The Center for Extended Magnetohydrodynamic Modeling

(Global Stability of Magnetic Fusion Devices)

S. Jardin—lead PI

a SciDAC activity…
Partners with:
TOPS
TSTT
APDEC

General Atomics: D. Brennan*

MIT: L. Sugiyama, J. Ramos*

NYU: H. Strauss

PPPL: J. Breslau, J. Chen, G. Fu, S. Klasky, W. Park, R. Samtaney

SAIC: D. Schnack, A. Pankin*

TechX*: S. Kruger

U. Colorado: S. Parker , D. Barnes*

U.Wisconsin: J. Callen, C. Hegna, C. Sovinec, C. Kim*

Utah State: E. Held

*new

Summary: CDX-U Simulation

- All toroidal modes of the q_0=0.92 CDX equilibrium are linearly unstable in resistive MHD model
  - n=1 is an internal kink mode
  - n>1 are resistive ballooning instabilities
  - Higher n modes have higher growth rates
- Nonlinear resistive MHD evolution beginning with just an n=1 perturbation disrupts within a sawtooth crash time
- Adding toroidal flow reduces the growth rate but does not stabilize
- Adding large parallel thermal conductivity reduces growth rate, but does not stabilize
- Adding the ω* term does not appreciably alter the growth rate

- **Realistic treatment of these modes requires a more complete extended MHD model—this is our present focus**

Future Directions

- Get serious about extended-MHD
  - Evaluate several sets of equations with different orderings
  - Efficient algorithms for solving the extended MHD equations with dispersive waves
- Working towards burning plasma problems
  - 7 critical problems identified that are of interest to ITER
- Improved infrastructure
  - Further expand common visualization packages
  - Unified data management system
- Integration Activities
  - Integrated calculation with RF
  - Hybrid calculation of neoclassical closures

Extended MHD Models

Model	Momentum Equation	Ohm's law	Whist-lers[1]	KAW[2]	GV[3]	Slow dynamics[4]
General	$mn\frac{d\mathbf{V}}{dt}=-\nabla(p_e+p_i)$ $+\mathbf{J}\times\mathbf{B}-\nabla\cdot(\Pi_{\parallel e}+\Pi_{\parallel i})-\nabla\cdot\Pi_i^{gv}$	$\mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}$ $+\frac{1}{ne}\left(\mathbf{J}\times\mathbf{B}-\nabla p_e-\nabla\cdot\Pi_{\parallel e}\right)$	Yes	Yes	Yes	Either
Generalized Hall MHD[5]	$mn\frac{d\mathbf{V}}{dt}=-\nabla(p_e+p_i)$ $+\mathbf{J}\times\mathbf{B}-\nabla\cdot(\Pi_{\parallel e}+\Pi_{\parallel i})$	$\mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}$ $+\frac{1}{ne}\left(\mathbf{J}\times\mathbf{B}-\nabla p_e-\nabla\cdot\Pi_{\parallel e}\right)$	Yes	Yes	No	No
Neoclassical-MHD	$mn\frac{d\mathbf{V}}{dt}=-\nabla(p_e+p_i)$ $+\mathbf{J}\times\mathbf{B}-\nabla\cdot(\Pi_{\parallel e}+\Pi_{\parallel i})-\nabla\cdot\Pi_i^{gv}$	$\mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}-\frac{1}{ne}\nabla\cdot\Pi_{\parallel e}$	No	No	Yes	Yes
Generalized resistive MHD[5]	$mn\frac{d\mathbf{V}}{dt}=-\nabla p+\mathbf{J}\times\mathbf{B}-\nabla\cdot\Pi_{\parallel}$	$\mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}$	No	No	No	No
Generalized drift[6]	$mn\frac{d\mathbf{V}}{dt}=-mn\mathbf{V}_{di}\cdot\nabla\mathbf{V}_{\perp}+\upsilon_{gv}$ $+nm\mu\nabla_{\perp}^2\mathbf{V}-\nabla\cdot(\Pi_{\parallel e}+\Pi_{\parallel i})$ $-\nabla(p_e+p_i)+\mathbf{J}\times\mathbf{B}$	$\mathbf{E}=-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}^*$ $-\frac{1}{ne}\left[\nabla_{\parallel}p_e+\nabla\cdot\Pi_{\parallel e}\right]$	No	Yes	Yes	Yes

Higher order modes present in Extended MHD models present new numerical challenges

Mode	Origin	Wave Equation	Dispersion	Comments
Whistler	in Ohm $\mathbf{J}\times\mathbf{B}$	$\frac{\partial^2 \mathbf{B}}{\partial t^2} = -\left(\frac{V_A^2}{\Omega}\right)^2 (\mathbf{b}\cdot\nabla)^2 \nabla^2 \mathbf{B}$	$\omega^2 = V_A^2 k^2 \left[1 + \frac{1}{\beta}(\rho_i k_\parallel)^2\right]$	•electron response •finite $k_\parallel$
KAW	in Ohm $\nabla_\parallel p_e$	$\frac{\partial^2 \mathbf{B}}{\partial t^2} = \left(\frac{V_A V_{th*}}{\Omega}\right)^2 (\mathbf{b}\cdot\nabla)^2 \nabla\times[\mathbf{bb}\cdot\nabla\times\mathbf{B}]$	$\omega^2 = V_A^2 k_\parallel^2 \left[1 + (\rho_s k_\perp)^2\right]$	•ion and e$^-$ response •finite $k_\parallel$ $k_\perp$
Parallel ion GV	η_4 term in $\nabla\cdot\Pi^{GV}$	$\rho\frac{\partial^2 \mathbf{V}_\perp}{\partial t^2} = -\eta_4^2 \nabla_\parallel^4 \mathbf{V}_\perp$	$\omega_{\frac{L\pm}{R\pm}} = V_A k_\parallel \left[\pm 1 \pm \frac{1+\beta}{2\sqrt{\beta}}(\rho_i k_\parallel)\right]$	•ion response •finite $k_\parallel$
Perp. ion GV	η_3 term in $\nabla\cdot\Pi^{GV}$	$\rho\frac{\partial^2 \mathbf{V}_\perp}{\partial t^2} = -\eta_3^2 \nabla_\perp^4 \mathbf{V}_\perp$	$\omega^2 = V_A^2 k_\perp^2 \left[1 + \frac{\gamma\beta}{2} + \frac{\beta}{16}(\rho_i k_\perp)^2\right]$	•ion response • finite $k_\perp$

How to solve the equations with dispersive waves in them?

1. The primary NIMROD approach (Sovinec) is to developing time-split implicit equations with only 2nd order spatial derivates by introducing auxiliary variables. (makes larger system and is only first order accurate in time)

2. Second NIMROD approach (Barnes) formulates the operator as a filter on the drives. More complex, but second order in time

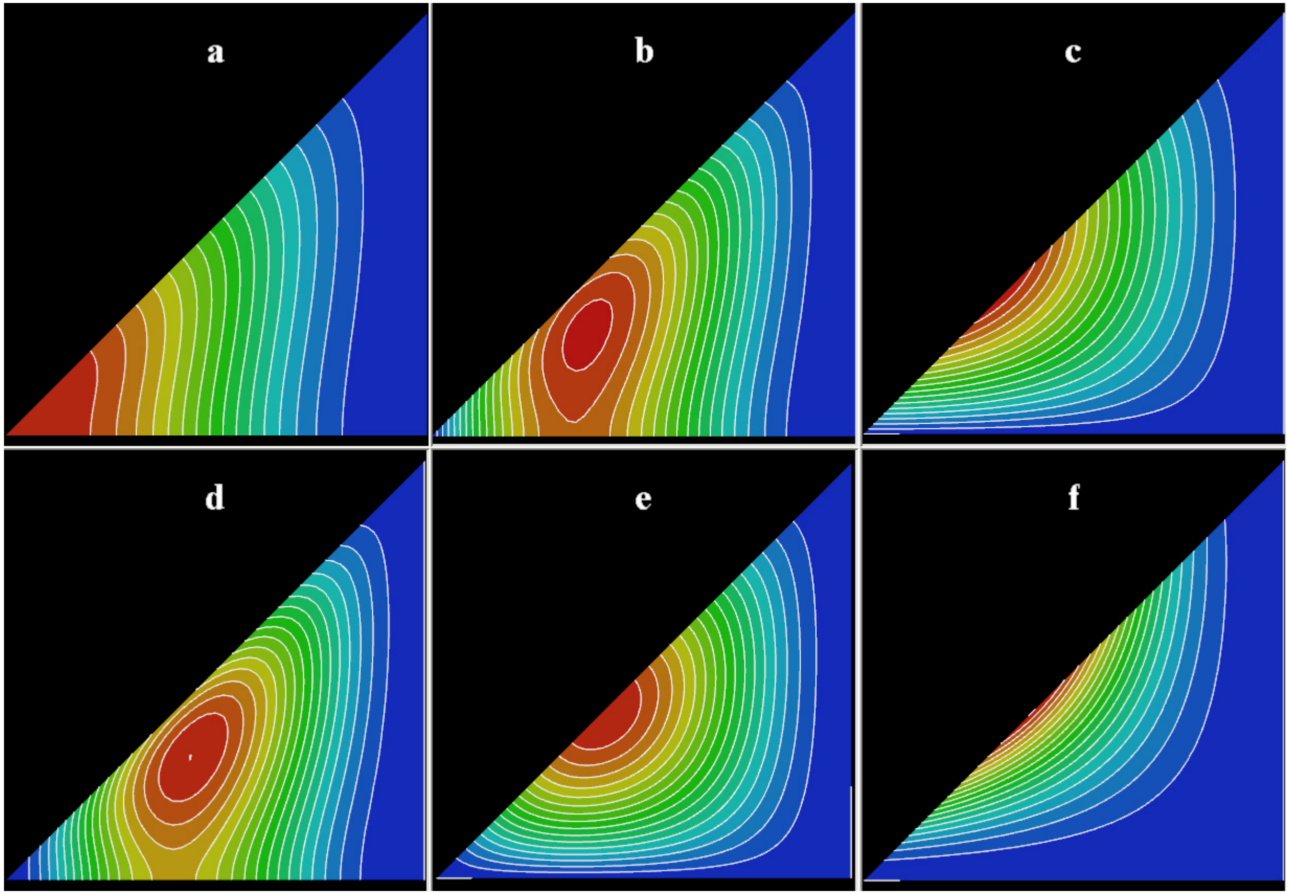
3. Current M3D approach is to use C^1 finite elements, that can be used to represent fourth order system directly. Leads to smaller matrices in implicit formulation that can be efficiently inverted.

$$a_i = g_{ij}\,\Phi_j$$

The M3D C^1 Trial Functions:

$$\phi = \sum_{i=1}^{20} a_i\,\xi^{m_i}\eta^{n_i} = \sum_{i=1}^{20}\sum_{j=1}^{18} g_{ij}\Phi_j\xi^{m_i}\eta^{n_i} = \sum_{j=1}^{18} v_j\Phi_j$$

$$v_j = \sum_{i=1}^{20} \xi^{m_i}\eta^{n_i} g_{ij}$$



These are the trial functions. There are 18 for each triangle.

The 6 shown here correspond to one node, and vanish at the other nodes, along with their derivatives. Function and first derivatives are continuous across element boundaries.

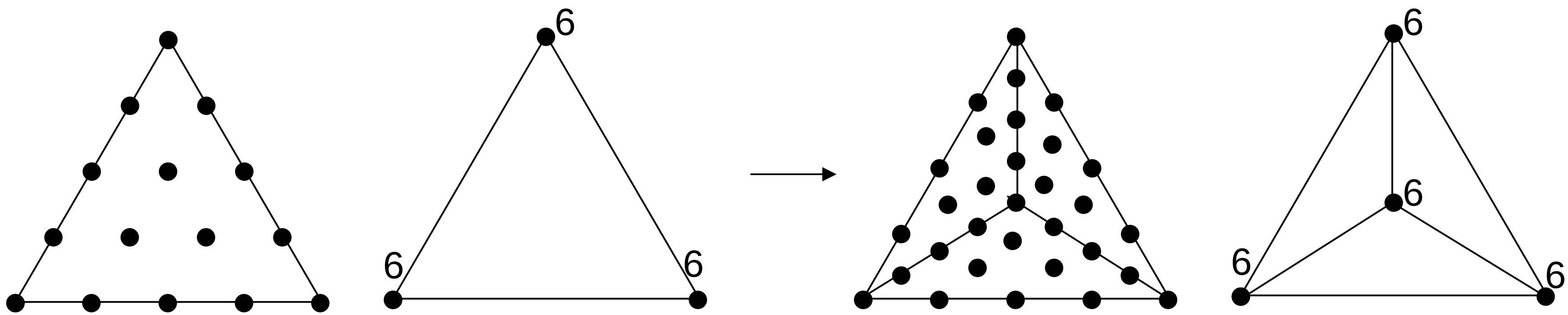
Each of the six has value 1 for the function or one of it's derivatives at the node, zero for the others.

Note that the function and it's derivatives (through 2nd) play the role of the amplitudes

Comparison of M3D C^1 element to other popular triangular finite elements

	Vertex nodes	Line nodes	Interior nodes	accuracy order h^p	UK/T	continuity
linear element	3	0	0	2	½	C^0
Lagrange quadratic	3	3	0	3	2	C^0
Lagrange cubic	3	6	1	4	4½	C^0
Lagrange quartic	3	9	3	5	8	C^0
M3D C^1	18	0	0	5	3	C^1

UK/T is the number of unknowns per triangle



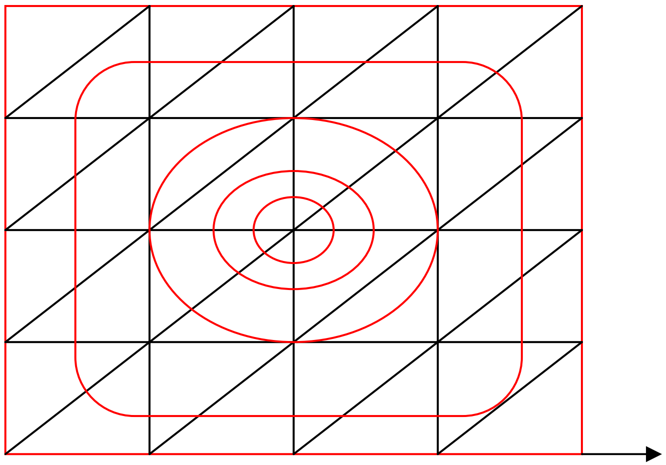
15 18 +16 +6

Test problem: Anisotropic Diffusion

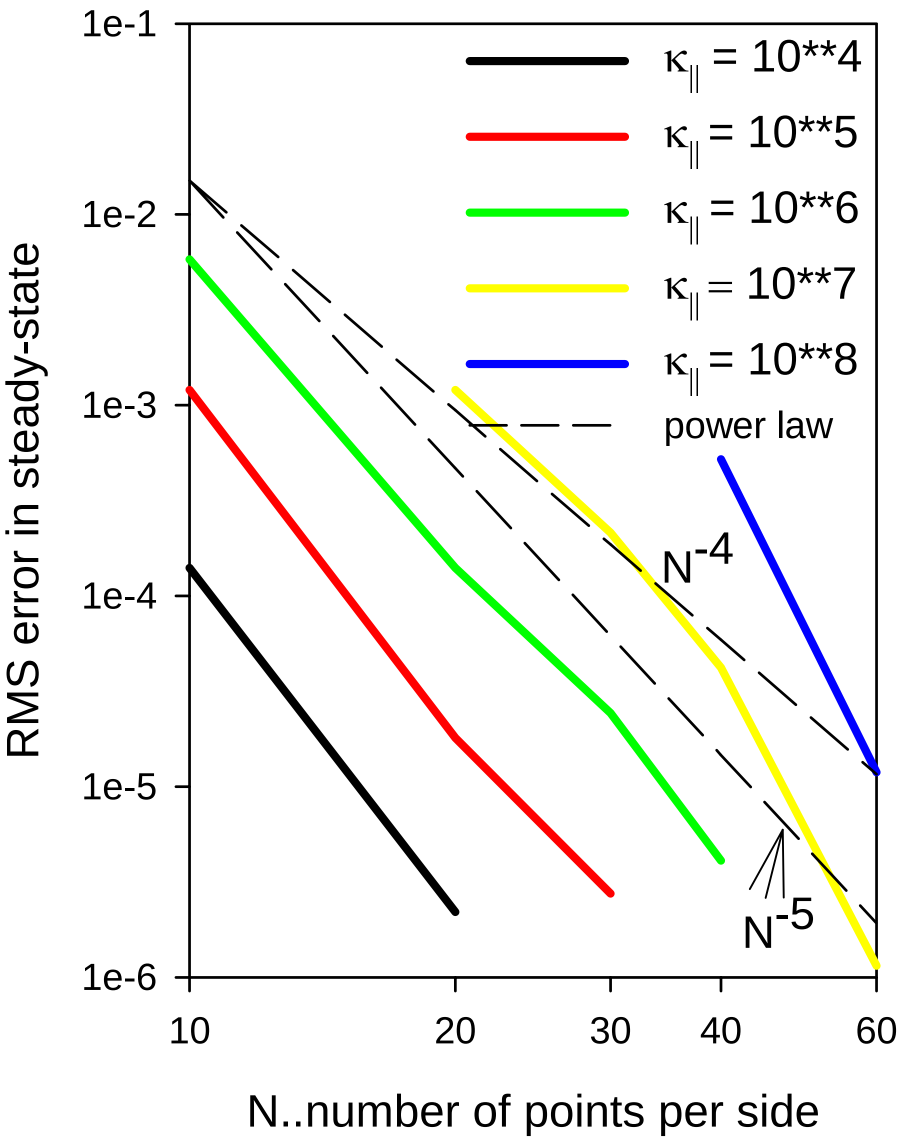
Shows greater than N^{-5} convergence

$$\frac{\partial \phi}{\partial t} = \nabla \bullet \left[\kappa_{\|} \frac{\vec{B}\vec{B}}{B^2} \bullet \nabla \phi \right] + \nabla \bullet \kappa \nabla \phi + S$$

Solve to steady state



$$S = \psi = \cos\frac{\pi x}{L} \cos\frac{\pi y}{L}$$



M3D-C^1 code has been set up in a general form, to allow non-trivial subsets of lower rank equations. Implicit 2-fluid equations unconditionally stable.

$$\begin{bmatrix} S_{11}^v & S_{12}^v & S_{13}^v \\ S_{21}^v & S_{22}^v & S_{23}^v \\ S_{31}^v & S_{32}^v & S_{33}^v \end{bmatrix} \bullet \begin{bmatrix} \phi \\ V_z \\ \chi \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^v & D_{12}^v & D_{13}^v \\ D_{21}^v & D_{22}^v & D_{23}^v \\ D_{31}^v & D_{32}^v & D_{33}^v \end{bmatrix} \bullet \begin{bmatrix} \phi \\ V_z \\ \chi \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^v & R_{12}^v & R_{13}^v \\ R_{21}^v & R_{22}^v & R_{23}^v \\ R_{31}^v & R_{32}^v & R_{33}^v \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ T_e \end{bmatrix}^{n}$$

$$\begin{bmatrix} S_{11}^p & S_{12}^p & S_{13}^p \\ S_{21}^p & S_{22}^p & S_{23}^p \\ S_{31}^p & S_{32}^p & S_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ T_e \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^p & D_{12}^p & D_{13}^p \\ D_{21}^p & D_{22}^p & D_{23}^p \\ D_{31}^p & D_{32}^p & D_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ T_e \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^p & R_{12}^p & R_{13}^p \\ R_{21}^p & R_{22}^p & R_{23}^p \\ R_{31}^p & R_{32}^p & R_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \phi \\ V_z \\ \chi \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^p & Q_{12}^p & Q_{13}^p \\ Q_{21}^p & Q_{22}^p & Q_{23}^p \\ Q_{31}^p & Q_{32}^p & Q_{33}^p \end{bmatrix} \bullet \begin{bmatrix} \phi \\ V_z \\ \chi \end{bmatrix}^{n}$$

Phase-I: Reduced resistive MHD: done: Plasma tilting test problem

$$\frac{\partial}{\partial t}\nabla^2\phi + \left[\nabla^2\phi, \phi\right] - \left[\nabla^2\psi, \psi\right] = \mu\nabla^4\phi$$

$$\frac{\partial \psi}{\partial t} + [\psi, \phi] = \eta\nabla^2\psi$$

Phase-II: Fitzpatrick-Porcelli model: done Taylor reconnection test problem.

$$\frac{\partial}{\partial t}\nabla^2\phi = \left[\phi, \nabla^2\phi\right] + \left[\nabla^2\psi, \psi\right] + \mu\nabla^4\phi$$

$$\frac{\partial V_z}{\partial t} = [\phi, V_z] + c_\beta [I, \psi] + \mu\nabla^2 V_z$$

$$\frac{\partial \psi}{\partial t} = [\phi, \psi] + d_\beta [\psi, I] + \eta\nabla^2\psi$$

$$\frac{\partial I}{\partial t} = [\phi, I] + d_\beta \left[\nabla^2\psi, \psi\right] + c_\beta [V_z, \psi] + c_\beta^2 \eta\nabla^2 I$$

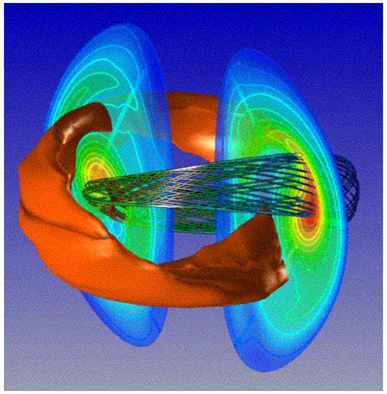
Outstanding Problems in MHD

- Full nonlinear sawtooth oscillation modeling in fusion-grade plasmas
- Tearing mode and neoclassical tearing mode excitation in high-beta plasmas
- Nonlinear evolution and control of resistive wall modes, including toroidal flows.
- Effects of Fast Ions
- Edge MHD-type instabilities
- Disruptions and Vertical Displacement Events
- Pellet Injection fueling of a Burning Plasma

For each of these, the goal is to have both a detailed 3D complete model that can be invoked, and to develop and test simplified models

Full nonlinear sawtooth oscillation modeling in fusion-grade plasmas

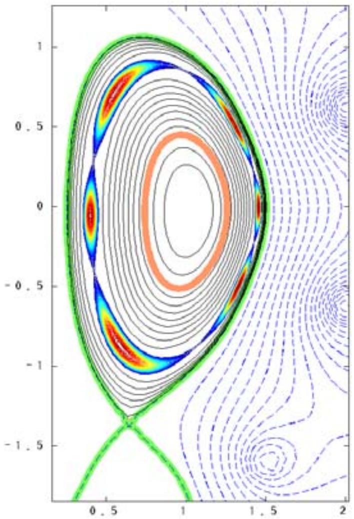
- What will be the period and inversion radius of the sawtooth oscillation in an ITER-class burning plasma, as a function of plasma current and pressure?
- What physics underlies complete and incomplete reconnection during the sawtooth?
- Under what conditions will the sawtooth instability trigger the onset of a meta-stable island (neoclassical tearing mode) or lead to a disruption?
- Can RF be used effectively to stimulate small sawteeth in ITER?



- Requires Extended MHD model (more terms than in resistive MHD) to get correct onset criteria, crash time
- Requires Hybrid description including energetic alpha particles in ITER
- Multiple space-scales due to small reconnection regions
- Free boundary with vacuum region required.
- Extensive validation program required.
- Coupling to RF modeling desired.

Tearing mode and neoclassical tearing mode excitation in high-beta plasmas

- What type of disturbance can cause the neoclassical tearing mode to form in an ITER-class tokamak?
- Under what conditions do “spontaneous NTMs” form?
- What will be the saturated island size as a function of plasma current and beta?
- What level of external current drive power is required to fully stabilize the NTM?



• Requires Extended MHD model with neo-classical closure

• Requires Hybrid description including energetic alpha particles to get correct ion response when islands are small

• Multiple time-scales due to slow growth rate of islands

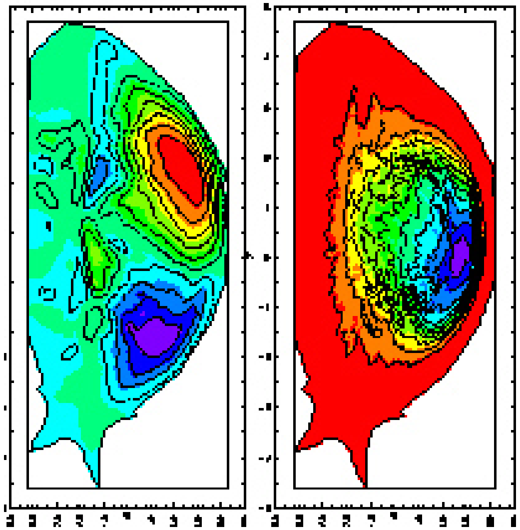
• Free boundary with vacuum region required.

• Extensive validation program required.

• Coupling to RF modeling desired

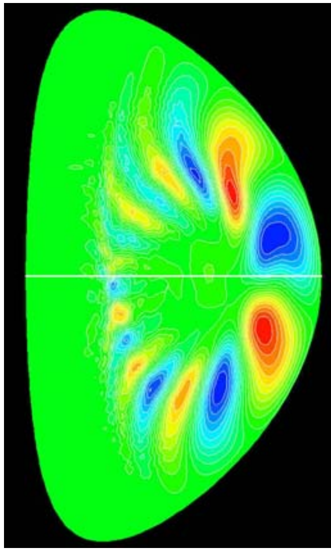
Nonlinear evolution and control of resistive wall modes, including toroidal flows

- What are the dominant toroidal flow and RWM damping mechanisms in present experiments, and how do these scale to ITER?
- What is an allowable error field in ITER as it relates to MHD stability limits?
- How much beyond the ideal-wall beta limit can an ITER-class plasma operate with a properly designed feedback system?



- Requires Extended MHD model to get correct damping,
- Requires Hybrid description including energetic alpha particles in ITER
- Multiple time-scales due to small growth rate of mode
- Free boundary with vacuum region and conductor required.
- Extensive validation program required.

Effects of Fast Ions

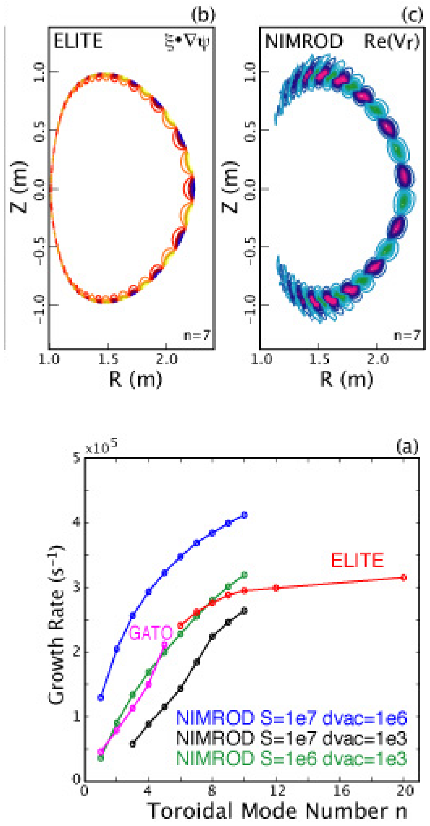


- How does the presence of fast ions alter the nonlinear MHD behavior of the sawtooth, the NTM, and the other MHD modes?
- What new instabilities are introduced in an ITER-class plasma by the fast ions, and what are the non-linear consequences in terms of anomalous alpha particle transport?
- Under what conditions will a large fraction of alpha particles be lost?

- Requires Hybrid description including energetic alpha particles in ITER
- Free boundary with vacuum region and conductor required.
- Extensive validation program required.

Edge MHD-type instabilities

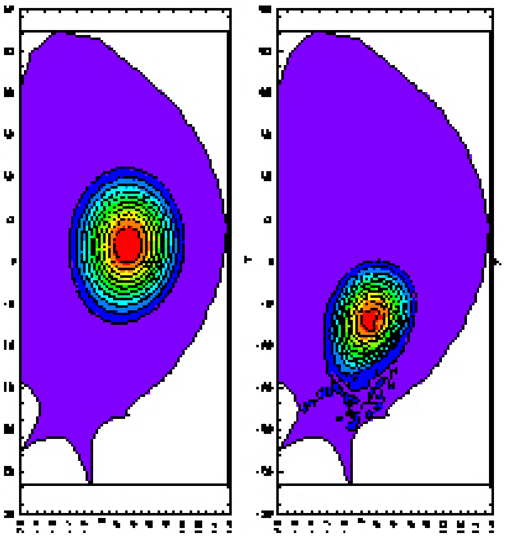
- What type of ELM behavior do we expect in an ITER-class plasma as a function of current and beta?
- What external mechanisms, such as pellet injection, RF current drive, or boundary modulation, are effective in increasing the ELM period and reducing the magnitude?
- Can non-linear, dynamical models of ELM evolution be effectively coupled with other detailed models of edge dynamics and transport?



- Requires Extended MHD model,
- Multiple time-scales due to difference between growth and crash times
- Free boundary with vacuum region and conductor required.
- Needs coupling with other Edge Physics
- Extensive validation program required.

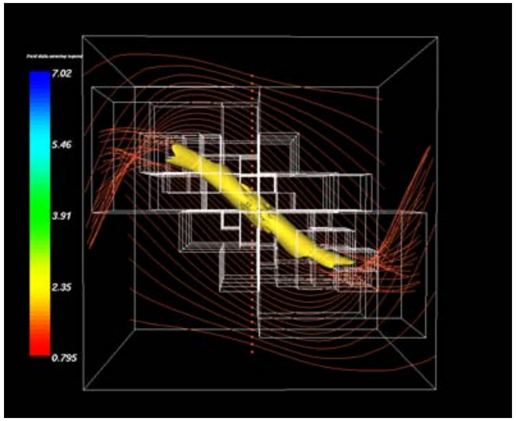
Disruptions and Vertical Displacement Events

- Under what circumstances will a chain of nonlinear events occur in an ITER-class plasma that result in a major disruption?
- What is the thermal quench time for each type of disruptive sequence, and where is the energy deposited?
- What are the mechanical forces on the ITER vacuum vessel for different types of disruptions, and how are these forces distributed spatially and temporally?
- Can we design a “killer pellet” or massive gas influx that, if initiated on an appropriate precursor signal, can significantly mitigate the effects of the disruption?



- Requires good anisotropic thermal conductivity model,
- Free boundary with vacuum region and conductor required.
- Needs integration of pellet model with MHD model
- Extensive validation program required.

Pellet Injection fueling of a Burning Plasma



- Can we reproduce the mass deposition differences between inside launch and outside launch as measured on the JET experiment?
- What mass deposition profile will we get for ITER as a function of pellet mass, injection velocity, and injection angle?
- Can we reproduce experiments performed on JET where Edge Localized Modes (ELMS) are induced by pellet injection, and project this to ITER

  - Multiple space-scales due to difference between pellet and device sizes
  - Anisotropic heat conduction required
  - Needs coupling with ablation physics
  - Needs coupling to full MHD and Edge models
  - Extensive validation program required.

Milestones—part 1

	Year 1	Year 2	Year 3
Model development	• Compare IGV stress in general ordering with drift 2-fluid model	• Clarify 2-fluid effects in reconnection: islands and 1/1 mode	• Compare two-fluid and non-local parallel closure for tokamak tearing-mode
M3D Code Development	• Implement C^1 elements in 2-fluid 2D form • Add collision effects to fast ions	• Extend C^1 elements to full 2-fluid linear 3D simulation • Field-aligned mesh and 2nd order FLR for thermal ions	• Extend C^1 elements to full 2-fluid non-linear 3D simulation • Optimize matrix solves and time advance
NIMROD Code Development	• Implement anisotropic ion stress (local operators) • Semi-implicit algorithm for Hall term • Upgrade hybrid option to high-order elements	• Implement and test nonlocal stress closures and compare with local models • Evaluate semi-implicit algorithms for full 2-fluid equations	• Optimize semi-implicit algorithms for two-fluid terms
AMRMHD Code Development[1]	• Complete flux-surface grid AMR code for ideal MHD in tokamaks, including requisite mapped-grid versions of AMR hyperbolic solver • Design and test 4th-order finite-volume solver for anisotropic diffusion	• Complete initial implementation of 4th order anisotropic diffusion solver for AMR • Complete flux-surface grid AMR code for resistive MHD	• Design and test flux-tube coordinate version of 4th order solver for anisotropic diffusion

Milestones—part 2

	Year 1	Year 2	Year 3
Visualization[2]	• Enhance the joint AVS-plotting package to allow viewing of all variables relevant to extended-MHD • Develop comparative utilities to focus on differences for use in code-comparison studies	• Develop streaming utilities to depots to facilitate rapid real time data transfer • Integrate the Logistical Runtime System (Logistical networking software) into the visualization routines.	• Integrate the magnetic island and other advanced viz tools into the visualization package. • Develop AVS collaborative viz using Logistic network technology, and client-server based minimum information methods
Applications	• Calculate 3D halo currents for a ITER disruption (M3D) • Apply non-local parallel heat flow to NTMs and disruptions • Sawtooth with 2-fluid model • Investigate fundamental physics issues in instabilities induced by pellet injection with AMR code[1] • Begin discussions to integrate RF code with MHD code if applicable	• Study toroidal flow damping due to error field • Perform a burning-plasma sawtooth simulation with 2-fluid and energetic particle effects. • High-n alpha-driven TAEs: linear stability • Compare inside and outside pellet simulations with JET data (AMR)[1]	• Nonlinear resistive wall modes with flow damping in DIII-D and NSTX • Tokamak tearing and NTM mode simulations • ELM simulations • High-n alpha-driven TAEs: nonlinear saturation and alpha particle transport • Project pellet injection simulations to ITER(AMR)[1]

Summary

- CDX-U Verification/Validation problem leading to interesting physics
  - Driver for developing better extended-MHD model
  - Common problem being addressed by both codes

- Emphasis in the near future is on Extended-MHD
  - Which set of equations to use
  - How to solve them
  - Understand the physics output
  - Implications for ITER-class burning plasmas