Summer Report for the PPPL Plasma Physics Internship

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Abstract

A Hamiltonian computer code was numerically verified for ions in a magnetic Field-Reversed Configuration (FRC) device. The computer code then was used to find times of heating for ions in a rotated magnetic field (RMF), check a formula for the change in magnetic moment, and study the paths of electrons through magnetic fields with and without the RMF. The time of heating for ions was measured for its dependence on the RMF phase and found to be uncorrelated. The change in magnetic moment during one non-adiabatic "bounce" of the ion was found to be sinusoidally dependent on the launch pitch and phase angles as predicted by Chen's formula, but the energy was not found to have the predicted exponential dependence. In the study of electron entrance and containment within the magnetic fields, electrons launched into the FRC configuration had a greater entrance rate without the RMF than with it.

I. Introduction

Theoretical calculations were used to check a computer code for particle trajectories in a rotating magnetic field. The Hamiltonian computer code, $RMF_{1.13}$, designed by Alan Glasser and Samuel Cohen allows the integration of six non-linear differential equations in order to study ion heating and trajectories in a three-dimensional Solov'ev field-reversed configuration. [4]

The Larmor radius and gyrofrequencies of the ions, and the drift velocities of ions were calculated using the magnitude of the magnetic field and compared against values measured on the code's graphical output. The Larmor radii and cyclotron gyrofrequencies were measured over a range of five different energies and five different magnetic fields, and the magnetic field was checked against the equations for the Hill Vortex fields. [4] Measurements of the drift velocities of ions due to curvature of the magnetic field and the gradients in the magnetic field were similarly compared to calculations using the measured magnetic field.

The time to heat ions to 1 KeV and 5 KeV was measured for different values of the RMF phase. Ion heating gives an indication of the efficiency of the magnetic field configuration to produce fusion-like conditions, around 10 KeV [4]. The ions were started at 100 eV and after a waiting period, the ion would rapidly gain kinetic energy due to scattering with the field gradients and magnetic nulls. [4] By testing for the dependence on the RMF phase, the uniformity and average of the time of heat could be measured.

The change in the magnetic moment, $\Delta\mu$, for ions in one reversal of direction in the FRC device was analyzed for the ϕ, θ and energy dependence. The quantity, $\mu = mv_{\perp}^2/2B$, is an adiabatic invariant when the rate of change of the magnetic field is not small compared to the gyrofrequency of the particle. [2] Here, the adiabaticity is violated when the particle reverses direction since the particle sees a large change in B. The electron is launched with θ around 45^o towards the end of the FRC device. The initial perpindicular energy is gradually converted to parallel energy as the magnetic field decreases, but as the electron turns the magnetic field sharply increases. Birmingham and Chen show that $\Delta\mu$ is dependent on the sine of both θ and ϕ at the launch, since both variables will affect the electron's phase in its gyrofrequency as it reaches the turn.

Electrons launched into Hill Vortex magnetic fields with and without rotating magnetic fields were tracked for containment in the FRC device. A large energywill launch the electron right through the device, but by varying the different launch parameters a lower energy will contain the electron for longer times. The Hill-Vortex fields used in the code have cusps, where the magnetic field extends outwards, making a convenient exit path for the electrons.

II. Numerical Comparisons

To check the computer code, the Larmor radius, gyrofrequencies and the grad B and curve B drifts were both measured on the code's graphical output and theoretically computed. I checked the Larmor radius for 10 cases by launching the ion at different radial distances and checking that the magnetic field matches the computed value.

The Larmor radius is defined as $v_{\perp}mc/qB_G$ which can be rewritten as

$$r_L = 1.02 * 10^2\sqrt{(\mu E)}/ZB \tag{1}$$

where the energy, E, is in electron volts and the magnetic field is in gauss. For an energy of 100 eV, at a radius of 8.0 cm, the Larmor radius is computed to be 0.21 cm, while a measurement of the code's output graph gives

Table 1: The benchmark check for the Larmor radii.

Energy	Radius	Measured Magnetic Field	Measured Larmor Radius	Calculated Larmor Radius	Error in Measurement
25	2.13	18250	0.035	0.040	0.001
50	2.13	18200	0.055	0.056	0.001
75	2.13	18200	0.070	0.068	0.002
100	2.13	18200	0.080	0.079	0.002
150	2.13	18150	0.095	0.097	0.002

0.25 cm. A table is shown below with the computed and measured values at different energies and radii. The gyrofrequencies of ions also were benchmarked against the known formula, $\Omega = qB_G/mc$, or in terms of the Larmor radius, $\Omega = v/r_L$. At 150 eV and 16000 gauss, the measured gyrofrequency was $7.99*10^7 \pm 0.40*10^7$ rad/sec while the calculated frequency was $7.66*10^7$ rad/sec. The magnetic field was measured at the midplane, where the magnetic field is in the z-direction, with $B_z = -B_0(1-r^2/r_s^2)$ [4].

Numerical drift velocity calculations near the z = 0 plane in a Hill vortex magnetic field were made using a Hamiltonian computer code and compared with traditional drift velocity formulas. The drift velocities due to the curvature of the magnetic field and the magnetic gradient are found with the following formulas. [2]

$$\mathbf{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2} \tag{2}$$

$$\mathbf{v}_r = \frac{mv_\parallel^2}{qB^2}\frac{\mathbf{R}_c\times\mathbf{B}}{R_c^2} \tag{3}$$

where R_c is the radius of curvature. The magnetic gradient is calculated looking at the dependence of the magnetic field on radius, which can be measuredby looking at the average change of the magnetic field over all the ion's trajectories within the run.

$$\nabla B = \frac{\partial B}{\partial r} = \frac{\Delta B}{\Delta r} \tag{4}$$

The accuracy of the measured drift velocities to the calculated ones are shown in Fig. 1 below. The drift velocity, $\Delta\phi/\Delta\tau$, was measured for the

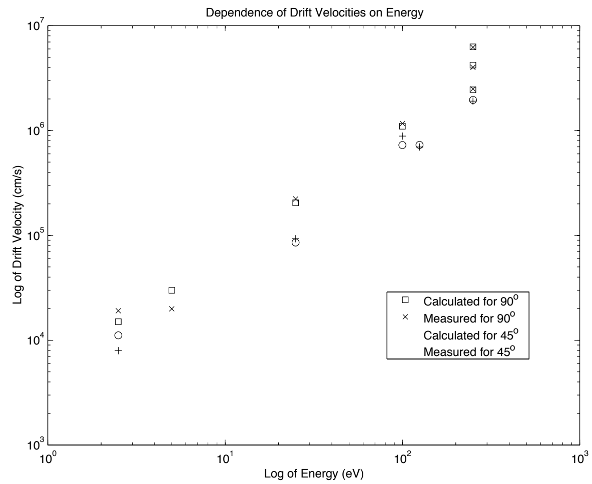

Figure 1: The logarithmic plot of the drift velocities for different energies. The three values for 250 eV and $\theta = 90^o$ are for different ϕ values.

longest interval in the same drift direction. The discrepancies between the measured and the calculated drift velocities were from 0.5

III. RMF Heating Times

Data was collected on the time ions take to reach 1 KeV and 5 KeV in an RMF plasma. The ion is started with 100 eV of energy in an odd parity rotating field with $b_e = 100 gauss$ and $b_0 = 20$. The phase of the RMF was varied to find the average time of heating. As seen in the graph below, the time of heating is independent of the phase, with an average time of heating of 0.116 ms to heat to 5 Kev, while it took 0.080 ms to heat to 1 KeV.

IV. $\Delta\mu$ Measurements

The change in magnetic moment, $\Delta\mu$, was measured for an ion launched the length of the FRC device and with a reversal of direction at the edge of the separatrix. The ion turns in compliance with μ conservation, but at the moment the particle finishes the turn, there is a large spike in μ caused by the increase in $v_\perp$. The ion's percent loss of magnetic moment depends on the gyrophase of the particle at the midplane: if ϕ is equal to 0 or $\pm\pi$ then change in μ is zero since the motion is reversible coming in and leaving the bounce. [1]

Birmingham derived an equation for the change in the magnetic moment that depends sinusoidally on the pitch and phase angles in the midplane,

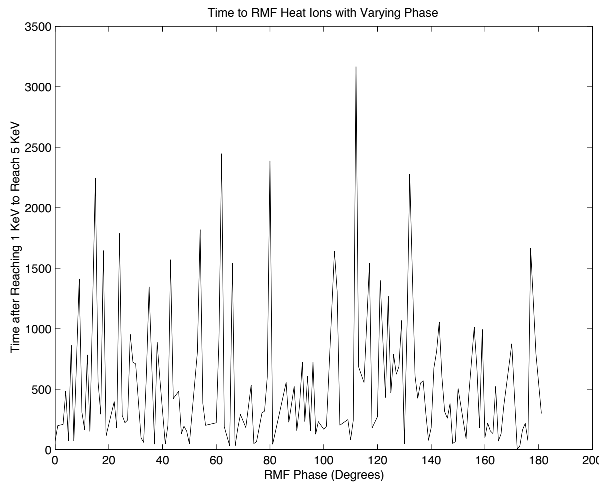

Figure 2: The time taken for the ion to heat to 5 KeV.

which Chen quotes as:

$$\Delta\mu = \frac{mv^2}{B_n}\frac{\pi sin(\beta_0)sin(\phi_0)}{2^{5/4}\Gamma(9/8)}(\frac{1}{\hat{\rho_n}})^{1/8}e^{-\frac{F_\mu}{\hat{\rho n}}} \tag{5}$$

Here ρ_n is the larmor radius mcv/qB_n, B_n is the magnetic field in the z-direction at the mid-plane, B_0 is in the radial direction, $\hat{\rho_n} = \rho_n/b_n\delta$ where δ is the characteristic half thickness and $b_n = B_n/B_0$. [1] F_μ is an integral defined by Birmingham and approximated as $F_\mu = 2/3+3\pi\mu^*/16+4\mu^{*2}/15$ where μ^* is the normalized magnetic moment. This approximation works at small μ^* (less than 0.3), but for this hair-pin turn the normalized magnetic moment is around 1. The equation is sinusoidally dependent on both the pitch and phase angles, and exponentially dependent on the square root of the energy ($\Delta\mu \simeq e^{-\frac{1}{\sqrt{E}}}$). [3]

The change in magnetic moment on one bounce was measured as both phi, theta, and the energy were varied, as shown in Fig. 4, 5, and 6 below. The magnetic moment for one ion was calculated and compared to Birmingham's equation. The particle was launched at r = 8.485 cm and z = 0 cm, with $\theta = 50^o$ and $\phi = 0.01^o$. The magnetic field at this point is 8798 gauss which checks with the Hill-Vortex formulas.

The energy's effect on $\Delta\mu$ does not show the predicted $e^{-\sqrt{E_0}}$ behavior. The electron's phase when entering the turn will depend on the launch, since a faster particle will get to the turn sooner, thus creating the sinusoidal

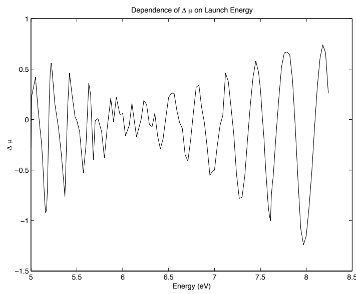

Figure 3: The energy dependence of the change in mu.

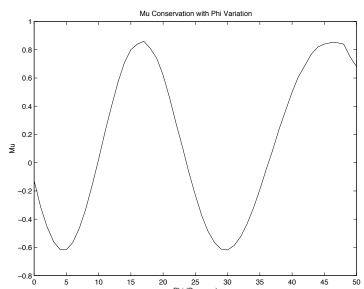

Figure 4: The sinusoidal dependence of $\Delta\mu$ on the phase angle.

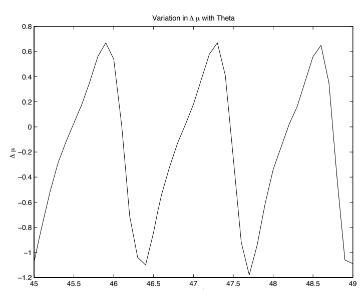

Figure 5: The pitch angle in the midplane, θ, produces approximately a sine wave in $\Delta\mu$.

behavior. The increase and decrease in the average $\Delta\mu$ with energy is most likely due to the sharpness of the electron's bounce. At 5 eV, the electron would make one bounce at around z = 15 cm, but at 7 eV the electron would bounce once, only come back to z $\simeq$ 10 cm and then bounce multiple times at z = 25 cm. The multiple bounces may affect the data from the first bounce, causing higher μ values.

V. Electron Trajectories

I attempted to contain electrons launched from a position outside the Hill Vortex magnetic fields by varying launch parameters in the computer code. The electrons were started from a position with $z = -90$ cm and $r = 0.05$ cm, at an energy of 10 KeV and a launch angle of $\theta = 1$ and $\phi = 0$. The program plotted the trajectories of the electrons for a maximum of 5000 τ and the time the electron remained in the device was recorded.

Initially the electrons would not enter the device unless very high energies were selected. The electrons tended to escape through the cusp of the magnetic fields or oscillate just outside the separatrix. Electrons would also travel along the outer field lines of the separatrix and then exit on the opposite cusp. By varying the launch parameters and reducing the energy, the best launch parameters and an appropriate range of energy could be selected. Without an added rotating magnetic field, 5 eV was the lowest energy an electron would enter the separatrix. At 100 eV, the electrons stayed in the FRC device for an average time of 0.196 ms. At 5 eV, the electrons would stay in the device as long as 0.445 ms. The largest pitch angle possible was 30 degrees, and any phase angle could be chosen. The electron had to be launched close to the central axis; the maximum radial distance was 0.001 cm. At lower energies, the electrons would travel along the cusp's magnetic field lines, reverse and then enter the separatrix. At energies higher than 5000 eV, the cusp could be avoided.

With the rotating magnetic fields, an electron needed an energy of 18 KeV to enter the separatrix. The preferred added magnetic field had $b_e = 50$, $b_o = 10$, odd parity and out of phase by an angle of 30^o with the initial Hill vortex fields. The preferred launch angles were $\theta = 1.2$ and $\phi = 20$ degrees, and a maximum radial distance of 0.0001 cm. The closure of the field lines with the added RMF field decreases the ability to enter the separatrix and increases the specificity of the launch parameters.

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