Reduced MHD "favors" straight field line coordinates for mode description.

Reason:

In straight field line coordinates, the safety factor q has no poloidal dependence on a given flux surface.

Particle guiding center theory "favors" orthogonal magnetic coordinates.

Reason:

Guiding center equations of motion have a convenient Hamiltonian structure in orthogonal magnetic coordinates.

Representation of the unperturbed axisymmetric magnetic field in orthogonal coordinates:

$$\mathbf{A} = A_\varphi \nabla\varphi + A_\theta \nabla\theta$$
$$\mathbf{B} = B_\varphi \nabla\varphi + B_\theta \nabla\theta$$

φ - toroidal angle

θ - poloidal angle

ψ - flux coordinate

Note that there is no $\nabla\psi$ - component in $\mathbf{A}$ or $\mathbf{B}$ in orthogonal coordinates.

Littlejohn Lagrangian in orthogonal magnetic coordinates:

$$L = \frac{e}{c}\left[A_\varphi + \frac{Mc}{e}v_\parallel \frac{B_\varphi}{B}\right]\dot{\varphi} + \frac{e}{c}\left[A_\theta + \frac{Mc}{e}v_\parallel \frac{B_\theta}{B}\right]\dot{\theta} + \frac{Mc}{e}\mu\dot{\varsigma} - \mu B - \frac{Mv_\parallel^2}{2}$$

Dynamical variables:

φ - toroidal angle

θ - poloidal angle

ψ - flux coordinate

$v_\parallel$ - parallel velocity

ς - gyroangle

μ - magnetic moment

Hamiltonian form:

$$L = P_\varphi \dot{\varphi} + P_\theta \dot{\theta} + P_\varsigma \dot{\varsigma} - H(P_\varphi; P_\theta; P_\varsigma; \theta)$$

$$P_\varphi \equiv \frac{e}{c}\left[A_\varphi + \frac{Mc}{e} v_\parallel \frac{B_\varphi}{B} \right]$$

$$P_\theta \equiv \frac{e}{c}\left[A_\theta + \frac{Mc}{e} v_\parallel \frac{B_\theta}{B} \right]$$

$$P_\varsigma \equiv \frac{Mc}{e}\mu$$

$$H \equiv \mu B + \frac{Mv_\parallel^2}{2}$$

Transformation from straight field line coordinates $\left(\underline{r};\underline{\theta};\underline{\varphi}\right)$ to orthogonal coordinates $\left(r;\theta;\varphi\right)$:

$$\underline{r} = r$$

$$\underline{\varphi} = \varphi$$

$$\underline{\theta} = \theta - \left(\frac{r}{R_0} + \Delta' + \int_0^r \frac{\Delta'}{r} dr \right) \sin\theta \equiv \theta - \hat{\varepsilon}\sin\theta$$

$\Delta' \equiv$ Shafranov shift

Mode representation (single poloidal component) in straight field line coordinates:

$$\Phi_{SAW} = \Phi_{nm}(\underline{r})\exp(-i\omega t + in\underline{\varphi} - im\underline{\theta})$$

Mode representation in orthogonal coordinates:

$$\Phi_{SAW} = \Phi_{nm}(r)\exp(-i\omega t + in\varphi - im\theta + im\hat{\varepsilon}\sin\theta)$$

Issue to address:

The case of large $m\hat{\varepsilon}$ presents a challenge for brute-force numerical implementation because insufficient poloidal resolution can break the Hamiltonian structure of wave-particle interaction.