Report for PPPL Summer 2001 Work

MHD Surface Waves in Liquid Metal

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Abstract

The Liquid Metal Experiment at the Princeton Plasma Physics Lab was upgraded for the author's Princeton Senior Thesis to make a precision measurement of the effects of a magnetic field on surface waves in liquid metal. It is topic of interest generally as a domain of resistive MHD physics and specifically as basic knowledge for the construction of liquid metal plasma facing components in a fusion reactor. A linear surface wave theory predicts that a magnetic field parallel to wave propagation will damp waves and that a perpendicular field will have no effect. Data taken for the author's senior thesis quantitatively confirmed that a perpendicular field has no effect, and qualitatively observed damping for the parallel field. Challenges of driving planar waves prevented a quantitative measurement of the damping rate. Progress this summer included deeper analysis of the data from the spring and work with the summer intern on improved wave driving techniques.

Background

The solution to the problem of linear surface waves propagating on the surface of a fluid, an exercise of the Euler fluid equation and linearization techniques, gives a well-known formula for the surface wave dispersion relation. The derivation can be found in most fluid mechanics textbooks, such as Landau and Lifshitz. The central approximation of the theory is that the amplitude of the waves is much less than the wavelength. That is, the waves must not be too steep. Nonlinear theories of surface wave propagation, most famously solitons, have been developed to understand the motion of more general waves.

Surface waves in a conducting fluid, such as gallium, are affected by a magnetic field. Because the fluid conducts electricity, the wave motion can distort the background magnetic field. That is, some of the energy of passing waves will be stored into bending the magnetic field. But the fluid is not perfectly conductive; the fluid has a small magnetic Reynolds number. The distortions will dissipate, providing a means for the wave energy to escape, via ohmic heating of the fluid.

In fact, only waves traveling parallel to an applied magnetic field will create these small bends in the magnetic field. Wave traveling perpendicular to the field will not. Thus, only parallel-traveling waves will lose energy as described above, while perpendicular- traveling waves propagate unimpeded. Mathematically, one can solve the Euler-MHD equations, in the approximation of small magnetic Reynolds number, and find an imaginary component of the wave number in the dispersion relation. This imaginary component implies that the waves will damp over space.

The goal of our experiments has been to measure this damping rate in a tank of liquid gallium and compare to this linear theory. The experiment, constructed by the author for his senior thesis, consists of a 50 cm square tank of gallium, filled with about 1 cm depth of gallium. A wave driver, which is a close relative of a speaker, connected to a paddle excites surface waves in the liquid. A pair of coils at each end of the tank provides a magnetic field of up to about 600 Gauss at the center of the tank. To diagnose the surface waves, an array of 7 lasers reflects off the mirror-like surface of the gallium and onto a screen, filmed by a CCD camera. Due to the surface waves, the laser spots move periodically on the screen. The time history of the laser spot motion, as recorded by the camera, gives a measurement of the amplitude and relative phase of the surface waves at the points of laser reflection. All components of the diagnostics, such as the tilt of the laser beams, the reflection points, and the height of the screen, are calibrated for each setup.

The experiments performed for the author's thesis confirmed that a perpendicular magnetic field does not affect the waves. For the parallel-traveling waves, however, the results were and are less conclusive. In the experiments performed, the lasers reflection points were separated by about 1 cm apart; the goal was a direct observation of a monotonic decrease in wave amplitude—wave damping—across the laser array. Instead, the measurements revealed that the wave amplitude fluctuated, seemingly randomly, from reflection point to reflection point. With an applied field, the amplitude of the waves at each point decreased, confirming qualitatively the MHD damping. However, the amplitude fluctuations prevented a direct, quantitative measurement of the damping rate.

At the time the thesis was submitted, the fluctuations stood unresolved, though two possible theories were presented. First, reflected waves from the far end of the tank might have combined with the progressing waves to create small standing waves. The nodes and anti-nodes of the standing waves would have created a non-monotonic amplitude profile. We did observe that a small pulse would return to the laser array after reflection from the back. The amplitude of the returned pulse was smaller due to natural (viscous, non-MHD) damping.

A second theory arose from experimental experience. The interface between the paddle and the gallium was impossible to scientifically control. It was the art of the experiment. It is possible that non-uniformities in the interface would have created a few points from which cylindrically symmetric—rather than planar—waves might have emanated. These cylindrically symmetric waves would have interfered with one another and the main planar wave from the paddle downstream at the measurement points, confounding the measurement.

Progress

The summer work sought to improve the experiment by eliminating or understanding the amplitude fluctuations. I examined the data for evidence of standing waves. In addition, I supervised the ERULF summer student, David Pace, in the improvement of and design of a better wave driving system. Summer work also closed the first chapter of the experiments. The summer provided time for general data analysis which the thesis deadline did not allow.

As mentioned above, standing waves possibly contributed to at least in part to the spatial amplitude fluctuations. The data from the spring experiments was re-examined to try to find the standing waves. Theoretically, if standing waves were present, one would expect to find a sinusoidal standing wave profile superposed with an exponentially decaying traveling wave profile. A human making a quick look at the experimental data could almost convince herself of the existence of these standing waves: the spacing of high and low amplitude points is roughly consistent with the wavelength of a standing mode.

A model of these standing effects was included into the data analysis. However, the spatial data, 7 points separated each by about 1 cm, is insufficient in quantity and resolution to discern the standing wave. This is a Nyquist-like limitation. The Nyquist frequency, an important concept of signal processing, is one half of the sampling rate. Frequencies above the Nyquist frequency will not be faithfully rendered. Compact discs, for example, necessarily store sound sampled at 44,100 Hz, roughly twice the highest frequency a human can hear. Here, since the typical experiment wavelength is about 1-2 cm, more than two data points will never lie within one wavelength. We cannot discern the standing wave. Practically, this explains why the data analysis code, which fits the data to the standing wave model, cannot converge on the standing wave parameters. Incidentally, this same Nyquist-like limitation also explains why the temporal data fitting routines do not converge for waves with frequencies above about 30 Hz: the camera runs at 60 Hz.

In addition to hunting for standing waves, the author also made more general analysis of the data. Notably absent from the thesis submitted on 30 April was a plot of the dispersion relation: since we ran experiments at multiple frequencies, this was an important addition. Plotting the dispersion relation revealed two effects. First, the textbook surface tension constant of gallium does not fit the measured dispersion relation, which exhibits a decrease in wavelength at high frequencies, consistent with a decrease in surface tension. Oxidation of the surface is the likely cause of this decrease in surface tension. Second, the wavelength increases a small amount with applied parallel magnetic field. This effect is not predicted in the linear theory—it is something which future experiments will have to confirm. The additional data analysis to find standing waves and the plots of the dispersion relations were included in a revision of the thesis.

David Pace also made progress on the improvement of the experiment itself. The paddle of the spring experiments was an unorthodox choice for fluid mechanics experiments. Instead of moving fluid bulk, such as the typical piston- or wedge-like wave drivers used in large flume fluid mechanics experiments, the spring experiments used a thin bar paddle that barely touched the surface. It created waves not by moving the fluid bulk but by pulling the surface up and down via surface tension. At the time, it seemed a good choice because it produced better, more uniform planar waves in water experiments. It also has theoretical appeal: because the waves we are creating are in all cases less than the surface tension length, surface tension will always play an important role in their creation—better to exploit it, or so we believed. The surface tension length is formed by comparing a liquid's surface tension constant to its density times gravity. It manifests, itself, for example, as the size of the meniscus on the side of a test tube.

However, with our experience of the challenge of creating a uniform interface between the gallium and paddle, we decided to try a new wave driving technique: a wedge-like paddle. David and Alan Paterson constructed the new paddle. In addition, they added other improvements to the wave driver, such as additional stabilization of the rod that the wave driver sits on and the rod connecting the wave driver to the paddle. These improvements seem to have worked: data taken after the author had left showed a monotonic decrease in the amplitude over the laser array, indicating that some of the wave driving problems have been solved.

Conclusion

The work on the liquid metal experiment in the summer of 2001 focused on the chief puzzle of the experiments from the spring of 2001: a spatial amplitude profile which, instead of simply monotonically decreasing away from the paddle, contained large amplitude fluctuations. These fluctuations prevented a quantitative measurement of the damping due to a parallel magnetic field.

We first attempted to find and discern standing waves in the spring 2001 data. It was found that the design of the diagnostic, 7 measurement points separated by about 1 cm each, was insufficient to discern any standing waves. Further data analysis was also made on the spring data, measuring dispersion relation. A revised thesis, including these additional pieces of data analysis, was submitted to Princeton.

The author also supervised David Pace with the improvement of the wave driving techniques of the experiment, including constructing a wedge-like paddle and better support for the wave driver and paddle. Test runs after the author left show that this has solved some of the wave driving challenges. Thus the experiment seems poised to make a measurement of the MHD damping rate due to a parallel magnetic field. As the wave driving technique improves, the liquid metal experiment will be able to do what in the author's opinion seem the most exciting experiments: observing the effects of applied magnetic field on nonlinear waves.

Bibliography

FOX, W. R. 2001. *Magnetohydrodynamic Surface Waves in Liquid Metal.* Senior Thesis, Princeton University.