Control of Three-Linkage Structure with a Dual Use Microthruster Array

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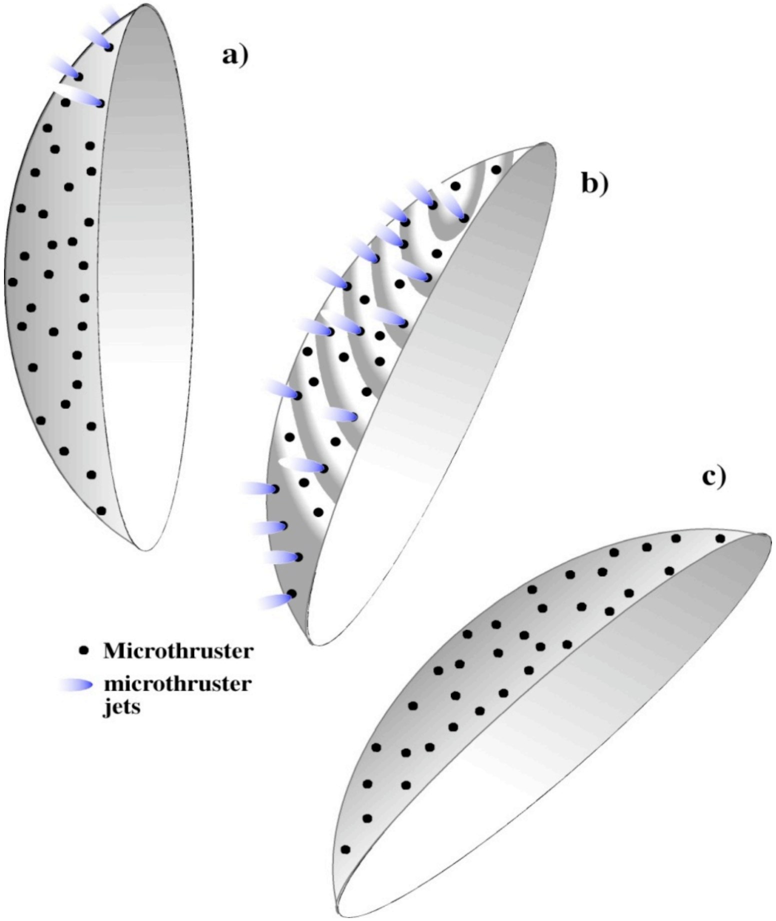
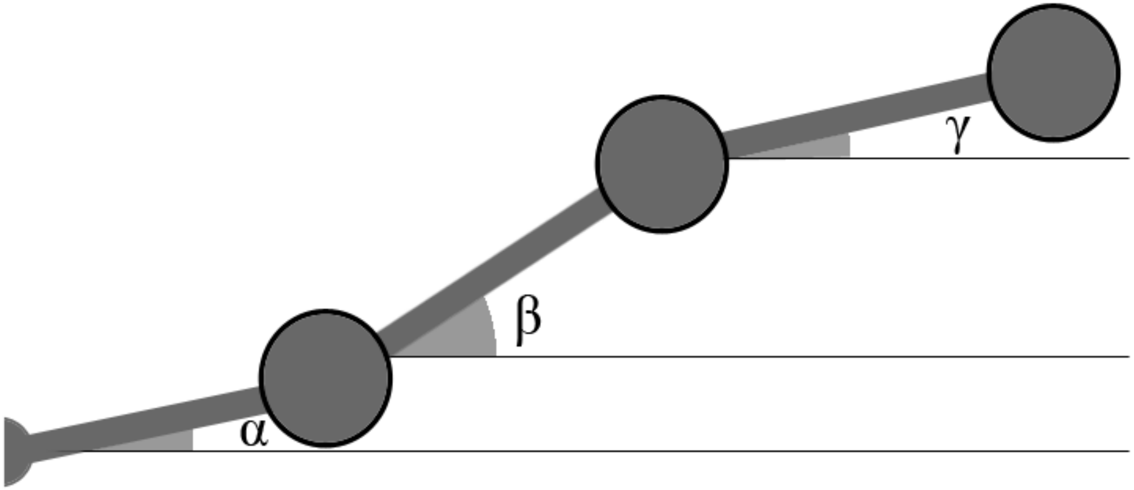
Motivation

- Use high specific impulse microthrusters to control large structures and damp vibrations
- Efficient
- No stress on system



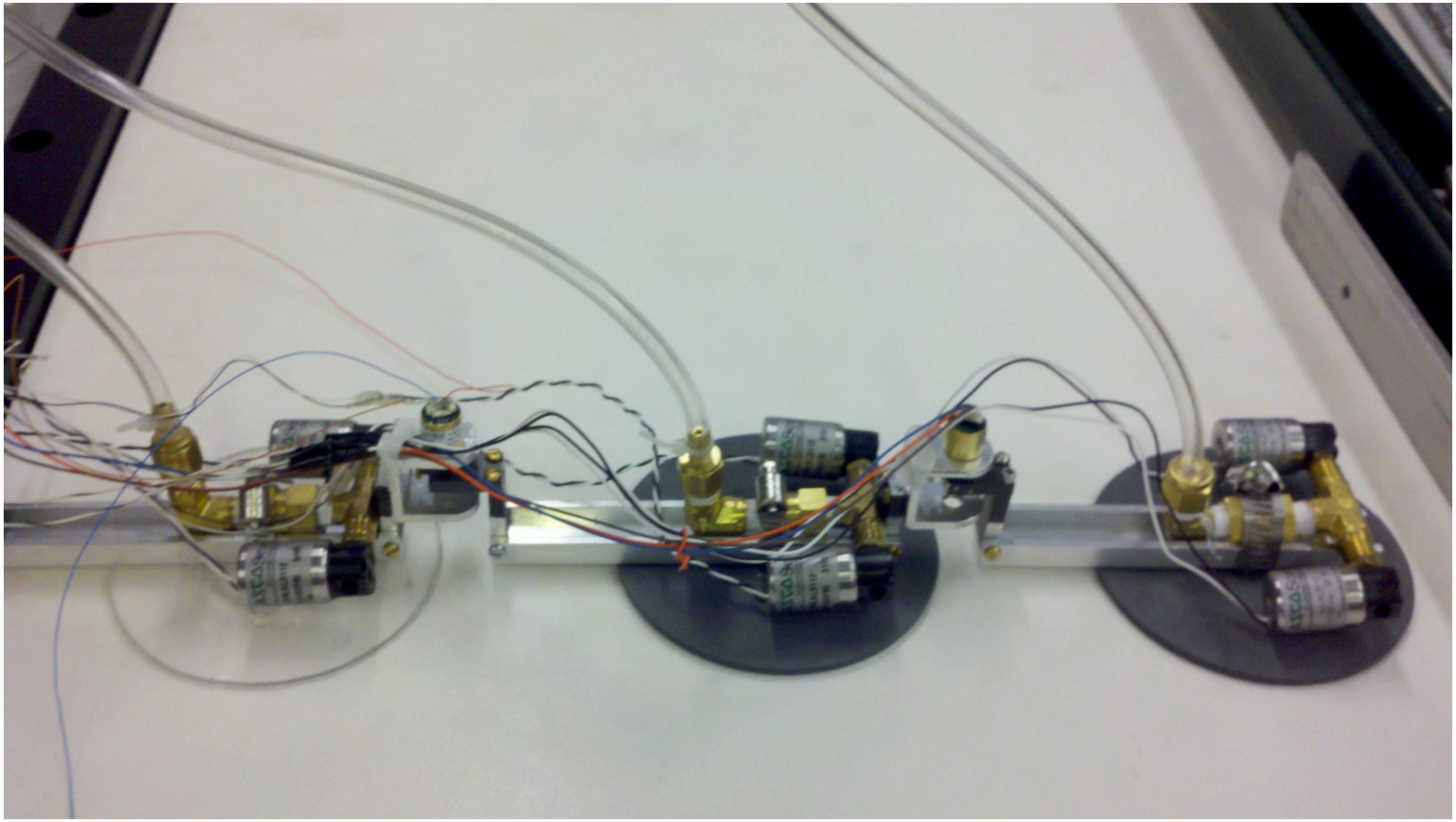
Modelling



- Equations of Motion depend on multiples and sums of $\cos(x)$, $\sin(x)$, $\cos^2(x)$, $\sin^2(x)$

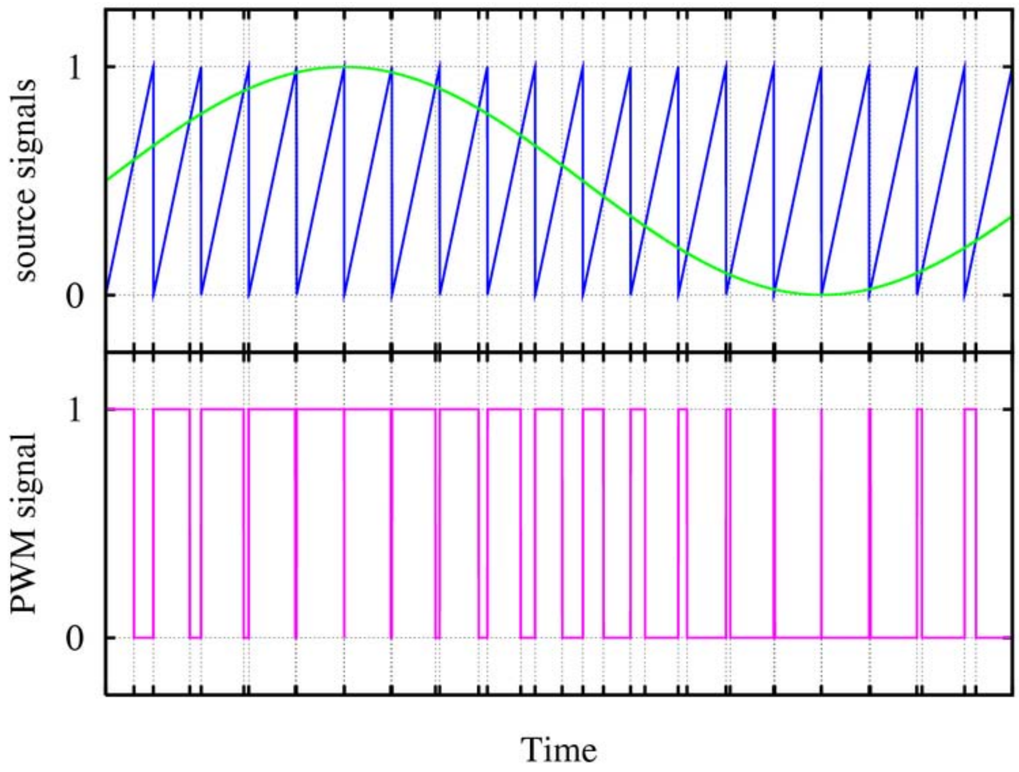
- Classical Control Methods such as PID controllers cannot be used on Multiple Input Multiple Output Nonlinear Systems

Experimental Setup



Pulse Width Modulation

- Method of Converting Analog signal to Digital signal
- PWM is needed due to on/off nature of thrusters



Input-Output Linearization (SISO)

$\dot{x} = f(x) + G(x)u \qquad y = h(x)$

- Cancel out nonlinearities to achieve 1:1 mapping of inputs to y

- Lie Derivative – Derivation of a tensor field B over a vector field A

$$L_A B = \nabla B \cdot A$$

$$\dot{y} = \nabla h \cdot \dot{x} = \nabla h \cdot (f + gu)$$

$$\dot{y} = L_f h(x) + L_g h(x) u$$

- Keep differentiating r times until coefficient of u is nonzero

Input-Output Linearization (SISO)

$$u = \frac{1}{L_g {L_f}^{r-1} h}\left(-{L_f}^{r} h + v\right) \Rightarrow y^r = v$$

- Setting v to track rth derivative of some reference signal + linear combination of errors in all lower order derivatives guarantees convergence on the condition that $y(0) = y_d(0)$

$$v = {y_d}^{r} - k_{r-1}\left(y^{r-1} - {y_d}^{r-1}\right) - \ \dots - k_0\left(y - y_d\right)$$

Extension to MIMO

- Method can be extended to multiple inputs/outputs

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_{g_j} L_f^{r_i - 1} h_i u_j$$

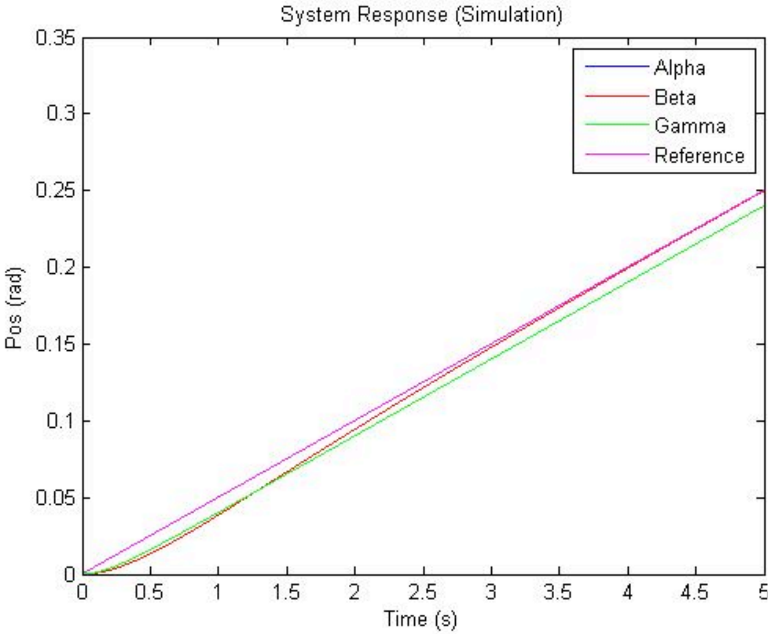
$$\begin{bmatrix} y_1^{(r_1)} \\ \cdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1 \\ \cdots \\ L_f^{r_m} h_m \end{bmatrix} + Eu \qquad \Rightarrow \qquad y_i^{r_i} = v_i$$

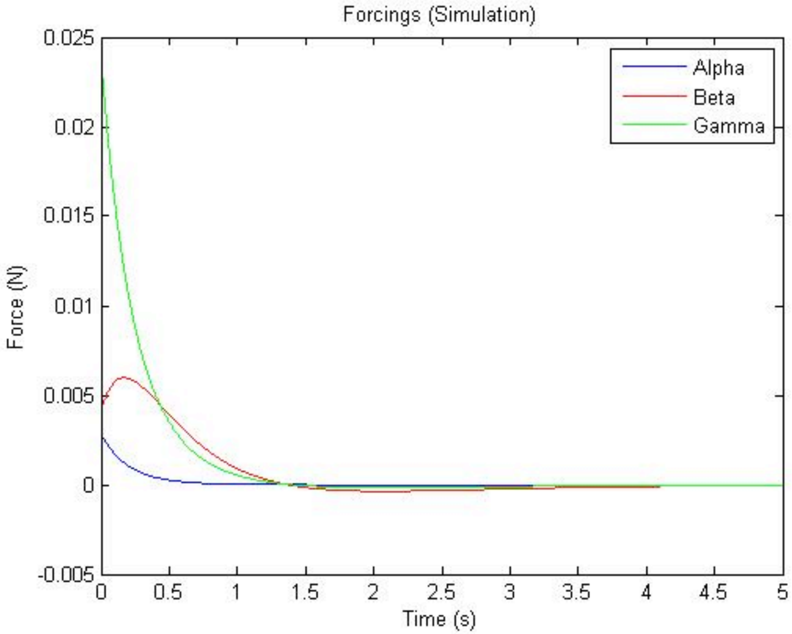
$$u = E^{-1} \left[\begin{bmatrix} v_1 \\ \cdots \\ v_m \end{bmatrix} - \begin{bmatrix} L_f^{r_1} h_1 \\ \cdots \\ L_f^{r_m} h_m \end{bmatrix} \right]$$

- 1 to 1 mapping of v to y again subject to same condition
- E is called the decoupling matrix
- IO Linearization controllers are thus sensitive to changes in the dynamics

Simulation Results

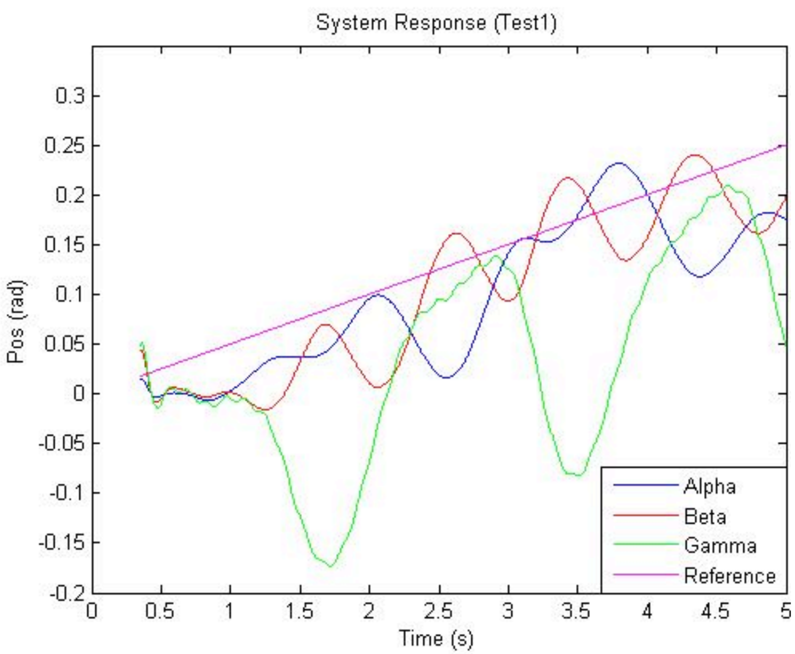
- Tested controller with an ideal model in MATLAB and Simulink software
- Ramp Input of constant velocity .05 rad/s

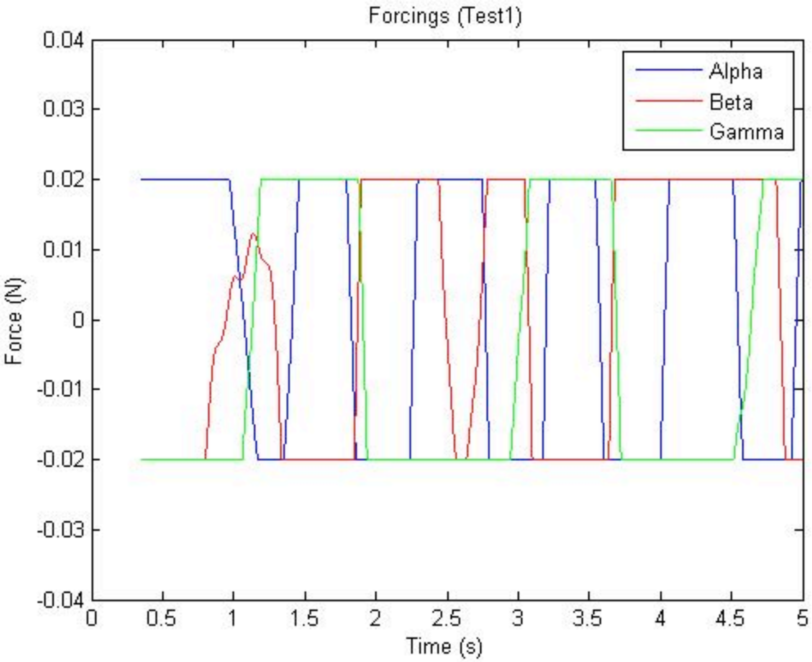




- Works well in the ideal case

Experimental Results

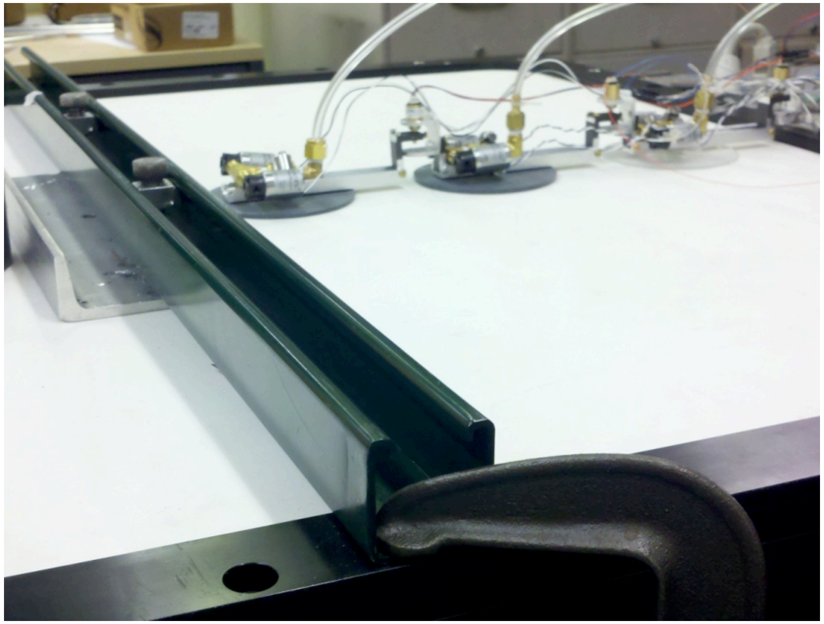




- General trend followed for .25 rad or 5s before disturbance from tubing tension becomes too large

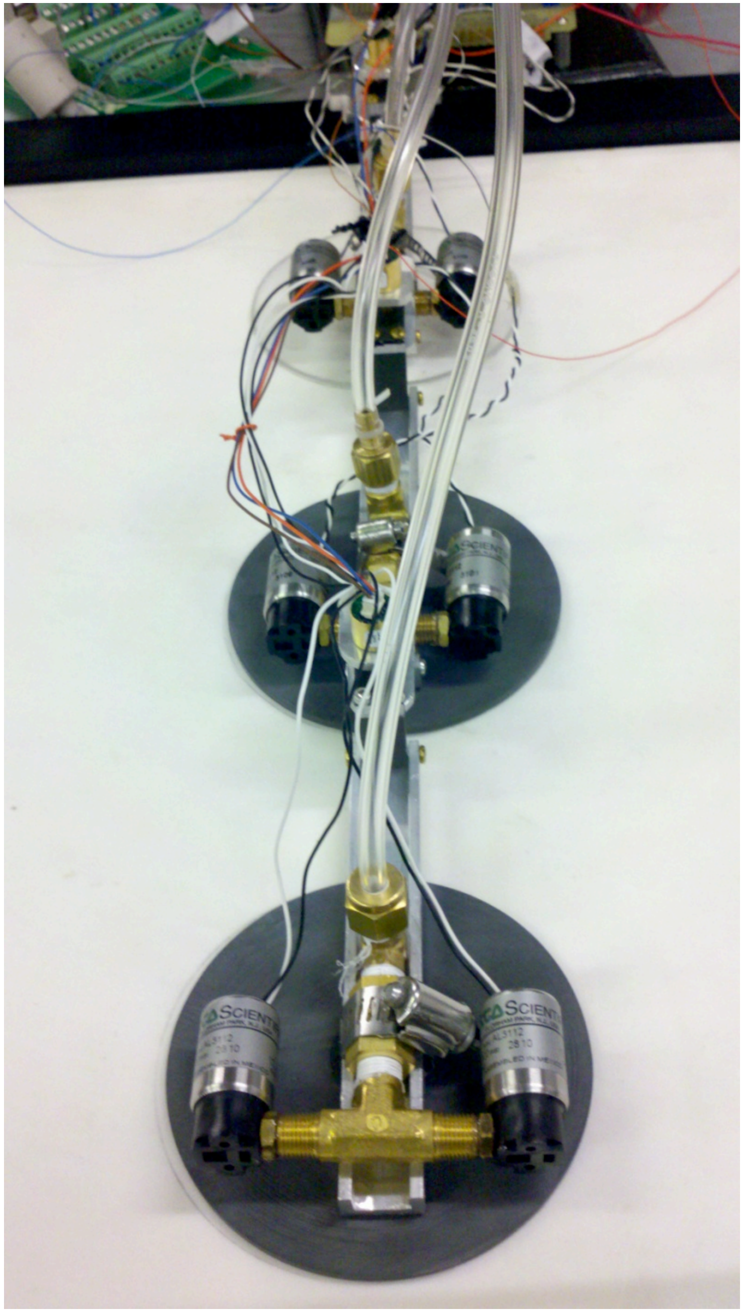
Sources of Error

- Table Curvature



  - Primary: Difficult to predict disturbance
  - Secondary:
    - Clamp used to limit this effect restricts space
    - Three arm structure is shorter than old structure

- Tension in Tubing and Wire
  - Disturbance becomes comparable to thruster forces past small angles
  - Further arms = more disturbance



Conclusions

- Input-Output Linearization produces good reference tracking for MIMO nonlinear systems
- External disturbances in experimental setup difficult to correct physically
- Controller is very sensitive to external disturbances

Further Exploration

- Implement second control loop to reduce external disturbances

- Improve experimental setup
  - Fix table curvature/Use new table
  - Reduce cable drag

- Reduce Noise
  - Kalman Filter

Thanks to:

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