Colorado Plan

- Add source and sink
- Add coarse-graining procedure
- Benchmark source and sink for single mode saturation
- Explore continuum method within the framework of CGP

Source and Sink in δf Method

- For alpha particles with distribution $f(\mathbf{x}, v, \lambda, t)$ $(\lambda = v_\parallel / v)$ in the presence of a particle source, annihilation due to charge exchange, slowing down, and pitch-angle scattering:

$$\frac{\partial f}{\partial t} + \mathbf{V}_H \cdot \nabla f + \dot{v}_H \frac{\partial f}{\partial v} + \dot{\lambda}_H \frac{\partial f}{\partial \lambda} - \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f}{\partial \lambda} - \frac{\nu}{v^2}\frac{\partial}{\partial v}[(v^3 + v_I^3) f]$$
$$= S(\mathbf{x}, v, \lambda) - \nu_a f.$$

- Alfvén waves are included in $\mathbf{V}_H$, $\dot{v}_H$ and $\dot{\lambda}_H$ by adding terms $\mathbf{V}_{H1}$, $\dot{v}_{H1}$ and $\dot{\lambda}_{H1}$—i.e.,

$$\mathbf{V}_H = \mathbf{V}_{H0} + \mathbf{V}_{H1},$$
$$\dot{v}_H = \dot{v}_{H1},$$
$$\dot{\lambda}_H = \dot{\lambda}_{H0} + \dot{\lambda}_{H1}.$$

  Since the zeroth-order Hamiltonian motion conserves energy, $\dot{v}_{H0} = 0$.

- Assume $f = f_0 + \delta f$, where the unperturbed distribution f_0 satisfies

$$\mathbf{V}_{H0} \cdot \nabla f_0 + \dot{v}_{H0} \frac{\partial f_0}{\partial v} + \dot{\lambda}_{H0} \frac{\partial f_0}{\partial \lambda} - \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f_0}{\partial \lambda} - \frac{\nu}{v^2}\frac{\partial}{\partial v}[(v^3 + v_I^3) f_0]$$
$$= S(\mathbf{x}, v, \lambda) - \nu_a f_0 \quad ,$$

  and δf satisfies

$$\frac{D\delta f}{Dt} = -\mathbf{V}_{H1} \cdot \nabla f_0 - \dot{v}_{H1} \frac{\partial f_0}{\partial v} - \dot{\lambda}_{H1} \frac{\partial f_0}{\partial \lambda} - \nu_a \delta f,$$

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \mathbf{V}_H \cdot \nabla f + \dot{v}_H \frac{\partial f}{\partial v} + \dot{\lambda}_H \frac{\partial f}{\partial \lambda} - \nu_d \frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial f}{\partial \lambda} - \frac{\nu}{v^2}\frac{\partial}{\partial v}[(v^3 + v_I^3) f].$$

A. Assuming particles are loaded according to the physical alpha distribution ($g = f$ (the marker distribution). Weight equation is conventional,

$$\dot{w} = \frac{1}{g}\left[-\mathbf{V}_{H1}\cdot\nabla f_0 - \dot{v}_{H1}\frac{\partial f_0}{\partial v} - \dot{\lambda}_{H1}\frac{\partial f_0}{\partial \lambda}\right].$$

The distribution g satisfies

$$\frac{Dg}{Dt} = S(\mathbf{x}, v, \lambda) - \nu_a g\ .$$

B. Alternatively, new particles can be added according to the physical alpha birth rate, but not removed according to the sink rate, (only removed when velocity is too low due to slowing down)

$$\dot{w} = \frac{1}{g}\left[-\mathbf{V}_{H1}\cdot\nabla f_0 - \dot{v}_{H1}\frac{\partial f_0}{\partial v} - \dot{\lambda}_{H1}\frac{\partial f_0}{\partial \lambda}\right] - \nu_a w.$$

The distribution g satisfies

$$\frac{Dg}{Dt} = S(\mathbf{x}, v, \lambda)\ .$$

In this case $g \neq f$. Source is implemented by adding new simulation particles at the birth speed. Sink term is implemented as a Krook term in the weight equation.

For marginal instabilities, $g \approx f_0$ is a good approximation. f_0 can be numerical.