GKM Model: a minimal set

Guoyong Fu

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Main equation (equivalent to perpendicular momentum equation derived from gyrokinetic equation)

$$-\frac{d}{dt}\nabla\cdot\left(\frac{1}{V_A^2}\nabla_\perp\Phi\right)+\boldsymbol{B}\cdot\nabla\frac{\boldsymbol{B}\cdot(\nabla\times\nabla\times\boldsymbol{A})}{B^2}+(\nabla A_\parallel\times\boldsymbol{b})\cdot\nabla\left(\frac{J_{\parallel 0}}{B}\right)$$

$$=\frac{1}{V_A^2}\left(\frac{3\mathbf{v}_t^2}{4\Omega^2}\right)\nabla_\perp^4\frac{d\Phi}{dt}+\boldsymbol{b}\times\sum_j\nabla\left(\frac{env_t^2}{2B\Omega^2}\right)_j\cdot\nabla\nabla_\perp^2\Phi-\sum_j\int(e\mathbf{v}_d\cdot\nabla f)_j\,d^3v$$

The LHS represent ideal shear Alfven wave equation without the ballooning term. On the RHS, the first term is ion FLR, the second is diamagnetic drift, and the third contains all other kinetic terms from both thermal ion and fast ions. The third term gives the energetic particle destabilization and ion Landau damping. This term also gives the usual ballooning terms from all species.

The main equation is closed by A_|| equation, parallel Ohm's law, electron density equation, and parallel momentum equation.

Assumes electron isothermal model !

$$\frac{\partial}{\partial t} A_{||} = -\nabla_{||}\Phi - E_{||}$$

$$E_{||} = -\frac{1}{en_e}\nabla_{||}\delta P_e$$

$$\frac{\partial}{\partial t} n_e = -\nabla \cdot (\mathbf{V_e} n_e)$$

$$\mathbf{V_e} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} - \frac{\delta \mathbf{J}_{||}}{e n_e} + v_{||,i}\mathbf{b}$$

$$\rho \frac{\partial}{\partial t} v_{||,i} = \mathbf{b} \cdot (\mathbf{J} \times \delta \mathbf{B} - \nabla \cdot \delta \mathbf{P_{th}})$$

Discussions

- This is a minimum set of GKM equations;
- It is nonlinear (partially);
- It recovers the dispersion for kinetic Alfven waves;
- It contains shear Alfven waves, sound waves, and GAM.
- It also contains the ITG.