RECENT RESEARCH PROJECTS

Allen Boozer

1. Plasma effects on location of outermost magnetic surface

PoP 12, 092504 (2005). Shielding unless $\square\square\square\frac{\square^2}{16}\frac{w}{a}\frac{a}{R}\left(\frac{a\square}{2\square}\right)\square 10^{\square 3}$ to $10^{\square 4}$ with $\square\square\mu\ m^4$.

2. Density limit in toroidal pure electron plasmas

PoP 12, 104502 (2005).

$\frac{n}{n_B}\square\square^2\frac{a}{R}$ axisymmetric and $\frac{n}{n_B}\square\frac{\square^2}{8m^2}\left(\frac{a}{R}\right)^2\frac{1}{\square_{mn}}$ non-axisymmetric. $n_B\equiv\frac{\square_0 B^2}{2m_e}$

3. Perturbed plasma equilibria using $\square W$ codes (with C. Nührenberg)

$\square_{mn}\equiv\left(\frac{\partial}{\partial\square}\frac{\square\vec{B}\cdot\vec{\square}\square\square}{\vec{B}\cdot\vec{\square}\square\square}\right)_{mn}$; island half-width $\frac{\square\square}{\square}\square 2\sqrt{\left|\frac{\square_{mn}}{m^2\square\square}\left(\frac{\square}{d\square/d\square}\right)\right|}$.

4. Resistive wall modes with plasma rotation and multimodes

(with J. Bialek and D. Maslovsky) Maslovsky & Boozer, PoP 12, 042108 (2005).

MAGNETIC RECONNECTION IN NON-TOROIDAL PLASMAS

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Reconnection is a breaking and connecting of magnetic field lines.

Requires a non-ideal magnetic evolution, $\vec{E}\cdot\vec{B}$ non-zero.

Evolution is ideal if $\vec{E}+\vec{v}\times\vec{B}=0$.

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Magnetic reconnection in nontoroidal plasmas, Boozer, Phys. Plasmas **12**, 070706 (2005)

Physics of Magnetic Confinement, Boozer, Rev. Mod. Phys. **76**, 1071 (2004).

WHY RECONNECTION IS IMPORTANT

Charged particles can move rapidly along magnetic field lines but slowly across, $m d\vec{v}/dt = q\vec{v} \times \vec{B}$. Changing the way magnetic field lines connect to objects (like the sun) can change the motion or the energy of a plasma (an ideal gas of charged particles).

RECONNECTION BREAKS A CONSERVATION LAW

Conservation Laws of Ideal Behavior

1. Tying of a conducting fluid to a magnetic field. (weak)
2. Preservation of the magnetic field lines. (strong, topic of talk)

Reconnection associated with the rapid transfer of energy from the magnetic field to the fluid. (easy, need not break a conservation law)

PRIMARY RESULTS

1. Maxwell equations imply any evolution $\vec{B}(\vec{x},t)$ is ideal locally.

2. Local ideality broken in laboratory plasmas on toroidal surfaces on which field lines close on themselves. *Textbook example of reconnection but not relevant to space or astrophysics.*

3. Separation between neighboring field lines normally increases exponentially with distance along lines, $\propto \exp(\ell / L_L)$. Leads to reconnection if $\ell > 20 L_L$.

4. A rapid transfer of energy from field to plasma need not imply non-ideal field behavior, as reconnection. For example, runaway electrons reduce dissipation and will produce a corona around any star with a large scale $\vec{B}$ exiting a convective zone.

I. GIVEN $\vec{B}(\vec{x},t)$ IS THE EVOLUTION IDEAL? Yes locally!

Electric field $\vec{E}(\vec{x},t)$ gives $\vec{B}$ evolution, $\partial\vec{B}/\partial t = -\vec{\nabla}\times\vec{E}$.

Where $\vec{B}$ is non-zero, any electric field can be represented as $\vec{E}+\vec{u}(\vec{x},t)\times\vec{B} = -\vec{\nabla}\Phi_u(\vec{x},t)$.

Parallel representation: $\vec{B}\cdot\vec{E} = -\vec{B}\cdot\vec{\nabla}\Phi_u$, or $\dfrac{d\Phi_u}{d\ell} = -\dfrac{\vec{E}\cdot\vec{B}}{B}$.

Perpendicular representation: $\vec{u} = \dfrac{(\vec{E}+\vec{\nabla}\Phi_u)\times\vec{B}}{B^2}$.

Gives same $\vec{B}$ evolution as $\vec{E}+\vec{u}\times\vec{B}=0$, which is ideal.

II. MAGNETIC FIELD NULLS

Near a null can write $\vec{B}(\vec{x},t) = \vec{\vec{\mathcal{B}}} \cdot \{\vec{x} - \vec{x}_0(t)\}$.

An arbitrary electric field can be represented as $\vec{E} + \vec{u} \times \vec{B} = -\vec{\nabla}\Phi_u$ near a null if $\left|\vec{\vec{\mathcal{B}}}\right|$ non-zero, called a point null.

Four equations, $B_x = B_y = B_z = 0$ plus $\left|\vec{\vec{\mathcal{B}}}\right| = 0$, for four unknowns (x,y,z,t) can generally be solved only at isolated points. These points correspond to the collision or separation of two point nulls.

Note a line null is split into a set of discrete point nulls by an arbitrarily small perturbation $\delta\vec{B}$.

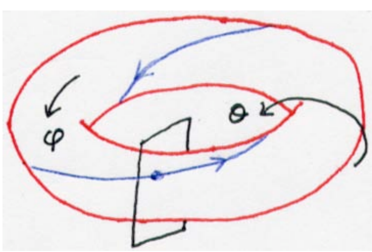
Nulls of $\vec{B}$ do not explain reconnection.

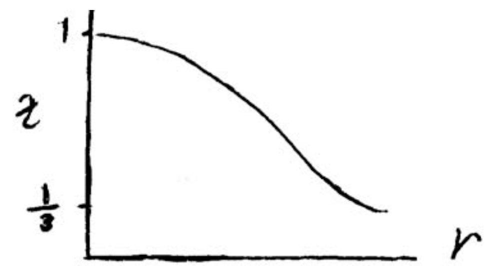
III. RECONNECTION IN TOROIDAL PLASMAS

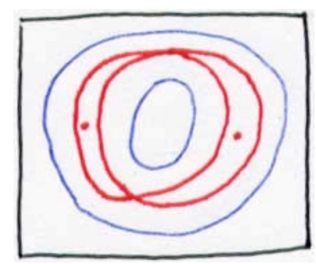
In toroidal laboratory plasmas, $\vec{B}$ lines close on themselves on isolated surfaces, the rational surfaces.

$$\frac{d\Phi_u}{d\ell} = -\frac{\vec{E}\cdot\vec{B}}{B} \text{ has no global solution if } V \equiv \oint \frac{\vec{E}\cdot\vec{B}}{B}\,d\ell \text{ non-zero.}$$

If V varies from field line to field line on a rational surface, the surface splits into islands, and reconnection occurs.





Field lines don't close on themselves in space, so island reconnection is not applicable there.

IV. EVOLUTION of $\vec{B}$ in NON-TOROIDAL SYSTEMS

A. Definition of Clebsch coordinates (ψ,α,ℓ): $\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\alpha$.

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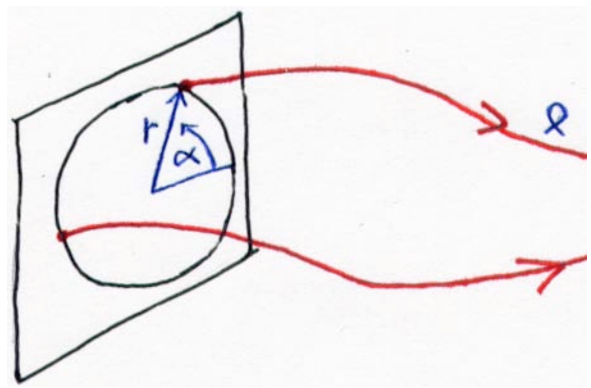
Define (r,α,ℓ) coordinates so $\vec{B}\cdot\vec{\nabla}r=0$ and $\vec{B}\cdot\vec{\nabla}\alpha=0$.



$\vec{B}=F(r,\alpha,\ell)\vec{\nabla}r\times\vec{\nabla}\alpha$, but $\vec{\nabla}\cdot\vec{B}=0$, so $\partial F/\partial\ell=0$.

Let $\partial\psi/\partial r=F(r,\alpha)$, then $\vec{B}=\vec{\nabla}\psi\times\vec{\nabla}\alpha$.

B. Magnetic evolution when Clebsch coordinates exist:

$$\left(\frac{\partial \vec{B}}{\partial t}\right)_{\vec{x}} = -\vec{\nabla}\times\vec{E} \text{ with } \vec{E}+\vec{u}\times\vec{B} = -\vec{\nabla}\Phi_u.$$

$$\vec{u} = \frac{\partial \vec{x}(\alpha,\beta,\ell,t)}{\partial t} = \left(\frac{\partial \vec{x}}{\partial t}\right)_c, \text{ velocity of } (\alpha,\beta,\ell) \text{ coordinates.}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \text{ with } \vec{A} = \alpha\vec{\nabla}\beta + \vec{\nabla}g.$$

$$\left(\frac{\partial \vec{A}}{\partial t}\right)_{\vec{x}} = \left(\frac{\partial \alpha}{\partial t}\right)_{\vec{x}}\vec{\nabla}\beta - \left(\frac{\partial \beta}{\partial t}\right)_{\vec{x}}\vec{\nabla}\alpha + \vec{\nabla}\left(\alpha\left(\frac{\partial \beta}{\partial t}\right)_{\vec{x}}\right) + \vec{\nabla}\left(\frac{\partial g}{\partial t}\right)_{\vec{x}}$$

$$\left(\frac{\partial \alpha}{\partial t}\right)_c = \left(\frac{\partial \alpha}{\partial t}\right)_{\vec{x}} + \vec{u}\cdot\vec{\nabla}\alpha = 0, \text{ which implies } \left(\frac{\partial \alpha}{\partial t}\right)_{\vec{x}} = -\vec{u}\cdot\vec{\nabla}\alpha.$$

$$\left(\frac{\partial \vec{A}}{\partial t}\right)_{\vec{x}} = -(\vec{u}\cdot\vec{\nabla}\alpha)\vec{\nabla}\beta + (\vec{u}\cdot\vec{\nabla}\beta)\vec{\nabla}\alpha + \vec{\nabla}\left[\alpha\left(\frac{\partial \beta}{\partial t}\right)_{\vec{x}} + \left(\frac{\partial g}{\partial t}\right)_{\vec{x}}\right]$$

Equivalently $\left(\frac{\partial \vec{A}}{\partial t}\right)_{\vec{x}} = \vec{u}\times\vec{B} + \vec{\nabla}\Phi_c$

Faraday's law implies $\vec{E} = -\left(\partial\vec{A}/\partial t\right)_{\vec{x}} - \vec{\nabla}\Phi$, so

$$\vec{E} + \vec{u}\times\vec{B} = -\vec{\nabla}\Phi_u.$$

V. REQUIREMENT FOR RECONNECTION

Need $\vec{u}$ or $\square_u$ in $\vec{E} + \vec{u} \,\square\, \vec{B} = \square\vec{\square}\,\square_u$ to be ill defined.

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1. In a torus $\square_u$ non-single-valued where field lines are closed.

2. Sometimes boundary conditions prevent a smooth $\square_u$.

3. Exponentially separating field lines make $\square_u$ and $\vec{u}$ ill behaved with enough exponentiations (about 20).

VI. EXPONENTIAL SEPARATION OF FIELD LINES

Generically neighboring field lines separate exponentially
$\square(\ell) = \square_0 \exp(\ell / L_L)$; Lyapunov length is L_L.

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Field lines equations $d\vec{x} / d\ell = \vec{B}(\vec{x}) / B = \hat{b}(\vec{x})$.

Trajectories given by $\vec{x}(\square, \square, \ell)$ with $\square = const.$ and $\square = const.$, where $\vec{B} = \vec{\square}\square \square \vec{\square}\square$.

Trajectory separation $\vec{\square}(\ell) = \left(\partial\vec{x} / \partial\square\right)\square\square + \left(\partial\vec{x} / \partial\square\right)\square\square$ with two independent separations, $\square\square$ and $\square\square$.

$\vec{B} \cdot (\vec{\square}_1 \square \vec{\square}_2) = const.$ Because, $d\vec{a}_\ell = (\partial\vec{x} / \partial\square) \square (\partial\vec{x} / \partial\square) d\square d\square$.

$$\frac{d\vec{\square}}{d\ell} = \vec{\square} \cdot \overleftrightarrow{K} \quad \text{where } \overleftrightarrow{K} \equiv (\overleftrightarrow{1} \square \hat{b}\hat{b}) \cdot \vec{\square}\hat{b} \cdot (\overleftrightarrow{1} \square \hat{b}\hat{b}).$$

If $\overleftrightarrow{K}$ were constant, $\vec{\square}(\ell) = \vec{\square}_0 \cdot \exp(\overleftrightarrow{K}\ell)$.

$\overleftrightarrow{K}$ has two non-zero eigenvalues. If one has a positive real part, two orthogonal separation directions exist:

$\vec{\square}_d(\ell)$ with exponentially diverging trajectories, μ $\exp(\ell / L_L)$.

$\vec{\square}_c(\ell)$ with exponentially converging trajectories, μ $\exp(\square\ell / L_L)$.

$$\left|\vec{\square}_c(\ell) \square \vec{\square}_d(\ell)\right| \; \mu \; 1/B(\ell).$$

Field lines separate exponentially unless $\overleftrightarrow{K}$ happens to be a perfectly antisymmetric tensor—requires careful design.

VII. RECONNECTION & EXPONENTIAL SEPARATION

With exponential separation, $\vec{u}$ and Φ_u of $\vec{E}+\vec{u}\times\vec{B}=-\vec{\nabla}\Phi_u$ are ill behaved.

$$\vec{B}\cdot\vec{\nabla}\Phi_u=\vec{E}\cdot\vec{B} \text{ implies } \frac{\vec{\delta}_c}{\left|\vec{\delta}_c\right|}\cdot\vec{\nabla}\Phi_u \propto e^{\ell/L_L};$$

Field line velocity $\vec{u}\propto\exp(\ell/L_L)$.

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To illustrate reconnection, assume:

1. Small perturbation $\delta\vec{B}$ is added to a system, $\vec{B}(\vec{x},t)=\vec{B}_0+\delta\vec{B}$, which twists field lines in a small region transverse to $\vec{B}_0$.
2. $\vec{B}_0$ is static and curl free.
3. Field lines of $\vec{B}_0$ e-fold apart many times within the system.

Assume transverse variations large and linearize equations:

1. Ohm's law $\Box\vec{E} + \Box\vec{v} \Box \vec{B}_0 = \Box_{\parallel}\Box\vec{j}_{\parallel}$.
2. Twist perturbation to vector potential $\Box\vec{A} = (\Box A_{\parallel} / B_0)\vec{B}_0$.
3. Force balance $\Box\partial\Box\vec{v} / \partial t = \Box\vec{j} \Box \vec{B}_0 + \Box\Box\Box^{2}\Box\vec{v}$.
4. Ampere's law $\vec{\Box}\ \Box\Box\vec{B} = \Box_0\Box\vec{j}$.

Obtain
$$\frac{\partial}{\partial \ell}\Box v_{\mathrm{A}}^2 \frac{\partial(\Box j_{\parallel} / B_0)}{\partial \ell}\Box\Box\ \Box\frac{\Box j_{\parallel}}{B_0}\Box = \frac{\partial^2(\Box j_{\parallel} / B_0)}{\partial t^2}.$$

Alfvén speed $v_A = \sqrt{B_0^2 / \Box_0\Box}$

Dissipation:
$$\Box\frac{\Box j_{\parallel}}{B_0}\Box = \Box\Box\frac{\Box_{\parallel}}{\Box_0} + \Box\Box\Box\Box^{2}_{\Box}\frac{\partial(\Box j_{\parallel} / B_0)}{\partial t} + \frac{\Box\Box_{\parallel}}{\Box_0}\Box^{4}_{\Box}\frac{\Box j_{\parallel}}{B_0}.$$

If $\quad = 0$, field line twist relaxes at v_A along the field lines.

With dissipation, assume v_A=*const*. and a very slow time variation compared to the time an Alfvén wave takes to propagate.

Then solution is also a solution to $\frac{\partial}{\partial \ell}\frac{\Box j_\parallel}{B_0} = \pm\sqrt{\frac{\Box_\parallel \Box}{\Box_0 v_A^2}}\,\Box^2_{\Box}\,\frac{\Box j_\parallel}{B_0}.$

Without dissipation $\Box j_\parallel / B_0$ must be constant along each field line, so if field lines exponentially separate $\frac{\vec{\Box}_c}{|\vec{\Box}_c|}\cdot\vec{\Box}\,\frac{\Box j_\parallel}{B_0}\ \mu\ e^{\ell/L_L}.$

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With dissipation, $\Box j_\parallel / B_0$ constant along $\vec{B}$ for $\ell << L_D$ and constant across $\vec{B}$ for $\ell >> L_D$, where $L_D / L_L \ \Box\ \ln(\Box_0 v_A L_L / \Box_\parallel \Box)^{1/4}\ \Box\ 20.$

Discussion of Reconnection through Exponentiation

An arbitrary evolution of $\vec{B}(\vec{x},t)$ tends to increase the number of Lyapunov lengths along some field lines.

Once a field line has a length of more than $20L_L$, a further tendency to increase would cause reconnection (diffusion of magnetic field lines) rather give more Lyapunov lengths.

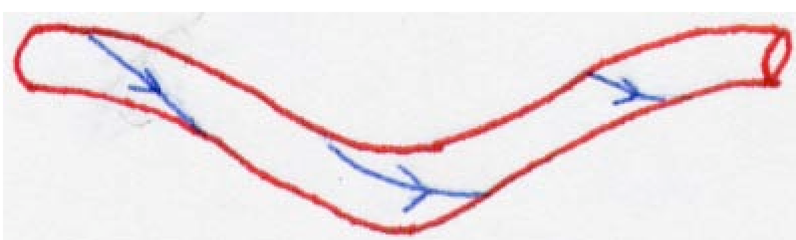
Note Ampere's law implies a magnetic field must have a significant curl within a Lyapunov length, L_L.

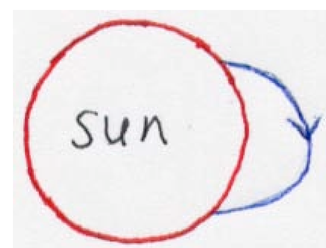
VIII. RAPID ENERGY TRANSFER FROM $\vec{B}$ TO PLASMA

Reconnection need not be implied by a rapid transfer of energy.

1. Ideal $\vec{B}$ evolution can give kinks and loss of equilibrium, which transfer energy to the plasma.

2. If $j_{\parallel}/\left(en\sqrt{T_e/m_e}\right)$ increases along $\vec{B}$, electron distribution suddenly switches from a near Maxwellian to a very high energy, or runaway, distribution for $j_{\parallel}/\left(en\sqrt{T_e/m_e}\right)$ □1.





Physics of Runaway Electrons (Dreicer 1960)

For a near Maxwellian, $f = f_M + \delta f$, kinetic eq. $\dfrac{\partial \delta f}{\partial t} - \dfrac{eE}{m_e}\dfrac{\partial f_M}{\partial \mathrm{v}} = -\nu_e \delta f$.

$\delta f = -\dfrac{e\mathrm{v}E}{T\nu_e} f_M$, so $j = e\int \mathrm{v}\,\delta f\, d^3\mathrm{v} = \dfrac{ne^2E}{m_e\nu_e}$, and $\dfrac{\delta f}{f_M} \sim \dfrac{j}{en\sqrt{T_e/m_e}}$.

For super thermal electrons $m_e d\mathrm{v}/dt = -eE - m_e\nu_e \mathrm{v}$, but $\nu_e \propto 1/\mathrm{v}^3$.

Runaway electons reach whatever energy is needed to carry current.

Runaway phenomenon reduces $E_{||}$ and hence dissipation.

Without runaway $E_{||} = \eta j_{||}$ with $\eta \propto n^0/T_e^{3/2}$.

Runaway Electron Effect in Solar Atmosphere

Scale of magnetic fields on sun is of order $10^4 km$.

Density scale height (due to gravity) above photosphere and below corona is $n/(dn/dr)=100km$ with T_e almost constant.

Magnetic field lines above sun form 10^4km loops exiting from convective zone. $j_{||}$ must be large with short correlation across $\vec{B}$.

$j_{||}$/B must be essentially constant along a magnetic field line since $\vec{\nabla} \cdot \vec{j} = 0$ and plasma pressure is too small to support $\vec{j} \times \vec{B}$ force.

If electrons remained Maxwellian, $j_{||} / en\sqrt{T_e / m_e}$ would increase by exp(10^4km/100km)= $3\text{x}10^{43}$ from bottom to top of loops.

PRIMARY RESULTS

1. Maxwell equations imply any evolution $\vec{B}(\vec{x},t)$ is ideal locally.

2. Local ideality broken in laboratory plasmas on toroidal surfaces on which field lines close on themselves. *Textbook example of reconnection but not relevant to space or astrophysics.*

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