An Investigation of the Effect of Coaxial Cables on the PFRC's RF System

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***Abstract*—The radio frequency heating system is an essential part of the Princeton Field Reversed Configuration. However, in the current configuration of the PFRC, the RF system run at roughly 30% efficiency. We aim to make this value significantly higher in future designs of the PFRC. Thus, this paper tackles the problem of understanding the effect that transmission lines have on the system's efficiency. In the end, the relationship between transmission lines and power efficiency is described through numerical simulations and mathematical justifications.**

I. Introduction

The Princeton Field Reversed Configuration (PFRC) is a compact fusion power reactor. As opposed to the mainstream large power plant design found in the fusion space, the PFRC hopes to configure a small, clean reactor that generates 1-10 MW. This reduced size allows the PFRC to be portable, which makes it useful in many unique and important applications, such as natural disaster relief and spacecraft propulsion.

A crucial part of the PFRC design is the RF system because it generates a rotating magnetic field (RMF). This RMF is what forms and heats the FRC. However, there currently exists two key shortcomings of the RF system. Firstly, it has been proven in previous literature that maximum heating occurs when the RMF frequency and the ion cyclotron frequency are comparable. Thus, since the ion cyclotron frequency is roughly 2 MHz, we hope to reduce the RMF frequency to that value. Secondly, in the current state of the PFRC, roughly 75% of the power is lost is the components of the circuit. Therefore, we wish to reduce this number as much as possible to have more power deposited in the plasma.

In this paper, there is a brief discussion on reducing this RMF frequency to 2MHz; however, the majority of it will focus on understanding how the coaxial transmission lines present in the RF system affect the power efficiency of the system.

II. PSpice Model

In order to analyze the PFRC's RF system, PSpice (a circuit analysis software) was used. Fig. 1 illustrates the schematic drawn in the software. It should be noted that this circuit only represents one half of the entire system. That is, the system starts off with a source outputting a sinusoidal signal that runs through a series of amplifiers. Then, the signal is divided through a hybrid splitter into two signals with equal magnitude and a phase difference of 90 degrees. The two identical halves of the circuit in which each of the signals propagates through are denoted as North/South (N/S) and Top/Bottom (T/B). The PSpice model depicts only one of these halves.

After the hybrid splitter, each signal travels through one more amplifier. This is denoted as V1 in the schematic. It should be noted that the amplifier has an internal impedance of 50 Ω this is symbolized by R1. Then, the signal is sent to a match box. This consists of two variable vacuum capacitors. One is connected in series with the input amplifier and has a tunable capacitance range of 10 pF to 650 pF (Cs), and one is connected in parallel with the input amplifier and has a tunable capacitance range of 1000 pF to 2000 pF (Cp). The signal is then split into two parallel arms. Each arm contains an antenna coil, which is used to generate the RMF in the FRC (L1 and L2 in the schematic) along with its associated parasitic resistance (R4 and R5). Furthermore, in the simulation, additional resistors R2 and R3 were added to symbolize the resistance of the plasma. Additionally, a fixed capacitor tank of 6 nF is connected in parallel with the input amplifier. Lastly, transmission lines are found in the system to connect between the different components of the circuit. To connect between the amplifier and the match box (T1) and to connect between the match box and the antennas (T2 and T3) RG-218 cables are used, and to connect between the match box and the 6 nF capacitor tank (T4) an RG-220 cable is used.

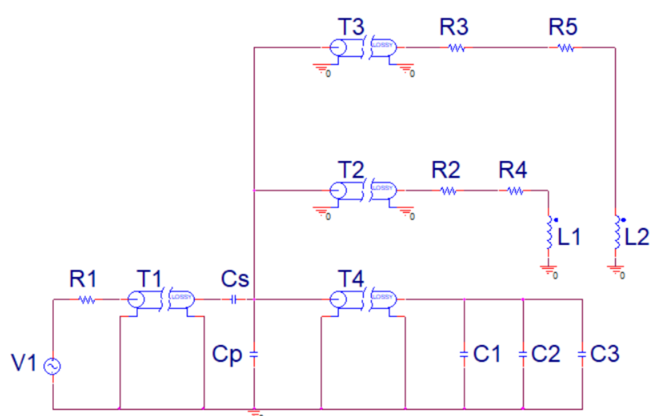


Fig. 1. PSpice schematic of the PFRC's RF system

III. Resonant Frequency of the RF System

The resonant frequency of a circuit is defined as the frequency at which the impedance of the system is the lowest, or equivalently, the frequency at which the amplitude of the current is maximum. This frequency is of great interest when studying the PFRC's RF system because maximum current implies maximum energy stored in the RF coils, since $E \propto I^2$ for an inductor.

As stated earlier, it has been shown in previous works that maximum heating occurs when the resonance frequency of the RF system is similar to the ion cyclotron frequency. Currently, the ion cyclotron frequency is 2 MHz, and previous simulations and diagnostics have shown that the current RMF frequency is around 8 MHz. In order to reduce the RMF frequency to 2 MHz an extra 6 nF capacitor tank was added in parallel to the system.

To understand why this helps achieve the goal, it is useful to make some simplifying assumptions of the model. If the transmission lines affect are neglected and they are considered as ideal wires, then the system reduces down to an RLC circuit. This means that the resonance frequency of the system can be approximated by that of an RLC circuit, which is known to be proportional to $\frac{1}{\sqrt{C}}$, where C is the parallel capacitance. Thus, when 6 nF of capacitance is added to the variable capacitor already present in the match box, the resonance frequency lowers to the ideal value.

IV. Methodology of Analyzing Power Efficiency in PSpice

In order for the plasma heating in the PFRC to be optimized, the power coupled to the plasma needs to be very high. Currently, the system runs at roughly 25% efficiency. However, once fully developed, the goal is for the PFRC to operate at over 90% efficiency. A large contributor of unwanted power consumption are the transmission lines. Transmission lines are cables that transfer electromagnetic waves from one place to another. They are very important to consider in this application, and in RF application in general, because of the high frequency of operation. Unlike a wire connection, the signal varies along a transmission line. To model a lossy transmission line in PSpice, an RLGC model is used. R defines the ohmic losses due to the metal conductor of the transmission line. L represents the inductance of the transmission line. G denotes the losses due to the dielectric. C marks the conductance of the transmission line. The model consists of a lumped element (shown in Fig. 2), which corresponds to a unit length, being repeated to form a transmission line. Note that the prime denotes values per unit length.

In the PFRC's RF system, coaxial cables (a prototypical cable) are used. Fig. 3 depicts the specifications of a general coaxial cable. The variables a and b are the radii of the inner conductor and outer conductor respectively, ρ_{ic} and ρ_{oc} denote resistivity of the inner conductor and outer conductor respectively, ρ_s denotes the resistivity of the dielectric, and ϵ_r denotes the relative permittivity of the dielectric. Due to its simplistic design, R, L, G, and C of a coaxial cable can be calculated analytically using Maxwell's equations. The result is that

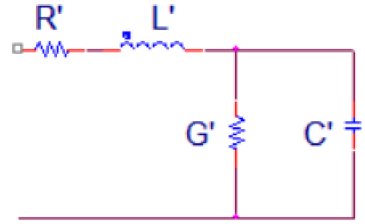


Fig. 2. Lumped-element in the Transmission Line Model

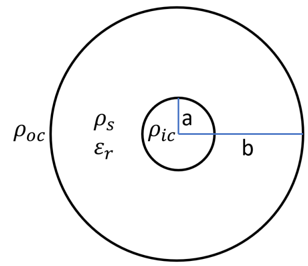


Fig. 3. Lumped-element in the Transmission Line Model

$$R' = R'_{ic} + R'_{oc} \approx \frac{\rho_{ic}}{2\pi a \delta_{ic}} + \frac{\rho_{oc}}{2\pi a \delta_{oc}} \tag{1}$$

$$L' = \frac{\mu}{2\pi} \ln \frac{b}{a} \tag{2}$$

$$G' = \frac{2\pi}{\rho_s \ln \frac{b}{a}} \tag{3}$$

$$C' = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \frac{b}{a}} \tag{4}$$

In the equation for resistance per unit length, δ_{ic} and δ_{oc} signify the skin depths of the conductors. The skin depth is defined as the depth in a conductor at which the current density is $\frac{1}{e}$ of the current density on the surface of the conductor. From this value, the area through which current flows can be approximated as a ring.

In the PFRC, the cables connecting the amplifier to the match box and the two cables connecting the match box to the antenna are RG-218 cables, and the cable connecting the match box to the 6 nF capacitor tank is an RG-220 cable. Table 1 presents the relevant parameters that were inputted into PSpice for each cable.

TABLE I
RLGC values of transmission line cables used in the PFRC's RF system

Transmission Lines	Transmission Line Parameters			
	R' (mΩ/ft)	*L' (nH/ft)*	*G' (p℧/ft)*	*C' (pF/ft)*
RG-218	12.84	76.2	0.153	31.2
RG-220	9.60	76.4	0.153	31.1

To ensure that a maximum amount power is transferred from the input amplifier to the system, the reflected signal should be minimized. Furthermore, reflected signals can cause damage to the amplifier, and it can cause voltage in the cables to be higher than expected. This is achieved when the source impedance is equal to the load impedance. In terms of the PFRC, this is achieved when the load impedance is equal to 50 + 0jΩ, as the input amplifier has a 50Ω internal resistance associated with it. The variable vacuum capacitors are used to achieve this goal. More specifically, using PSpice, series and parallel capacitance values (Cs and Cp) are determined which satisfy the impedance matching criteria.

In the following sections, after finding the appropriate Cs and Cp combination for impedance matching, the effect of transmission line lengths on power efficiency is investigated, assuming 100kW of power is delivered to the system.

V. Power Efficiency of Varying Transmission Line Lengths

As previously stated, observing the effect of various transmission line lengths on the RF system in very important because transmission lines are a large contributor of power loss in the system. When using PSpice to investigate variations in transmission line T1, T2 and T3 were fixed to a length of 6ft, T4 was fixed to 3ft, and the effective resistance of the plasma was fixed to 100mΩ. T1 was varied from 5ft to 30ft. In order to investigate the effectiveness of heating, the power dissipation in the plasma and the magnetic field generated are noted. To approximately calculate the magnetic field, the formula

$$B = \frac{2\pi * I}{5 * (l + w)} \tag{5}$$

is used where I is the root mean square current in the antenna, l in the length of an RMF coil, and w is the width of the RMF coil.

As can be seen in Fig. 4, the length of T1 is positively linearly related to the power lost in it. This linearity can be explained by the fact that for short transmission lines, the difference in current between the receiving and sending ends are negligibly different at varying lengths. Therefore, the resistance of the cable is effectively the only variable when looking at power dissipation, and it is directly proportional to the length. It should be noted that the power lost in T1 is very small regardless of the length of the transmission line ($< 1\%$). Thus, it appears to not be a major point of concern when looking to reconfigure the PFRC.

The magnitude of the RMF field and the length of T1 exhibit a negative linear relationship. This can be explained by the fact that the power dissipating in T1 in linearly increasing, as shown by Fig. 4; therefore, the current in the antenna is linearly decreasing, which in turn, implies the magnetic field generated is linearly decreasing by equation 5. Once again, it should be pointed out that the variance in the magnitude of the generated RMF in quite small across the different lengths of T1. Thus, once again we see that the length of T1 is not significant.

Next, the effects of varying T2 and T3 symmetrically in length are observed, while holding T1 and T4 constant. To do this, T1 is set to 20ft, T4 is set to 3ft, the plasma resistance is set to 100mΩ, and T2 and T3 are varied from 5.5ft to 7 ft. It should be noted that T2 and T3 are manipulated together because they are identical cables, configured in an identical manner in the RF circuit. Thus, there effect on the system will be the same. We can observe from Fig. 6 and Fig. 7 that the trends found by varying T2 and T3 are very similar to that of varying T1. More specifically, when looking at the power dissipation we see a positive, linear relationship between the sum of the power dissipated in T2 and T3 and their lengths. Moreover, when looking at the magnetic field generated versus the length of T2 and T3, we see a negative linear relationship. This phenomenon occurs because of the same points discussed for T1. However, unlike T1, when varying T2 and T3 the power dissipation in the cables and the variance in the magnetic field are significant values. Thus, shrinking the length of T2 and T3 is of major importance.

Lastly, the effects of varying T4 were investigated. The set up was to fix T1 to a length of 20ft, T2 and T3 to 6ft, the plasma resistance was set to 100mΩ, and T4 was varied from 2.5ft to 3.5ft. Once again the same trends were seen in the power dissipation and magnetic field plots (Fig. 8 and Fig. 9 respectively) for T4 as were noted in the other cables.

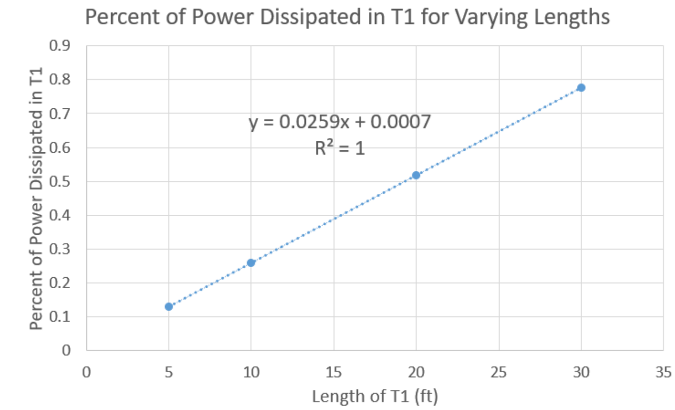
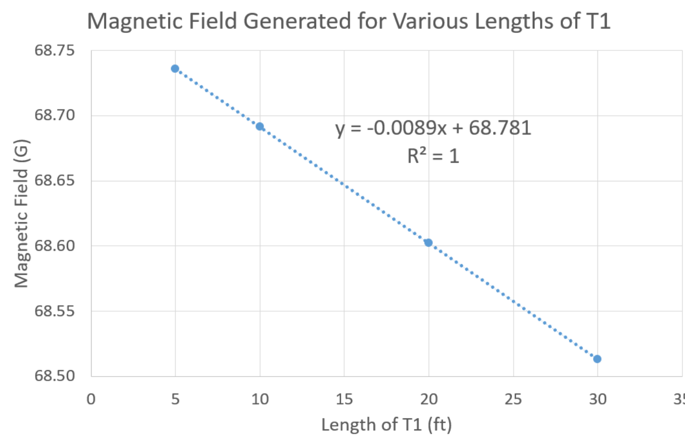


Fig. 4. CHANGE



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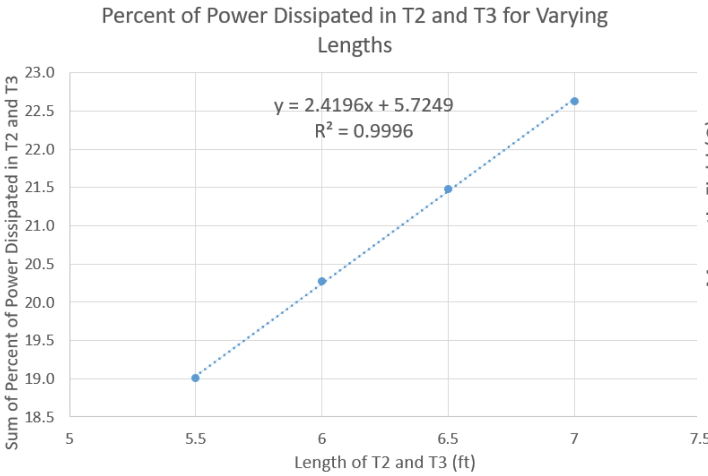

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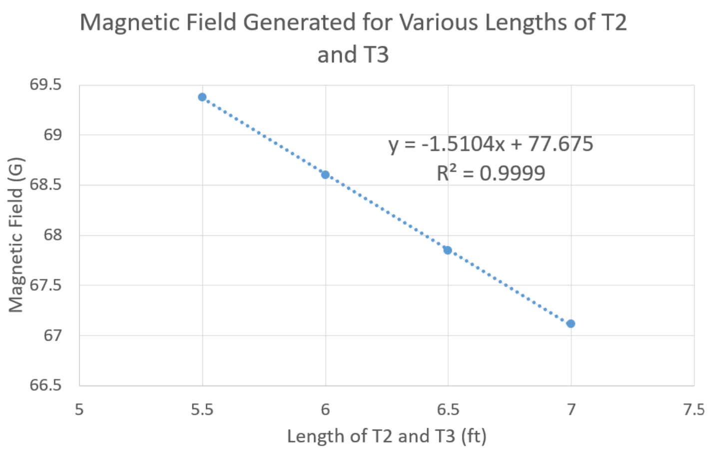
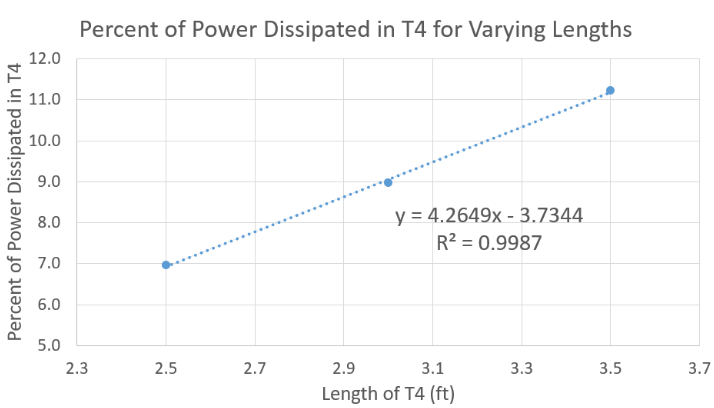


Fig. 7. CHANGE

Furthermore, like T2 and T3, for T4, the magnitude of loss in the cable and the range of the magnetic field are noteworthy. Thus, T4 should as be a focus of improvement for future PFRC configurations.



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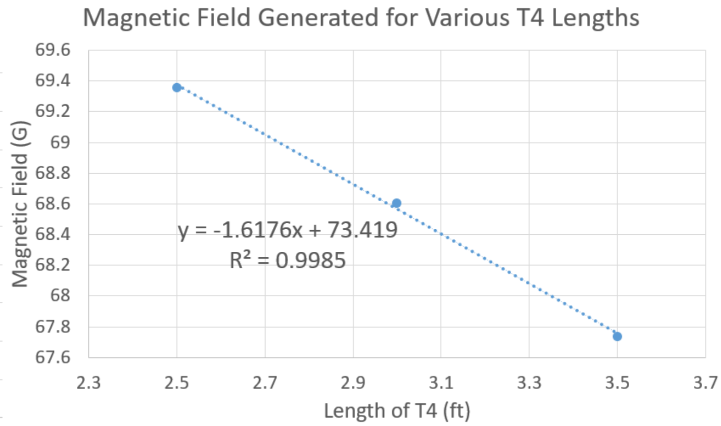

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VI. Power Efficiency for Varying Plasma Resistances

An investigation of the relationship between the power efficiency of the PFRC's RF system and the resistance of the plasma is a point of interest in the current state of the PFRC. To conduct this investigation, three combinations of transmission line lengths were selected and the total power lost in the circuit's transmission lines as well as the power dissipated in the plasma was noted for plasma resistances of 50mΩ, 100mΩ, 150mΩ, and 200mΩ for each combination.

The first combination of transmission line lengths observed consists of T1 being 20ft, T2 and T3 being 6ft, and T3 being 3ft. These numbers were chosen because they are roughly the average lengths of cables that still enable the system to operate at 2MHz and satisfy the impedance matching criteria with the given range of the variable capacitors in the system. The second combination consists of T1 being 20ft, T2 and T3 being 4.667ft, and T4 being 4ft. This combination attempts to minimize T2 and T3 and then select T1 and T4 based on the frequency and impedance matching goals. Lastly, the third combination of lengths consists of T1 being 20ft, T2 and T3 being 8ft, and T4 being 1.5ft. This combination represents the opposite of combination two because it represents a circuit which minimizes T4. Note that a combination that minimizes T1 is not of concern here because as previously shown, T1 has little significance on the power efficiency of the entire system.

Figures 10-12 illustrate the affect of the plasma resistance on the power dissipation in the circuit. We see that as the plasma resistance increases for all three combinations there is an increase in the power dissipated in the plasma and a symmetric decrease in power lost in the transmission lines.To mathematically model the increase of power dissipated in the plasma, the components of the RF circuit need to be combined using the basic series and parallel addition of impedances formula; then, the current at the resistors R2 and R3 can be found; finally, multiplying the current by the impedance of the resistors gives the power. Upon doing this it can be seen that the power supplied to the plasma will be of the form $\frac{aR}{R+b}$, where R is the resistance of R2, or equivalently R3, and a and b are constants. It can be observed that the regression equation for each of the three combinations varies only slightly. Thus, from this we can conclude that as long as a resonant frequency of 2MHz is achieved and the impedance matching criteria is met, the individual lengths of the cables are not significant.

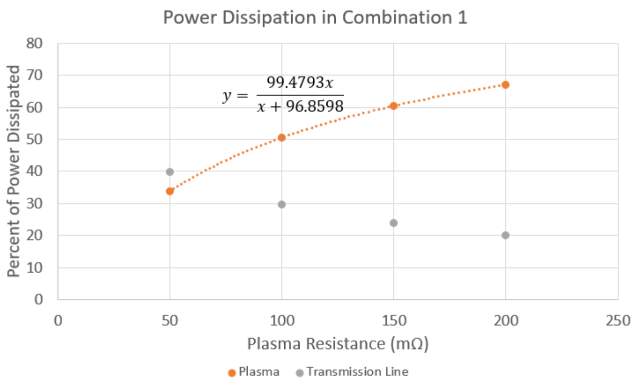


Fig. 10. CHANGE

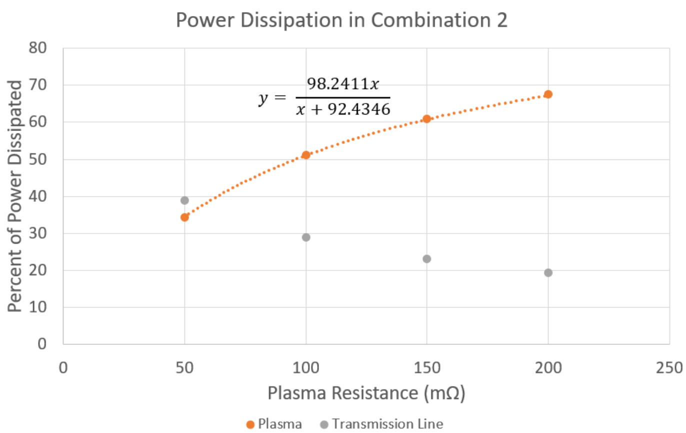


Fig. 11. CHANGE

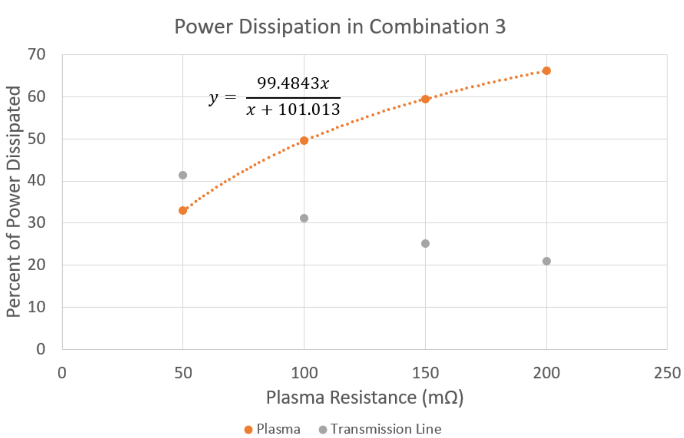


Fig. 12. CHANGE

When observing the magnetic field as a function of the plasma resistance in the RF system, we see that as the plasma resistance increases the magnitude of the magnetic field generated decreases. Intuitively, this makes sense because as the plasma resistance increases the power dissipated in the plasma is increasing. Therefore, there is less current going through the antenna, and as previously discussed, current is proportional to the magnitude of the RMF generated. To quantify this relationship, the formula to relate the power supplied to the plasma as a function of plasma resistance can be used. Since, current is equal to power divided by impedance and current is linearly proportional to the magnitude of the magnetic field, the magnetic field will be of the form $\frac{a}{R+b}$. We once again see that across the different combinations the magnetic field does not significantly vary. Therefore, once again we can see that with regards to power efficiency, the individual cable lengths do not matter. Instead, whether the combination of lengths satisfies the impedance matching and resonant frequency criteria established is what is important.

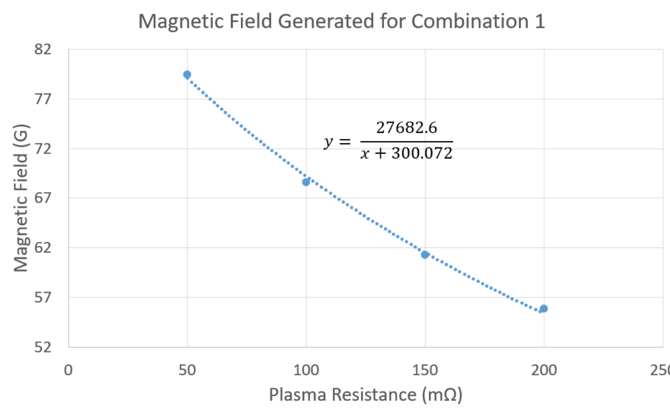
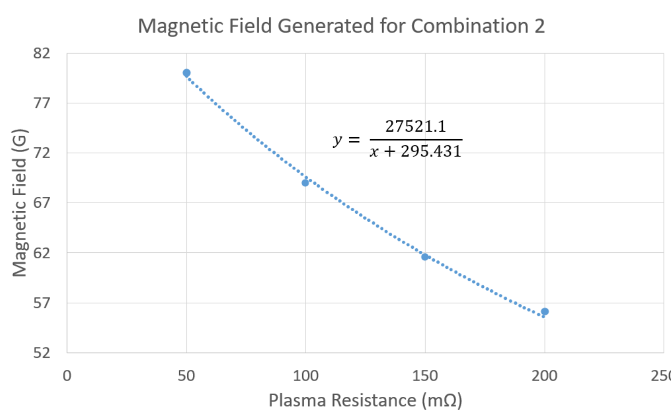


Fig. 13. CHANGE



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VII. Conclusion

In this paper, an overview of the Princeton Field Reverse Configuration's Radio Frequency Heating system was given. Then, a discussion the resonant frequency of the system and the use of parallel capacitors to reduce the resonant frequency

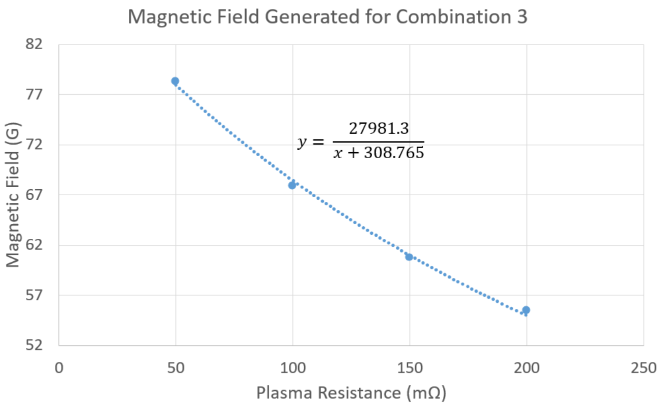

Fig. 15. CHANGE

was presented. Lastly, the effect of length of transmission lines on the power efficiency of the system was analyzed. It was found that the length of an individual transmission line and the power lost in it are linearly, positively correlated. However, the individual lengths are of little importance on if the entire circuit satisfies the impedance matching criteria and has the appropriate resonant frequency. Future research should look into the physical constraints of the components of the system and verifying these results analytically.

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