Design Aid Diagram for FRC Fusion-research Devices:

Research paths defined by dimensionless variables

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I. FRC PARAMETERS AND EQUATIONS

To explore FRC reactor physics properly, experimental devices should be designed to have their pertinent dimensionless parameters approaching those of their target reactor. Among the dimensionless parameters considered most relevant are ν^*, collisionality – to address transport issues – , and s, a measure of the degree of MHD-like *vs.* kinetic-like behavior – to address stability questions. ν^* is defined by the ratio of the machine size, κr_s, to the Spitzer-scattering mean-free-path (mfp), $\lambda_S \equiv \lambda_C/4ln\Lambda$, where λ_C = 90° Coulomb-scattering mfp, κ = FRC elongation, and r_s = separatrix radius. The parameter s is approximated as the number of ion gyro-radii, ρ_i, between the FRC O-point radius, r_o, and r_s: $s \sim 0.3 r_s/\rho_i$. Both s and ν^* are determined by relatively few basic machine parameters, *e.g.*, primarily external magnetic field, B_e, and r_s, with the plasma species temperatures, $T_{e,i}$, remaining the only free parameters. Plasma temperatures would be set in a reactor by the desired fusion power density.

The equations used to find s and ν^* are (in the commonly used mixed units, n (cm^{-3}), r_s (cm), B_e (G), T (eV) and m_i in amu):

$$\langle\beta\rangle = 4.03 \times 10^{-11} \Sigma \langle nT \rangle / B_e^2 \quad (1)$$

$$\phi \sim \pi r_s^2 B_e/4 \quad (2)$$

$$s = 3 \times 10^{-3} r_s B_e/(m_i T_i)^{1/2} \quad (3)$$

$$\nu^* = 2.5 \times 10^{-14} n \kappa r_s / T^2 \quad (4)$$

Equation (1) is used to eliminate density from the other equations. (Note that this creates a T^3 in the equation for ν^*.) Equation (2) assumes reasonable plasma profiles. What remains are equations that contain κ, B_e, r_s, $\langle\beta\rangle$, m_i, and T. In the following we assume singly charged ions, $m_i = 1$, $\kappa = 5$, $\langle\beta\rangle = 0.7$, $T_e = T_i$ and $n_e = n_i$. These choices have little impact on the overall picture, due to the limited ranges these parameters may have and the log-log scales on the diagrams.

When considering a reactor, an important issue is the confinement of fusion products. Ignited operation requires, at a minimum, that the gyro-radii of the fusion products, ρ_{fp}, be considerably small than r_s. For $r_s/\rho_{fp} \geq 6$, the minimum required magnetic flux for confining 3.5 MeV α particles is $\phi_m \geq 2.8 \times 10^5 E/B_e \sim 0.7$ Wb at $B_e = 15$ kG, with E the α-particle energy in eV and

ϕ in $G\ cm^2$. (1 Wb = $10^8\ G\ cm^2$.) Driven reactors, ones that do not rely on direct plasma heating by fusion products, do not have this constraint.

Ignited D-T reactors are estimated have $r_s \sim 2.5$ m, $B_e \sim 15$ kG, and temperature and density near 10 keV and $4 \times 10^{14}\ cm^{-3}$, respectively, yielding $s \sim 30$ and $\nu^* \sim 10^{-2}$. A driven, $D - He_3$ reactor would require about 10× higher temperature but could have a proportionally smaller r_s, $B_e \sim 50$ kG, resulting in $s \sim 8$ and $\nu^* \sim 10^{-5}$. (Note that s for the α particles is still 2, enough to allow appreciable energy deposited in the plasma.) An ignited $p - B_{11}$ reactor, if possible, would have $r_s \sim 2.5$ m and $B_e \sim 150$ kG, with $s \sim 40$ and $\nu^* \sim 10^{-5}$.

The final parameter that describes the FRC reactor system is the energy confinement time, embodied in the diffusivity, D. For classical collisional transport, $D = \rho_i^2 \nu_S = r_s^2/\tau_E$, with ν_S the Spitzer scattering frequency. D can be rewritten in terms of s and ν^*,

$$D = 0.1 \frac{r_s v_t}{\kappa} \frac{\nu^*}{s^2}, \tag{5}$$

where v_t is the ion thermal velocity. The resulting classical D's are 0.6 m^2/s for the ignited, 10-keV, D-T, 2.5-m device and 0.003 m^2/s for the driven, 100-keV, $D - He_3$, 25-cm device. Both the ignited D-T and driven $D - He_3$ FRC reactors described above require a (thermal) diffusivity less than a factor of 10 above classical. Non-classical mechanisms could increase transport and D. At their expected ν^* and s values, ignited reactors face extreme stability issues while driven reactors would face challenging confinement requirements. Equation (5) indicates that confinement and stability issues are not strictly independent.

Other dimensionless parameters should be considered in the design of next-step FRC research devices. For example, particularly important to pulse-length requirements and flux sustainment is the Lundquist number.

II. THE DESIGN AID DIAGRAM (DAD)

Figure 1 shows iso-contours of the parameters ϕ, $\nu^* T^3$, and $sT^{1/2}$ plotted in the $r_s - B_e$ plane. Also plotted are points representing four operating FRCs (TCSU, MRX-CT, PFRC and PHD) and proposed new ones, including that of the Tri-Alpha Corporation, a $p - B_{11}$-fueled beam-heated device. Research devices intended to explore stability or transport separately would have a wide area of the $r_s - B_e$ plane available, depending on the temperature achievable. However, if simultaneous reactor-like values for s and ν^* were necessary for more realistic experiments, then temperature is no longer a free parameter and a constraint is found from equations (1)-(4):

$$B^{4/5} r_s = 243 (\frac{s^6}{\nu^*})^{1/5} (m^3 \kappa \langle\beta\rangle)^{1/5}, \tag{6}$$

The value of $(s^6/\nu^*)^{1/5}$ is about a factor of 1.5 higher for ignited operation than for driven. Iso-contours of equation (6) are shown for both the ignited and driven base cases and approximately bisect figure diagonally.

What can be learned from the diagram? Clearly, higher ion temperatures and larger $r_s - B_e$ — nearly equivalent to larger ϕ — are necessary to achieve reactor-like s and ν^* values simultaneously. An important conclusion is that tests at reactor-relevant s and ν^* can be performed in devices with B_e and r_s 3-5 times smaller than required for reactors or to contain α particles. One may note that three of the four operating FRCs have chosen a development path that takes the shortest line to reach the $r_s B_e^{4/5}$ criterion.

FRC research programs must attack fundamental issues and simultaneously improve plasma parameters by many orders of magnitude. Should FRC research programs make large steps in size and power to a next-step device? At least one program, the privately financed Tri-alpha device, intends to do so. In contrast, many questions in the RMF_o program can be addressed at a smaller scale. Examples are: 1) the predicted closure of magnetic field lines by odd parity has not been proven; 2) the heating mechanism for electrons has not been proven; 3) the heating regime of ions has not been reached; 4) methods for non-invasive measurement of internal fields have not been developed; and 5) the stabilizing characteristics of RMF_o have not been demonstrated. All these could be done at a small scale, 1/3 and 2/3 of the way towards the $r_s B_e^{4/5}$ criterion on the design aid diagram. Subsequent experiments could explore energy confinement and stability at reactor-like s and ν^*.

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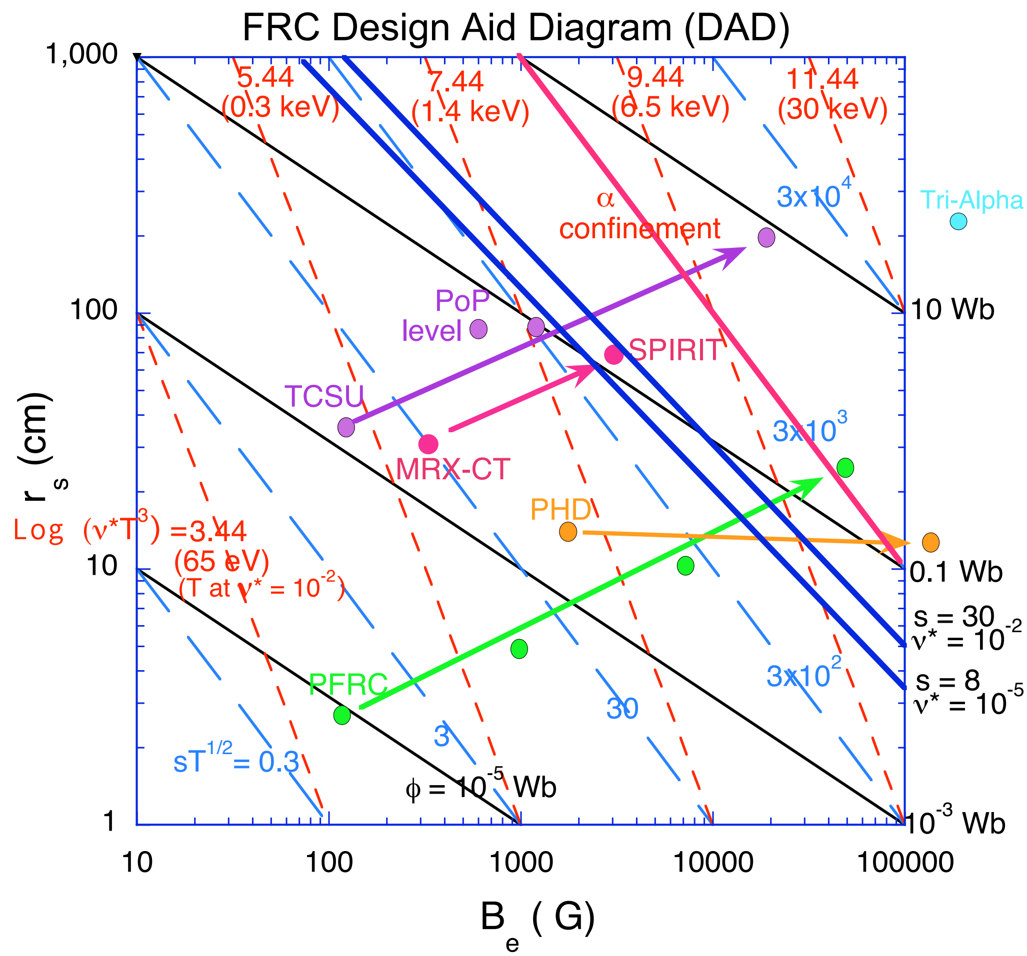

FIG. 1: Design aid diagram showing contours of constant ϕ, $sT^{1/2}$, and $log(\nu^* T^3)$ in the $r_s - B_e$ plane. Positions of current and proposed FRC devices are shown. Note, for calculating s, T is the ion temperature. Also shown are iso-contours of constant $(s^6/\nu^*)^{1/5}$ for two generic FRC reactors, ignited D-T and driven D-He$_3$, described in the text.