Particle Simulation of Magnetically Confined Plasmas

W. W. Lee
Princeton Plasma Physics Laboratory
Princeton University
Princeton, NJ 08543

Present Co-workers:
Gyrokinetics: Z. Lin, T. S. Hahm, J. Lewandowski
Beams: H. Qin, R. C. Davidson

OUTLINE

- Progress in Particle Simulation
- Gyrokinetic Particle Simulation of Microinstabilities
- Future of Gyrokinetic Particle Simulation
- Particle Simulation of Relativistic Beams
- Summary and Conclusions

Abstract

- Particle Simulation + Massively Parallel Computers

  –*a dynamite combination: [Reynders, ..., Lin]*

  –*local, explicit, scalar*

- Particle Simulation is a Powerful Tool:

  –for Tokamaks and Stellarators
  (microturbulence, neoclassical and MHD physics)

  –for High Energy Particle Beams
  (space charge effects)

- Advantages of Particle Simulation

  –Minimal deviation from the original kinetic equations
  (linear and nonlinear kinetic effects)

  –Minimal numerical restrictions due to recent advances
  (large time step, large grid spacing, low numerical noise)

Particle Simulation of the Vlasov-Maxwell System

- The Vlasov equation,

$$\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + \mathbf{v} \cdot \frac{\partial F}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right) \cdot \frac{\partial F}{\partial \mathbf{v}} = C(F).$$

- Particle Pushing,

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{v_j}, \qquad \frac{d\mathbf{v}_j}{dt} = \frac{q}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}_j \times \mathbf{B}\right)_{\mathbf{x}_j}.$$

- Klimontovich-Dupree representations,

$$F = \sum_{j=1}^{N} \delta(\mathbf{x} - \mathbf{x}_j)\delta(\mathbf{v} - \mathbf{v}_j),$$

- Poisson's equation : $\mathbf{E} = -\nabla\phi$

$$\nabla^2\phi = -4\pi \sum_{\alpha} q_\alpha \sum_{j=1}^{N} \delta(\mathbf{x} - \mathbf{x}_{\alpha j})$$

- Ampere's Law and Faraday's Law

Particle Simulation of the Vlasov-Maxwell System (cont.)

- Vlasov equation is solved in Lagrangian coordinates

  –*Nonlinear PDE* $\Rightarrow$ *Linear ODE : Particle Pushing*

- Maxwell equations are solved in Eulerian coordinates

  –*Linear PDE*

- Collisions are treated as sub-grid phenomena

  –*Monte-Carlo processes*

- Suitable for high frequency short wavelength physics, e.g.,

$$\omega \approx \omega_{pe} \quad k\lambda_D \approx 1$$

- Disparate spatial and temporal scales for physics of

$$\omega \approx \omega_*, \quad k\rho_s \approx 1$$

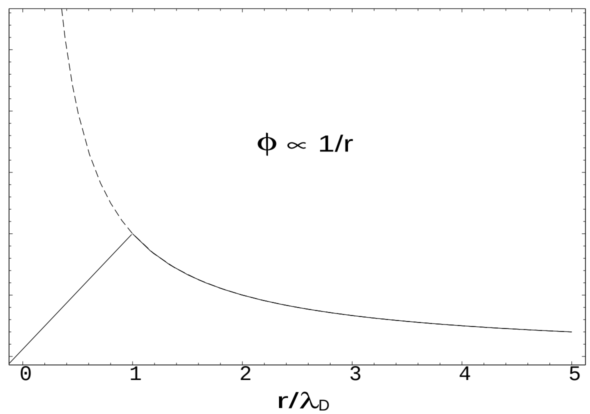
- Enhanced numerical noise (N: no. of particles)

$$\delta n/n \gg 1/\sqrt{N}$$

Progress in Particle Simulation

- Early attempts [*Buneman (1959); Dawson (1962)*]
- Finite-Size Particles and Particle-in-Cell Simulation [*Dawson et al. BAPS (1968) and Birdsall et al. BAPS (1968)*]

  –Coulomb potential is modified for a finite size particle due to Debye shielding – no need to satisfy $1/(n\lambda_D^3) \ll 1$.



- Number of calculations for N particles

  –N^2 for direct interactions and $N log N$ for PIC
- Collisionless Simulations [*Langdon et al. (1971)*]
- Collisions are re-introduced via Monte-Carlo methods [*Shanny, Dawson & Greene* (1976)]

Progress in Particle Simulation (cont.)

- Numerical Properties

  –Grid spacing imposed by Debye shielding [Langdon (71)]:

$$\Delta x < \lambda_D$$

  –Time step imposed by high freq. oscillat'ns [Langdon (79)]:

$$\omega_{pe}\Delta t < 1$$

  –Time step imposed by fast electrons [Langdon (79)]:

$$k v_{te}\Delta t < 1$$

  –Noise enhanced by Debye shielding [Okuda et al. (71)]:

$$\frac{\delta n}{n} \approx \frac{1}{\sqrt{N}(k\lambda_D)}$$

Progress in Particle Simulation (cont.)

- Implicit Schemes [Mason (1982); Denavit (1982); Langdon et. al (1982)]

  –Instability: $\omega_{pe}\Delta t > 1$

  –Inaccuracy: $kv_{te}\Delta t = (\omega_{pe}\Delta t)(k\lambda_D) > 1$

  –for $k\lambda_D \ll 1 \Rightarrow \omega_{pe}\Delta t > 1$ **but keeping** $kv_{te}\Delta t < 1$

- Culprits: plasma waves originated from space charge effects
- Quasineutral waves are the waves of interest in tokamaks
- Reduced Vlasov-Maxwell equations:

  –Gyrokinetic ordering:

$$\frac{\omega}{\Omega_i} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\rho}{L_{eq}} \sim \frac{\delta B}{B} \sim \frac{e\phi}{T} \sim O(\epsilon); k_\perp \rho \sim O(1)$$

  –Gyrophase average

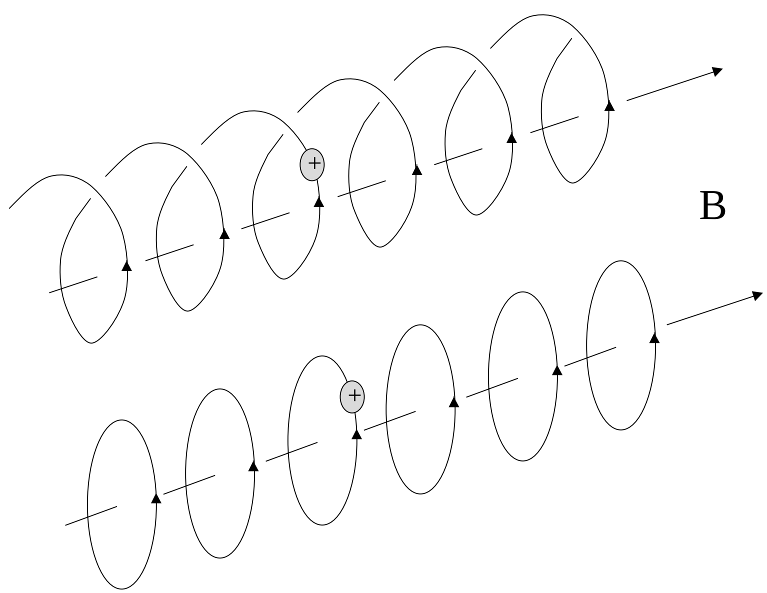
Gyrokinetic Theory

- Linear theory : Rutherford and Frieman (1968);
  Taylor and Hastie (1968); Catto (1978)
- Nonlinear Theory:
  –Frieman and Chen (1982) – in Fourier k-space
  –Lee (1983) - in real space
- Nonlinear Theory – Lie perturbation methods:
  –Dubin et al. (1983) - electrostatic slab
  –Hahm (1988) - electrostatic toroidal
  –Hahm et al. (1988) - electromagnetic slab
  –Brizard (1989) - electromagnetic toroidal and reduced MHD
  –Qin et al.(1999) - compressional-Alfven and Bernstein waves
  –Qin et al. (2000) - pressure balance
  –Qin et. ql. (Sherwood 2000) - Gyro-gauge theory

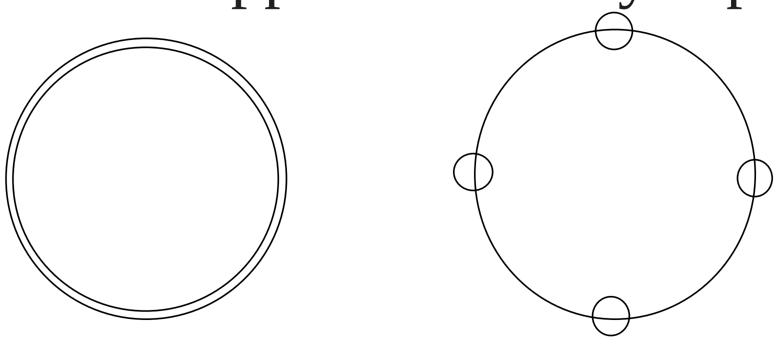
Gyrokinetic Particle Simulation

[Lee, PF ('83); Lee, JCP ('87)]

- Gyrophase-averaged Vlasov-Maxwell equations for low frequency microinstabilities.
- The spiral motion of a charged particle is modified as a rotating charged ring subject to guiding center electric and magnetic drift motion as well as parallel acceleration.



- A charged ring is further approximated by 4-point average,



valid for $k_\perp \rho_i \leq 2$.

- Debye shielding is replaced by polarization shielding in the gyrokinetic model giving rise to quasineutral simulation,

$$\nabla^2 \phi = -4\pi\rho \quad \Rightarrow \quad (\frac{\rho_s}{\lambda_D})^2 \nabla_\perp^2 \phi = -4\pi e(\overline{n}_i - n_e),$$

- Equations of Motion

$$\frac{d\mathbf{R}}{dt} = U\hat{\mathbf{b}} + \mathbf{v}_d - \frac{c}{B}\frac{\partial\overline{\phi}}{\partial\mathbf{R}} \times \hat{\mathbf{b}}, \quad \frac{\mu}{B} \equiv \frac{v_\perp^2}{2B} = const.,$$

$$\frac{dU}{dt} = -[\hat{\mathbf{b}} + \frac{U}{\Omega}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial\mathbf{R}})\hat{\mathbf{b}}] \cdot (\mu\frac{\partial}{\partial\mathbf{R}} lnB + \frac{q}{m}\frac{\partial\overline{\phi}}{\partial\mathbf{R}}),$$

Gyrokinetic Particle Simulation (cont.)

- Numerical Properties of a Gyrokinetic Plasma

–Grid spacing imposed by cold electron response

$$\Delta x < \rho_s; \quad (\rho_s/\lambda_D \approx 100)$$

–Time step imposed by cold electron response $(\omega_H \equiv \frac{k_\parallel}{k_\perp}\frac{\lambda_D}{\rho_s}\omega_{pe})$

$$\omega_H \Delta t \ll 1; \quad (\omega_{pe}/\omega_H \approx 1000)$$

–Time step restricted by streaming of thermal electrons:

$$k_\parallel v_{te} \Delta < 1$$

–Noise enhanced by ω_H:

$$\delta n/n \approx 1/\sqrt{N}(k\rho_s).$$

- We need to get rid of cold electron response to lift these restrictions.

Gyrokinetic Particle Simulation (cont.)

Methods to further improve the numerical properties

- Adiabatic electron response ($\delta n_e/n_0 \approx e\phi/T_e$) can achieve all of the above, but by completely forfeiting non-adiabatic electron dynamics from simulation, e.g., wave-particle interactions.
- Split weight δf simulation scheme can relax all the numerical restrictions:

  –Grid Spacing: $\Delta x \gg \rho_s$

  –Time Step: $\omega_* \Delta t < 1$ (or $\omega_A \Delta t < 1$), $k_{\|} v_{te} \Delta t \gg 1$;

  –Noise: $\delta n/n \approx 0 \rightarrow 1/\sqrt{N}$.
- With this scheme, accuracy now dictates the number of particle used in the simulation.

The Perturbative (δf) Particle Simulation Scheme *PPPL*

[Dimits and Lee, JCP (1993); Parker and Lee, FPB (1993)]

- The Vlasov equation,

$$\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + \mathbf{v} \cdot \frac{\partial F}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E} \cdot \frac{\partial F}{\partial \mathbf{v}} = 0.$$

- For $F = F_o + \delta f$,

$$\frac{d\delta f}{dt} = -\frac{dF_o}{dt}$$

- Let $W = \delta f / F$ to obtain

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{E}, \quad \frac{dW}{dt} = -(1-W)\mathbf{v} \cdot \frac{\partial}{\partial \mathbf{x}}(\frac{q\phi}{T_e}),$$

$$\delta f = \sum_{j=1}^{N} W_j \delta(\mathbf{x} - \mathbf{x}_j)\delta(\mathbf{v} - \mathbf{v}_j).$$

- Easy access to both linear and nonlinear regimes
- Gyrokinetic Poisson's equation remains unchanged.

Split Weight Perturbative (δf) Particle Simulation Scheme

[Manuilskiy and Lee, PoP(2000)]

- Let $\delta f_e = (e\phi/T_e)F_{0e} + \delta h_e$ to obtain

$$\frac{d\delta h_e}{dt} = -\frac{\partial}{\partial t}\frac{e\phi}{T_e}F_{0e} + \frac{\mathbf{v}}{2}\cdot\left[\frac{\partial}{\partial \mathbf{x}}(\frac{e\phi}{T_e})^2\right]F_{0e}$$

- For $w^{NA} = \delta h_e/F$,

$$\frac{dw^{NA}}{dt} = \frac{1-w^{NA}}{1+e\phi/T_e}\left[-\frac{\partial}{\partial t}\frac{e\phi}{T_e} + \frac{\mathbf{v}}{2}\cdot\frac{\partial}{\partial \mathbf{x}}\left(\frac{e\phi}{T_e}\right)^2\right].$$

- Modified Poisson's equation and Charge Conservation

$$\left(\lambda_D^2\nabla^2 - 1\right)\frac{e\phi}{T_e} = \int \delta h_e d\mathbf{v} - \delta n_i,$$

$$\lambda_D^2\nabla^2\left(\frac{\partial}{\partial t}\frac{e\phi}{T_e}\right) = -\frac{\partial}{\partial \mathbf{x}}\cdot\int \mathbf{v}\delta h_e d\mathbf{v},$$

$$\delta h_e = \sum_{j=1}^{N} w_j^{NA}\delta(\mathbf{x}-\mathbf{x}_j)\delta(\mathbf{v}-\mathbf{v}),$$

Split-weight scheme for finite-β plasmas

[W. W. Lee, J. L. V. Lewandowski and T. S. Hahm, Sherwood(2000)]

- The governing gyrokinetic Vlasov equation,

$$\frac{dF_\alpha}{dt} \equiv \frac{\partial F_\alpha}{\partial t} + v_\parallel \hat{\mathbf{b}} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + c\frac{\mathbf{E}^L \times \hat{\mathbf{b}}_0}{B_0} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial F_\alpha}{\partial v_\parallel} = 0,$$

$$\hat{\mathbf{b}} = \hat{\mathbf{b}}_0 + \frac{\delta \mathbf{B}}{B_0}, \quad \hat{\mathbf{b}}_0 = \frac{\mathbf{B_0}}{B_0}$$

$$\delta \mathbf{B} = \nabla A_\parallel \times \hat{\mathbf{b}}_0,$$

$$\mathbf{E} = E_\parallel^T \hat{\mathbf{b}}_0 + \mathbf{E}^L,$$

$$\mathbf{E}^L = -\nabla \phi, \quad E_\parallel^T = -\frac{1}{c}\frac{\partial A_\parallel}{\partial t}.$$

Finite-β simulation (cont.)

- Gyrokinetic Poisson's equation & Ampere's law,

$$(\frac{\rho_s}{\lambda_D})^2 \nabla_\perp^2 \phi = -4\pi e(n_i - n_e),$$

$$\nabla_\perp^2 A_\| = -\frac{4\pi}{c} J_\|,$$

$$n_\alpha = \int F_\alpha dv_\|, \quad J_\| = \sum_\alpha q_\alpha \int v_\| F_\alpha dv_\|.$$

- The generalized Ohm's law

$$[\frac{n_e}{n_i}\omega_{pe}^2 + \frac{n_i}{n_0}\omega_{pi}^2 - c^2\nabla_\perp^2] E_\| / 4\pi e$$

$$= \hat{\mathbf{b}} \cdot \nabla \int v_\|^2 (F_i - F_e) dv_\| - v_A^2 \hat{\mathbf{b}} \cdot \nabla \int (F_i - F_e) dv_\|$$

Finite-β Simulation, (cont.)

- $F_\alpha = F_{0\alpha} + \delta f_\alpha$, $\delta f_e = \psi F_{0e} + \delta h_e$.
- $E_\| = -\hat{\mathbf{b}} \cdot \nabla \psi = -\hat{\mathbf{b}} \cdot \nabla \phi - (1/c)\partial A_\| / \partial t$
- Generalized Ohm's law

$$[1 - \nabla_\perp^2]\psi = -\beta \int v_\|^2 (\delta f_i - \delta h_e) dv_\| + \int (\delta f_i - \delta h_e) dv_\|.$$

- Equations for $\partial \psi / \partial t$

$$(\beta \frac{m_i}{m_e} - \nabla_\perp^2) \frac{\partial \psi}{\partial t} = \beta \frac{\partial}{\partial x_\|} \int v_\|^3 (\delta f_i - \delta h_e) dv_\|$$

$$- \frac{\partial}{\partial x_\|} \int v_\| (\delta f_i - \delta h_e) dv_\|$$

- Split Weight: $w^{NA} = \delta h_e / F_e$

$$\frac{dw^{NA}}{dt} = \frac{1 - w^{NA}}{1 + \psi} \left[-\frac{\partial \psi}{\partial t} + \frac{v_\|}{2} \frac{\partial \psi^2}{\partial x_\|} \right].$$

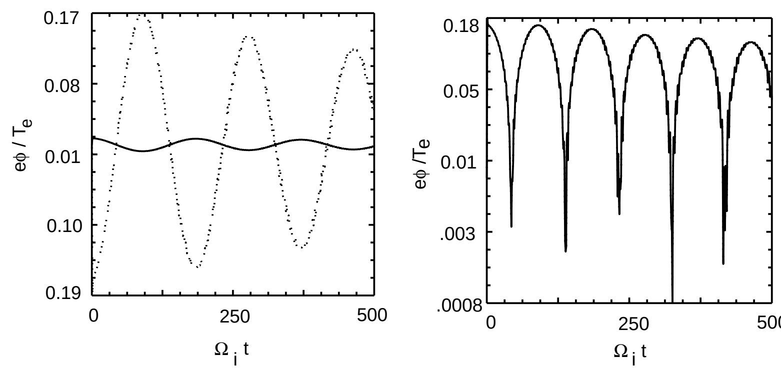
Finite-β Simulation, (cont.)

Shear-Alfven Waves

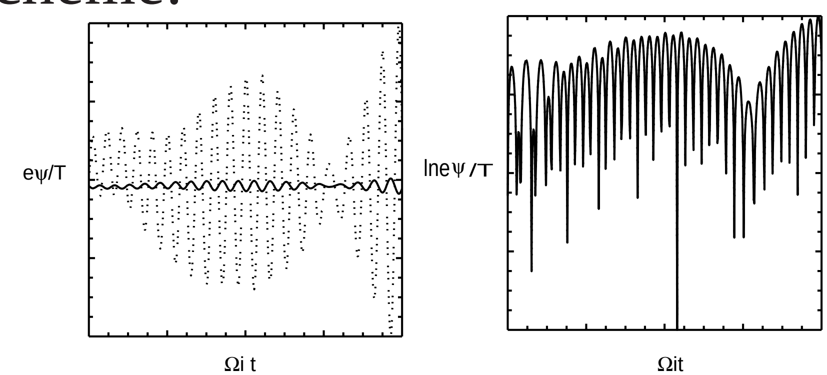
- One-Dimensional Simulation:

$$k_\perp \rho_s = 0.8, \beta = 10\%, k_{||}/k_\perp = 0.01, T_e/T_i = 1$$

- Split weight scheme: $\omega \Delta t \approx 0.033, k_{||} v_{te} \Delta t \approx 0.344$



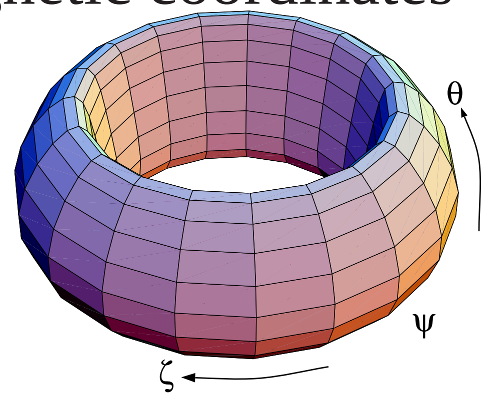
- Original δf scheme:



Gyrokinetic Particle Simulation Codes

⇒1,2&3D electrostatic and electromagnetic (Darwin) slab codes

⇒3D electrostatic global toroidal code in magnetic coordinates



- GTC [Lin et al. *Science* (1998)]

- Basic formalism for GTC
  - –magnetic coordinates (ψ, θ, ζ) [Boozer, 1981]
  - –guiding center Hamiltonian [White and Chance, 1984]
  - –non-spectral Poisson solver [Lin and Lee, 1995]

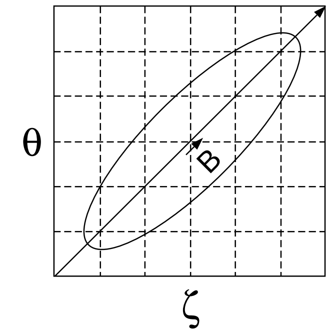
- Advanced features for GTC
  - –general geometry (e.g., stellarator)
  - –profile effects (e.g., low-aspect ratio, steep gradient)
  - –capable of simulating full poloidal cross section (*global*) or thin annulus box (*local*)

Global Field-line Coordinates

[Z. Lin, in preparation]

- Microinstability wavelength: $\lambda_\perp \propto \rho_i$, $\lambda_\| \propto qR$

  –grid # $N \propto a^2$, a: minor radius



$$\Delta\zeta \propto \rho$$

- Most global codes: w/o field-line coordinates

  –grid # $N \propto a^3$

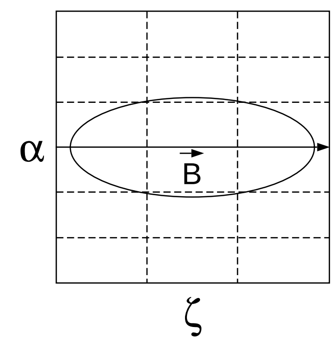
- GTC global code: use field-line coordinates

$$(\psi, \alpha, \zeta), \qquad \alpha = \theta - \zeta/q$$

  –grid # $N \propto a^2$

  –larger time step: no high $k_\|$ modes

  –order of magnitude saving of computing time for reactor size simulations



$$\Delta\zeta \propto R$$

- Field-line coordinates in flux-tube codes [Dimits and Beer]

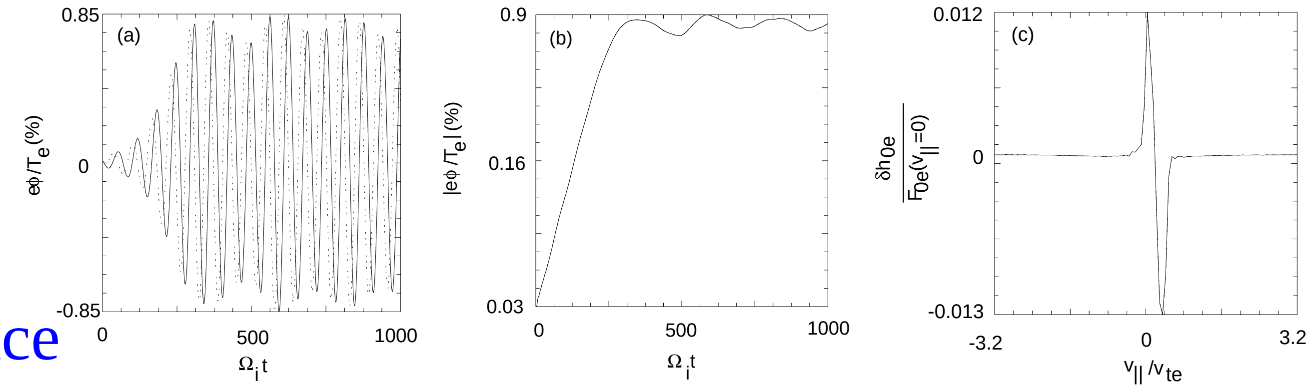
Gyrokinetic Particle Simulation of Microinstabilities

- Codes:
  - –1&2D slab codes
  - –3D global toroidal code in general geometry
- Physics:
  - –wave-particle interactions: nonlinear saturation
  - –zonal flows: reduction of fluctuation and transport
  - –collisional effects: enhancement of transport due to weak ν_{ei}, ν_{ii}
- Scaling trends:
  - –Mixing Length Rule: $\gamma_L/k_\perp^2$,
  - –Resonance Broadening: $\Delta\omega_{NL}/k_\perp^2$,

Nonlinear Saturation: Three Mode Coupling in Slab Geometry

- 1D drift instability: (linear: $\omega \Delta t \approx 0.72, k_{||} v_{te} \Delta t \approx 2.7$)

Trapping of resonant particles in velocity space



[Parker and Lee ('93); Manuilskiy and Lee ('00)]

- 2D drift instability: $E \times B$ trapping of resonant particles
- 2D drift instability with collisions: saturation level caused by $E \times B$ trapping is greatly enhanced by weak collisions.
- 2D ITG drift instability: $E \times B$ trapping of resonant particles

[Lee et al., ('84), Federici et al. ('87), Lee and Tang ('88)]

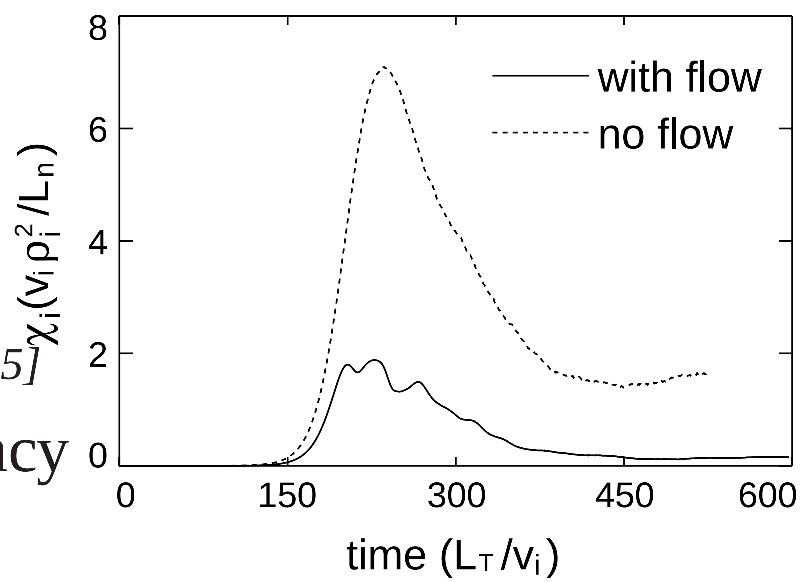
Gyrokinetic Particle Simulation of Microinstabilities (cont.)

Global Toroidal Simulation

- Without zonal flow *[Lee and Santoro, PoP (1997)]*
  - –Nonlinear saturation : $E \times B$ trapping of resonant particles
  - –Energy cascade to low (m,n) modes
  - –Consistent with Resonance Broadening Scaling (Dupree):

$$\chi_i \propto (k_\perp \rho_s)^2 |e\phi/T_e|^2 (1/k_{||}\rho_s)(cT/eB)$$

- With zonal flow: Reduction of Turbulent Transport *[Z. Lin et al., Science* **281***, 1835 (1998)]*
  - –Reduction of radial decorrelation length and fluctuation level *[Biglari et al., '90, Hahm and Burrell, '95]*
  - –Broadening of mode frequency



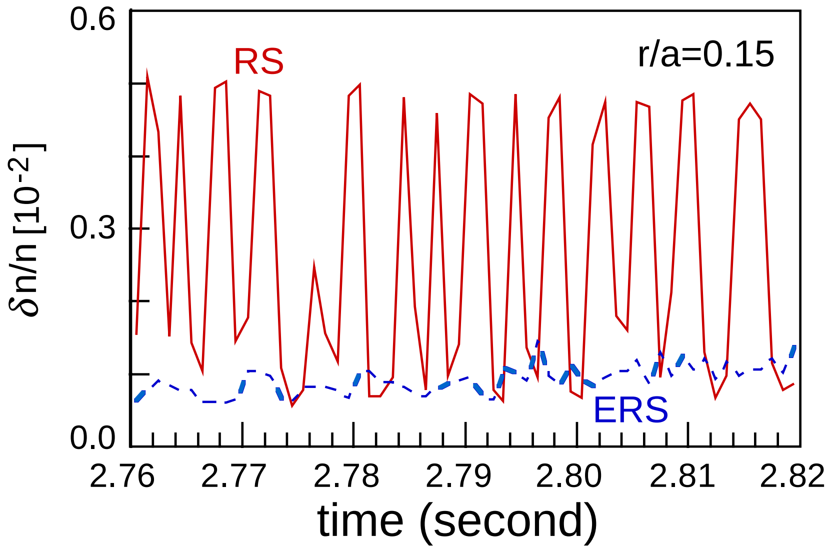
Gyrokinetic Particle Simulation of Microinstabilities (cont.)

Collisional Effects on Toroidal ITG modes

- Weak collisions have negligible effects on linear growth, but can enhance transport due to collisional damping of the zonal flow,i.e.,

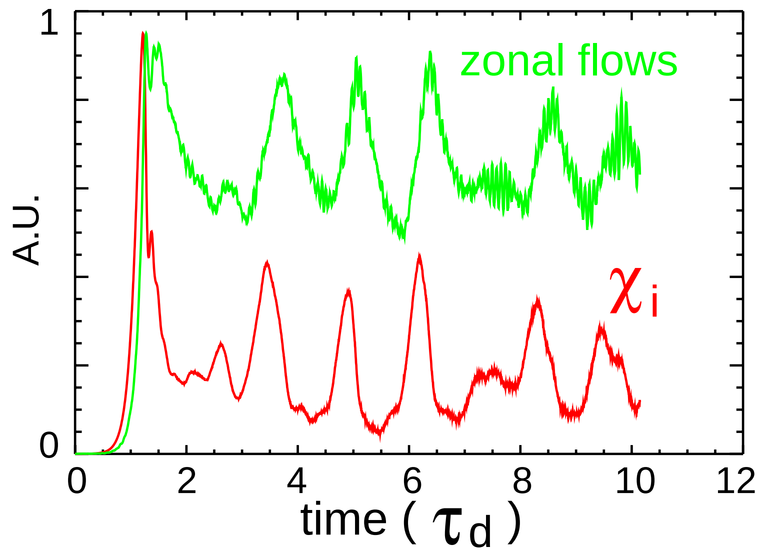
$$\chi_i \propto \nu^*$$

- Bursting Behavior



[Mazzucato, et al., PRL, 1996]

large bursts of fluctuation in TFTR RS plasmas observed period ~ collisional flow damping time



[Lin, et al., 1999]

collisional damping of zonal flows causes bursts of turbulent transport in gyrokinetic simulations

Gyrokinetic Particle Simulation of Neoclassical Transport

- Neoclassical δf scheme: $f = f_0 + \delta f$
  f_0: local Maxwellian; δf: deviations due to magnetic drift

  [Lin, Tang, and Lee, 1995; Sasinowski and Boozer, 1995]

- Advantages: reduced noise; steady state; Forker-Planck collision operators conserving momentum and energy

- Formal derivation and validation of collisional δf method: source and sink; nonlinear collision operators

  [Chen and White, 1997]

- Neoclassical χ_i near magnetic Axis

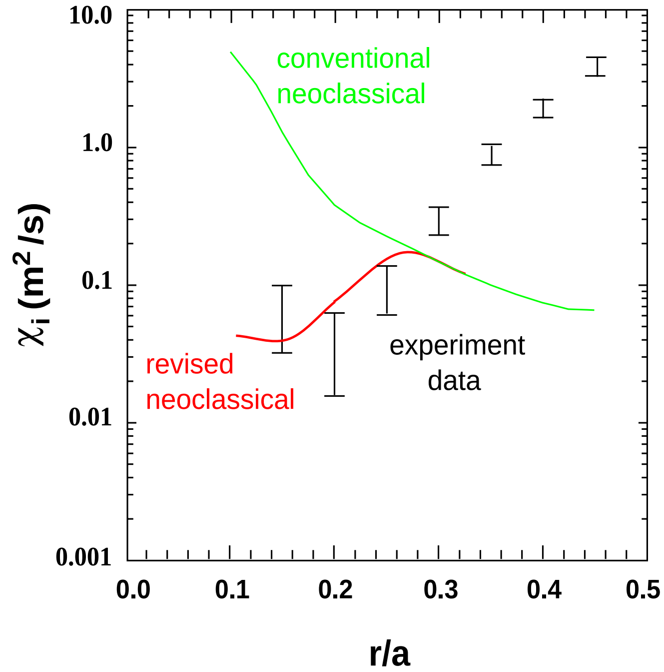
  [Lin, Tang, and Lee, 1997]

- Widely used in stellarator neoclassical transport calculation

Neoclassical χ_i Near Magnetic Axis

- Ion thermal transport levels in ERS plasmas observed to fall below the neoclassical theory "irreducible minimum level"
- Simulations lead to improved calculation of neoclassical χ_i

  [Lin, Tang, and Lee, Phys. Rev. Lett. , 1997]



- Physical picture:
  - –orbit size reduced due to $\rho_p \sim r$
  - –higher energy particle: larger reduction
  - –outward energy flux reduced
  - –χ_i decreases for smaller minor radius

Plans for Gyrokinetic Particle Simulation

- Trapped electron physics:
  - Field-line coord. + Split-weight + Adiabatic field pusher

$$k_{\|} v_{te} \Delta t >> 1, \quad \omega \ll \omega_{be}$$

- Finite-β modified microinstabilities
  - Shear-Alfven waves: split-weight for passing electrons
- Gyrokinetic MHD: neoclassical drive and turbulence drive
  - Compressional Alfven Waves *[Qin et al., 1999]*
  - Pressure Balance *[Qin et al. to appear]*
- Wave Heating - high frequency gyrokinetics *[Qin et al. 1999]*

–IBW physics

Perturbative Particle Simulation of Space Charge Dominated Relativistic Beams

- δf simulation model *[Lee, Qian, and Davidson, 1997]*
  - –based on KV equilbrium: $F_0 \propto \delta(W)$
- Non KV equilibrium:
  - –simulation: *[Stoltz, Davidson, Lee, 1999]*
  - –theory: *[Davidson, Qin, Channell, 1999]*
- 3D multi-species NL δf code: *[Qin, Davidson, Lee, 1999]*

  Beam Equlibrium, Stability, Transport (BEST) Code
- e-p instability (two-stream)
  - –theory: *[Davidson et al, 1999]*
  - –simulation: *[Qin, Davidson and Lee, 1999 and 2000]*
- Darwin model: nonradiative simulation for chamber transport

  [Lee, Qin, and Davidson, 2000]

Darwin Model for Beam Physics

$$\mathbf{F}_j^{foc} = -\gamma_j m_j \omega_{\beta j}^2 \mathbf{x}_\perp,$$

$$\{\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - [\gamma_j m_j \omega_{\beta j}^2 \mathbf{x}_\perp + e_j \nabla_\perp(\phi - \frac{v_z}{c} A_z)]\cdot\frac{\partial}{\partial \mathbf{p}_\perp}$$

$$-e_j \frac{\partial}{\partial z}(\phi - \frac{v_z}{c} A_z)\frac{\partial}{\partial P_z}\} f_j(\mathbf{x}, \mathbf{p}_\perp, P_z, t) = 0,$$

$$\nabla^2 \phi = -4\pi \sum_j e_j \int d^2 p_\perp dP_z f_j(\mathbf{x}, \mathbf{p}_\perp, P_z, t),$$

$$\nabla^2 A_z = -\frac{4\pi}{c} \sum_j e_j \int d^2 p_\perp dP_z v_z f_j(\mathbf{x}, \mathbf{p}_\perp, P_z, t).$$

$$\mathbf{v}_\perp = \mathbf{p}_\perp / \gamma_j m_j$$

$$v_z = (P_z - \frac{e_j}{c}\delta A_z)/\gamma_j m_j$$

Darwin Model for Beam Physics (cont.)

$$\frac{d\mathbf{x}_{ji}}{dt} = \mathbf{v}_{ji},$$

$$\frac{d\mathbf{p}_{\perp ji}}{dt} = -\gamma_j m_j \omega_{\beta j}^2 \mathbf{x}_{\perp ji} - e_j \nabla_\perp (\phi - \frac{v_{zji}}{c} A_z).$$

$$\frac{dP_{zji}}{dt} = -e_j \frac{\partial}{\partial z}(\phi - \frac{v_{zji}}{c} A_z).$$

$$\frac{dw_{ji}}{dt} = -(1 - w_{ji}) \frac{1}{f_{j0}} [\frac{\partial f_{j0}}{\partial \mathbf{p}_\perp} \cdot \delta(\frac{d\mathbf{p}_{\perp ji}}{dt}) + \frac{\partial f_{j0}}{\partial P_z} \delta(\frac{dP_{zji}}{dt})],$$

$$\delta(\frac{d\mathbf{p}_{\perp ji}}{dt}) \equiv -e_j \nabla_\perp (\delta\phi - \frac{v_{zji}}{c} \delta A_z),$$

$$\delta(\frac{dP_{zji}}{dt}) \equiv -e_j \frac{\partial}{\partial z}(\delta\phi - \frac{v_{zji}}{c} \delta A_z),$$

Darwin Model for Beam Physics (cont.)

$$\delta f_j = \sum_{i=1}^{N_{sj}} w_{ji}\delta(\mathbf{x} - \mathbf{x}_{ji})\delta(\mathbf{p}_\perp - \mathbf{p}_{\perp ji})\delta(P_z - P_{zji}),$$

$$\nabla^2 \delta\phi = -4\pi \sum_j e_j \delta n_j,$$

$$\nabla^2 \delta A_z = -\frac{4\pi}{c} \sum_j \delta j_{zj},$$

$$\delta n_j = \int d^2 p_\perp dP_z \delta f_j(\mathbf{x}, \mathbf{p}_\perp, P_z, t) = \sum_{i=1}^{N_{sj}} w_{ji} S(\mathbf{x} - \mathbf{x}_{ji}),$$

$$\delta j_{zj} = e_j \int d^2 p_\perp dP_z v_{zj} \delta f_j(\mathbf{x}, \mathbf{p}_\perp, P_z, t) = e_j \sum_{i=1}^{N_{sj}} v_{zji} w_{ji} S(\mathbf{x}-\mathbf{x}_{ji}).$$

Darwin Model for Beam Physics (cont.)

PPPL

Equilibrium Distribution

$$f_{b0}(r, \mathbf{p}_\perp, P_z) = \frac{\hat{n}_b}{\gamma_b (2\pi \gamma_b m_b T_b)^{3/2}}$$

$$\times \exp\{-\frac{p_\perp^2/2\gamma_b m_b + \gamma_b m_b \omega_{\beta b}^2 r^2/2 + e_b(\phi_0 - \beta_b A_{z0})}{T_b}\}$$

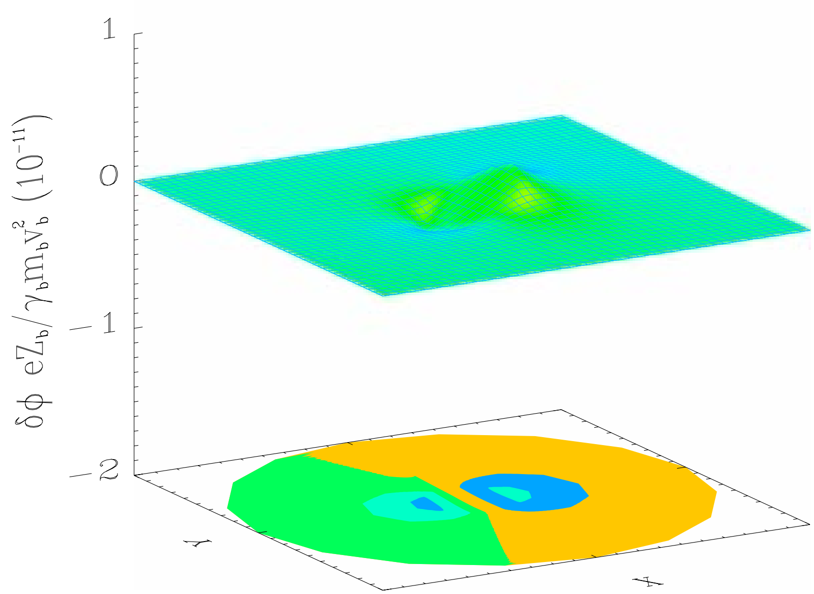
$$\times \exp\{-\frac{(P_z - \gamma_b m_b \beta_b c)^2}{2\gamma_b^3 m_b T_b}\},$$

Adiabatic Pusher

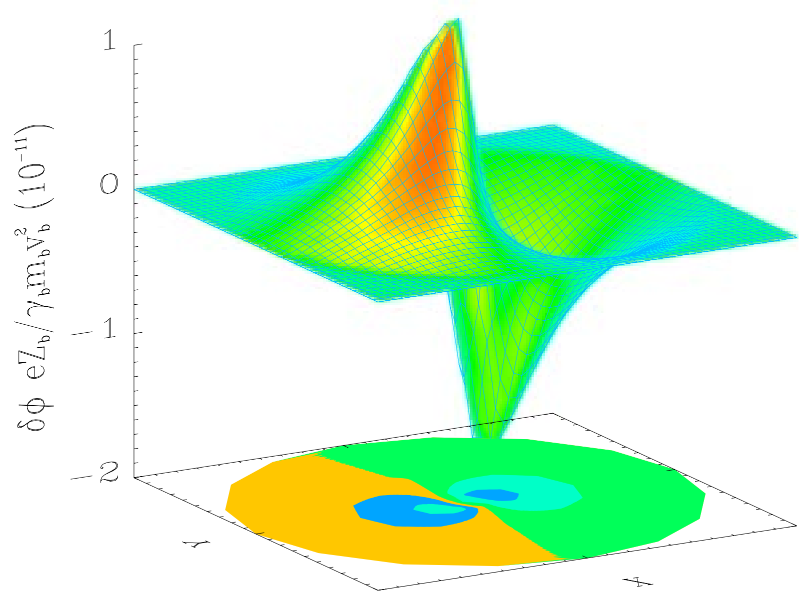
- Fast electron motion in $x - y$ plane limits the time step.
- Treat electrons as charged strings.
- Push ions and solve field equations in the time scale of the frequency of interest, i.e., less often.

Two-Stream Instability

When a background electron component is introduced in a proton storage ring, the dipole surface mode can be destabilized.



$\omega_{\beta b} t = 0$



$\omega_{\beta b} t = 200$

Electrostatic/Magnetostatic Model

[H. Qin, R. C. Davidson and W. W. Lee, Phys. Lett. A (in press)].

Summary and Conclusions

- Particle simulation finally emerges as one of the most promising tools for plasma physics research
- With thousand-processor parallel computers at NERSC and ACL/LANL, we can investigate:

MFE: $(0.1 - 1) \times 10^9$ particles with 10,000 time steps $\sim$ (30 - 300) hours on 256PE / T3E-900

– Transport scaling for large tokamaks

– NSTX simulation

– NCSX design

IFE: $(1 - 10) \times 10^6$ particles with 1,000,000 time steps

– two stream instability

– filamentation instability