CIRCUIT EQUATION FORMULATION OF RESISTIVE WALL MODE FEEDBACK STABILIZATION SCHEMES AND APPLICATION TO ACTIVE COIL DESIGN IN DIII-D DEVICE

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1. Introduction

Various experimental observations near the beta limit in tokamaks[1,2] have indicated that ideal external kinks are converted into Resistive Wall Modes (RWM‘s) under the influence of surrounding conducting material such as the vacuum vessel or a conducting shell. This mode conversion is promising for reducing disruptions, since a slow RWM can be stabilized by a magnetic feedback system.

Recently, a new formulation for RWM feedback stabilization has been developed to analyze the relationship between the hardware elements and the plasma response in a systematic manner. Details are given in ref [3,4,5]. Basically, this formulation utilizes concepts from electric circuit theory. As shown in Figure 1, the coupled elements are (1) the perturbed plasma current, (2) the poloidal passive shell system, and (3) the active coil system as lumped-parameter electric circuits obeying the usual laws of linear circuit theory. An inductance matrix describes the interactions between the coupled circuits. The resulting simple set of coupled linear differential equations are in a form familiar to both physicists and engineers. With this formulation, the physics for control of $n \geq 1$ modes can be compared with the theoretical and experimental results of $n = 0$ vertical position control. This circuit equation approach is convenient for designing a RWM stabilization feedback coil system.

2. Analysis with cylindrical geometry

A key element of the new approach is an “effective” self inductance for the plasma circuit defined as

$$L_1^{eff} \equiv L_1 \Big(\frac{\gamma_\infty^2 \tau_A^2}{\gamma_\infty^2 \tau_A^2 + \frac{2f^2}{\beta^0}} \Big), \tag{1}$$

where L_1^{eff} represents the plasma driving term for the $n \geq 1$ MHD instability, γ_∞ is the ideal MHD growth rate without the shell, τ_A is the Alfven time constant, $f = m - nq_a$, and β^0 is the parameter to represent the current peakingness. For a flat current profile, $\beta^0 = 1$, $L_1^{eff}/L_1 = 1 - f$ and ideal kink mode is unstable for $0 < f < 1$ so that $L_1^{eff} \simeq 0$ near the MHD marginal condition $f \simeq 1$. It is to be noted that L_1^{eff}/L_1 is equivalent to the magnetic decay index for $n = 0$ vertical positional stability and $L_1^{eff}/L_1 \simeq 0$ corresponds to the onset of the vertical positional instability due to the non-circularity.

From the MHD-Maxwell equations for a coupled plasma-conducting wall, the following circuit equations are obtained describing the interaction of a feedback circuit with a resistive wall mode [4,5]:

$$L_1^{eff} I_1 + M_{12} I_2 + M_{13} I_3 = 0,\psi \tag{2}$$

$$(\gamma\tau_2) M_{21} I_1 + (\gamma\tau_2 + 1) L_2 I_2 + (\gamma\tau_2) M_{23} I_3 = 0,\psi \tag{3}$$

$$(\gamma\tau_3) M_{31} I_1 + (\gamma\tau_3) M_{32} I_2 + (\gamma\tau_3 + 1) L_3 I_3 \;=\; V_3 \tau_3.\psi \tag{4}$$

Here, M_{ij} is the mutual inductance between circuit i and j, $\tau_2 = L_2/R_2$, and $\tau_3 = L_3/R_3$. The V_3 is the voltage to the active power with the feedback sensor signals from flux loops or eddy current on the shell.

Let us discuss a solution of the circuit equations with a total flux sensor mounted just outside the shell to measure the flux leaking through the shell(this is similar to the DIII-D system). In this case,

$$V_s \tau_3 = G_t (M_{12} I_1 + L_2 I_2 + M_{32} I_3).\psi \tag{5}$$

Using standard feedback analysis techniques with Equations 2, 3 and 4, the growth rate with the limit of $\tau_2 \gg \tau_3$ is given by [5]

$$\gamma\tau_2 \approx \Gamma\tau_2 \Big[1 - G_t \Big(\hat{M}_{32} - \hat{M}_{31}\hat{M}_{12}/\hat{L}_1^{eff}\Big)\Big], \Gamma\tau_2 = \frac{-1}{1 - \hat{M}_{12}\hat{M}_{21}/\hat{L}_1^{eff}} \tag{6}$$

where $\hat{M}_{jk} \equiv M_{jk}/L_j$, G_t is the feedback gain, and $\Gamma\tau_2 > 0$ is the growth rate of the RWM in the absence of the active feedback circuit.

The important combination is $\hat{M}_{23} - \hat{M}_{13}\hat{M}_{21}/\hat{L}_1^{eff}$. This term represents a measure of the shielding of the field from the active coil by the conducting shell. When the active coil energized with current I_3 produces the flux $\psi = I_3 M_{13}$ at the plasma surface, which drives a circuit current $I_1 = I_3 M_{13}/L_1^{eff}$ on the plasma surface. This, in turn, creates a flux $\psi_{21} = I_3 M_{13} M_{21}/L_1^{eff}$ at the passive shell. To have negative feedback signal on the shell, the flux ψ_{21} should be larger than the direct flux at the shell due to the control field, $I_3 M_{23}$. Since $L_1^{eff} < 1$, the relation $\hat{M}_{23} - \hat{M}_{13}\hat{M}_{21}/L_1^{eff}$ can be satisfied. This can be viewed as a criterion for the minimum amount of direct coupling M_{31} relative to the shielding effect. For finite τ_3/τ_2 the roots of the circuit equation dispersion relation can become complex at finite gain and the oscillatory component is comparable to the the growth rate component, which indicates that the mode identification may become difficult with finite mode rotation. The role of this shelding relationship remains intact.

3. DIII-D active feedback system

Figure 2 shows the hardware arrangement for active RWM feedback stabilization in the DIII-D device. Three sets of segmented loops sensing the total flux are mounted just outside of the vacuum vessel at the mid-plane, and at upper and lower poloidal locations. At each poloidal location, there are 6 independent loops equally spread in the the toroidal direction. Three set of active coils are located outside the DIII-D toroidal coils. One coil set at the mid-plane is the existing error field correction coil and other sets are planned to be installed as 6 toroidally segmented sections poloidally extended up/down symmetrically. The L/R time constant of the vacuum vessel, τ_2, is 2-5 ms and the active coil L/R time constant, τ_3, is 15 ms. The power supply is capable up to 50 hz with 4 kA which can produce 40 gauss on the plasma surface with off-axis coils.

Figure 3(a) shows the eddy current pattern of an $n = 1$ RWM on the DIII-D vacuum vessel based on the experimental configuration. The calculation was done with the PPPL-VACUUM code [6]. The current pattern (corresponding to I_1) shows the n=1 current vortex dominantly on the outboard side and the perturbation at the inboard side is small.

The cause of RWM is due to the ohmic flux loss of this current pattern. Thus, it has been often stated that the active coil compensates the flux loss on the shell surface regardless of the direct influence at the plasma surface. However, as pointed out in Section 2, the critical element is the flux due to the active field on the plasma surface (corresponding to $M_{31}I_3$). In the toroidal geometry, the helical flux may be represented by a 2-dimensional normal magnetic field pattern. Figure 3(b) shows the normal magnetic field on the plasma surface produced by the eddy current on the vacuum vessel (corresponding to $M_{12}I_2$). This normal field pattern is the essence of the kink stabilization. Thus, the active coil should produce this pattern as closely as possible. Otherwise, the active field will add unnecessary field harmonics and require more power to achieve the same effect.

The current ratio of the mid-plane and off-axis coils is optimized to produce the desired pattern using the cross-correlation between the pattern of Fig 3(b) and the pattern generated by the active coils. The ratio of $I_{mid}/I_{off-axis} = 4/10$ produces the field pattern shown in Fig 3(c) with a cross correlation coefficient $C_{cor} =$ 0.74. Further improvement can be made if the upper/lower coil is extended over the existing mid-plane coils, since the extensive poloidal coverage of the existing mid-plane coil limits the fine tuning of field pattern near the mid-plane. The mid-plane coil only produces a pattern $C_{cor} = 0.28$ and the off-axis only produces a field pattern with $C_{cor} = 0.39$. Thus, the installation of the upper/lower active coils is highly desirable for minimizing the required power and reducing the unnecessary field harmonics.

4. Summary

The circuit equation formulation of RWM feedback systems has been used as a tool to aid in the design of the DIII-D RWM feedback system. The relationship between the plasma contribution and the hardware circuitry can be studied in a qualitative manner. The further quantitative analysis will be made by defining the mutual inductance in toroidal geometry, which is under progress.

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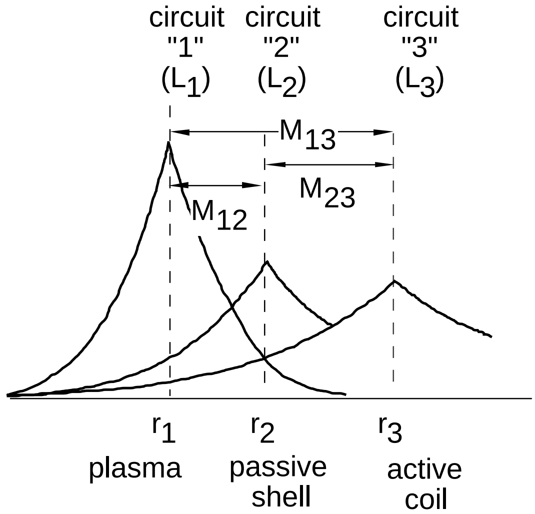

Fig. 1. RWM Circuit Geometry

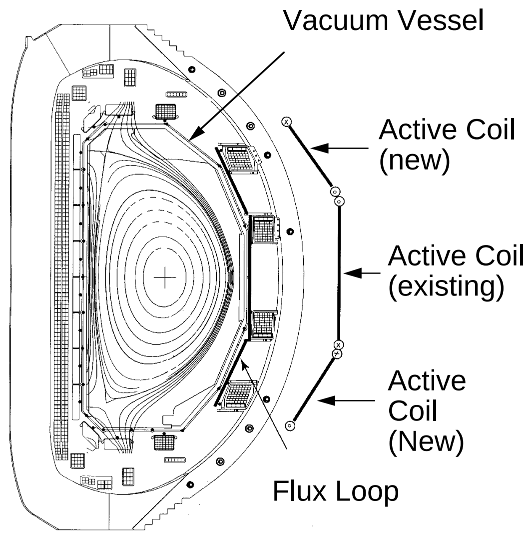

Fig. 2. DIII-D RWM Feedback System

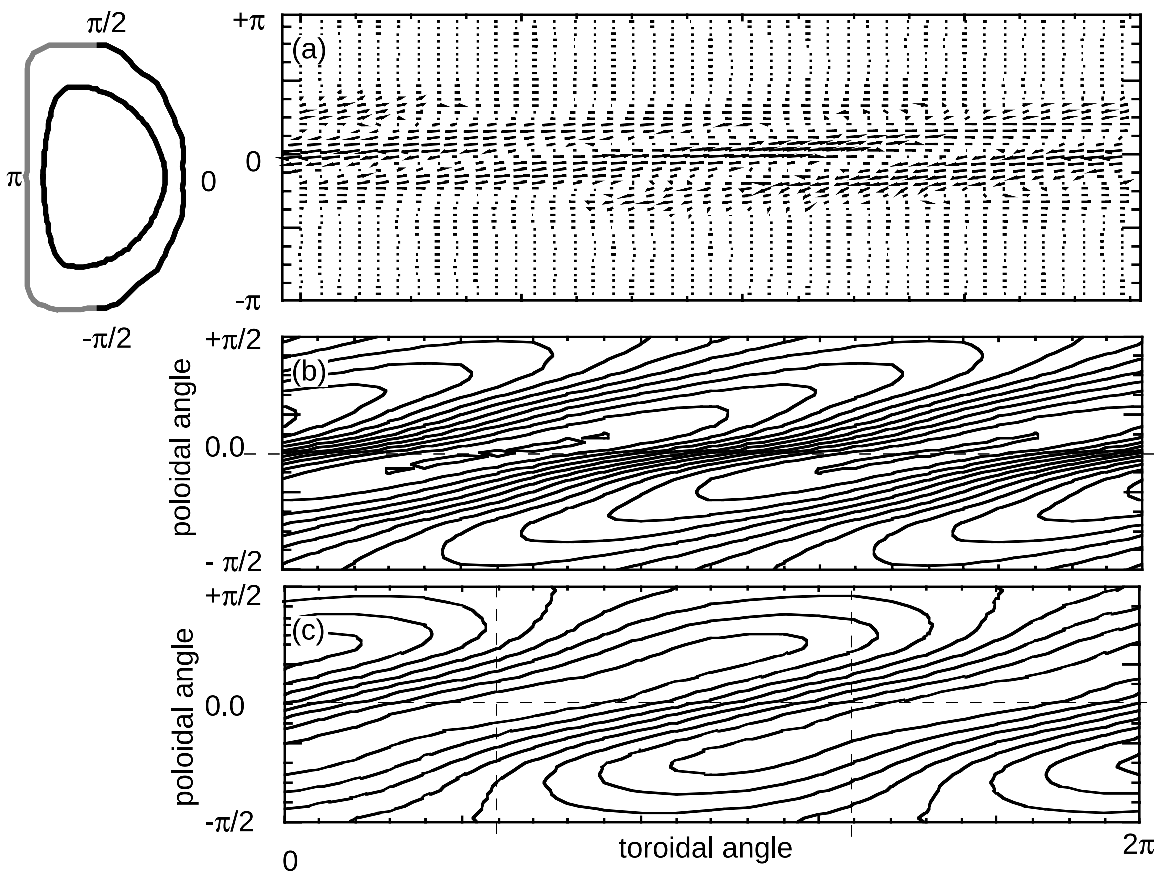

Fig. 3. Eddy current and Normal Magnetic Field Pattern