Investigation of Langmuir-probe-characteristic behavior at high bias frequency

Sarah Newbury
Harvard University

Samuel Cohen
Princeton Plasma Physics Laboratory

Jack Matteucci
Princeton University

(Dated: September 10, 2015)

Langmuir probe characteristics recorded for magnetized DC hollow-cathode neon plasmas yield increasing values for electron temperature and density with increasing frequency of the applied probe bias voltage. The extent to which stray capacitance to the probe plays a role was investigated. Langmuir probe characteristics were recorded at frequencies ranging from 10 Hz to 8 kHz while the applied voltage to the probe was sawtooth modulated thus the rate of change of the bias voltage is constant. From these characteristics, the plasma temperature and density, as well as estimates for the capacitance in the wires, were calculated. The consistency and accuracy of these calculations were analyzed and found to indicate that the stray capacitance is the major factor contributing to the effects observed at higher bias frequencies. Similar experiments were performed with the PFRC-2 Langmuir probe apparatus, and the results of these experiments confirm the significant role of the stray capacitance. Contributions from electron thermalization, ion transit time, Debye length/probe radius, gyroradius, and displacement current at high bias frequencies have also been considered, but were found to be unlikely to produce the observed effects.

I. BACKGROUND

The Langmuir probe is a diagnostic widely used in low temperature plasma to determine plasma density and temperature. However, temperature and density measurements are found to be anomalously high when the bias voltage is swept at an excess of 500 Hz, both when compared to simple estimates and low frequency bias sweeps. In their investigation, Matteucci and Taunay have considered the effects of the bias frequency on electron thermalization, ion transit time, Debye length/probe radius, gyroradius, and displacement current. They have concluded that these effects are unlikely to result in the observed measurements of increased plasma parameters at high bias frequencies [1].

This paper investigates the possibility that the stray capacitance in the circuit connecting the probe tip to the power supply leads to these anomalous high-frequency measurements. As the electric potential of the wires in the circuit varies with time, the measured current deviates from the actual current at the probe by an amount given by the relation $I = C * \frac{dV}{dt}$, where C is the stray capacitance of the circuit. Since this deviation scales with the bias frequency, it is a good possible candidate for explaining the irregular measurements made with the Langmuir probe at high bias frequencies. Furthermore, it may be possible to eliminate the effects of this deviation by varying the bias voltage to the probe according to a sawtooth modulation and recording characteristics during one half period of the voltage sweep, such that $\frac{dV}{dt}$ is constant while recording the characteristic. This process would apply a constant, measurable offset to the probe current.

A. Langmuir Probe Analysis

Various plasma parameters can be obtained from the analysis of a Langmuir probe characteristic. Specifically, the plasma potential and the ion saturation current are measured via the IV characteristic, and subsequently the electron temperature and density can be calculated using relations established by probe theory. Additionally, the current offset measured between characteristics with positive and negative $\frac{dV}{dt}$ can be used to estimate the capacitance in the circuit connecting the probe tip to the power supply. See the Appendix for a description of these processes.

The characterization of the ion current is a particularly subtle part of cylindrical Langmuir probe analysis. Following the expression given for the ion current for a cylindrical probe in D.K. Owens' lecture on Langmuir probes [2], a square root function was used to estimate the ion current. The major assumptions upon which this expression is based include thermal distribution of electrons, neutral plasma outside the sheath, cold ions, and a probe radius comparable to the Debye length. All of these conditions reasonably apply to the regime in which measurements were made for this project.

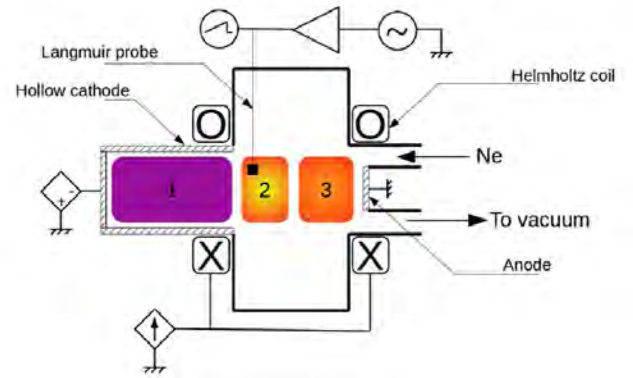

FIG. 1. Schematic of experimental apparatus. The numbered and colored areas represent different regions of the plasma. The stray capacitance that this project investigates is in the probe and the wires that connect the probe to the oscillator.

II. EXPERIMENTAL METHOD AND APPARATUS

In order to assess the possible contribution of the stray capacitance to discrepancies between IV characteristics taken at low and high bias frequencies, characteristics were recorded at a range of bias frequencies. Each characteristic was taken for identical conditions, and during each, $\frac{dV}{dt}$ was held constant.

Identical sets of measurements were taken with both a planar and cylindrcal probe inserted into a hollow cathode DC Neon discharge, with no applied radio frequency or magnetic fields. A schematic of the apparatus which was utilized to measure the bias voltage and the current flowing through the probe is shown in Figure 1. A Kepco BOP served as the AC voltage source, and the current was measured using a current probe. The characteristics recorded by the current probe were sent to an oscilloscope, which averaged them 100 times.

For each set of measurements, the bias voltage to the probe was varied at 8 different frequencies according to a sawtooth modulation. Two characteristics were recorded at each frequency: one while the voltage swept up and another while it swept down. The applied voltage was 500 V $\pm$ 10 V and the pressure was 50 mT $\pm$ 1 mT. These characteristics were used to compute the plasma temperature and electron density for different bias frequencies.

An additional set of measurements was recorded with the planar probe, for the purpose of estimating the stray capacitance in the circuit. The voltage was swept at 4 different frequencies across the ion saturation region of the characteristic, and data was recorded for the duration of one full period of the voltage sweep. Two sets of data were recorded at each frequency. For these measurements, the applied voltage was 450 V and the pressure was 20 mT.

Measurements were also recorded with the probe inserted into a PFRC-2 helicon plasma of Hydrogen gas. For these measurements, the bias voltage to the probe was varied at 6 different frequencies, and the following 6 measurements were taken at each frequency: 1) the current measured when the circuit was disconnected, in order to account for the offset of the probe 2) the current measured when the bias voltage to the probe was held constant, to determine where to position the recorded characteristic on an IV plot 3) a Langmuir probe characteristic recorded while $\frac{dV}{dt}$ was held constant and positive; 4) a characteristic recorded while $\frac{dV}{dt}$ was held constant and negative; 5) a characteristic recorded with nonconstant $\frac{dV}{dt}$ (bias voltage oscillated sinusoidally); 6) the ion saturation region of a characteristic recorded during one full period of the bias voltage with sawtooth modulation, for the purpose of computing the stray capacitance. For these measurements, the RF power to the plasma was 140 W. An external magnetic field was applied in the z-direction by sending current through a set of main magnetic coils, while a magnetic mirror configuration was produced by sending current through a set of nozzle coils, as illustrated in Figure 2. The current to the main magnets was 90 A and the current to the nozzle coils was 300 A. The pressure in the main chamber (left chamber in figure, where plasma is generated) was 1.77 mT, and the pressure in the expansion region (right chamber in figure, where probe is located) was 1.42 mT.

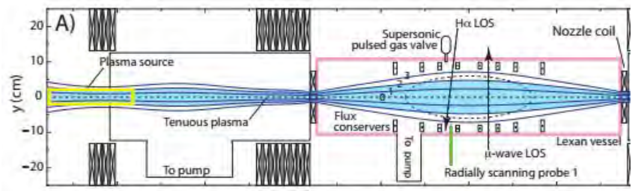

FIG. 2. The chamber on the left is the main chamber, where the plasma was generated. A magnetic mirror configuration was created in the subsequent chamber, known as the expansion region, through the passage of current through the nozzle coils on either side of this region. The probe used to take measurements for this project is labeled "radially scanning probe 1".

III. RESULTS AND DISCUSSION

A. Hollow Cathode DC Discharge

The plasma temperature was calculated based on each characteristic recorded at a given bias frequency with a given sign of $\frac{dV}{dt}$. For all calculated temperatures, the corresponding uncertainty for the calculation was determined from both the uncertainty of the linear fit to the log of the electron current provided by MATLAB, and the uncertainty inherent in selecting at which voltage the measured current is equal to the ion saturation current.

The results for the planar probe are plotted in Fig-

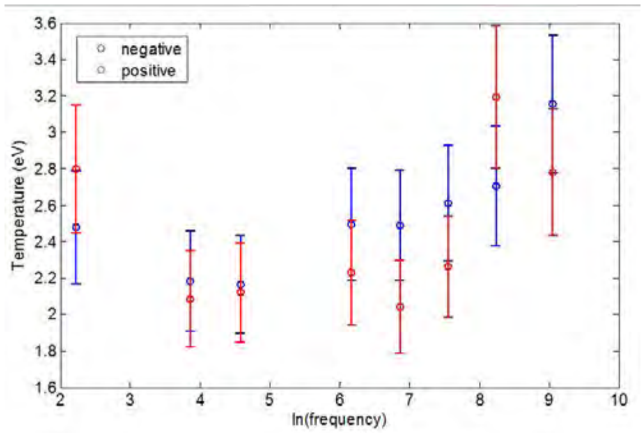

FIG. 3. Plot of the calculated temperature based on characteristics recorded with the planar probe in a hollow cathode DC discharge versus the natural log of the bias frequency at which it was recorded. 'Negative' and 'positive' refer to the sign of the rate of change of the bias voltage while the characteristic was recorded.

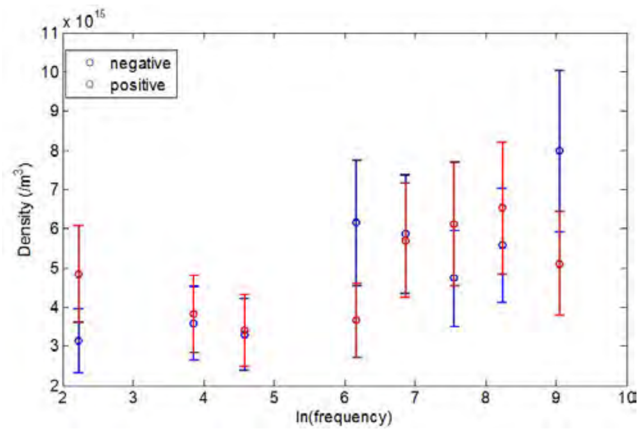

FIG. 4. Plot of the calculated electron density based on characteristics recorded with the planar probe in a hollow cathode DC discharge versus the natural log of the bias frequency at which it was recorded. 'Negative' and 'positive' refer to the sign of the rate of change of the bias voltage while the characteristic was recorded.

ure 3. The plasma temperature remains within a range of about 1.2 eV across different bias frequencies. This level of variation in temperature is negligible for the purposes of this project, which aims to explain jumps by an order of magnitude in the temperature at high bias frequencies, similar to those observed by Matteucci and Taunay. Furthermore, the variation is within the uncertainty bars of all the measurements. Therefore, these results serve as a good indication of the prominent role of the capacitance in the discrepancies observed at high bias frequencies. Additionally, the signal generator used to control the variation of the bias voltage failed to produce a perfect sawtooth wave at high frequencies, which is likely the cause of the slight increase in temperature observed at the highest bias frequency.

The corresponding electron densities for each recorded characteristic are plotted in Figure 4. Relative consistency is again observed across the different bias frequencies, for the variation of the electron density is within the uncertainty bars of all the measurements. It is possible that slightly higher densities are observed at higher bias frequencies, but the magnitude of the uncertainties of the measurements makes this trend unclear. The relative consistency of the calculated electron densities does not constitute as strong evidence for the large role of the capacitance at high frequencies as the consistency of the temperatures does, because consistent densities across different bias frequencies were observed even when the effects of the capacitance were not eliminated [1].

The analogous results for data taken with the cylindrical probe are shown in Figures 5 and 6. The temperatures again remain relatively consistent across different bias frequencies, with the notable exception of the highest frequency, 7.85 kHz, with positive $\frac{dV}{dt}$. This discrepancy is most likely an effect of either the malfunction of the signal generator at high frequencies, an error in the analysis performed, or some plasma mechanism.

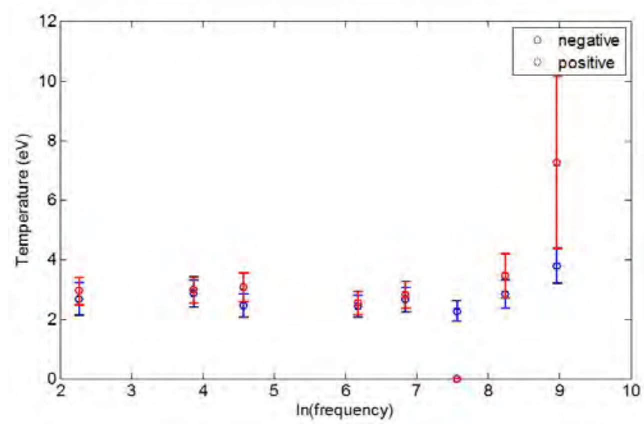

FIG. 5. Plot of the calculated temperature based on characteristics recorded with the cylindrical probe in a hollow cathode DC discharge versus the natural log of the bias frequency at which it was recorded. Data point at 1.92 kHz and positive $\frac{dV}{dt}$ set to zero because the data wasn't properly recorded.

The calculated electron densities remain relatively consistent with the exception of those calculated for the two highest frequencies. The possible causes of this discrepancy are identical to those listed above for the anomalously high calculated temperature.

Estimates for the capacitance of the wires were also computed based on data taken with the planar probe inserted into the hollow cathode DC discharge. The results are shown in Figure 7. The accuracy of these estimates may be evaluated by comparing them with the value of around 310 pF, the capacitance of the wires measured with a capacitance meter. However, small deviations of the measured results from this value should be expected due to the fact that the wires in conjunction with

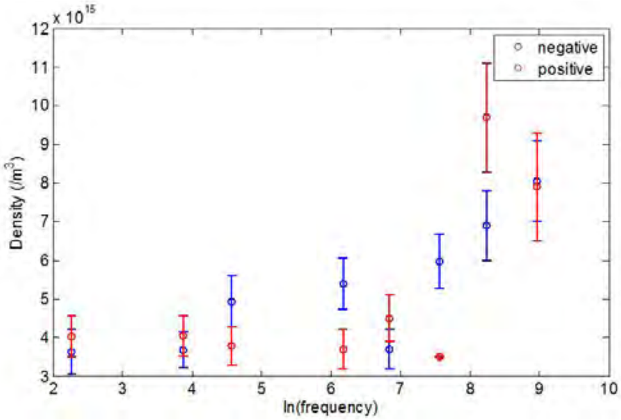

FIG. 6. Plot of the calculated electron density based on characteristics recorded with the cylindrical probe in a hollow cathode DC discharge versus the natural log of the bias frequency at which it was recorded. Data point at 1.92 kHz and positive $\frac{dV}{dt}$ arbitrarily set to $3.5 * 10^{15}$ because the data wasn't properly recorded.

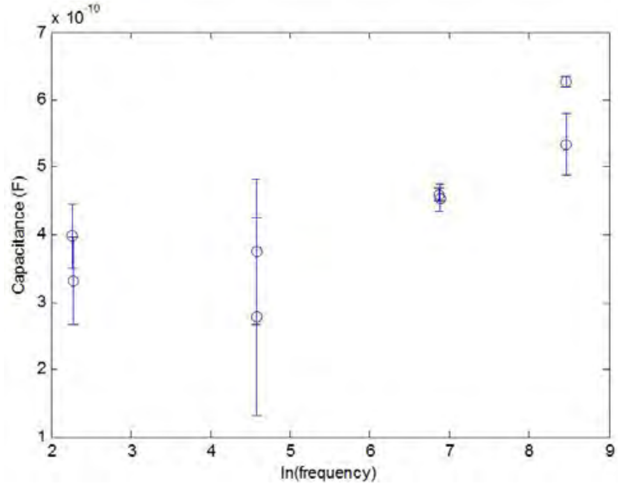

FIG. 7. Plot of the stray capacitance calculated based on data recorded with the planar probe in a hollow cathode DC discharge versus the natural log of the bias frequency at which it was recorded.

the probe apparatus may carry additional capacitance to that which is measured with the meter in the wires alone. Deviations in the measured values could also have arisen from error in the meter, or small drift of the current probe during one sweep of the voltage. The slight increase in the capacitance at the highest frequency is reminiscent of the similar effect observed for plasma temperature and electron density, and has the same possible causes.

The uncertainty for each estimate was determined based on the uncertainty inherent in selecting the voltage at which the difference between the two corresponding values of current was computed, and on the noise in the measured current. The larger uncertainties of the estimates made at lower bias frequencies are likely a result of the larger proportion of the noise to the shift in measured current due to the change in the sign of $\frac{dV}{dt}$, since this shift is smaller at lower bias frequencies.

The relative consistency and accuracy of these results provide further evidence to support the notion that the capacitance affects Langmuir probe characteristic behavior at high bias frequencies.

B. PFRC Helicon Plasma

Values for the plasma temperature and electron density were calculated for each characteristic recorded at a given frequency. The results are shown in Figures 8 and 9, with the 'up' and 'down' values corresponding to a sawtooth-modulated bias voltage and constant positive and negative $\frac{dV}{dt}$, respectively, while the characteristic was recorded; and the 'sine' values corresponding to sinusoidal modulation of the bias voltage and non-constant negative $\frac{dV}{dt}$ while the data was recorded.

The 'up' and 'down' values for the plasma temperature are relatively consistent across different frequencies, approximating 2 eV. However, the 'sine' values are not as consistent, ranging from 2-6 eV with no clear trend. Furthermore, at frequencies of around 495 Hz ($ln(frequency) \approx 6.2$) and above, the 'sine' values are consistently higher than both the 'up' and 'down' values. The uncertainties for these values were determined based on the same criteria used for the data taken with the probe in the hollow cathode DC discharge.

These results provide clear evidence that the capacitance largely contributes to anomalous measurements at high bias frequencies. The discrepancies between the 'up' and 'down' values and the 'sine' values are similar to those previously observed in Langmuir probe data [1], and must result from the capacitance, since the process of recording the 'up' and 'down' measurements adds only a constant offset to the current measured, eliminating the effects of the capacitance on the analyzed data.

The corresponding 'up' and 'down' values for the electron density were also relatively consistent, approximating $1.2 * 10^{16}/m^3$. The 'sine' values are likewise consistent, with the exception of the two values corresponding to bias frequencies of 900 Hz ($ln(frequency) \approx 6.8$) and 1900 Hz ($ln(frequency) \approx 7.5$), which are anomalously low.

Estimates for the stray capacitance of the system were computed based on the data recorded at each bias frequency with the cylindrical probe inserted into the PFRC-2 helicon plasma. The results are shown in Figure 10. The accuracy of the results may again be evaluated by comparing the values to the capacitance of the wires measured with the capacitance meter, which in this case was around 630 pF. Still, deviations of the measured results from this value should be expected for the same reasons mentioned in the discussion of the hollow cathode DC discharge data above.

The computed estimates for the capacitance are rela-

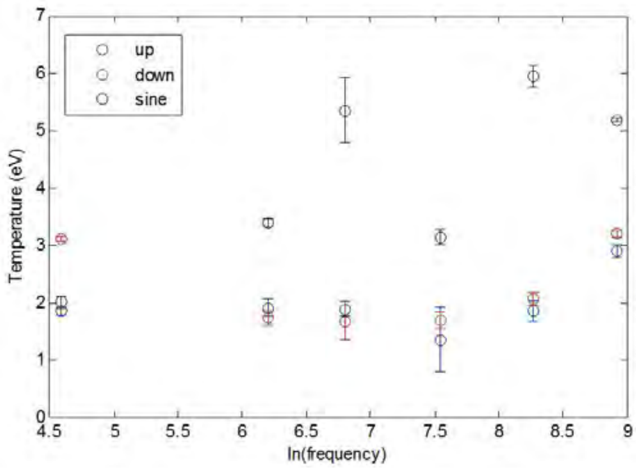

FIG. 8. Plot of plasma temperature calculated based on data recorded with the cylindrical probe in PFRC-2 helicon plasma versus the natural log of the bias frequency at which it was recorded.

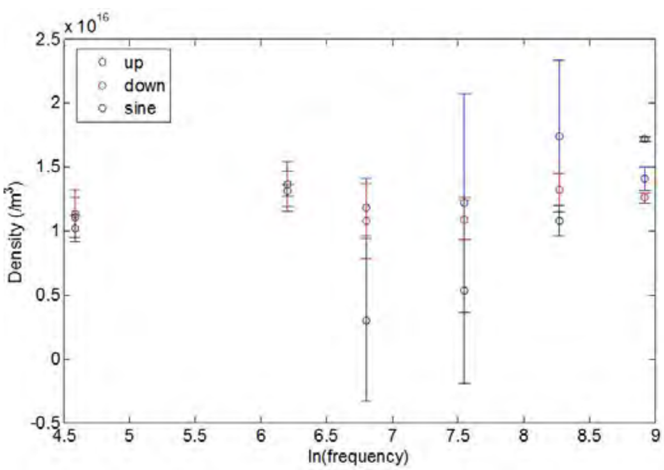

FIG. 9. Plot of electron density calculated based on data recorded with the cylindrical probe in PFRC-2 helicon plasma versus the natural log of the bias frequency at which it was recorded.

tively accurate and consistent across all bias frequencies but the lowest one, 98 Hz. Still, the high uncertainty corresponding to this estimate renders its deviation from the other values statistically insignificant.

Like the results from the hollow cathode DC discharge data, the overall consistency of these confirm that the capacitance is the major factor interfering with the measurement of the current at high bias frequencies, and thus producing the observed effects.

IV. CONCLUSION

An attempt was made to investigate the extent to which stray capacitance in the circuit contributes to the

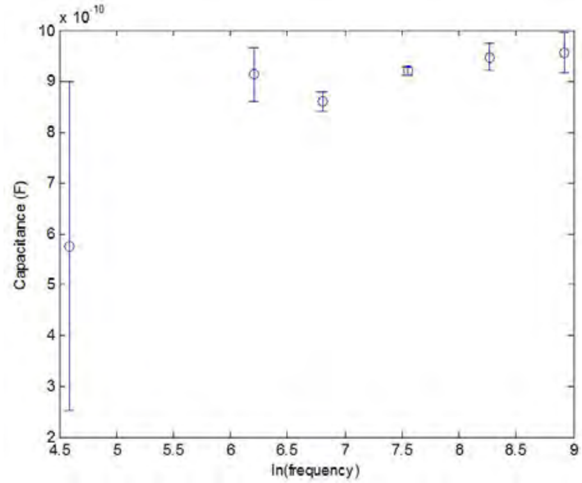

FIG. 10. Plot of stray capacitance calculated based on data recorded with the cylindrical probe in PFRC-2 helicon plasma versus the natural log of the bias frequency at which it was recorded.

discrepancies in plasma parameters observed at high bias frequencies. The bias voltage of the Langmuir probe was varied according to a sawtooth modulation, and characteristics were recorded while the rate of change of the voltage was held constant. Measurements were also taken that allowed for estimation of the stray capacitance of the measurement system. Data was recorded using both a cylindrical and planar probe inserted into a hollow cathode DC discharge, and a cylindrical probe inserted into the PFRC-2 helicon plasma.

For each set of data, the calculated temperatures and densities remained relatively consistent even as the bias frequency increased, indicating that the capacitance is a major factor contributing to the discrepancies previously observed at high bias frequencies. In particular, the significantly higher temperatures corresponding to data recorded with sinusoidal modulation of the bias voltage clearly illustrate how the capacitance can affect the analysis of data recorded at high bias frequencies.

The estimates for the capacitance of the system computed for each data set were accurate within a reasonable range and relatively consistent, providing further evidence to confirm our understanding of the manner in which the capacitance impacts current measurement.

The findings of this project provide evidence for the stray capacitance as an explanation of the discrepancies that have been observed in Langmuir probe characteristics at high bias frequencies. Furthermore, they show that this capacitance can be accounted for by recording characteristics while $\frac{dV}{dt}$ is held constant. These results are relevant to future work to be completed with the PFRC-2 Langmuir probe apparatus, for they suggest that accurate data can be recorded, even at high bias frequencies of up to a few kHz. This may allow for data to be taken with the probe while the rotating magnetic field (RMF) is applied, making available a regime that

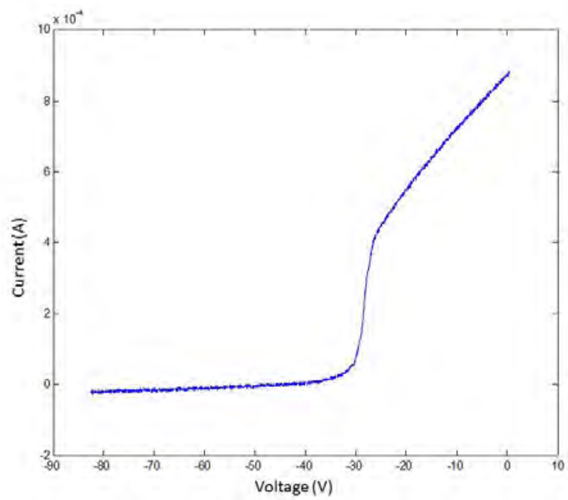

FIG. 11. Example of typical Langmuir probe characteristic.

has thus far been inaccessible to the probe.

V. REFERENCES

[1] Matteucci, J.V.H. and Taunay, P.-Y. C.R.*Effect of the bias frequency on Langmuir probe characteristic in a DC hollow cathode discharge.* 26 February 2015.

[2] Owens, D.K. *Lecture IV: Langmuir Probes.*

[3] Ruzic, David. *Electric Probes for Low Temperature Plasmas.* American Vacuum Society. 1994.

VI. APPENDIX: DETERMINATION OF PLASMA PARAMETERS FROM LANGMUIR PROBE CHARACTERISTIC

A typical Langmuir probe characteristic is shown in Figure 11. Following convention, the current is defined to be that flowing from the probe into the plasma.

At the plasma potential, only electrons reach the probe because of their high mobility compared to ions. Thus, the plasma potential can be taken to be the potential at which the electron saturation region starts, and can be found by determining the point at which the second derivative of the characteristic is zero (the concavity of the characteristic changes from positive to negative at the start of the electron saturation region). Following this same reasoning, the electron saturation current is taken to be the current measured when the probe is at the plasma potential.

The absolute value of the ion saturation current can be determined from data recorded with a cylindrical Langmuir probe by utilizing the relation $i_- = \sqrt{\frac{M}{2*m*\pi}} * \frac{1}{j} * i_+$, where i_- and i_+ are the absolute values of the electron and ion saturation currents, respectively; M is the mass of the ion; m is the electron mass; and j is a factor determined by the radius of the probe and the Debye length, which varies for different bias voltages and plasma temperatures [2]. For these calculations, j was determined based on the numerically calculated values given in David Ruzic's *Electric Probes for Low Temperature Plasmas* [3]. This approximation for the j factor is valid for this experiment because the probe radius is comparable to the Debye length. A typical plasma density $n \approx 10^{15}\ m^{-3}$ and plasma temperature $T \approx 2$ eV yields $\sqrt{\frac{kT}{4\pi*ne^2}} \approx$ 0.33 mm for the Debye length, while the probe radius is 0.25 mm. From the assumption that the electron and ion saturation currents in the properly positioned characteristic are positive and negative, respectively, follows the relation: $i_{-r} - i_{+r} = (\sqrt{\frac{M}{2*m*\pi}} * \frac{1}{j} + 1) * i_+$, where i_{-r} and i_{+r} represent the recorded values for the electron and ion saturation currents in the raw characteristic. Thus, the absolute value of the ion saturation current can be determined from the raw characteristic.

For planar probe data, the analysis proceeds in a similar manner, with the sole difference being that the relation between the electron and ion saturation currents does not involve the j factor, but is simply given by $i_- = \sqrt{\frac{M}{2*m*\pi}} * i_+$.

A. Calculation of Plasma Temperature and Electron Density

In order to obtain an estimate for the plasma temperature based on a given recorded Langmuir probe characteristic, one must first determine where to position the characteristic on an IV plot such that it accurately represents the current flowing from the probe into the plasma, and does not include any other sources of current or offsets inherent in the equipment used. There are two different possible methods for determining the appropriate position of the characteristic that were used in this project. Both assume constant rate of change of the voltage during the time in which the characteristic is being recorded. One is to compute the ion saturation current using the method described above, and position the characteristic such that the current at the bias voltage for which the j factor was calculated is equal to $j * i_+$. The analogous process for planar probe data involves positioning the characteristic such that the minimum value of current recorded is equal to i_+. This method is potentially unreliable, especially for cylindrical probe data, since the value of j itself depends on the plasma temperature. For this project, this method was used in the analysis of the data recorded for the hollow cathode DC discharge. In the cylindrical probe data analysis, the j factor was determined based on the temperature derived from applying planar probe analysis to the data.

A more straightforward method for determining the proper position of the characteristic is to separately record the current measured while the voltage is held

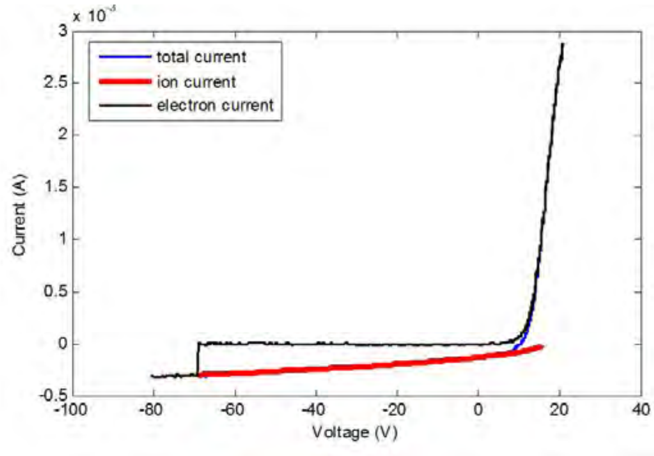

FIG. 12. The total current is the properly positioned characteristic. The ion current is the best fit of a square root function to the ion saturation region of the characteristic, which was here taken to range from -70 V to the plasma potential. The electron current is the difference between the total current and the ion current.

constant at some value, thus ensuring that there are no contributions from the stray capacitance. The characteristic is then positioned such that the current at that same voltage equals this recorded value. This method was used in the analysis of the data recorded for the PFRC-2 helicon plasma.

Once the characteristic has been positioned properly, the plasma temperature can be determined from the equation: $ln(I_e) = (\frac{1}{T_e}) * V + offset$, where I_e represents the current due only to electrons and the offset depends on the floating potential of the probe and the ion saturation current [2]. The electron current can be found by subtracting the ion current away from the recorded characteristic. For a cylindrical probe, the ion current is taken to be the best square root fit to the ion saturation region of the characteristic. This fit is based upon the assumption that the probe radius is comparable to the Debye length. The fit was made based on two coordinate pairs of voltage and current: one corresponding to the ion saturation current at the minimum voltage of the sweep, and the other to zero current at the plasma potential. The fit was determined in MATLAB. For a planar probe, the ion current is taken to be constant and equal to the ion saturation current. The plot of the natural logarithm of the electron current versus the voltage of the probe typically has a well-defined linear region. The plasma temperature is then found by fitting a line to this region, also using MATLAB, and taking the reciprocal of the slope of this line. Figures 12 and 13 exemplify this process.

Given the plasma temperature, the electron density of the plasma can easily be computed using the equation:

$$n_e = \frac{i_+}{e * A * \sqrt{\frac{Z * e * T_e}{M}}}$$

where A is the area of the probe and Z is the ionization

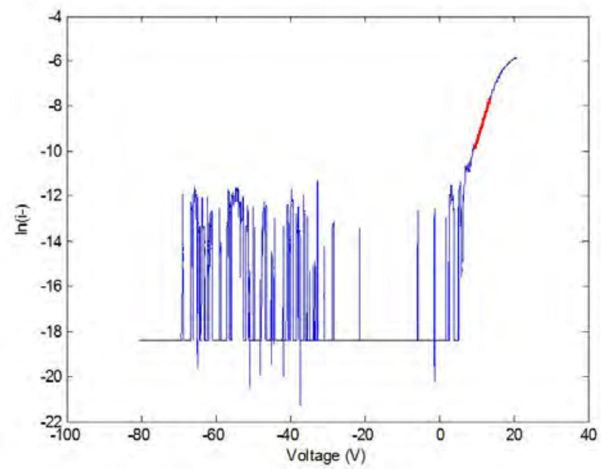

FIG. 13. Plot of the natural log of the electron current. The best fit line used to find the temperature is shown in red. The temperature was taken to be the reciprocal of the slope of this line.

number of the ions [2].

B. Calculation of Capacitance

The capacitance in the circuit connecting the probe tip to the power supply can be computed using data recorded during one full period of a sweep of the bias voltage, provided that the voltage is sawtooth modulated. Data recorded under these conditions will contain two values of current corresponding to each voltage. The difference between these two values must be equal to $2 * C * \frac{dV}{dt}$, and so the capacitance can be determined from the raw data by dividing this difference by twice the rate of change of the voltage. Figure 14 provides an example of the type of data that would be recorded to calculate the capacitance with this method.

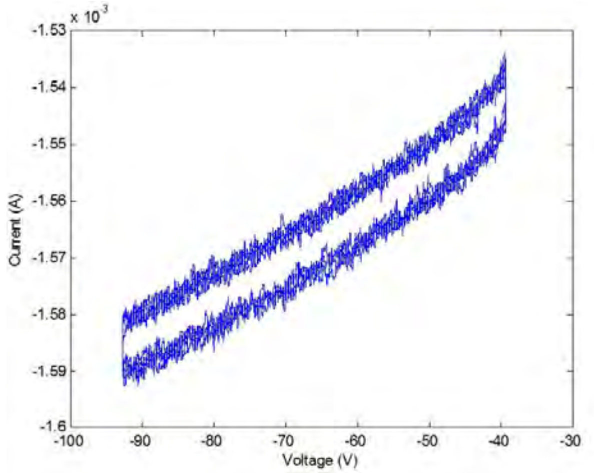

FIG. 14. Plot of the ion saturation region of a Langmuir probe characteristic, recorded during one full period of the sweep of a sawtooth-modulated voltage. The difference between two values of current recorded at the same voltage can be used to find the capacitance via $C = \frac{difference}{2 * \frac{dV}{dt}}$.