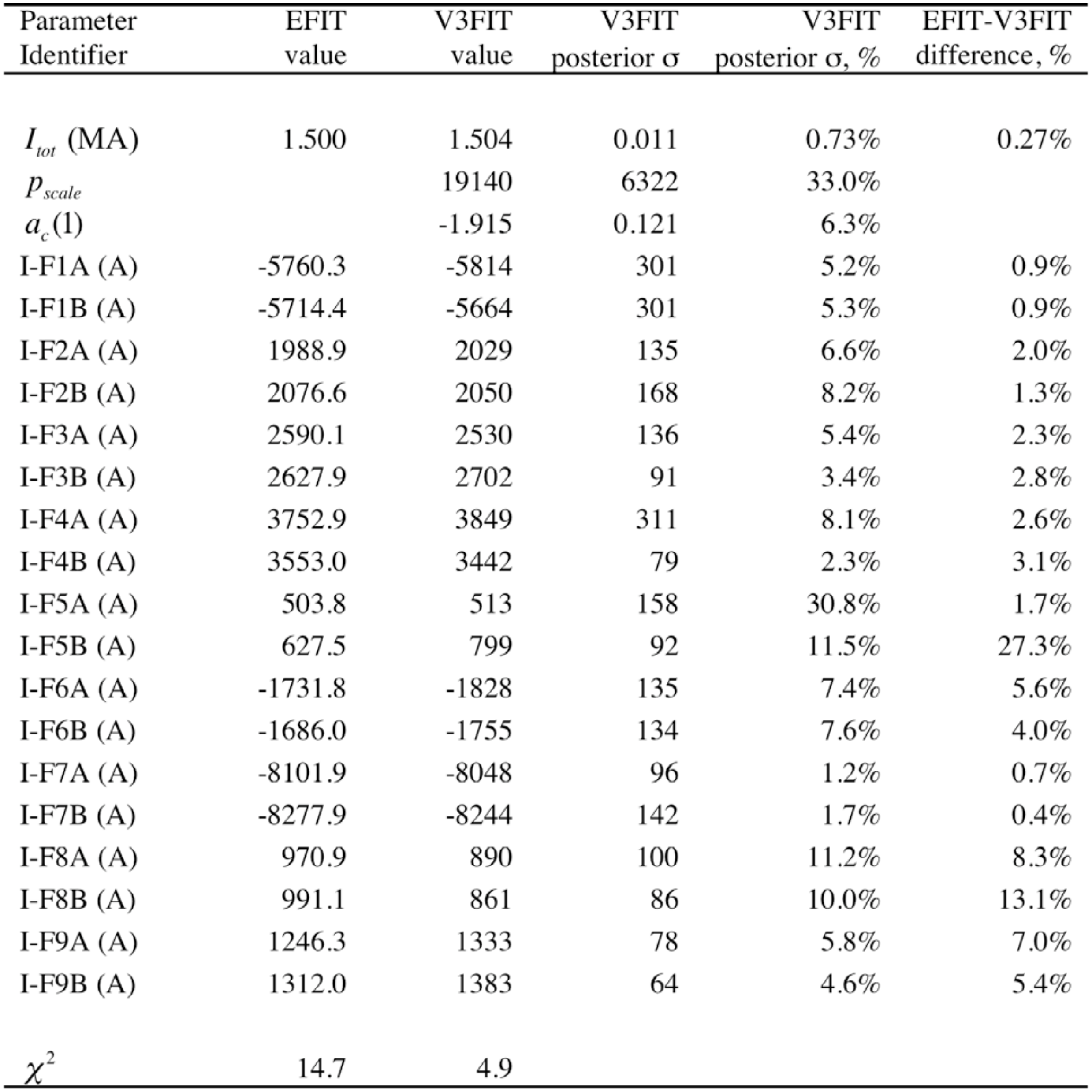


Conclusions

- V3FIT reconstruction algorithm converges as expected.
- V3FIT behaves correctly when noise is added to signals.
- VMEC / V3FIT agrees with EFIT – axisymmetric forward problem.
- V3FIT can reconstruct axisymmetric equilibria using real data – comparable to EFIT.
- V3FIT is proving useful for stellarator equilibrium reconstruction.
- The Jacobian contains lots of useful information:
  - Posterior parameter confidence limits
  - Signal effectiveness
- Confrontation with real experimental data is leading to improvements in both the V3FIT code and in the magnetic diagnostics.

Acknowledgements

- United States Department of Energy
- Auburn University
  - Mark Cianciosa
  - Greg Hartwell
  - Jonathan Hebert
  - Jorge Muñoz
  - John Shields
  - Adam Stevenson
- Los Alamos National Laboratory
  - John Finn and Chris Jones



Parameter Identifier	EFIT value	V3FIT value	V3FIT posterior σ	V3FIT posterior σ, %	EFIT-V3FIT difference, %
I_{tot} (MA)	1.500	1.504	0.011	0.73%	0.27%
p_{scale}		19140	6322	33.0%	
$a_c(1)$		-1.915	0.121	6.3%	
I-F1A (A)	-5760.3	-5814	301	5.2%	0.9%
I-F1B (A)	-5714.4	-5664	301	5.3%	0.9%
I-F2A (A)	1988.9	2029	135	6.6%	2.0%
I-F2B (A)	2076.6	2050	168	8.2%	1.3%
I-F3A (A)	2590.1	2530	136	5.4%	2.3%
I-F3B (A)	2627.9	2702	91	3.4%	2.8%
I-F4A (A)	3752.9	3849	311	8.1%	2.6%
I-F4B (A)	3553.0	3442	79	2.3%	3.1%
I-F5A (A)	503.8	513	158	30.8%	1.7%
I-F5B (A)	627.5	799	92	11.5%	27.3%
I-F6A (A)	-1731.8	-1828	135	7.4%	5.6%
I-F6B (A)	-1686.0	-1755	134	7.6%	4.0%
I-F7A (A)	-8101.9	-8048	96	1.2%	0.7%
I-F7B (A)	-8277.9	-8244	142	1.7%	0.4%
I-F8A (A)	970.9	890	100	11.2%	8.3%
I-F8B (A)	991.1	861	86	10.0%	13.1%
I-F9A (A)	1246.3	1333	78	5.8%	7.0%
I-F9B (A)	1312.0	1383	64	4.6%	5.4%
χ^2	14.7	4.9			