Reasons for Proposing a New Benchmark

- M3D and NIMROD results from 1st benchmark agree with each other but not with experiment. Better fidelity to experiment should yield better validation.
  - Replace current source with loop voltage.
  - Replace pressure source with ohmic heating.
  - Use a much more realistic profile for $\kappa_\perp$.
  - Allow resistivity to track evolving temperature profile.
  - Use constant Prandtl number.

- Beginning with an analytically specified equilibrium will make it possible to publish the benchmark as a standard test problem available to other nonlinear MHD codes.

Specification of Analytic Equilibrium

Quantity	Value
Major radius R_0	0.341 m
Minor radius a	0.247 m (aspect ratio = 1.38)
Ellipticity κ	1.35
Triangularity δ	0.25
Central temperature (T_e=T_i)	100 eV
Normalized central pressure $\mu_0 p_0$	2.5×10^{-4} (implies $n_0 = 1.8 \times 10^{-19}$ m^{-3})
α Parameter in pressure equation*	0.1
Vacuum value g_0 of $R \cdot B_T$	0.042 T·m
Effective ion charge Z_{EFF}	2.0
Loop voltage V_L	3.1741 V (implies $q_0 \approx 0.82$)

$$*p(\psi) = p_0\left[\alpha\tilde{\psi} + (1-\alpha)\tilde{\psi}^2\right], \text{ where } \tilde{\psi} \equiv \frac{\psi - \psi_{\text{limiter}}}{\psi_{\text{axis}} - \psi_{\text{limiter}}}.$$

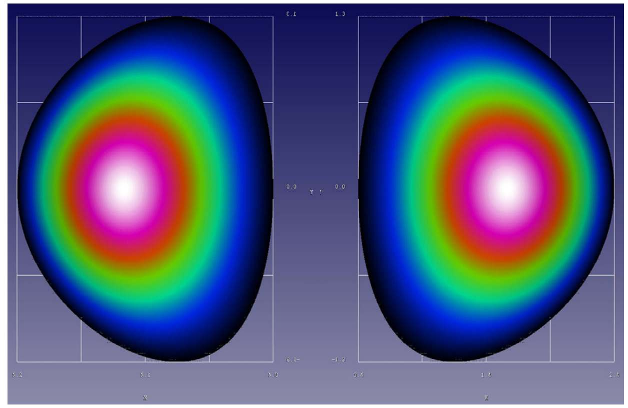
Use equilibrium code to solve Grad-Shafranov equation, with profile of heat conduction coefficient χ computed self-consistently to keep temperature constant given profile, energy supplied by applied V_L.

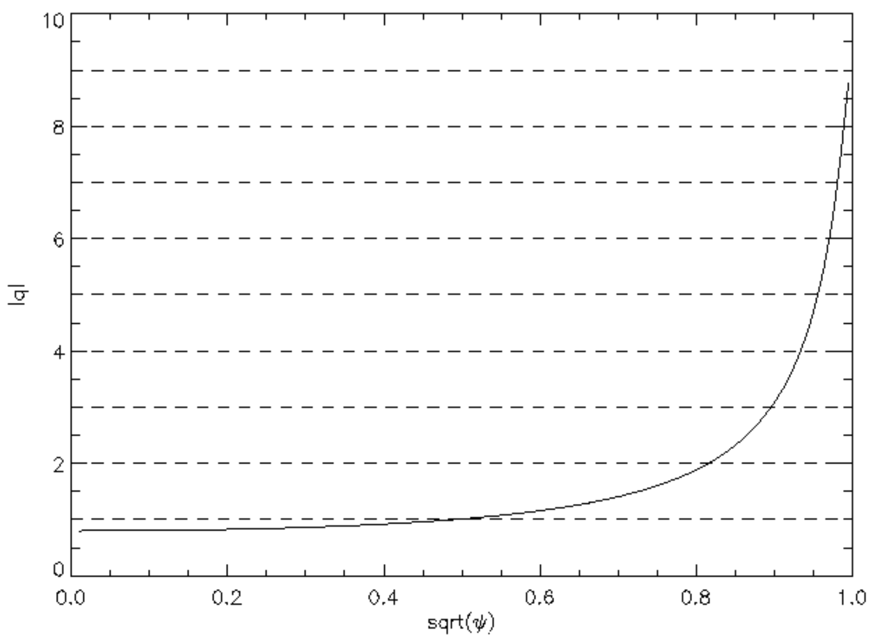
Form of New Equilibrium

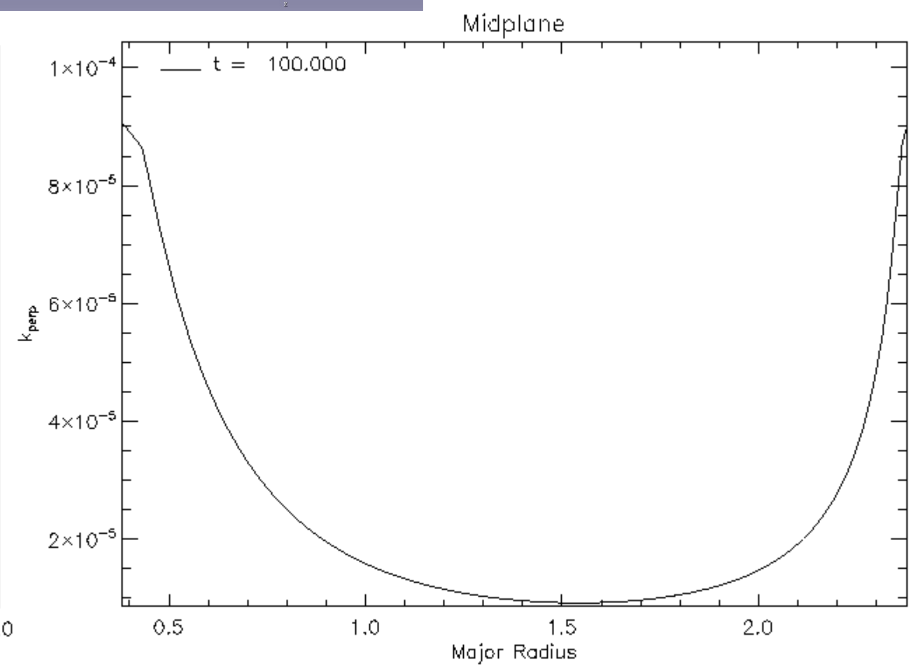
$$R(\theta) = R_0 + a\cos\left[\theta + \delta\sin(\theta)\right]$$
$$z(\theta) = a\kappa\sin(\theta)$$

$$T(\psi) = T_0\tilde{\psi},$$

$$n(\psi) = \frac{p}{2k_B T} = \frac{p_0}{2k_B T_0}\left[\alpha + (1-\alpha)\tilde{\psi}\right]$$







q_{min} = 0.8023

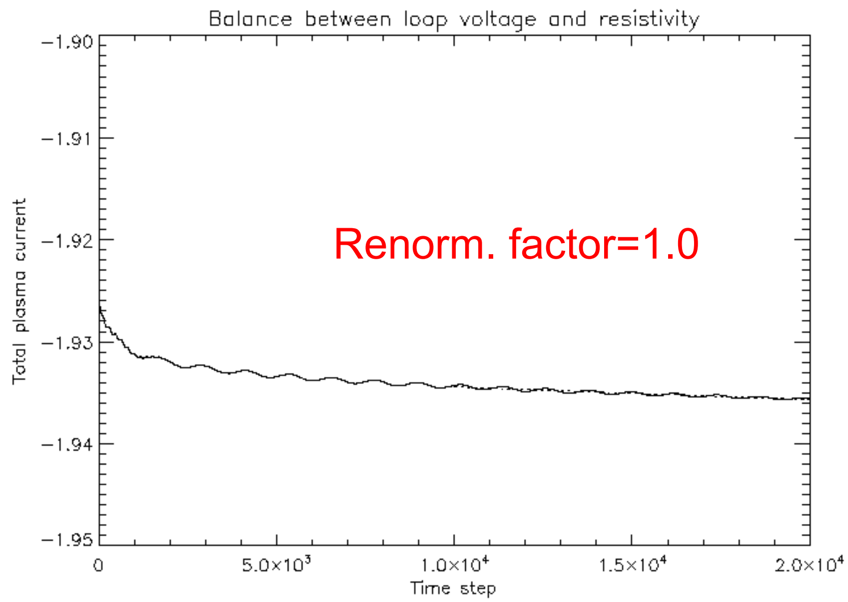
Minimum value: 9.21×10^{-6}

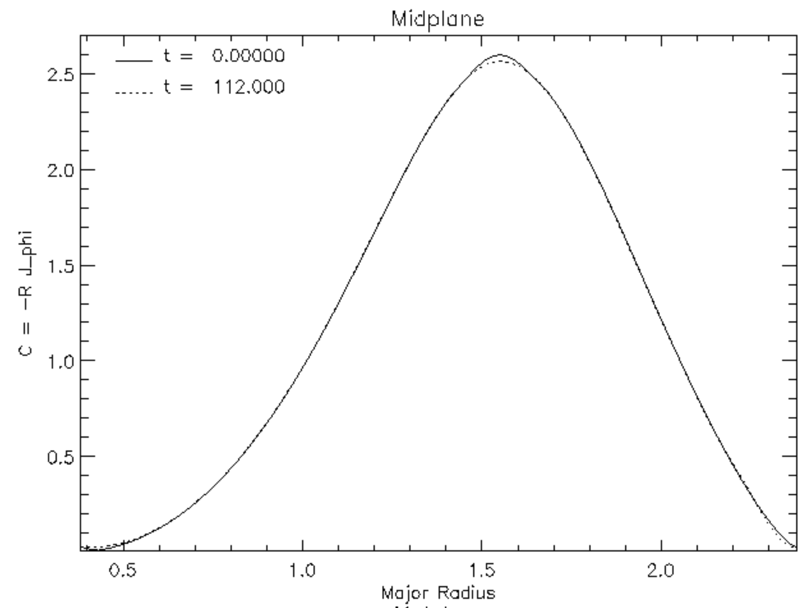
Old case: pkkk $\equiv 9.09 \times 10^{-4}$

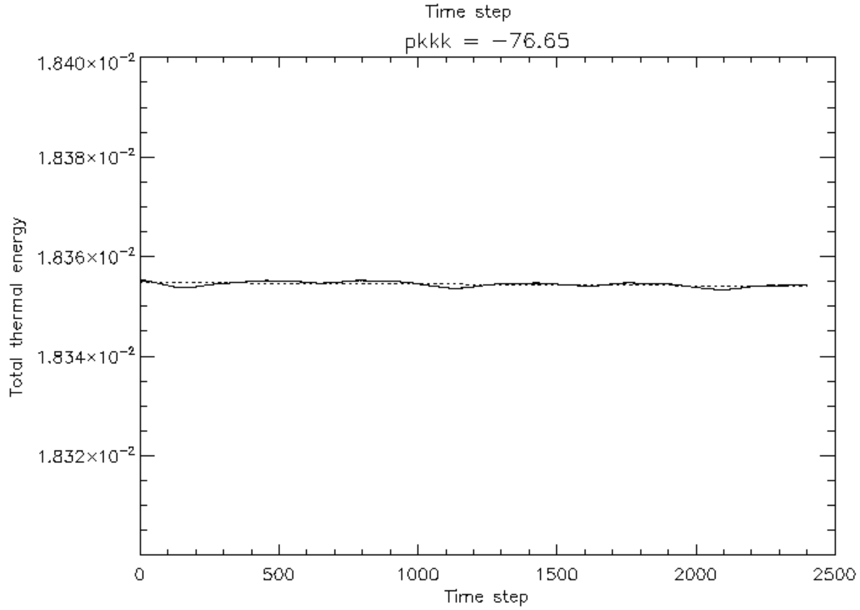
Transport Coefficients

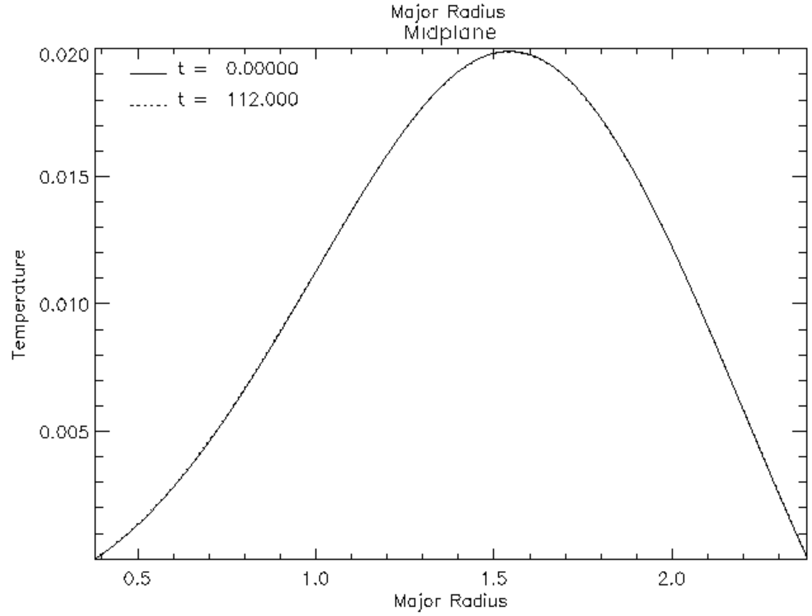
- Evolving Spitzer resistivity $\eta(\boldsymbol{x},t) \propto T^{-3/2}$ with cutoff 100x initial central value; initial central $S = 1.94 \times 10^4$.
- Constant Prandtl number 10 (evolving axisymmetric viscosity).
- Perpendicular heat diffusivity $\kappa_\perp$ read from self-consistent steady state computed with equilibrium code; central value renormalized to about 2.03 m^2/s to maintain steady-state.
- Parallel heat conduction as in previous case ($v_{Te} = 6\ v_A$).

Conservation properties



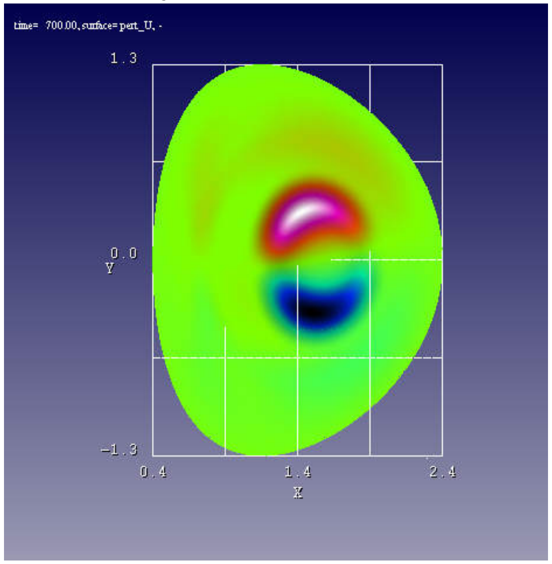




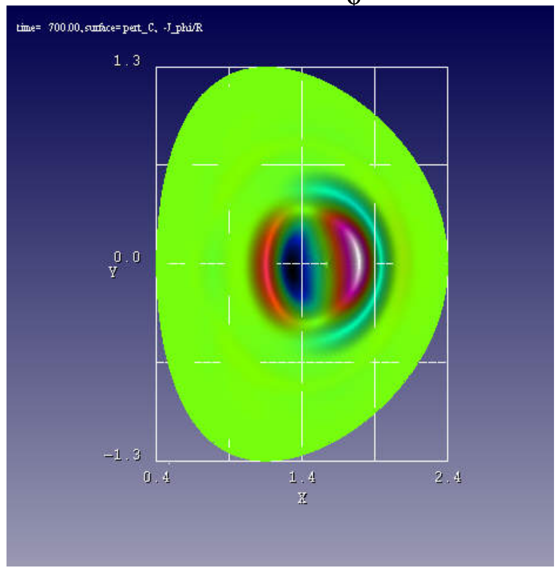


n=1 eigenmode

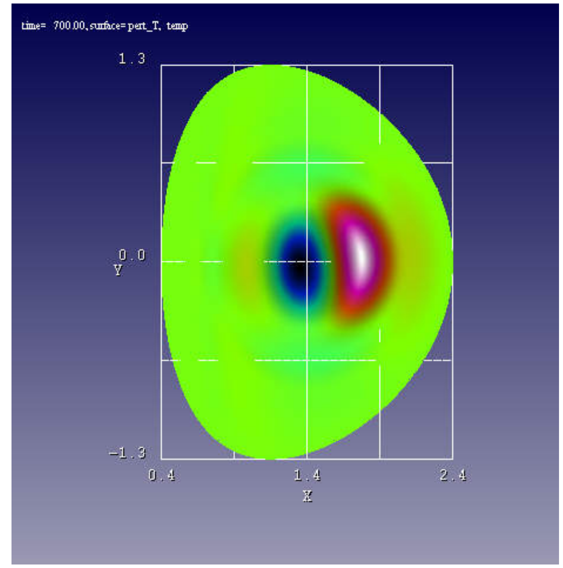
Velocity stream function U



C = $-RJ_{\phi}$



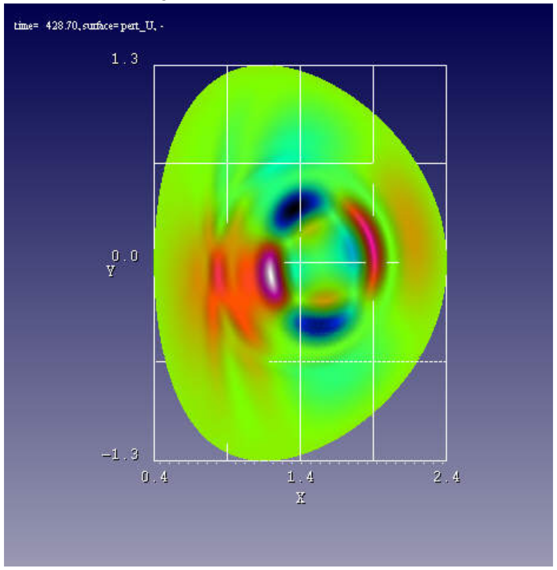
Temperature



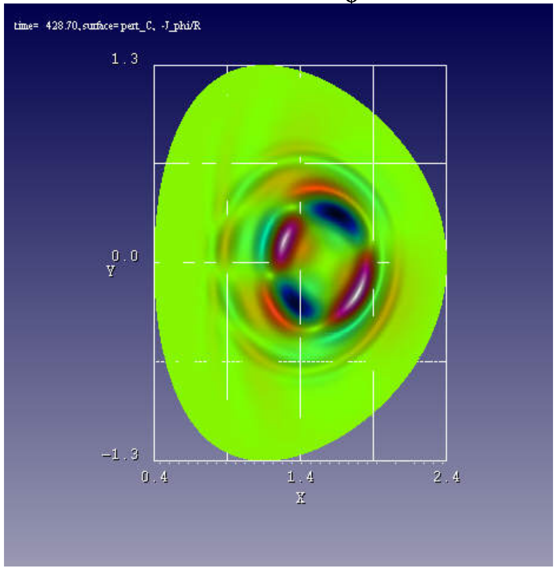
1,1 mode; $\gamma\tau_A \approx (1.415 \pm 0.0005) \times 10^{-2}$

n=2 eigenmode

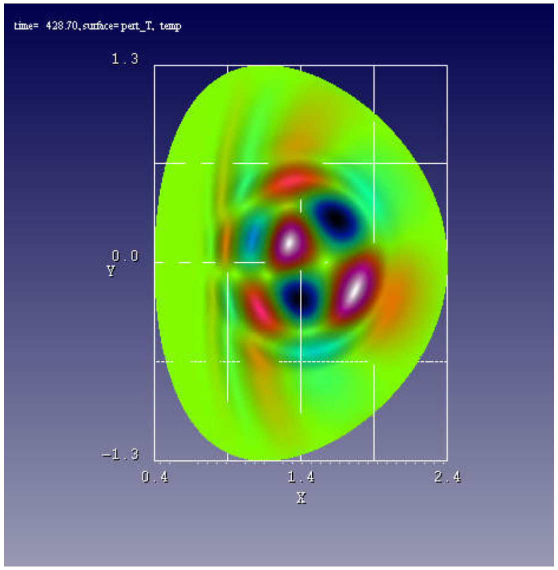
Velocity stream function U



C = $-RJ_{\phi}$



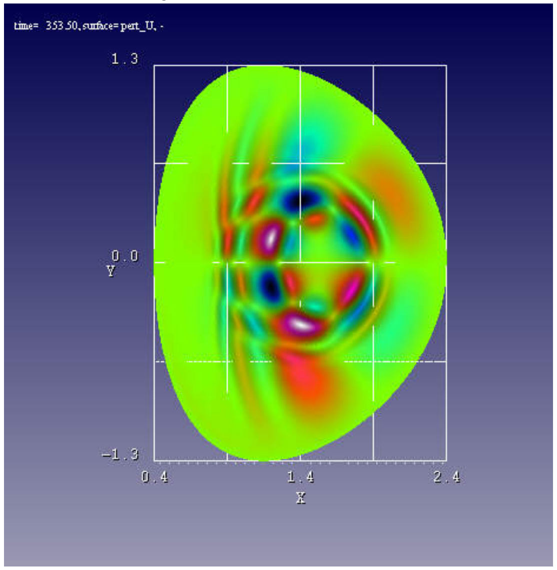
Temperature



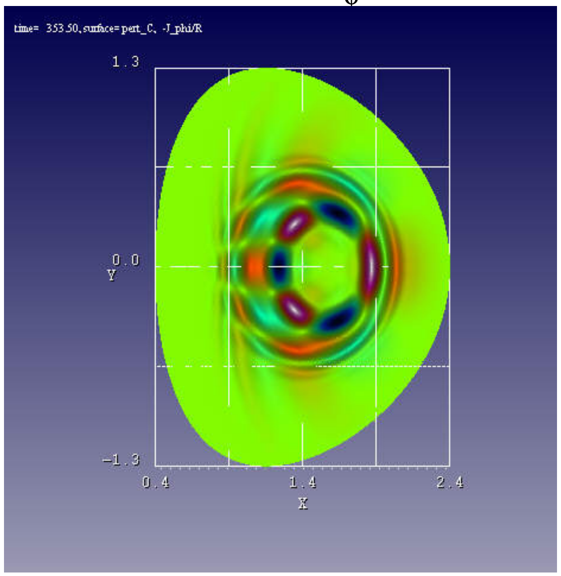
2,2 mode; $\gamma\tau_A \stackrel{?}{\approx} (3.9 \pm 0.5) \times 10^{-4}$

n=3 eigenmode

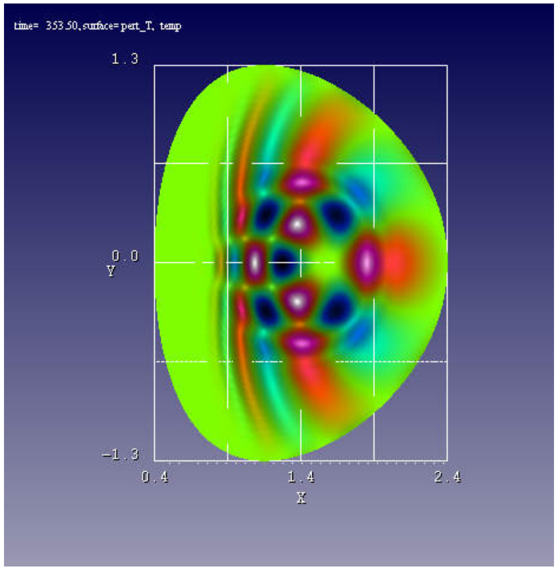
Velocity stream function U



C = -RJ$_\phi$



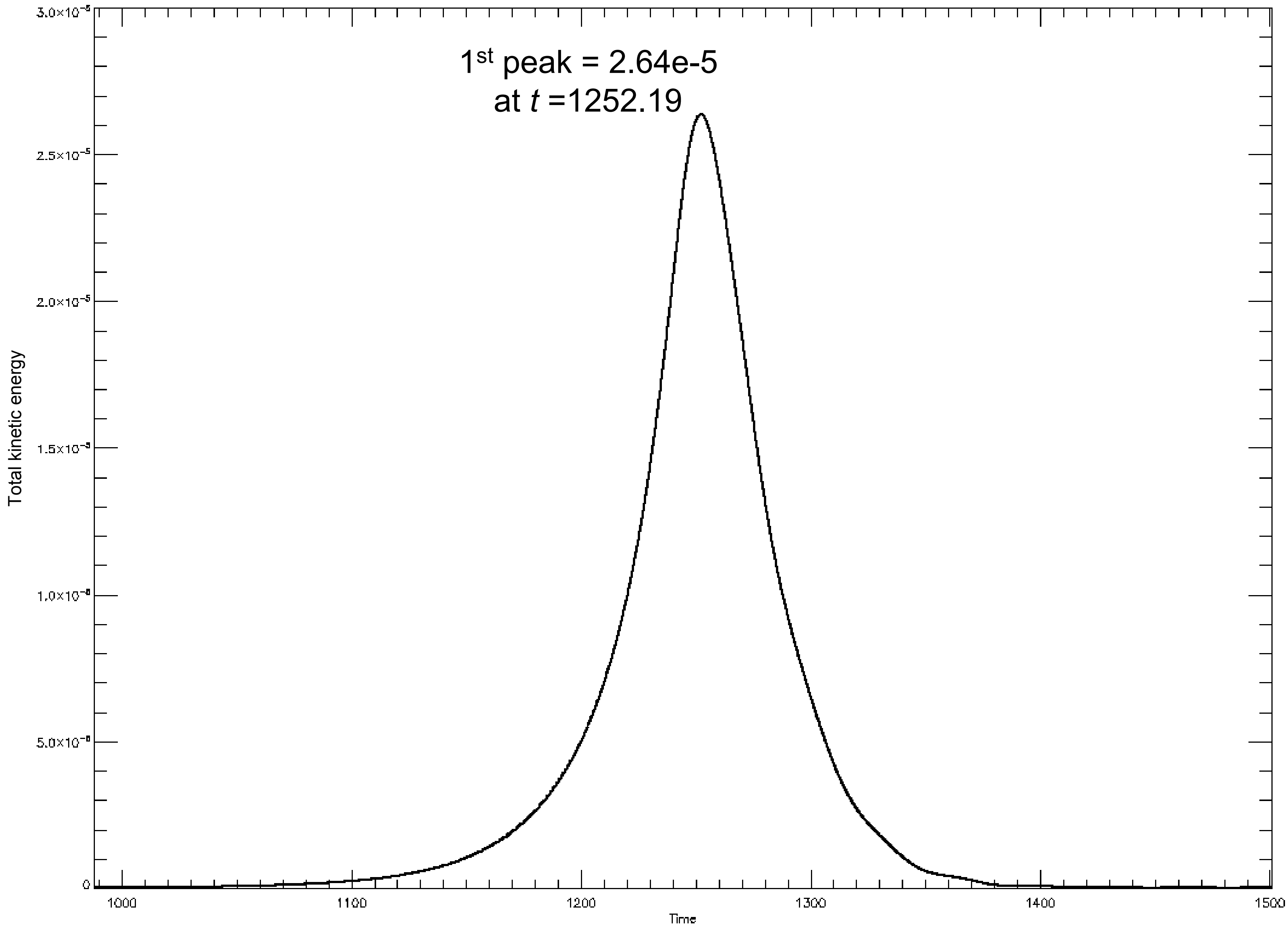
Temperature



3,3 mode; stable

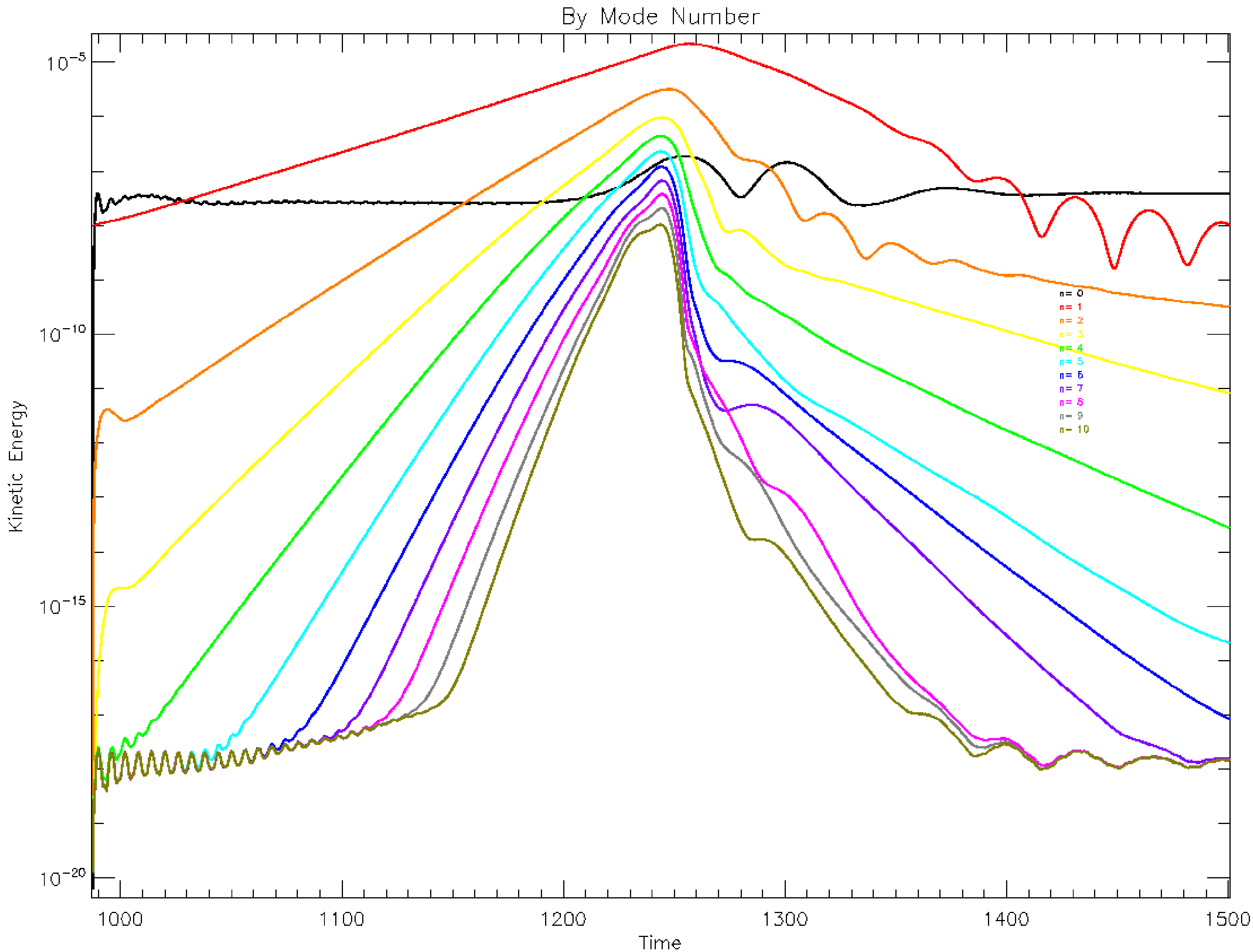
Nonlinear results

24 planes, 81 radial zones, sym 5 on 144 Franklin cores

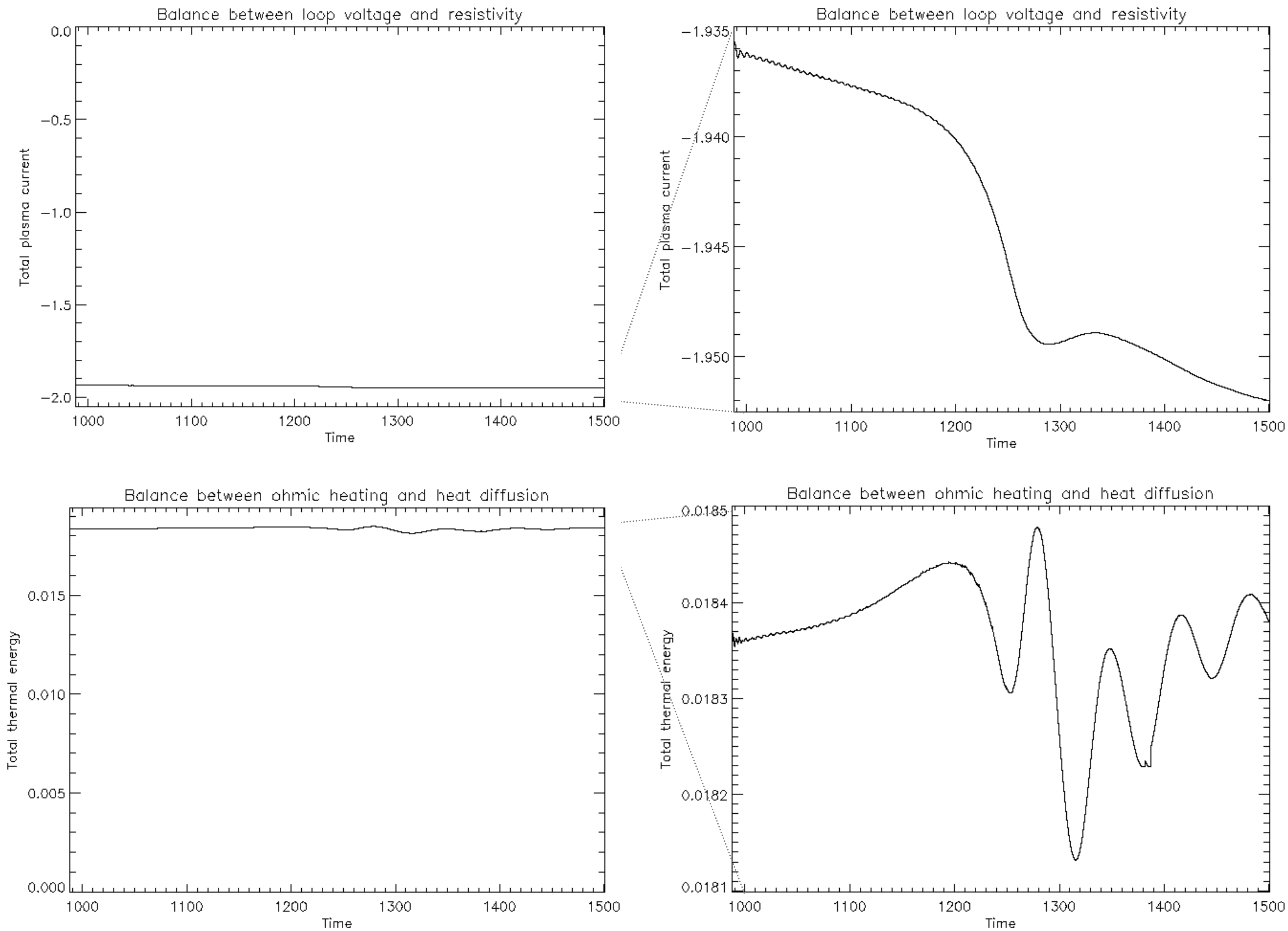


Nonlinear Mode History

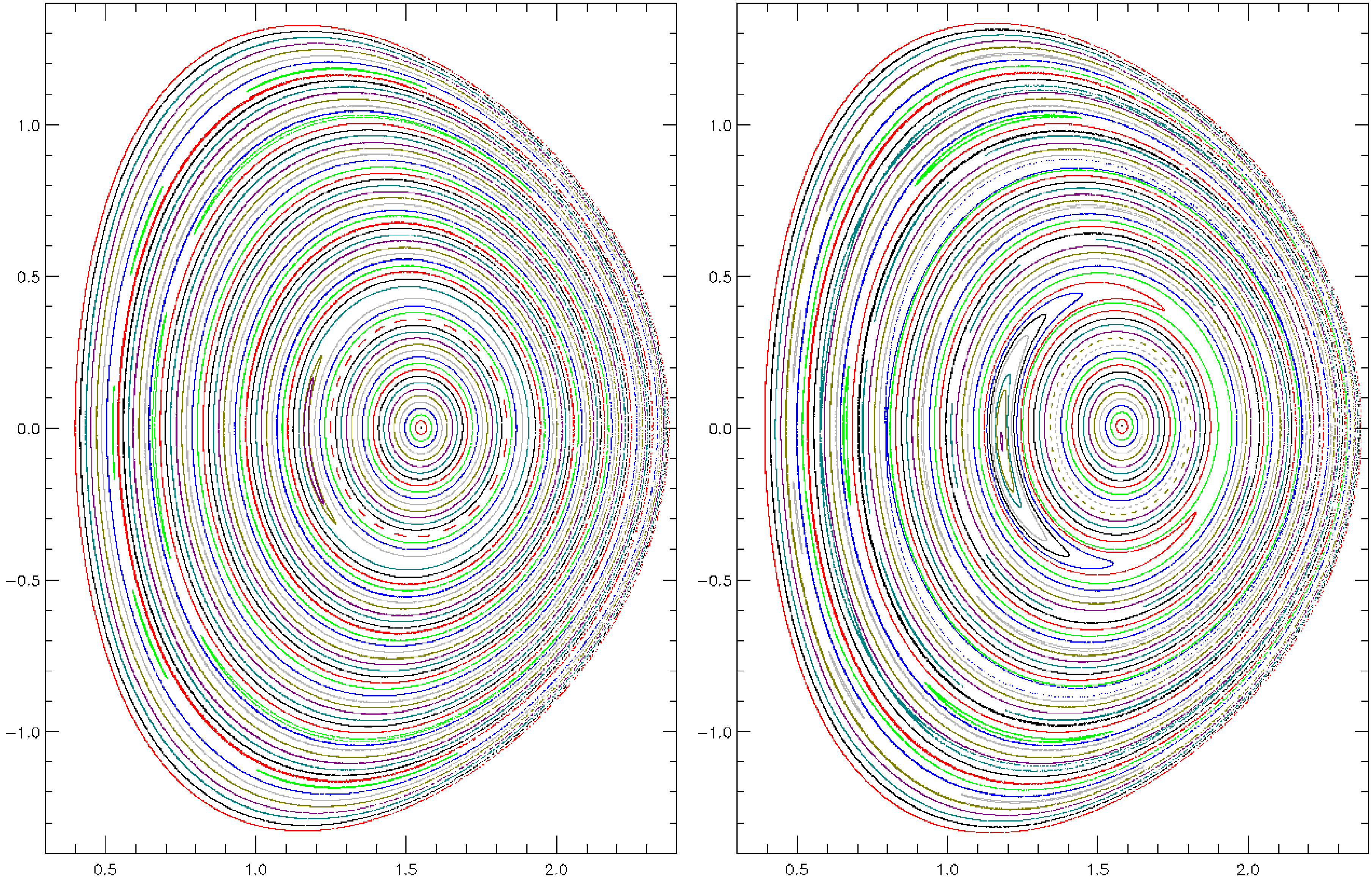
24 planes, 81 radial zones, sym 5 on 144 Franklin cores



Nonlinear Conservation



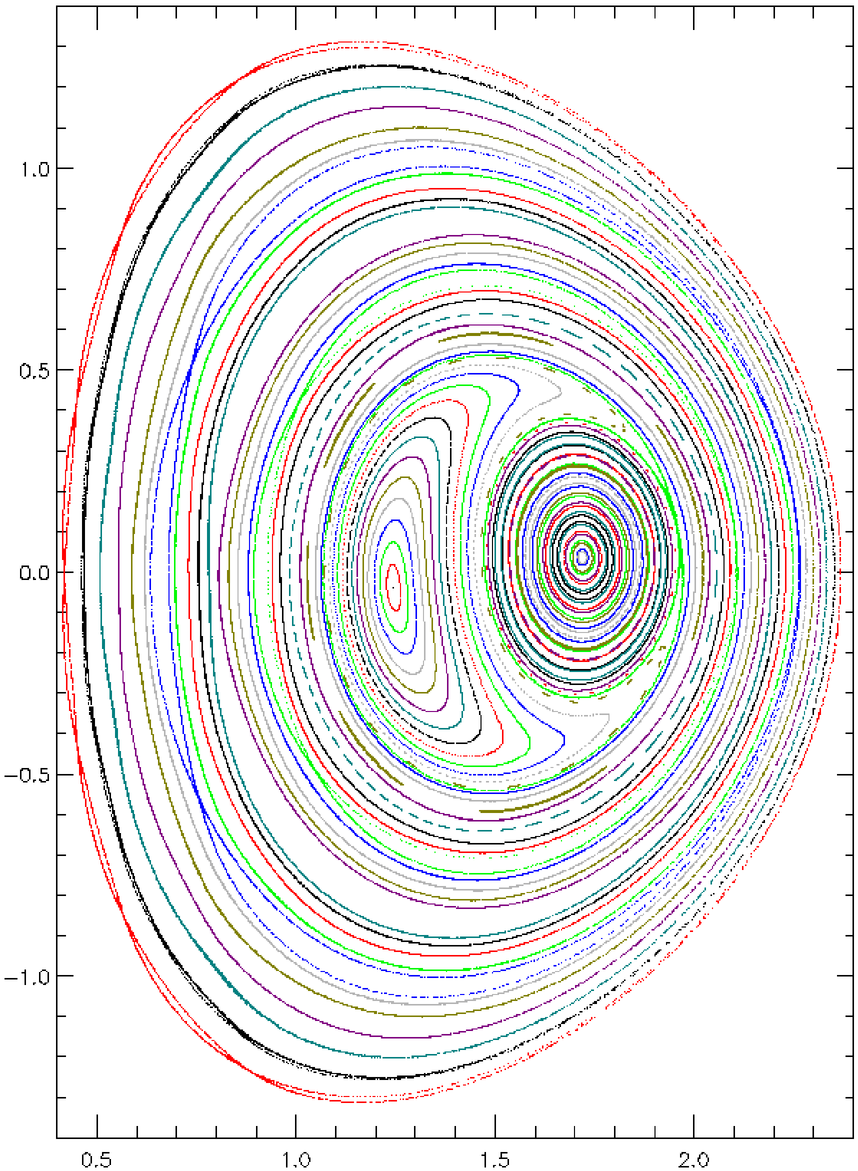
Poincaré Plots



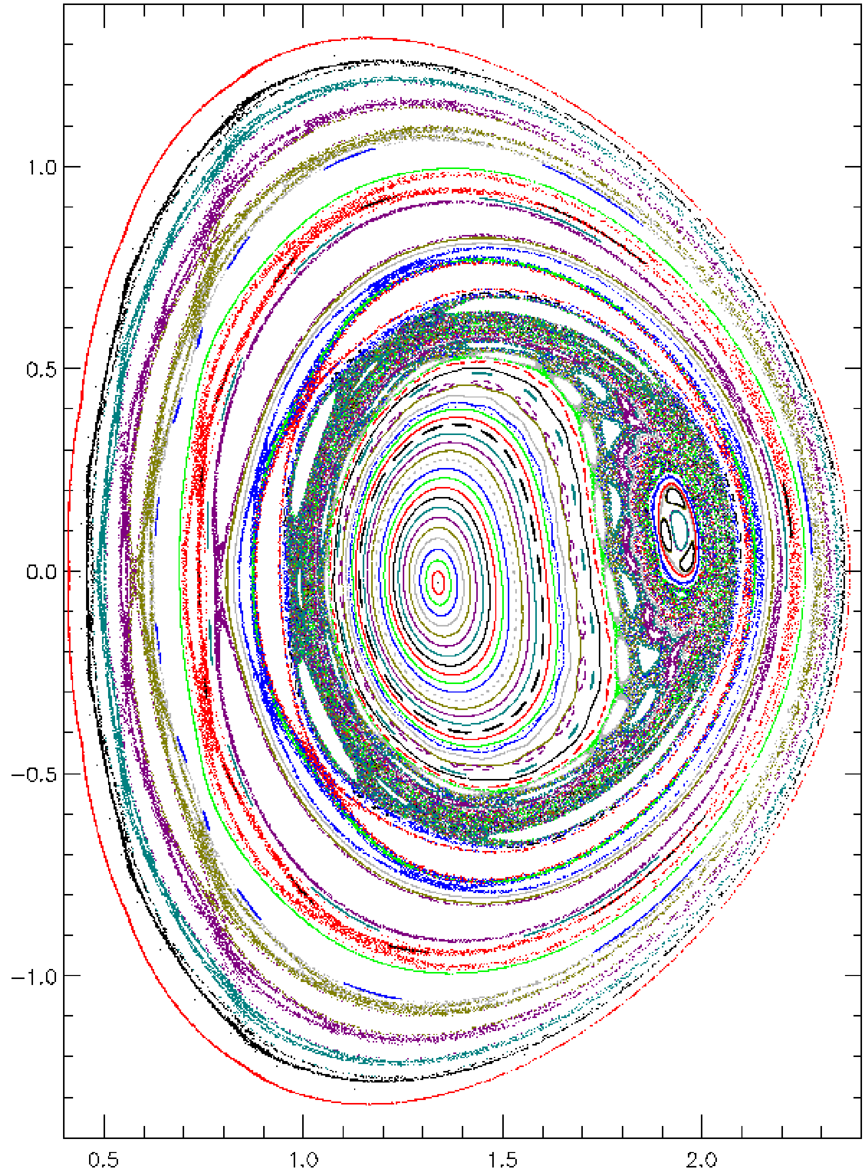
t = 987.80; q_{min} = 0.8022

t = 1109.55; q_{min} = 0.7965

Poincaré Plots

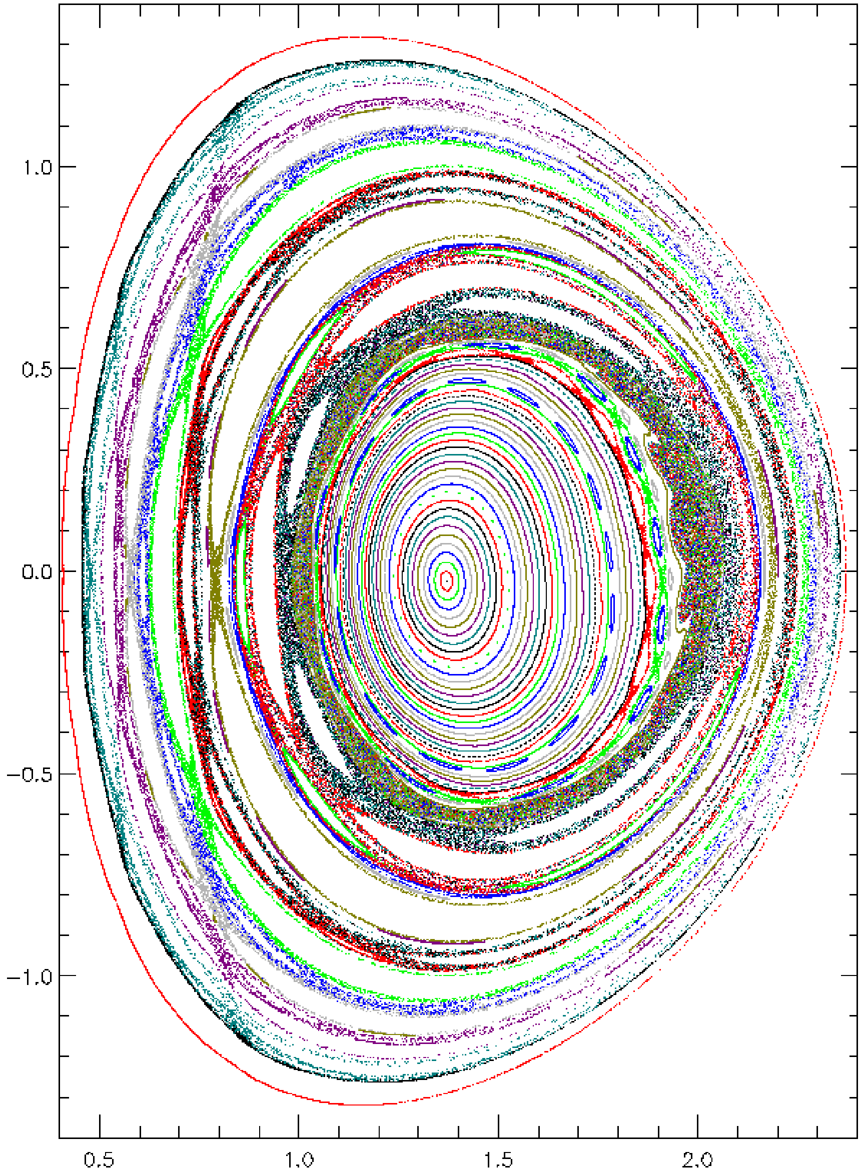


t = 1207.05; q_{min} = 0.7955

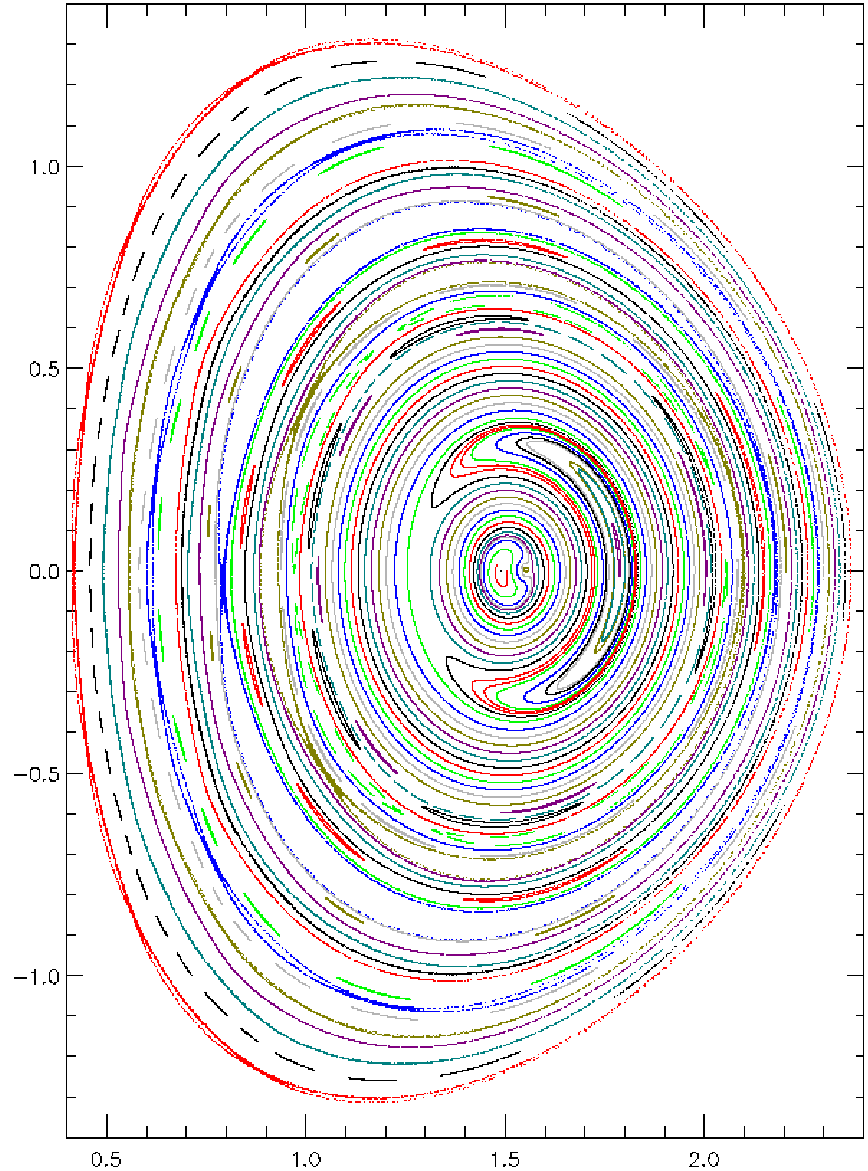


t = 1244.55

Poincaré Plots

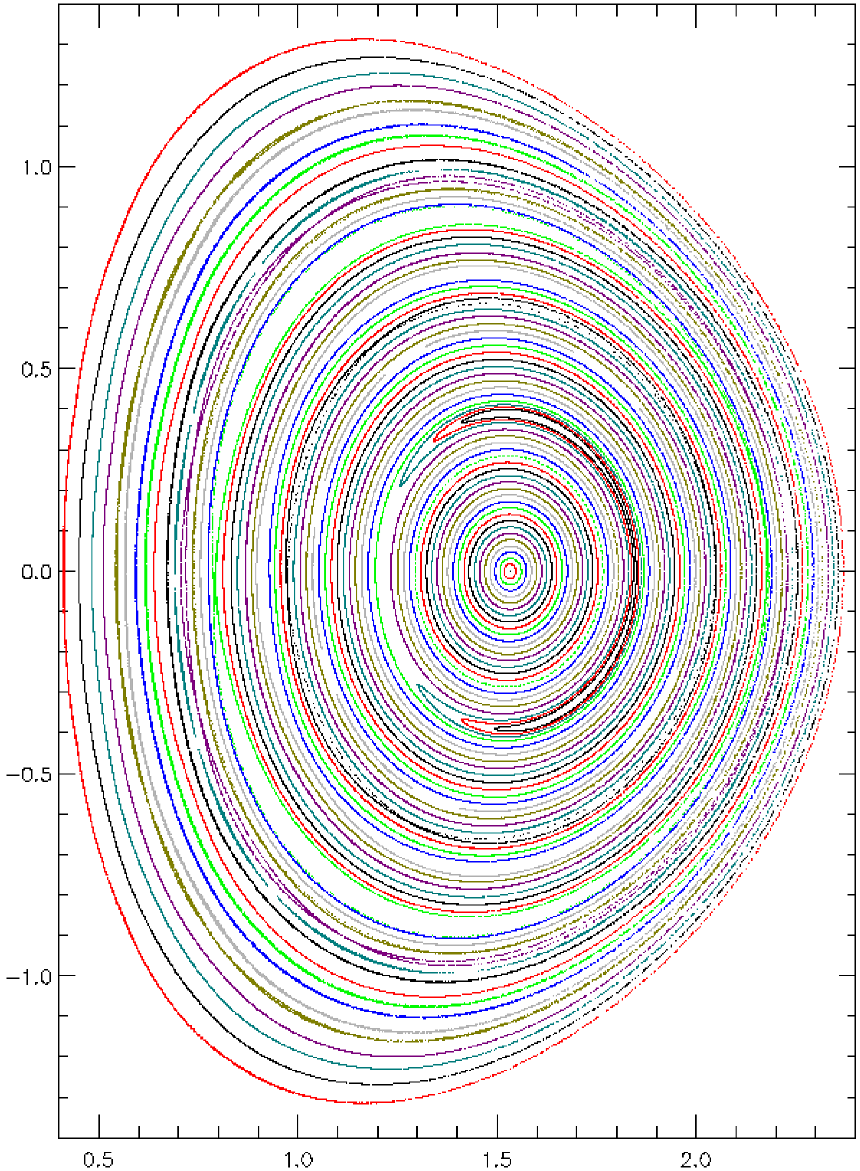


t = 1252.05; q_{min} = 1.0329

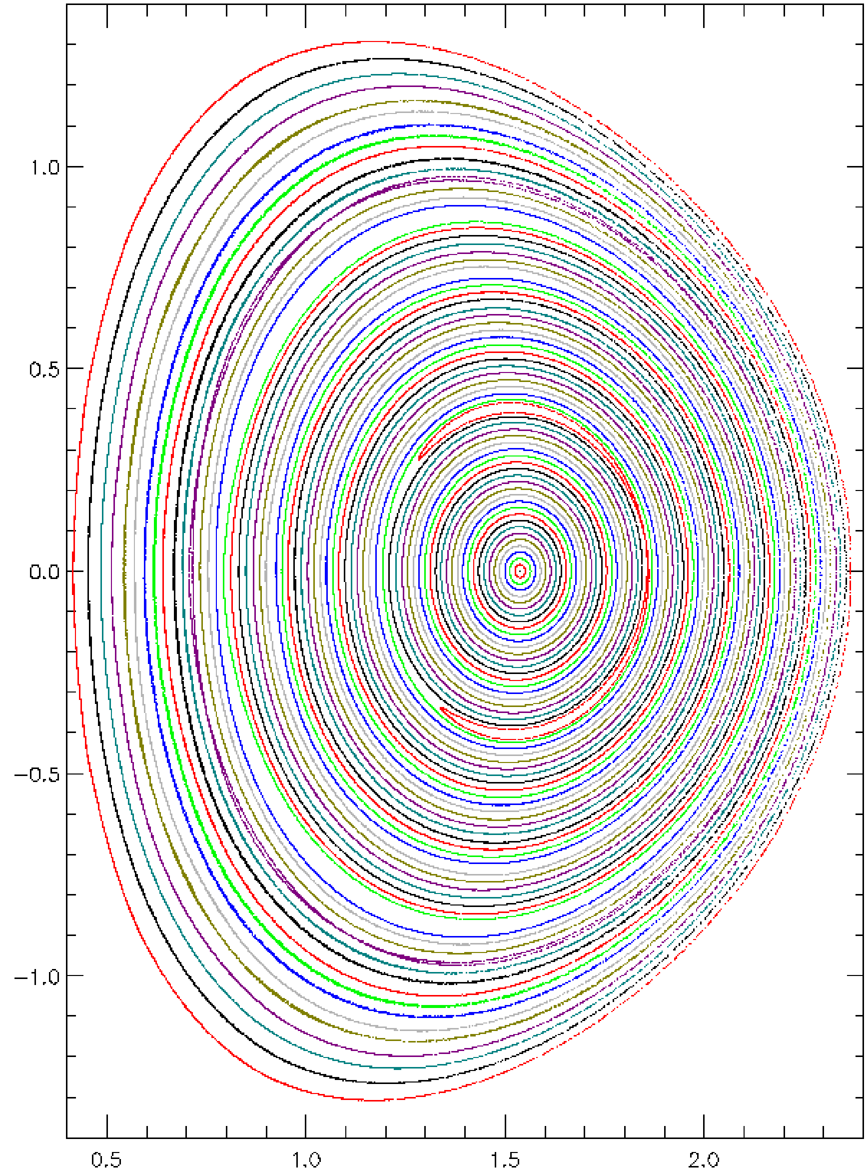


t = 1357.05; q_{min} = 0.9953

Poincaré Plots

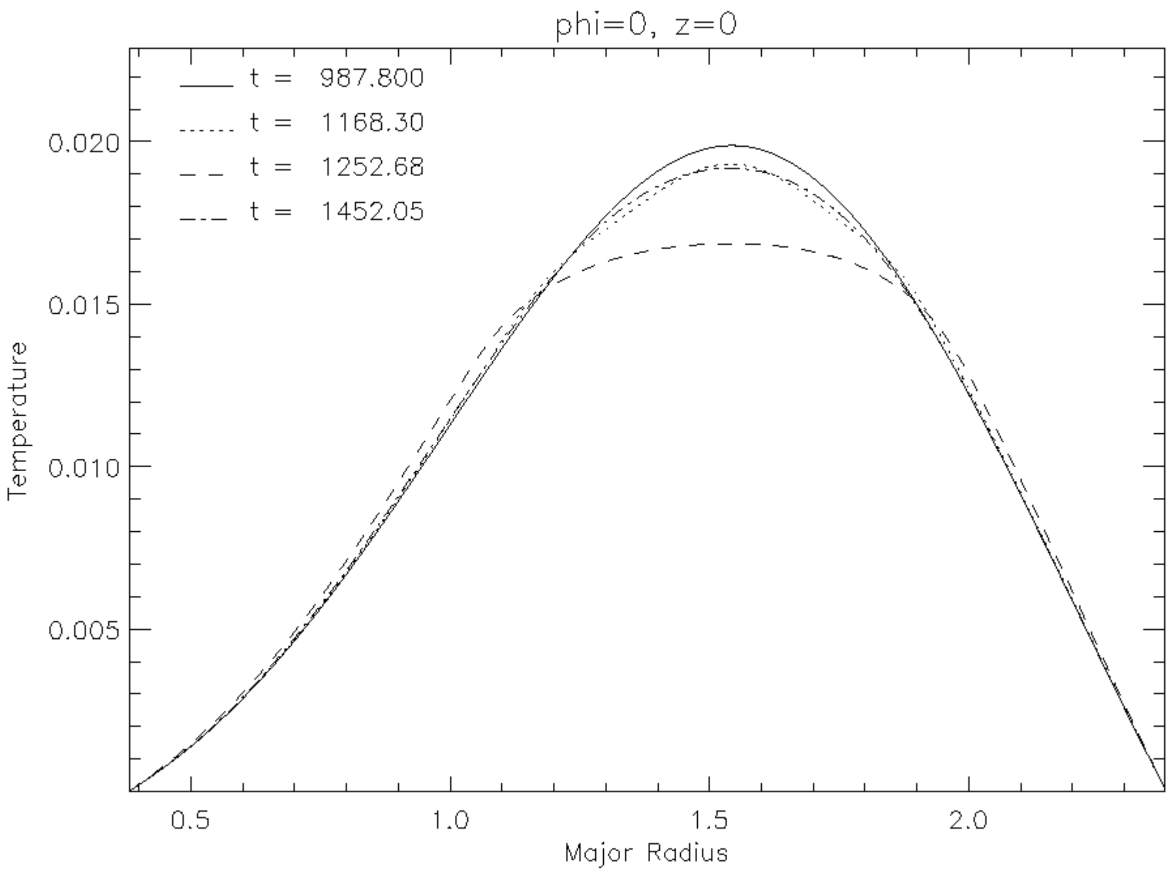


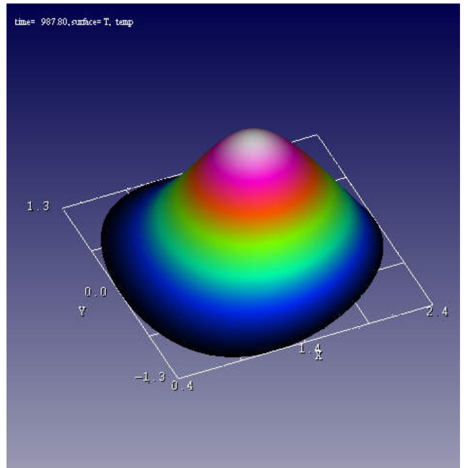
t = 1452.05; q_{min} = 0.9518



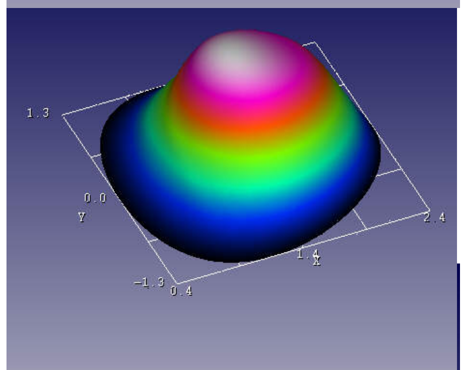
t = 1531.43; q_{min} = 0.9211

Temperature History

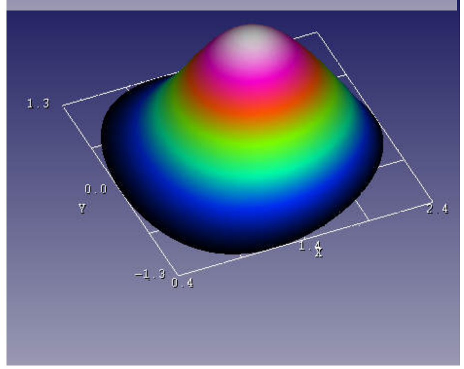




t = 987.8



t = 1252.68



t = 1452.05