A Review of “Field Reversal by Rotating Waves”

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Equations of Motion

Using the same definitions for w, p, ω, and τ from the paper[1], the equations of motion in the rotating coordinate system are

$$\frac{d^2u}{d\tau^2} - (p+1)u = (p+2)\,\frac{dv}{d\tau}$$

$$\frac{d^2v}{d\tau^2} - (p+1-w^2)v = -(p+2)\,\frac{du}{d\tau} + wc_z$$

$$\frac{dz}{d\tau} = -wv + c_z$$

Which are identical to those given in the paper (although the paper did not include the motion in z in terms of the rotating coordinates).

The c_z term originally arises from integration of the equation

$$\frac{d^2z}{d\tau^2} = w\,\frac{d}{d\tau}(x\sin(\tau) - y\cos(\tau))$$

Which comes from applying the Lorentz Force using the rotating B-field and its consistent E-field as defined in the paper.

The general solution to these equations of motion are

$$u = U_0\sin(\omega_0\tau+\alpha_0) + U_1\sin(\omega_1\tau+\alpha_1)$$
$$v = V_0\cos(\omega_0\tau+\alpha_0) + V_1\cos(\omega_1\tau+\alpha_1) + \frac{wc_z}{w^2-p-1}$$
$$z = Z_0\sin(\omega_0\tau+\alpha_0) + Z_1\sin(\omega_1\tau+\alpha_1) - \frac{(p+1)cz}{w^2-p-1}\tau + d$$

$$V_0 = \frac{\omega_0^2+p+1}{\omega_0(p+2)}U_0$$
$$V_1 = \frac{\omega_1^2+p+1}{\omega_1(p+2)}U_1$$

$$Z_0 = -\frac{w(\omega_0^2+p+1)}{\omega_0^2(p+2)}U_0$$
$$Z_1 = -\frac{w(\omega_1^2+p+1)}{\omega_1^2(p+2)}U_1$$

This solution is equivalent to the one given in the paper, except that the paper does not include the c_z term in v. This was also noted by W. Hugrass in his comments on the paper[2].

Relevance of C_z

$B = 1.0\text{e-}2$ T, $B_0 = 1.0\text{e-}3$ T, $\omega = 2.0\text{e}7\ s^{-1}$, $\alpha_0 = 0.0$, $\alpha_1 = 0.0$, $U_0 = 1.0\text{e-}4$, $U_1 = 1.0\text{e-}4$
(electron)

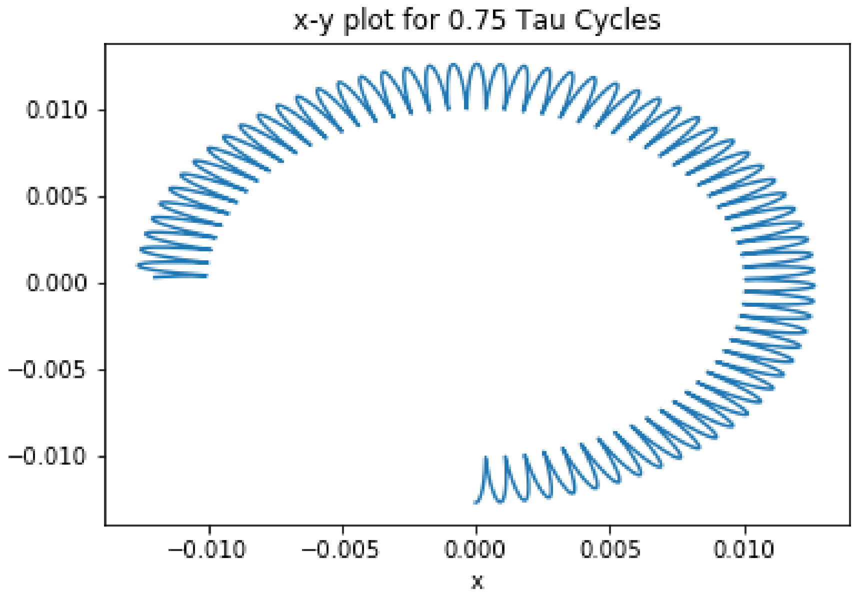

Fig 1: c_z = 1.0 m

Initial Position (m):
x: 0.0
y: -0.0126579503657
z: 0.0
Initial Velocity(m/s):
xvel: 435208.730658
yvel: 0.0
zvel: -2261539.67044

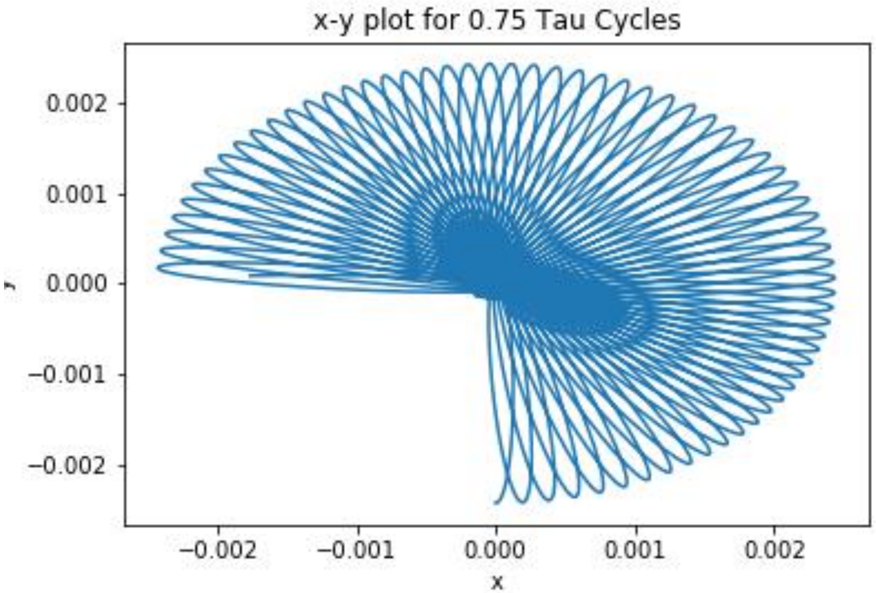

Fig 2: c_z = 0.1 m

Initial Position (m):
x: 0.0
y: -0.00243342400386
z: 0.0
Initial Velocity (m/s):
xvel: 230718.203422
yvel: 0.0
zvel: -2279663.24974

$B = 1.0\text{e-}2$ T, $B_0 = 1.0\text{e-}3$ T, $\omega = 2.0\text{e}7\ s^{-1}$, $\alpha_0 = 0.0$, $\alpha_1 = 0.0$, $U_0 = 1.0\text{e-}2$, $U_1 = 1.0\text{e-}5$ (electron)

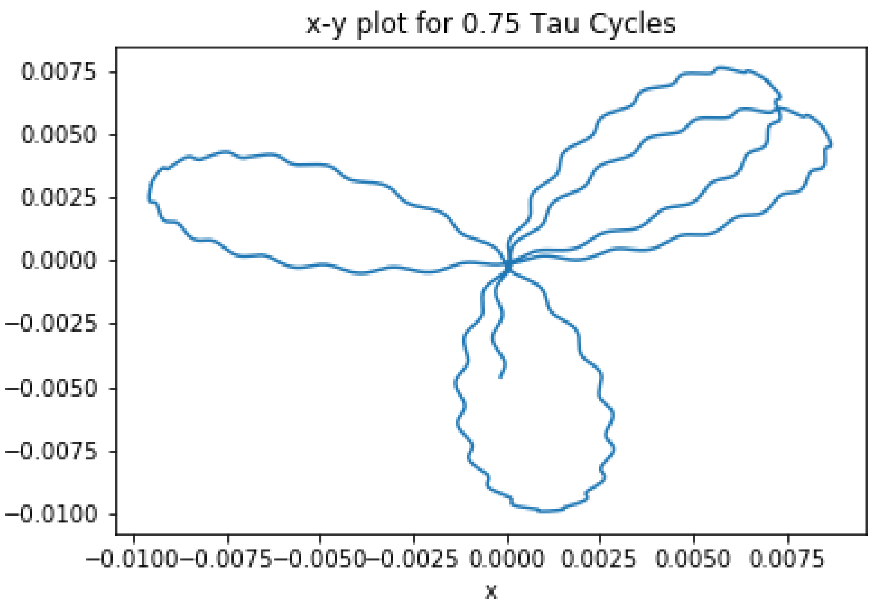

Fig 3: c_z = 0.0 m

<u>Initial Position (m)</u>:

x: 0.0

y: -0.000105307477089

z: 0.0

<u>Initial Velocity (m/s)</u>:

xvel: 576432.513864

yvel: 0.0

zvel: -185204.27961

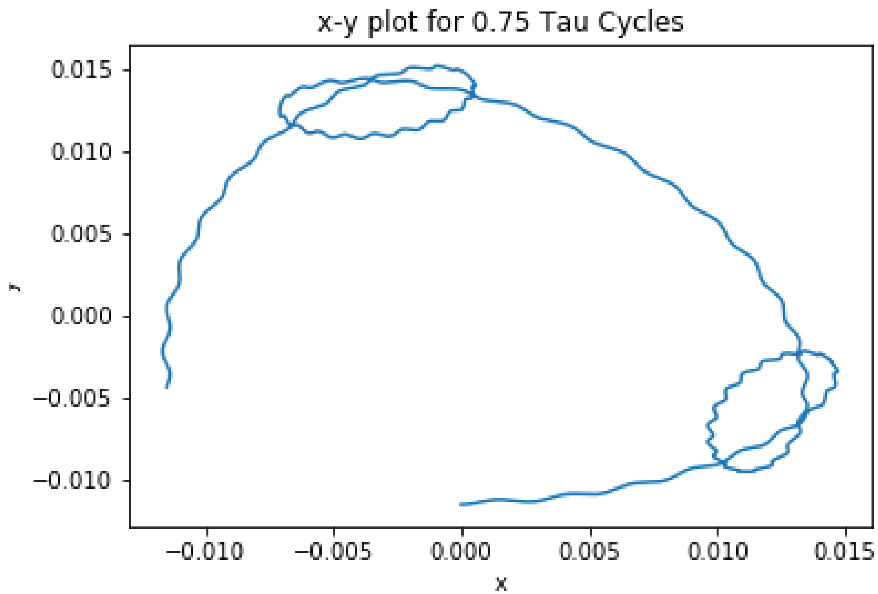

Fig 4: c_z = 1.0 m

<u>Initial Position (m)</u>:

x: 0.0

y: -0.0114658923235

z: 0.0

<u>Initial Velocity (m/s)</u>:

xvel: 803644.210793

yvel: 0.0

zvel: -165066.969277

$B = 1.0e\text{-}3$ T, $B_0 = 1.0e\text{-}2$ T, $\omega = 2.0e7\ s^{-1}$, $\alpha_0 = 0.0$, $\alpha_1 = 0.0$, $U_0 = 1.0e\text{-}3$, $U_1 = 1.0e\text{-}3$
(electron)

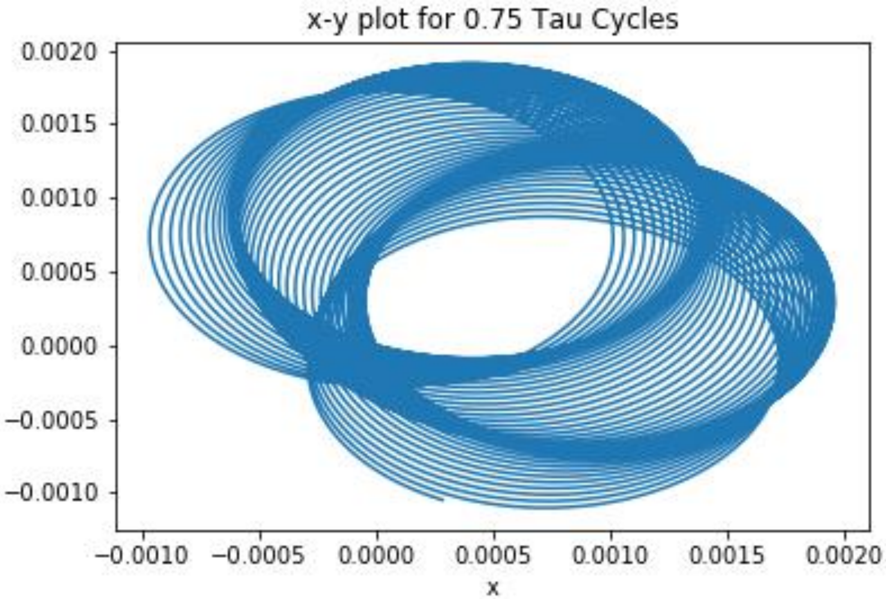

Fig 5: c_z = 0.0 m

Initial Position (m):
x: 0.0
y: -0.00028140298669
z: 0.0

Initial Velocity (m/s):
xvel: 1780454.18493
yvel: 0.0
zvel: -49490.348521

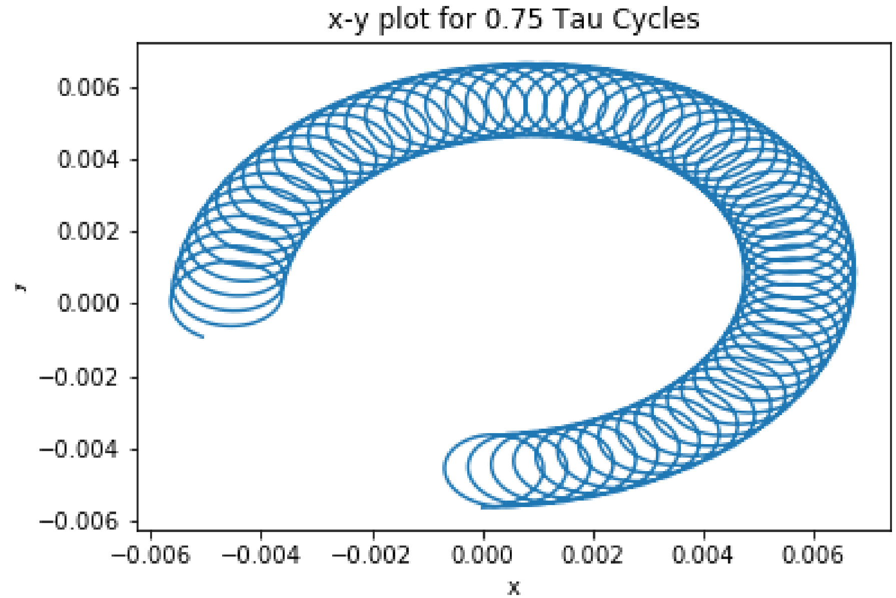

Fig 6: c_z = 0.1 m

Initial Position (m):
x: 0.0
y: -0.00563478492566
z: 0.0

Initial Velocity (m/s):
xvel: 1887521.82371
yvel: 0.0
zvel: 1009010.26996

When comparing figures 1 and 2, having a great enough value for c_z ensures that the electron's orbit stays far away from the origin, which is helpful for producing toroidal current.

Figures 3 and 4 put even greater emphasis on why c_z is important to include. In figure 3, the electron lags considerably far behind the rmf, while the electron in figure 4 does not.

Even in the case where the axial magnetic field dominates the rmf, as portrayed in figures 5 and 6, including c_z shows that there still are electron orbits which have a coherent motion of ωr.

It is also interesting to see how c_z affects the initial conditions (t = 0) in Cartesian coordinates, which are given by

$$x(0) = u(0) = U_0\sin(\alpha_0) + U_1\sin(\alpha_1)$$
$$y(0) = v(0) = V_0\cos(\alpha_0) + V_1\cos(\alpha_1) + \frac{wc_z}{w^2-p-1}$$
$$z\,(0) = Z_0\sin(\alpha_0) + Z_1\sin(\alpha_1) + d$$

$$\frac{dx}{dt}(0) = \frac{du}{dt}(0) \;-\; \omega v(0) = (\omega_0 U_0 - V_0)\cos(\alpha_0) + (\omega_1 U_1 - V_1)\cos(\alpha_1) - \frac{\omega w c_z}{w^2-p-1}$$
$$\frac{dy}{dt}(0) = \frac{dv}{dt}(0) + \omega u(0) = (-\omega_0 V_0 + U_0)\sin(\alpha_0) + (-\omega_1 V_1 + U_1)\sin(\alpha_1)$$
$$\frac{dz}{dt}(0) = -\omega w v(0) + \omega c_z = -\,\omega w\,V_0\cos(\alpha_0) - \omega w V_1\cos(\alpha_1)) - \frac{\omega(p+1)cz}{w^2-p-1}$$

From the equations, we can see that a large c_z corresponds to a large initial value in y, and a large and negative velocity in the x and z directions. In the limit, the particle follows a large circular orbit synchronous to the rmf.

Importance of Scattering for Electron Heating in the Rotamak Scheme

Single particles can only follow orbits determined by their initial conditions, and cannot be heated further by the rmf alone. However, random scattering allows for particles to be kicked into new orbits, potentially increasing how much current can be generated.

For our scattering model, we assume that the kinetic energy of the electron is conserved (electron-ion collisions), and that the deflection angle is completely random. The scattering events occur randomly, but at an average rate given by

$$\nu = \frac{\mathrm{n}\mathrm{e}^4}{16\pi\varepsilon_0^2\mathrm{m}^2\mathrm{v}^3}\ln\Lambda$$

Where n is the plasma density (in m^{-3}) and $\ln\Lambda = 10$ is assumed. Due to the inverse dependence in the electron velocity cubed, scattering events become more sparse as the electron temperature increases.

$B = 1.0\text{e-}2$ T, $B_0 = 1.0\text{e-}3$ T, $\omega = 2.0\text{e}7\ s^{-1}$, $n = 1.0\text{e}19\ m^{-3}$
(electron)

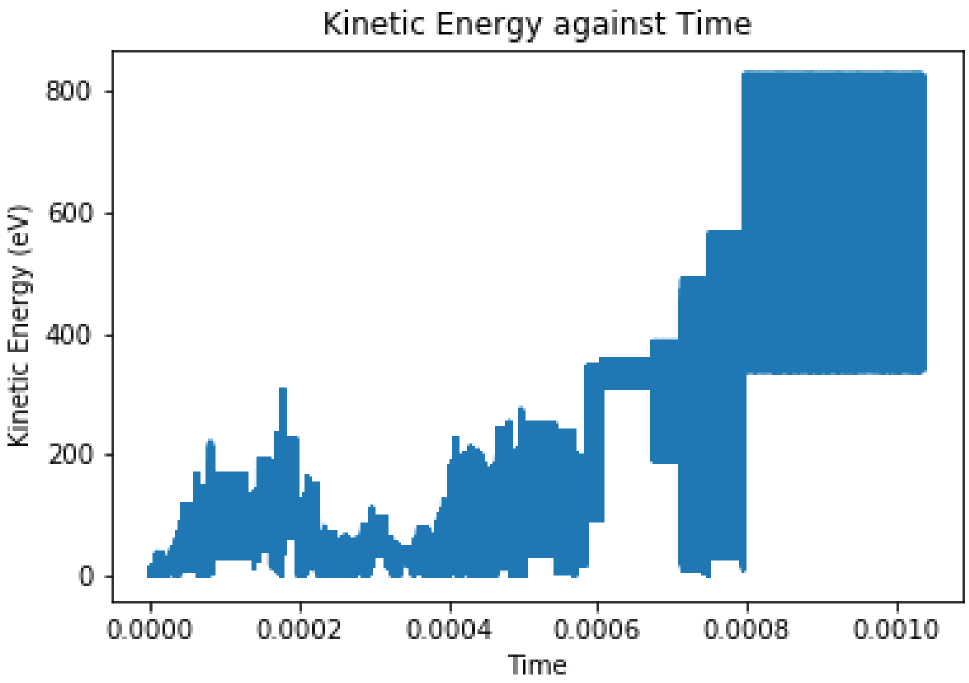

Fig 7: KE of the electron tends to increase over time

<u>Initial Position (m)</u>:

x: 0.0

y: 0.0

z: 0.0

<u>Initial Velocity (m/s)</u>:

xvel: 1.1e6

yvel: 1.1e6

zvel: 1.1e6

In fig 7, instead of using the analytic solution for the orbits, I employed a numerical method, since it is very difficult to find the correct orbit starting with initial position and velocity in Cartesian coordinates when using the analytic solution. I also checked that the numerical method was consistent with the analytic solution by giving it the same initial conditions in Cartesian coordinates for the orbits shown in the previous figures.

Initially, the electron’s kinetic energy is roughly 10eV. With this scattering model, its kinetic energy tends to increase by an order of magnitude after 1ms.

Although the scattering model is crude, it does suggest that scattering is an important part of the mechanism for electron heating in a Rotamak.

Conclusion

With the introduction of c_z we find that the axial magnetic field does not necessarily inhibit motion transverse to it, as orbits do exist where the axial field dominates, but the particles are still synchronous to the rmf. C_z also reveals orbits that never go through the origin, thus producing more overall current.

Adding a simple model of electron-ion scattering also shows how particles can be heated in a Rotamak, thus potentially increasing current/power efficiency.

References

[1] FISCH, N.J., WATANABE, T., NucL Fusion 22 (1982) 423.

[2] HUGRASS, W.N., Comments on the Paper by N.J. FISCH, T. WATANABE ‘Field Reversal by Rotating Waves'