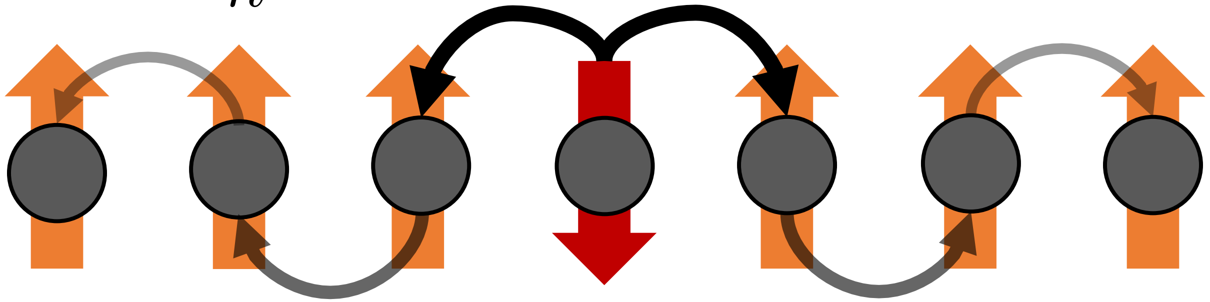
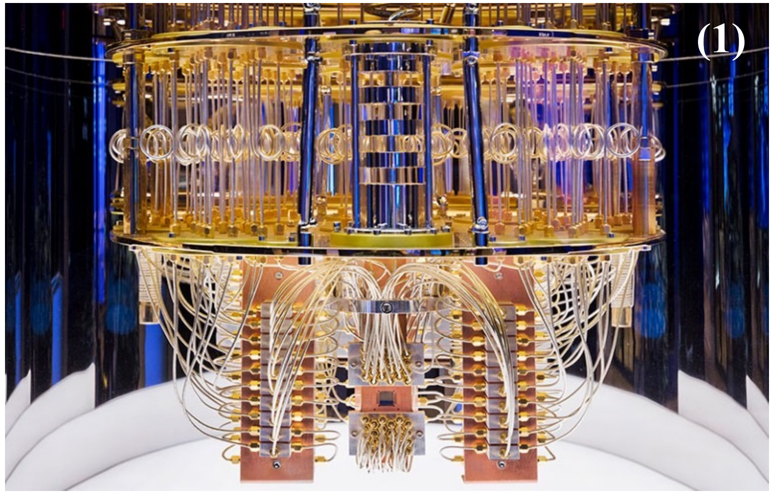
The Journey of Simulating Two Excitation Heisenberg Spin Chains

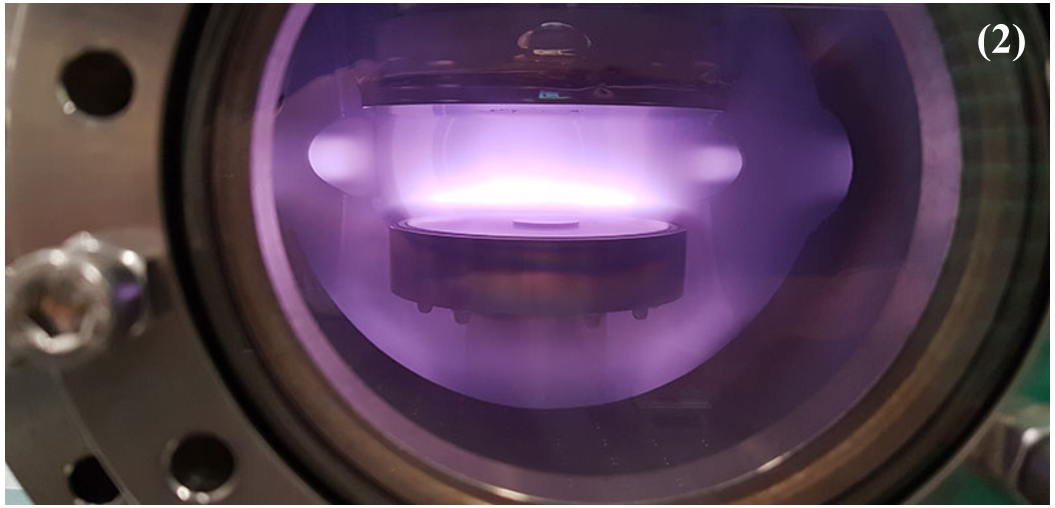
By Kit H. Bernard Foster

Background: Heisenberg Spin Chains

$$H = -J \sum_{n}^{N} \frac{1}{2}\left(S_n^+ S_{n+1}^- + S_n^- S_{n+1}^+\right) + \Delta S_n^z S_{n+1}^z$$







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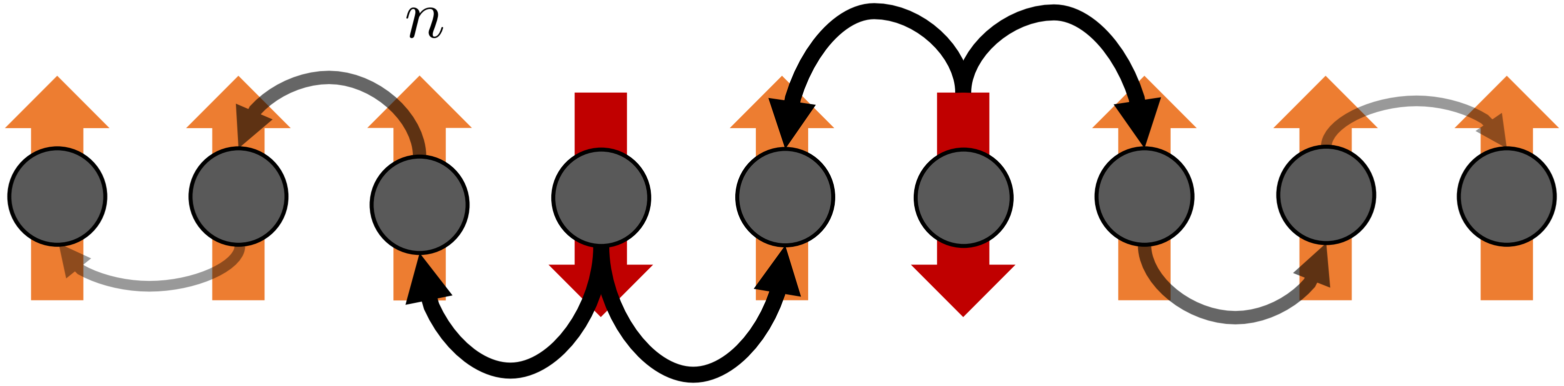
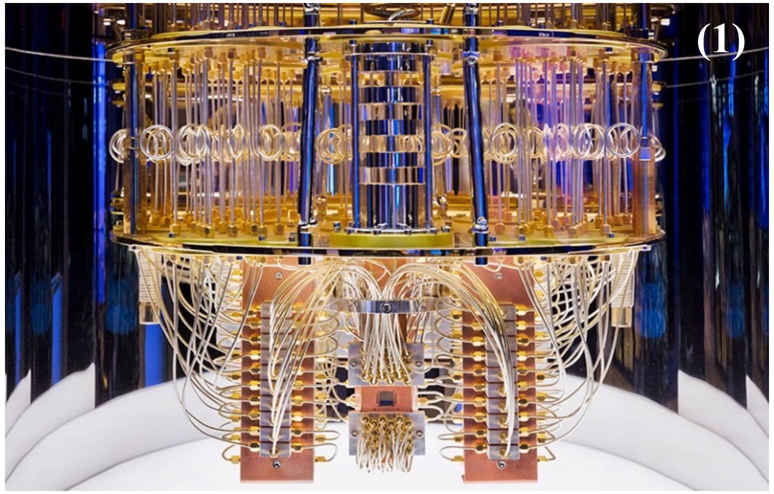
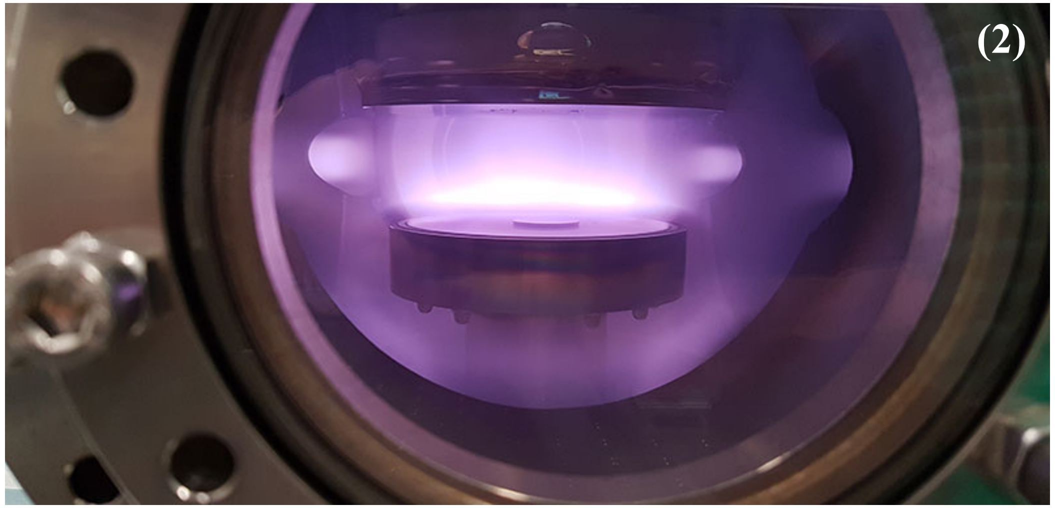






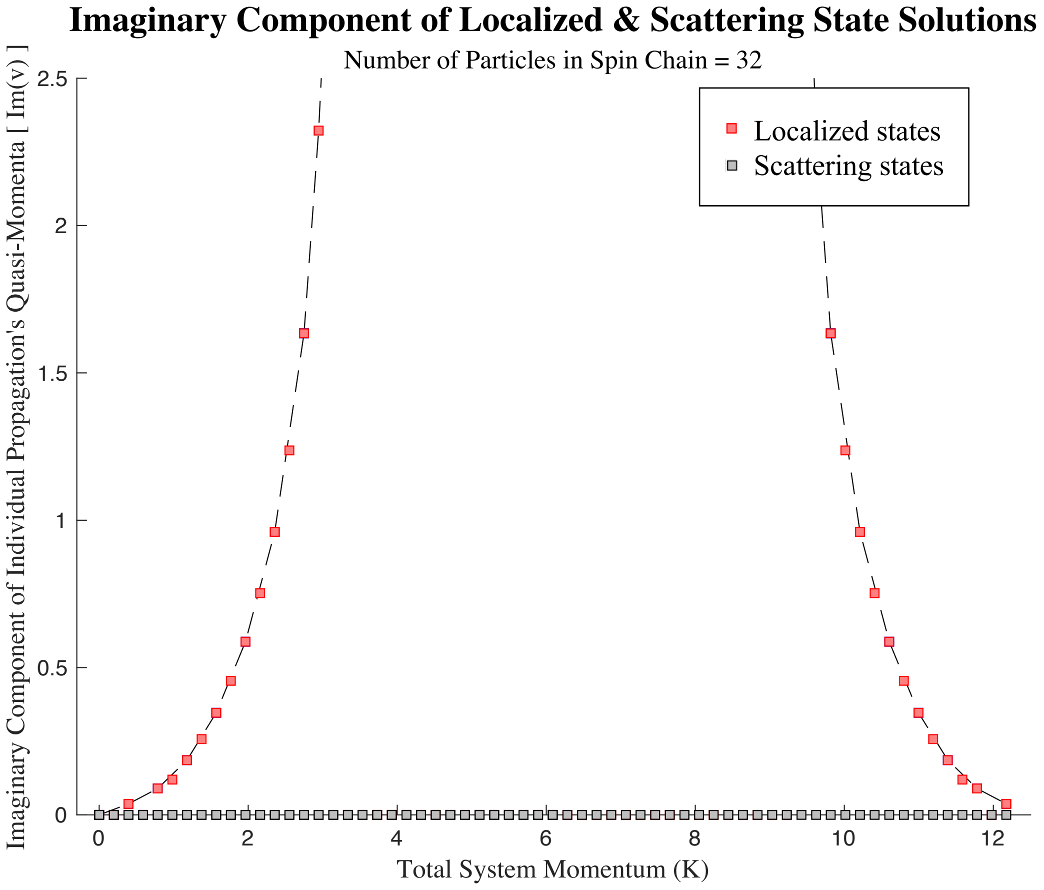
Research Goals – Finding Stability

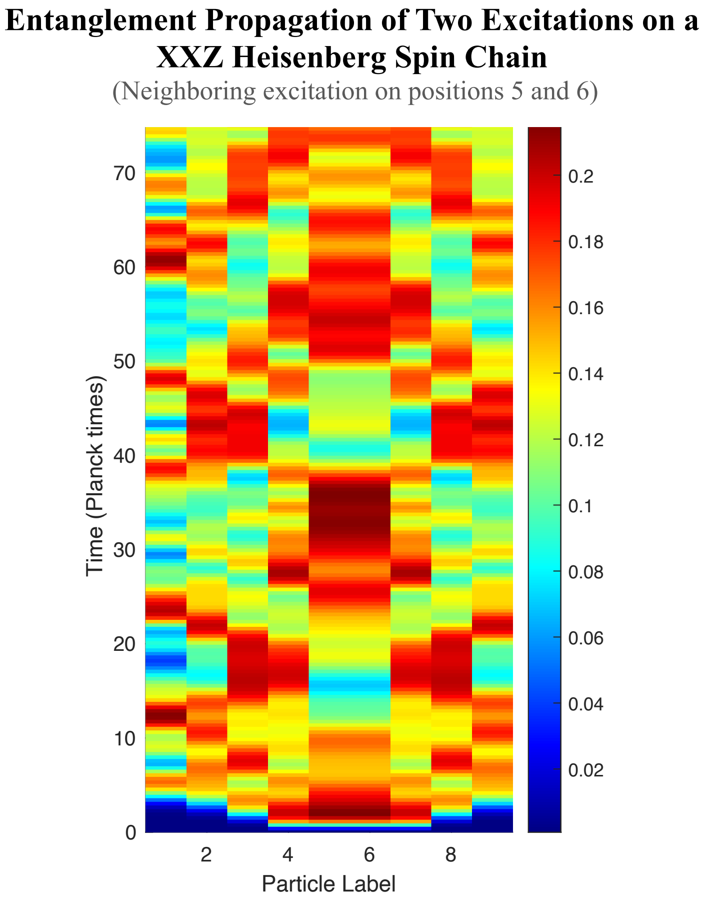
1. Develop simulations that yield qualitative understanding and can handle many spins over long lifetimes.

2. Generate a new analytical model for quantitatively understanding two excitation stability.

Properties of Localized Excitations

for Systems with Two Excitations Propagating

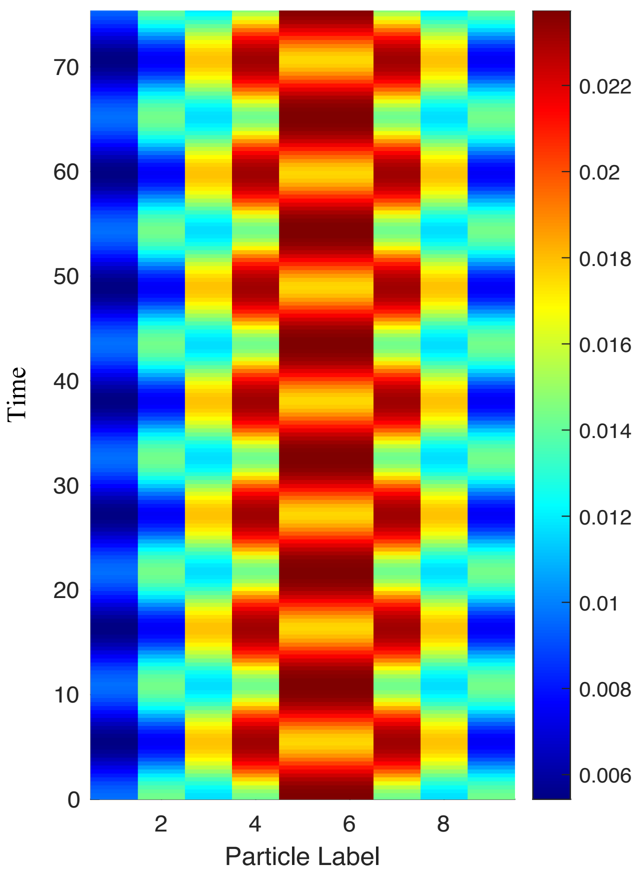




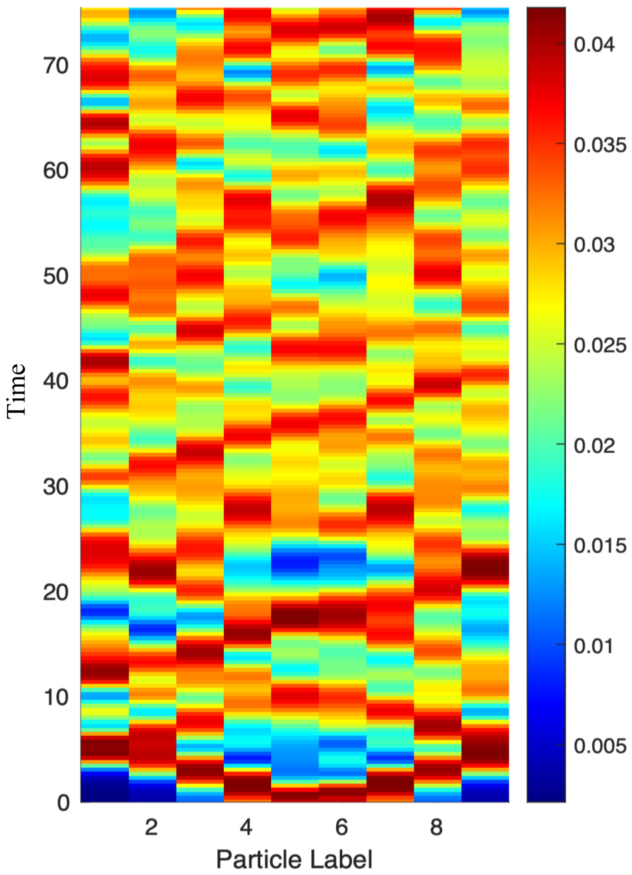
Qualifying Localized States' Stability

Entanglement Propagation of Two Excitations on XXZ Heisenberg Spin Chains

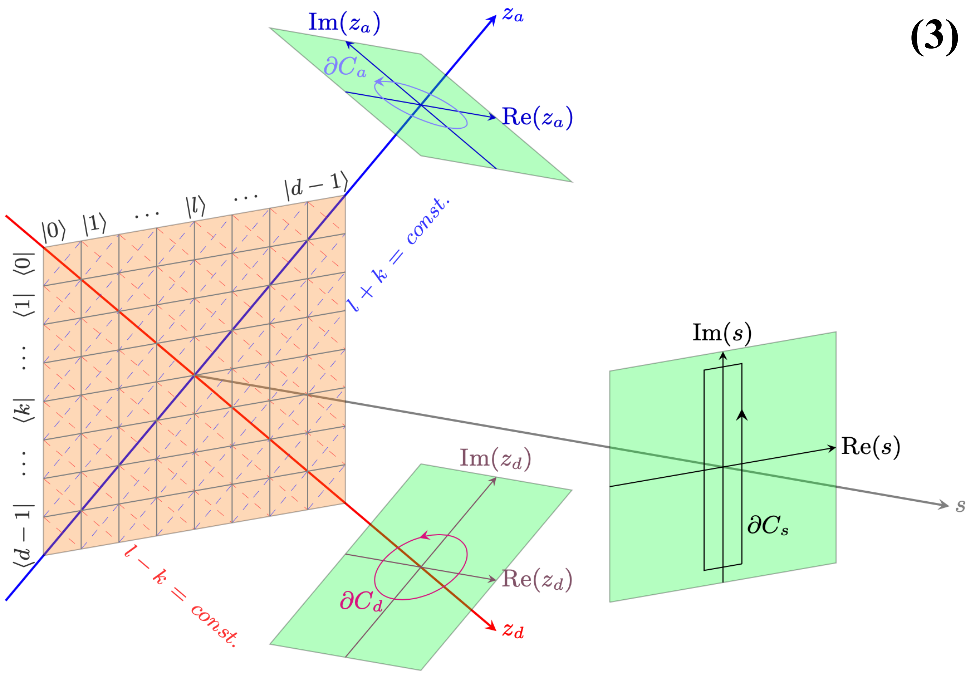
Only Localized Eigenstates Considered
(Neighboring excitation on positions 5 and 6)



Only Scattering Eigenstates Considered
(Neighboring excitation on positions 5 and 6)



QCTF – Analytically Propagating



(3)

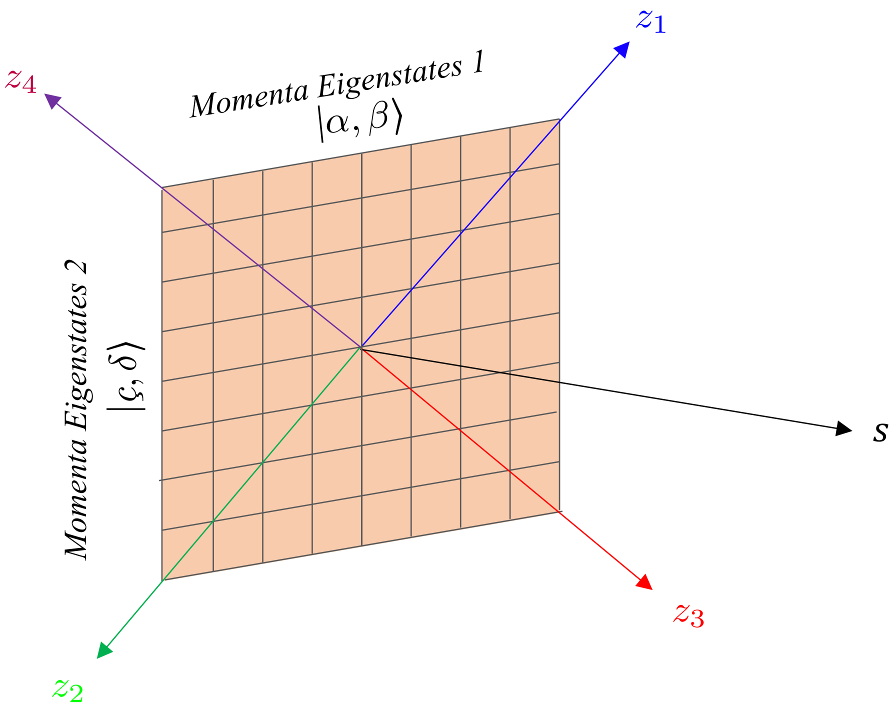
Single Excitation QCTF Outcome

$$\tilde{\mathcal{Q}}_q(s) = \frac{1}{N^4} \sum_{\substack{m_1,m_2 \\ m_3,m_4 \\ p\neq q}} \left(s - \frac{i\mathbf{J}}{\hbar} \left(\cos\left(\frac{2\pi}{N} m_2\right) - \cos\left(\frac{2\pi}{N} m_1\right) - \cos\left(\frac{2\pi}{N} m_4\right) + \cos\left(\frac{2\pi}{N} m_3\right) \right) \right)^{-1} e^{\frac{2i\pi}{N}\left(q(m_1-m_3)+p(m_4-m_2)\right)}.$$

Two Excitation QCTF – Central Insights

a) Maintain same entanglement measure as 1 excitation QCTF by parameterizing momenta eigenstates.

b) Use 4 orthogonal structural



c) Adjust measure to recreate final cancellations.

Two Excitation QCTF – Analysis

Current Entanglement formulation

$$\tilde{Q}_M(s) = \sum_{1\le\alpha<\beta}^{N} \sum_{1\le\varsigma<\delta}^{N} \left(4\text{x} \sum_{\lambda_1\le\lambda_2}^{N-1}\right)\left[s - J\frac{i}{\hbar}(E_1 - E_2' - E_3'' + E_4''')\right]$$
$$a(\alpha,\beta)a^*(\psi_0)a'^*(\varsigma,\delta)a'(\psi_0)a''^*(\alpha,\beta)a''(\psi_0)a'''(\varsigma,\delta)a'''^*(\psi_0)$$

Points of improvement for future formulations

1. High level analysis is required.

2. Advanced numerical programming is required.

3. Eigenstates are inseparable in this formulation.

Conclusion

1. Algorithmically solved for two excitation spin chain dynamics regardless of chain size.

2. Simulated spin chain evolution using numerical eigenstate solutions and the Schrödinger equation.

3. Found a QCTF formulation that reveals entanglement dynamics analytically

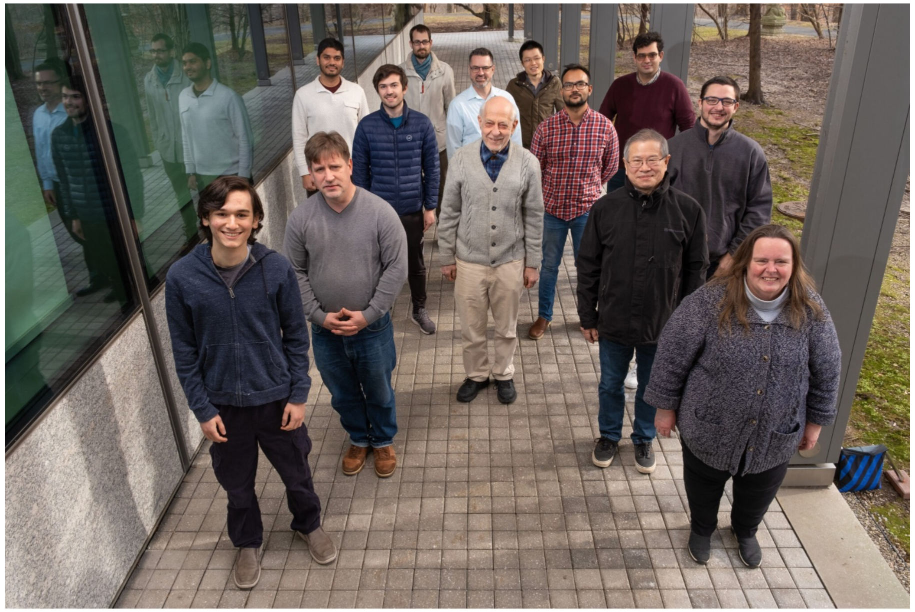
Future Work

1. Find an appropriate simplification of QCTF for analyzing bound states.

2. Analyze a two-particle correlation measure using QCTF methods.

Acknowledgements

Thank you Peyman Azodi and Professor Herschel Rabitz for advising this project.



Thank you PPST 2023 Internship Program for funding this project as well as Samuel Cohen for heading the program.

PPST PROGRAM IN PLASMA SCIENCE & TECHNOLOGY

Lastly, thank you to my mothers for supporting me!

Citation

(1) Ball, P. First 100-Qubit Quantum Computer Enters Crowded Race. *Nature* **2021**, 599, 542. URL: https://www.nature.com/articles/d41586-021-03476-5 (accessed 2023-07-27)

(2) University of York Plasma Institute, *Low Temperature Plasmas,* **2017**, URL: https://www.york.ac.uk/physics-engineering-technology/ypi/research/ltp

(3) P. Azodi and H. A. Rabitz, Exact Entanglement Propagation Dynamics in Integrable Heisenberg Chains, **2023** (unpublished results)