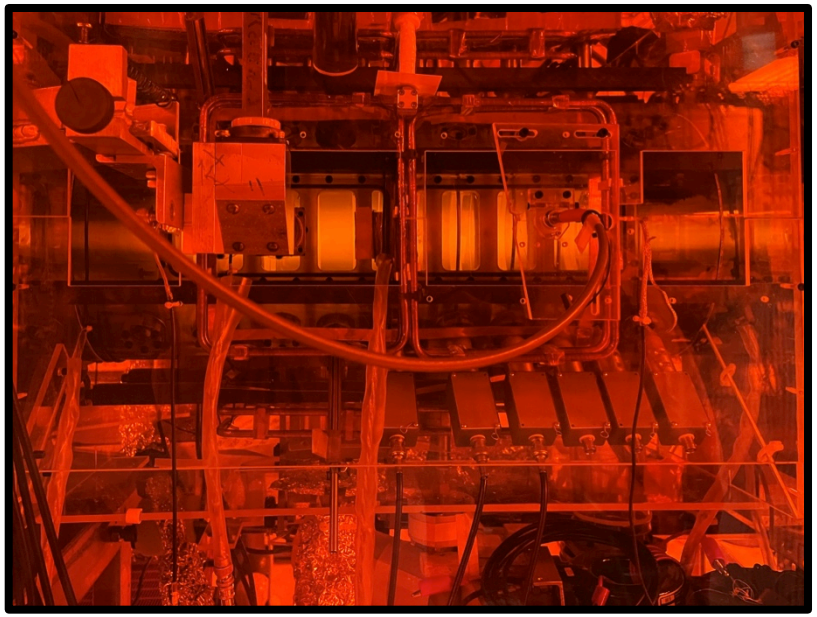
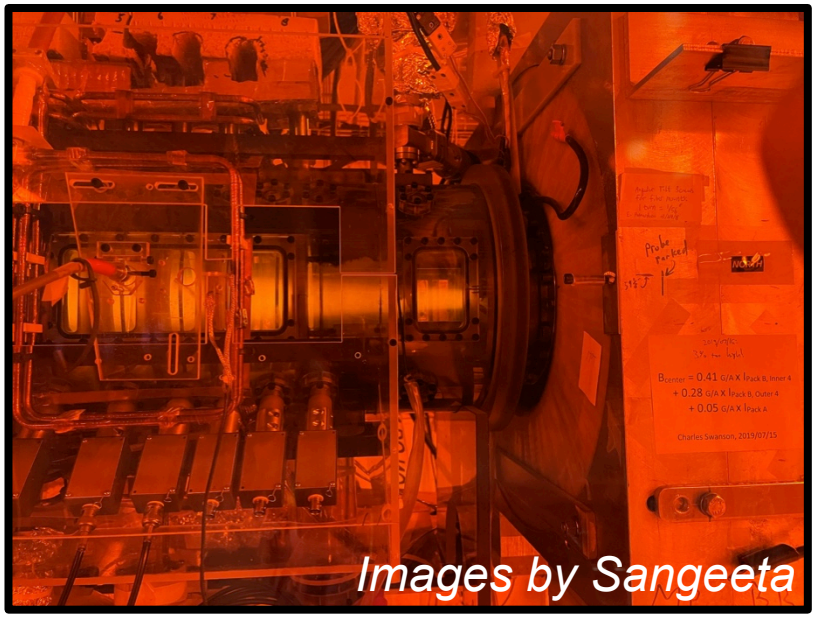
Weekly Updates 2

Thursday, June 23

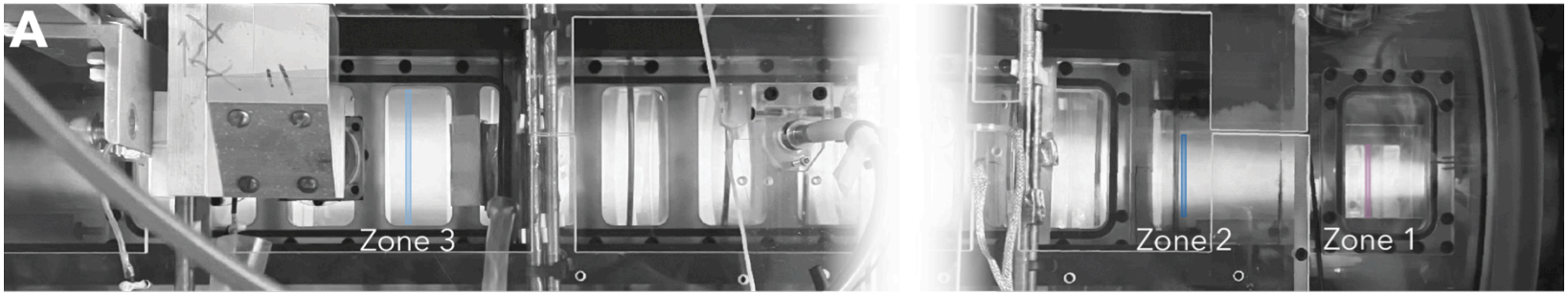
Devdigvijay Singh

Measuring Electron Densities via Abel Inversions

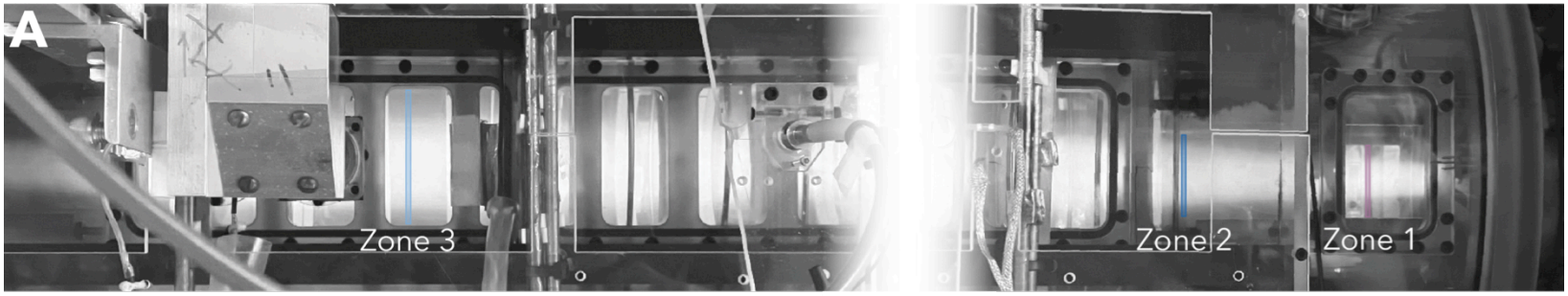




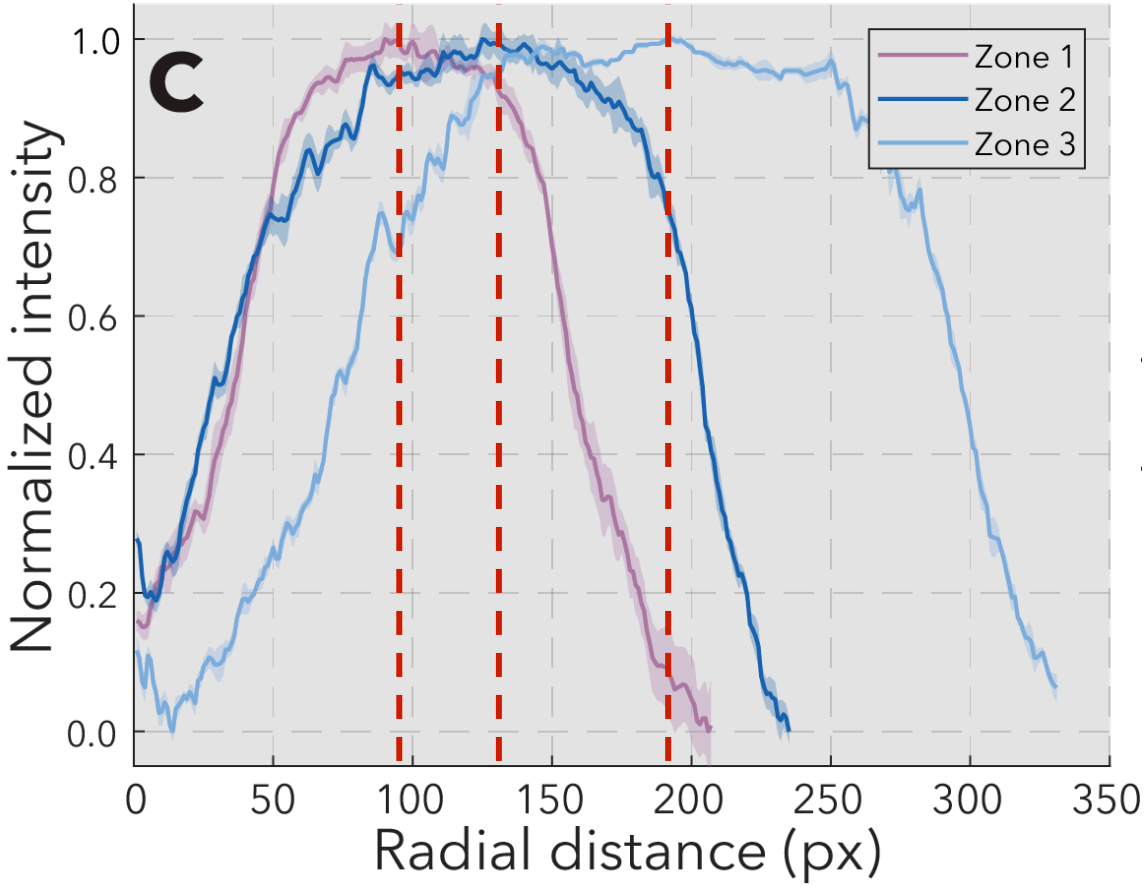
Use radial intensity distribution to infer bulk electron density distributions



Defining plasma zones

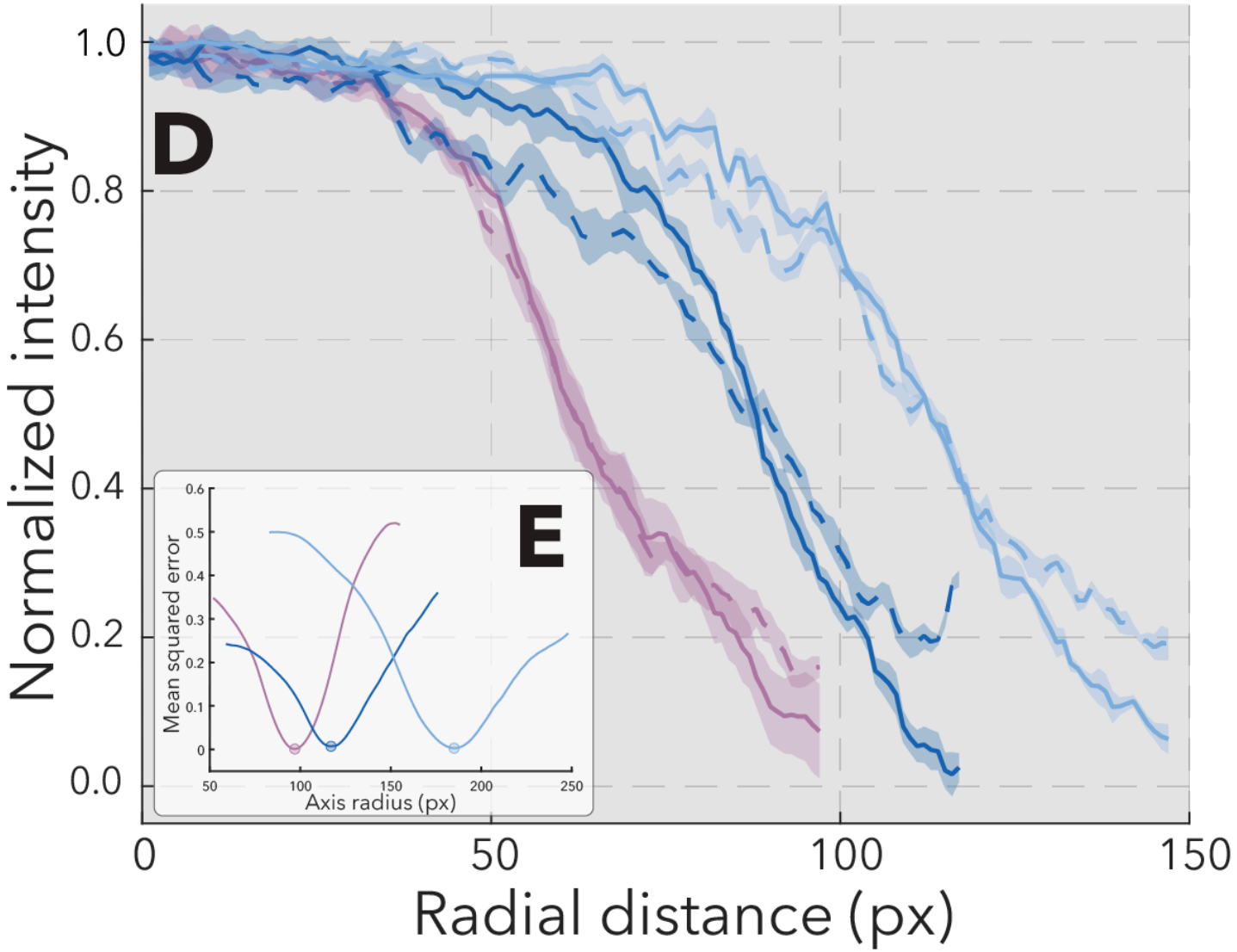


Intensity profiles

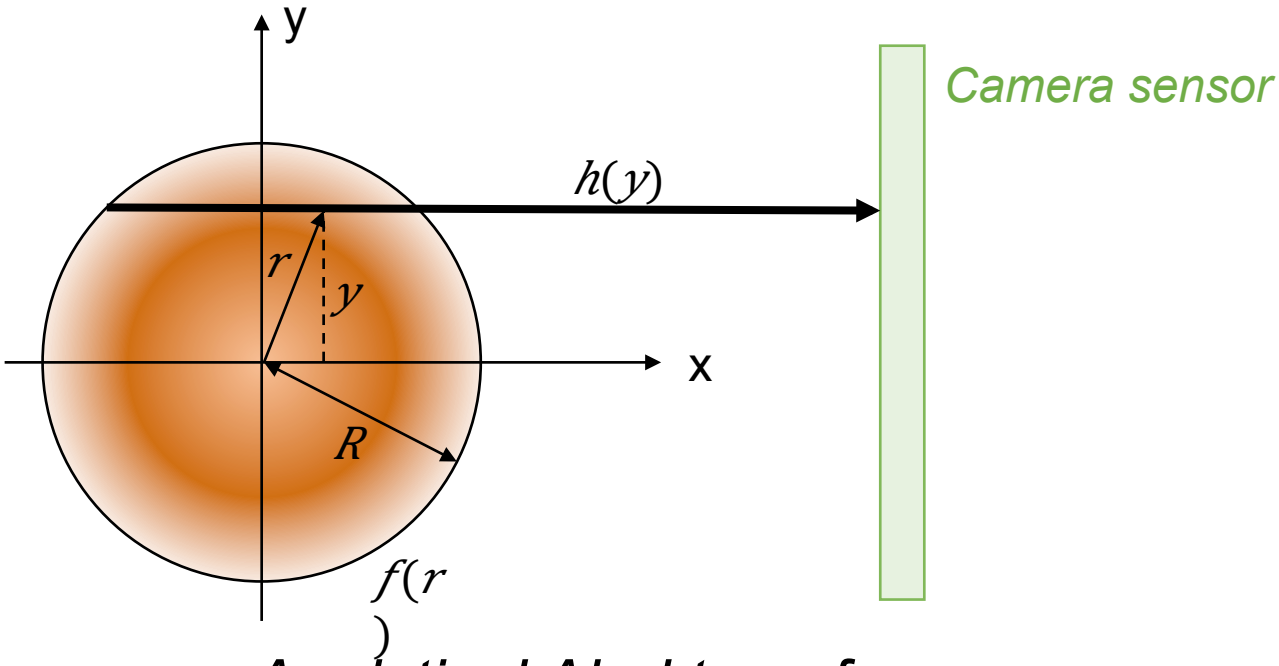


! *Abel transforms sensitive to errors in center location*

Determining plasma center axis



Analytical Abel Inversions



Analytical Abel transforms

$$h(y)=2\int_{y}^{R} f(r)r/\sqrt{r^2-y^2}\;dr$$

Forward

$$f(r)=-1/\pi \int_{r}^{R} dh(y)/dy\; 1/\sqrt{y^2-r^2}\;dy$$

Inverse

! *Difficult to take accurate derivatives of noisy data*

Numerical Abel Inversions

Analytical Abel transforms

$$h(y)=2\int_{y}^{R} f(r)r/\sqrt{r^2-y^2}\;dr$$ Forward

$$f(r)=-1/\pi \int_{r}^{R} dh(y)/dy\; 1/\sqrt{y^2-r^2}\;dy$$

Inverse

! *Difficult to take accurate derivatives of noisy data*

Cosine reconstruction (Pretzler, 1991)

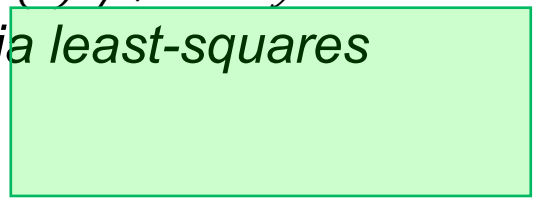
$$f_0(r)=1$$

$$f_n(r)=1-(-1)^n \cos(n\pi r/R)$$

$$H(y)=2\sum_{n=0}^{N} A_n \int_{y}^{R} f_n(r)r/\sqrt{r^2-y^2}\;dr$$

Reconstruction order 

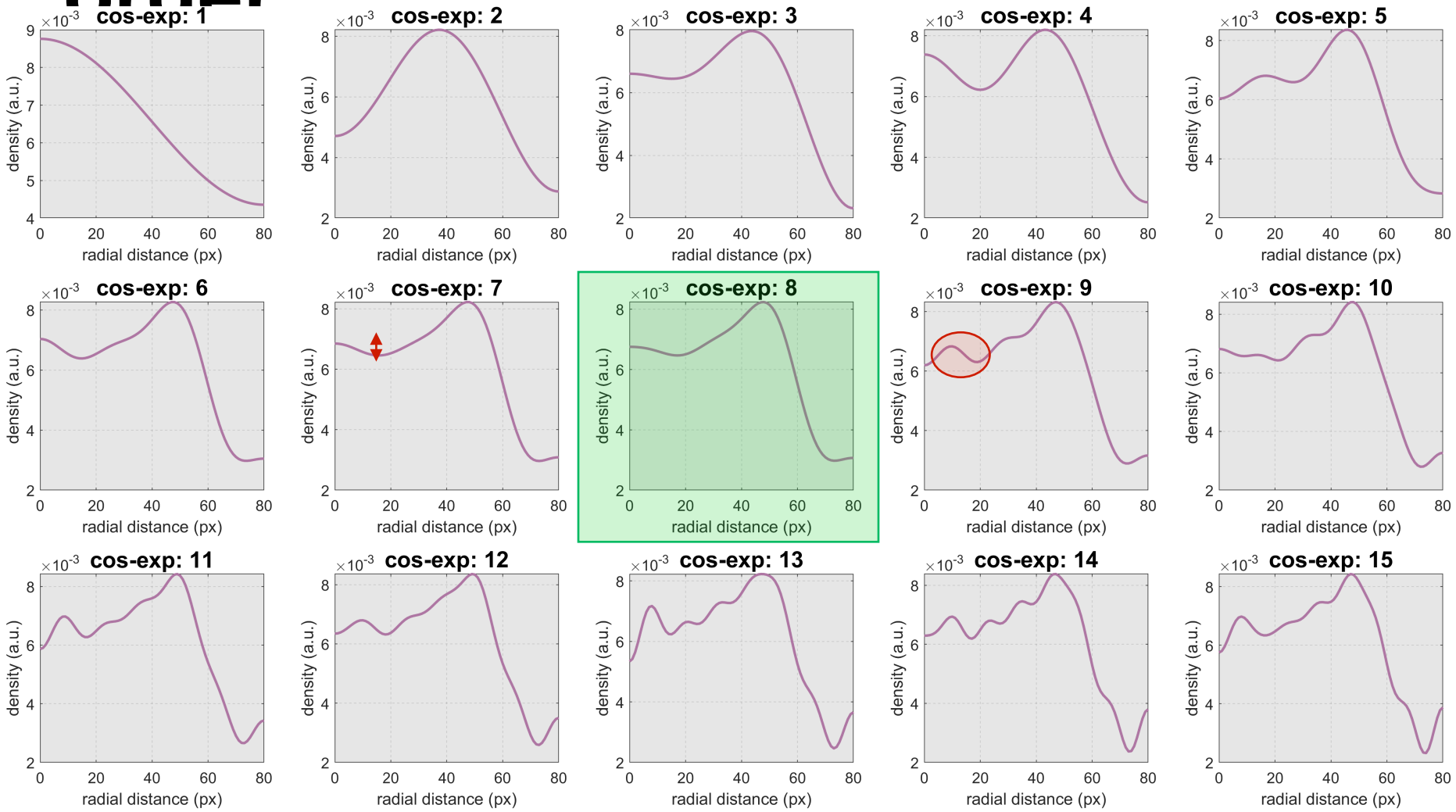
Fit A_n via least-squares



$h_n(y)$: precomputed integrals to reduce computational intensity

Choosing cosine expansion order

Tradeoff between accurate reconstructions and high-order artifacts



Monte Carlo Error Estimation

Current error estimates are based on number of observations and reconstruction order

Every data point has error
Abel inversions are highly non-linear based on individual datum

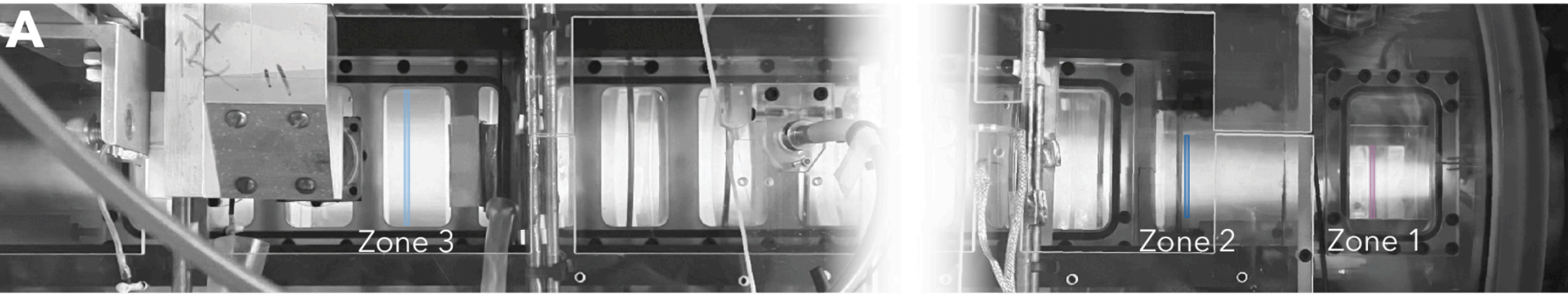
Use Monte Carlo methods to estimate error

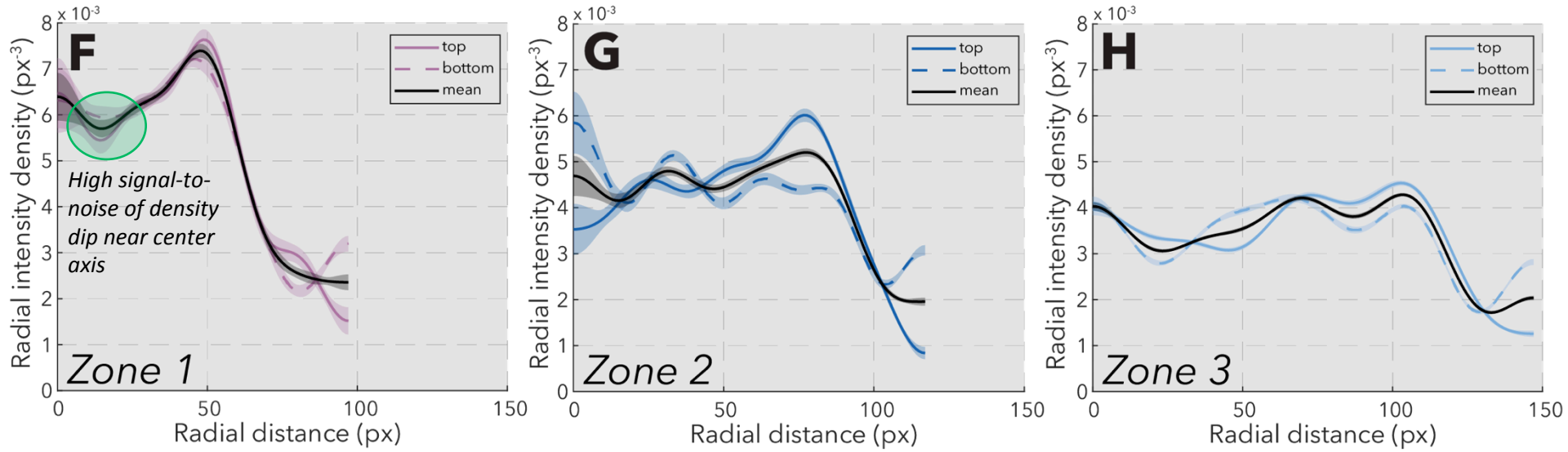
Algorithm routine:

```
for i = 1 to num_trials
    for j = 1 to num_data_points
        generate x↓j with a normal distribution using μ↓j and σ↓j
    compute and store Abel inversion

for each computed data point in f(r):
    compute mean and standard deviation as error estimate
```

Abel Inversion Results





Next steps: *Interpret results, use faster shutter speeds to reduce effects of plasma rotation and instabilities*