Towards a Unified Theory on the Limits of Control in Quantum Systems

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Abstract

In this work we provide an explicit equivalence between results from the quantum circuit and quantum control models of quantum computation. While quantum computers are believed to have significant advantages over classical computers in performing a computation, different theoretical and physical models yield different measures of such advantages. The quantum circuit model measures the number of discrete gates from a fixed set that are required to perform a final computation, whereas the quantum control model measures a physical time required for a system to continuously evolve under a controllable Hamiltonian configuration. We show that the minimum time within a fully controllable system defined by a set of drift and control Hamiltonians is equivalent to a minimum gate count in a circuit computation relative to an oracle, and attempt to establish this as a preliminary framework to connect future results.

Declaration

I declare that I have not violated the Honor Code during the composition of this work. This paper represents my own work in accordance with University regulations.

Contents

1 Introduction

The massive field of quantum information sciences began with the realization that information sciences, and the world itself, is quantum in nature and depends upon quantum effects. A quantum computer is a physical device that would be able to harness such effects to perform computations, and it has been hypothesized to be able to have significant advantages over a classical computer in certain fields (coined as *quantum supremacy* in 2012 [1]). With the recent report that Google AI Quantum achieved such quantum supremacy in late 2019 [2], research and interest in the potential of quantum computers is sure to grow even more in the coming years.

One of the most obvious advantages a quantum computer is expected to have is in the speed in which it is able to perform a computation - thus, exploring any aspect of quantum information sciences involves a consideration of the time required in some quantum process. Ascertaining the extent to which quantum supremacy could be achieved has led to a concept called the *quantum speed limit* in some regimes [3]. However, with an increasing number of subfields and directions of research, the quantum speed limit has been studied and discussed in many different contexts that nevertheless refer to the same general principle.

It is the goal of this paper to provide a preliminary framework for unifying results from two of the main branches of research: an implementation-inspired field of quantum optimal control, and a more conceptually-based field of quantum circuit theory. We start with an overview of the main concepts and previous work done in the relevant fields. Then, we use these results to provide a preliminary framework under which settings in the two fields could be considered equivalent. Finally, we conclude with a few pathways for future research. Throughout this paper, original mathematical results will be proven in-depth, but standard results from quantum mechanics and quantum computation will be presented without rigorous proof as deemed necessary. Any uncited quantum-computing results are inspired by standard textbook *Quantum Computation and Quantum Information* [4], but have been modified and translated to fit the framework presented in this work.

2 Prior Work

2.1 Quantum Computing

We begin with an overview of necessary concepts from quantum mechanics and quantum computing.

A classical computer manipulates bits, each of which is in one of two binary states. On the other hand, a quantum computer manipulates *qubits*, each of which can be in a superposition of the two states. In general, we can define an N-dimensional quantum state:

Definition 2.1. *An* *N-dimensional quantum state* $|\psi\rangle$ *is a superposition of* N *basis vectors in* $\mathbb{C}^N$*, written as a unit vector in the form* $|\psi\rangle = \sum_{j=1}^{N} \alpha_j |j\rangle$ *such that* $\alpha_j \in \mathbb{C}, \sum_{j=1}^{N} |\alpha_j|^2 = 1$.

Definition 2.2. *A* *qubit* *is a system represented by a 2-dimensional quantum state.*

A quantum computer operates on some N-dimensional quantum system in order to perform a computation. There are two ways to do so: through a unitary transformation or through a measurement.

Definition 2.3. *The* *inner product* *of two* N*-dimensional quantum states* $|\psi\rangle = \sum_{j=1}^{N} \alpha_j |j\rangle$ *and* $|\phi\rangle = \sum_{j=1}^{N} \beta_j |j\rangle$ *is defined as* $\langle\psi|\phi\rangle = \sum_{j=1}^{N} \alpha_j^* \beta_j$*. We write* $\||\psi\rangle\|^2 = \langle\psi|\psi\rangle$.

Definition 2.4. *A* *unitary transformation* *is a linear map* $U : \mathbb{C}^N \to \mathbb{C}^N$ *such that* $U^\dagger U = I_N$*; note that a unitary transformation preserves inner products, such that if* $|\psi'\rangle = U|\psi\rangle$ *and* $|\phi'\rangle = U|\phi\rangle$ *then* $\langle\psi'|\phi'\rangle = \langle\psi|U^\dagger U|\phi\rangle = \langle\psi|\phi\rangle$.

As a result, a unitary transformation maps an N-dimensional quantum state to another N-dimensional quantum state. Whereas a unitary transformation can change the state of a quantum system, this precise state is inaccessible; instead, one can only access properties called *observables.*

Definition 2.5. *An* *observable* *is a physical quantity of a quantum system that can be measured (accessed).*

If an observable of a quantum system can attain N possible values, then the state of the N-dimensional quantum system can be written with respect to an orthonormal eigenbasis, where each value is an eigenvalue corresponding to an eigenvector.

Definition 2.6. *A* *measurement in the observable basis* *collapses a quantum state* $|\psi\rangle = \sum_{j=1}^{N} \alpha_j |j\rangle$*, written in the eigenbasis corresponding to the relevant observable, into one of the* N *eigenvectors* $|j\rangle$*, with the transformation* $|\psi\rangle \to |j\rangle$ *occurring with probability* $|\alpha_j|^2$*; note that in general a measurement does not correspond to a unitary transformation.*

Once a quantum system is measured and collapses into some $|j\rangle$, the value j of the observable is revealed. Quantum computations are done by a series of unitary transformations followed by relevant measurements. As a result, there are two closely related processes of interest with respect to quantum computing: *preparation* of a quantum state from a given initial state, and *application* of a given unitary transformation on a quantum system. In this paper we focus our discussions on the second process; however, we mention the relevance of the first process as well. Before we delve deeper into two specific models for studying these processes, we direct the reader to [4, 5] for further background on quantum computation and quantum information sciences.

2.2 Quantum Circuit Model

The most common model in quantum computing is the quantum circuit model. As per definition (2.2), a single qubit is a 2-dimensional quantum system, with basis states represented by $|0\rangle$ and $|1\rangle$.

Definition 2.7. *The state of a 2-qubit (easily generalizable) quantum system is called* <u>*separable*</u> *if it can be written as the tensor product of two quantum states* $|\psi\rangle \otimes |\phi\rangle$.

Definition 2.8. *A system is said to be in an* <u>*entangled*</u> *state if it is not separable.*

A 2-qubit system is written as $|\psi\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle$; in general, an n-qubit quantum system can be written in a 2^n-dimensional basis, and is therefore represented by an $N = 2^n$-dimensional vector.

Definition 2.9. *A* <u>*standard qubit measurement*</u> *is a measurement of a single qubit within an n-qubit quantum system, yielding a value of either 0 or 1 corresponding to either $|0\rangle$ or $|1\rangle$, respectively.*

The state of the resulting n-qubit system becomes separable, and the collapse occurs in the following way:

Remark 2.1. *Without loss of generality, assume we measure the first qubit within an n-qubit quantum system. Then, the state of the remaining $n-1$ qubits can be represented using 2^{n-1} basis vectors $v_1, \ldots, v_{2^{n-1}}$ such that $|\psi\rangle = \alpha_1|0v_1\rangle + \cdots + \alpha_{2^{n-1}}|0v_{2^{n-1}}\rangle + \beta_1|1v_1\rangle + \cdots + \beta_{2^{n-1}}|1v_{2^{n-1}}\rangle$. Measuring the first qubit collapses the system into either $|\psi\rangle = |0\rangle \otimes \frac{\alpha_1|v_1\rangle + \cdots + \alpha_{2^{n-1}}|v_{2^{n-1}}\rangle}{|\alpha_1|^2 + \cdots + |\alpha_{2^{n-1}}|^2}$ (if the measurement yields a value of 0), or $|\psi\rangle = |1\rangle \otimes \frac{\beta_1|v_1\rangle + \cdots + \beta_{2^{n-1}}|v_{2^{n-1}}\rangle}{|\beta_1|^2 + \cdots + |\beta_{2^{n-1}}|^2}$ (if the measurement yields a value of 1).*

Definition 2.10. *A* <u>*k-qubit gate*</u> *is a unitary transformation $G : \mathbb{C}^{2^k} \to \mathbb{C}^{2^k}$ acting on k qubits. Note: if a k-qubit gate G acts on the first k qubits of an n-qubit system, then we write $G := G \otimes I_{n-k}$, and accordingly for any other set of k qubits.*

A quantum circuit is a model of quantum computation in which an n-qubit quantum system is acted upon by a (discrete) series of gates and standard qubit measurements. A generic quantum circuit prepares a quantum state from a given initial state; if the circuit consists of only gates (no measurements), the transformation is unitary and therefore can be represented by a single unitary U.

Definition 2.11. *A set of gates $\mathcal{G}$ is said to be exactly universal if any (finite) unitary transformation U can be written as the product of a finite sequence of gates from $\mathcal{G}$.*

Definition 2.12. *A set of gates $\mathcal{G}$ is said to be universal (or approximately universal) if for all $\epsilon > 0$ and any (finite) unitary transformation U, there exists a finite sequence of gates from $\mathcal{G}$ that approximates U to within ϵ (the precise definition of 'approximate' varies; one commonly used definition is that if the approximation is denoted $U_{\mathcal{G}}$, then $\|U_G - U\|^2 = sup_{|\psi\rangle \in \mathbb{C}^{2^n}} \|(U_G - U)|\psi\rangle\|^2 < \epsilon$).*

Many results have been proven throughout the years regarding universal gate sets. A few key results are presented here:

Theorem 2.1. *(Knill 1995 [6]) Let S_k be the set of all k-qubit gates. Then, $S_1 \cup S_2$ is exactly universal.*

Theorem 2.2. *(Barenco et. al. 1995 [7]) Let $CNOT$ be the 2-qubit gate represented by the matrix* $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. *Then, $\{CNOT\} \cup S_1$ is exactly universal.*

Remark 2.2. *There are known many finite universal gate sets, including those consisting of only one single-qubit gate and one 2-qubit gate [4].*

Remark 2.3. *A set of gates $\mathcal{G}$ is said to be n-universal if it is universal for all S_n.*

The definition of universality merely requires a finite sequence of gates to approximate a given unitary U; however, no restrictions are placed on the length itself.

Theorem 2.3. *(Solovay-Kitaev [8]) Let $\mathcal{G}$ be a universal gate set, and let U be any (finite) unitary. Then, the number of gates in $\mathcal{G}$ needed to approximate U to within ϵ scales polynomially in $\log \frac{1}{\epsilon}$.*

This important theorem yields that all universal gate sets are effectively equivalent.

Before we move on to the quantum control model of quantum computation, we remark on a few details worthy of note:

Definition 2.13. *An ancilla qubit is a pre-initialized (usually to $|0\rangle$) qubit added to a quantum system that becomes unneeded in a final measurement.*

Theorem 2.4. *(Stinespring [9]) A generic quantum circuit C (allowing both gates and measurements) on an n-qubit quantum system may not be unitary due to measurements. However, there* *exists some $k \leq 2n$ such that by adding k ancilla qubits, C can be obtained by reducing an $n+k$-dimensional unitary operation U acting on an $n+k$-qubit quantum system to the original n-dimensional subsystem.*

Theorem 2.5. *(Principle of Deferred Measurement) For any generic quantum circuit C, there exists an equivalent circuit C' obtained by delaying all measurements until the end.*

This theorem, combined with Stinespring's result, shows that a generic quantum circuit on n qubits can be viewed as a single unitary operation on at most $3n$ qubits. As a result, within the quantum circuit model of quantum computation, we ask for the minimum number of gates required to implement a desired n-qubit unitary operation U, allowing for ancilla qubits:

Definition 2.14. *Given a universal gate set $\mathcal{G}$ and an n-qubit unitary U, $C_{\mathcal{G}}(U)$ is the minimum number of gates from $\mathcal{G}$ required to exactly implement U, and $C_{\mathcal{G},\epsilon}(U)$ is the minimum number of gates required to approximate U to within ϵ.*

Sometimes, quantum computations on a quantum circuit are performed with the help of 'black-box' operations called oracles:

Definition 2.15. *An <u>oracle</u> f is an operation applied to a quantum circuit at unit cost [10].*

These 'black-box' operations do not reveal their inner nature, and as such may require a large number of gates from $\mathcal{G}$ to implement; however, when we compute $C_{\mathcal{G}}(U)$ relative to an oracle f, we ignore this contribution. One famous problem in which quantum oracles are used is the Hidden Subgroup Problem [11], which is necessary in many regimes, including Shor's polynomial factoring algorithm that has long been considered an important evidence of quantum supremacy, since integer factorization is conjectured to be non-efficient on a classical computer [4].

Remark 2.4. *(Setup for Hidden Subgroup Problem) Let A be a subgroup of B, and X be a finite set. A function $f : B \to X$ is said to hide A if f is constant only within cosets of A. In the Hidden Subgroup Problem, f is given as an oracle.*

While seemingly far-fetched, the relevance of quantum oracles will become evident in the main result of this paper after we discuss the continuous, quantum control model of quantum computation.

2.3 Quantum Control Model

The quantum circuit model of quantum computation is a highly theoretical model that uses discrete gates. When analyzing the *scaling* of $C_{\mathcal{G},\epsilon}$, the Solovay-Kitaev theorem allows us to ignore how $\mathcal{G}$

is formed. However, as any physical realization of a quantum computer is not discrete but rather continuous, it is not as easy to ignore the makeup $\mathcal{G}$, or even to describe $\mathcal{G}$ effectively.

In this sense, instead of a number of discrete gates, we explore a physical *time* required to implement a certain unitary operation. As a result, the concept of a *quantum speed limit* has been developed. Some versions of this 'speed limit' are explicitly dependent on a specific model or on constraints imposed by a proposed physical implementation [3]. However, other speed limits are more fundamental in nature, and will be the subject of this section. One common candidate for a physical implementation is an NMR-based quantum computer [12], which uses nuclear spins as qubits. The following quantum control model is based upon this proposed implementation, but applies also to generic quantum systems.

Definition 2.16. *The Hamiltonian, denoted H, is a Hermitian operator that completely describes the time-evolution of a quantum system.*

Within this description, the state of a closed quantum system evolves under the Schrodinger equation [10]:

Theorem 2.6. *Let the physical Planck constant be normalized to $\hbar = 1$. Then, the state $|\psi\rangle$ of a quantum system evolves according to the Schrodinger equation $\frac{d}{dt}|\psi(t)\rangle = -iH(t)|\psi(t)\rangle$. Solving this equation for time-independent H yields the solution $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$.*

Definition 2.17. *In the bilinear control model, the Hamiltonian $H(t)$ of a quantum system is controlled by writing $H(t) = H_0 + \sum_j f_j(t)H_j, f_j(t) \in \mathbb{R}$.*

Definition 2.18. *The drift Hamiltonian H_0 is the Hamiltonian that describes the free evolution of a quantum system (without external control), representing the uncontrollable interactions between the qubits that make up the physical implementation of the quantum computer.*

Definition 2.19. *The control Hamiltonians H_j are Hamiltonians that can be applied to a quantum system with controllable time-dependent amplitudes $f_j(t)$.*

Remark 2.5. *Note that for any Hamiltonian H and time t, e^{-iHt} is unitary, and (trivially) since the unitary group is closed, any product of unitary transformations is unitary.*

As a result, given the initial state of a quantum system $|\psi(0)\rangle$, for any time t the transformation from $|\psi(0)\rangle$ to $|\psi(t)\rangle$ is unitary. Therefore, we can define $|\psi(t)\rangle = U(t)|\psi(0)\rangle$. The quantum control model of quantum computing aims to find suitable values for $f_j(t)$ such that $U(t) = U$ for some desired unitary U.

Definition 2.20. *Given a set of drift and control Hamiltonians, the* <u>*minimum time*</u> *required to implement a desired unitary operation U is the smallest $T = T_U$ such that there exist $f_j(t), t \in [0, T_U]$ with $U(T_U) = U$.*

For an in-depth review of quantum speed limits relating to the quantum control model, we refer the reader to [3, 13], and for work in the field done by the present author, we refer to [14, 15]. Here, we present some of the major results that will become relevant in conjunction with discussions on the quantum circuit model.

One benefit of having access to a time T, as opposed to a number of discrete gates, is that it allows explicit comparisons and analyses to be made with respect to physical implementations. As per definition (2.6), a measurement forces the state of a quantum system to collapse. When performing a quantum computation, it is necessary for such collapses to only occur when desired.

Definition 2.21. *A quantum system is said to be* <u>*coherent*</u> *if its time-evolution is predictable under the Schrodinger equation.*

However, any physical implementation undergoes a process called quantum decoherence, where the system effectively loses coherence and therefore becomes unpredictable, and as such causing unwanted collapses.

Definition 2.22. *The* <u>*coherence time*</u> *or* <u>*relaxation time*</u>*, denoted t_{rel}, is the time it takes for a quantum system to lose coherence.*

As a result, given a relaxation time t_{rel} associated with the qubits in a quantum system, it is desirable to know the unitaries U for which the minimum time $T_U < t_{rel}$.

We now present algebraic and geometric models to describe a quantum control system.

Definition 2.23. *An N-dimensional quantum system defined by a set of drift and control Hamiltonians $\{H_j\}$ is* <u>*exactly fully controllable*</u> *if for all N-dimensional unitaries U, there exists some finite T and values $f_j(t)$ such that $U(T) = U$. The values $f_j(t)$ are said to be* <u>*control functions*</u> *that* <u>*drive*</u> *the unitary $U(t)$.*

Definition 2.24. *A quantum system, as above, is (approximately)* <u>*fully controllable*</u> *if for all U and $\epsilon > 0$, there exists finite T such that $\|U(T) - U\| < \epsilon$, where $\|U\|$ is defined as in (2.12).*

Definition 2.25. *The* <u>*unitary group*</u> *in N dimensions is denoted $U[N]$, and the* <u>*special unitary group*</u> *is denoted $SU[N]$, which have determinant 1.*

Theorem 2.7. *(Ignore global phase [4]) Measurement ignores the global phase in a quantum system, so $U \in SU[N]$ is effectively equivalent to $e^{i\theta}U \in U[N]$ for all $\theta \in [0, 2\pi)$.*

Definition 2.26. *The reachable set corresponding to a set of Hamiltonians $\{H_j\}$ is the subgroup of $SU[N]$ generated by $\{U_j = e^{-iH_j}\}$.*

Standard theorems from algebra yield the following result:

Theorem 2.8. *The reachable set of the control Hamiltonians forms a subgroup $R_C \subset SU[N]$, and as such partitions $SU[N]$ into disjoint right cosets of R_C.*

Furthermore, using the continuity of $U(t)$ derived from the Schrodinger equation (2.6) yields the following result as well:

Theorem 2.9. *Let $U = U(T) \in SU[N]$. The time-evolution $\{U(t) \mid 0 \leq t \leq T\}$ with $U(0) = I_N, U(T) = U$ defines a continuous path in $SU[N]$ from I_N to U.*

A major result in quantum control theory connects two theorems above:

Theorem 2.10. *A quantum system is exactly fully controllable if and only if the reachable set of the drift and control Hamiltonians is $SU[N]$. [13].*

Then, given a fully controllable quantum system, the next question involves the minimum time T_U for an arbitrary unitary U. For a full review, we refer to [3]. One key result in quantum speed limits relates the optimality of the time T with a distance defined by a metric on $SU[N]$:

Theorem 2.11. *Given a set of drift and control Hamiltonians, there exists a metric d on $SU[N]$ such that if $U \in SU[N]$, T_U is the minimum time if and only if the length of the path $\{U(t) \mid 0 \leq t \leq T_U\}$ is minimal with respect to d [16, 17].*

As a result, although the details of this proof is beyond the scope of this paper, we obtain an equivalence between the minimum time T_U and the length of a path in $SU[N]$.

Theorem 2.12. *Given a reachable set R_C, movement within a single right coset of R_C occurs instantaneously.*

Theorem (2.12) is a major result in the subfield of bang-bang quantum optimal control [18].

3 Main Result: Relating $C_{\mathcal{G}}(U)$ and T_U

The quantum circuit model gives insight into a minimum number of gates $C_{\mathcal{G}}(U)$ given a universal gate set $\mathcal{G}$, while the quantum control model yields a minimum time T_U given a Hamiltonian control system $\{H_j\}$. It is desirable to relate these two quantities.

3.1 Proof of Main Result

Theorem 3.1. *Fix some n, and define $N = 2^n$. Let $U \in U(N)$, and $\{H_0, H_1, \ldots, H_j\}$ be a set of N-dimensional Hamiltonians defining a fully controllable quantum system with reachable set R_C. Then, there exists an n-universal gate set $\mathcal{G}_n(H_0)$ such that computing T_U is equivalent to computing $C_{\mathcal{G}_n(H_0)}(U)$ relative to an oracle f that hides the right cosets of R_C.*

Proof. By Theorem (2.11), T_U is the minimum time if and only if the path $\{U(t)\}$ is minimal with respect to some metric d. By Theorem (2.8), $SU[N]$ is partitioned into disjoint right cosets of R_C, and the portion of T_U within a given right coset is 0 by Theorem (2.12). Since by Theorem (2.9) the path is continuous, all portions T_U occur as finitely many discrete transitions between right cosets, and so the path $\{U(t)\}$ can be viewed as a discretized path between the cosets $R_C I_N$ and $R_C U$, where by definition of a coset any transition occurs by application of the drift Hamiltonian H_0 alone for some non-zero time. This yields an equivalence relation that identifies all elements in $SU[N]$ that are in the same right coset of R_C. Thus, identification of which right coset a given unitary $U(t)$ can be written as a function described in remark (2.4). This specific function satisfies the condition that f is an oracle, namely form definition (2.15) that it is applied at unit cost, because the length of the path within a coset is 0. Now, let since we are in a fully controllable quantum system, there exists a universal gate set. Theorem (2.6) showed that for time-independent H the unitary evolution for some time t is simply e^{-iHt}; therefore, any unitary between right cosets is of the form e^{-iH_0t} for some t. Then, we see that choosing $\alpha = \sqrt{2}$, for example, yields that $e^{-iH_0\alpha}$ generates all such unitaries, and so the gate set $\mathcal{G} = \{e^{-iH_0\alpha}\} \cup R_C$ is universal. Since gates R_C are applied at unit cost via the oracle f, we can define the reduced n-universal gate set $\mathcal{G}_n(H_0) = \{e^{-iH_0\alpha}\}$, which by Theorem (2.3) is equivalent to any other universal gate set. Therefore, this concludes the proof that computing T_U is equivalent to computing $C_{\mathcal{G}_n(H_0)}(U)$ relative to f. $\square$

4 Conclusions and Future Work

While Theorem (3.1) only applies for a fixed number of qubits n, in general the Hamiltonians of an n-qubit quantum system $\{H_j\}$ can be described as an extendable family of Hamiltonians $\{H_{j,n}\}$ for all n [13], such as by having each Hamiltonian be a tensor product of single-qubit Pauli operations [19, 20, 21].

As a result, given a family of unitaries $\{U_n\}$, computing explicit times $\{T_{U_n}\}$ yields a scaling relation that is equivalent to the scaling of $C_{\mathcal{G}_n(H_{0,n})}$. However, while the equivalence between minimum number of gates and minimum time has been given, only one direction has been proven to transform a fully controllable set of Hamiltonians into a universal gate set relative to an oracle. It would be favorable to be able to describe the opposing direction as well, by identifying classes of control systems that are equivalent relative to a given oracle and universal gate set.

It is also worth noting that a given family of unitaries $\{U_n\}$ within a family of fully controllable quantum systems represents the solution to a decision problem in the quantum complexity class BQP relative to the oracle defined by the family $\{H_{0,n}\}$ if $\{T_{U_n}\}$ scales polynomially in n [10]. It would therefore also be favorable to explore families of Hamiltonian systems that are equivalent to universal gate sets without oracles.

In this work, we have compiled concepts and results from two models of quantum computation to provide a framework for understanding precise scenarios in which they are equivalent. While theoretical studies in quantum circuit and quantum complexity theory involve a given universal gate set, more experimentally-driven studies regarding physical implementations of quantum computing, such as quantum control theory, involve settings that are different from both each other and from the circuit-based framework. It is hoped that this paper has begun to identify the importance and possibility of allowing different subfields of quantum information sciences to communicate methods in a unified framework, such that the impact of a result in one subfield can readily be translated and applied in another.

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