

Anisotropic pressure and magnetic perturbations in tokamaks

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-Introduction:

-Externally-imposed nonaxisymmetric perturbations $\delta\mathbf{B}$ can strongly affect tokamaks, causing mode-locking, disruptions, or with benign effects such as ELM control, important issues for ITER.

-Imposing such a perturbation $\delta\mathbf{B}$ in a tokamak produces a nonambipolar “ripple” current [1-3] $\langle j^r \rangle = e\Gamma$, which produces a torque $\tau_\zeta \approx \langle j^r \rangle B_p R / c$ on the plasma.

-Less appreciated is that:

-scalar pressure gives 0 nonambipolar $\langle j^r \rangle$, hence 0 torque, so calculations using scalar-pressure equilibria are non-self-consistent.

-However, $\delta\mathbf{B}$ also produces a pressure anisotropy, $p_{\parallel} \neq p_{\perp}$, which produces both j^r and in-surface currents $j^{\theta, \zeta}$. The latter produce a self-consistent response to $\delta\mathbf{B}$, which can shield or amplify the imposed $\delta\mathbf{B}$.

-Here, we analytically compute p_{\parallel} , p_{\perp} , and from these, the perturbed currents and self-consistent $\delta\mathbf{B}$ from this effect.

-The radial current recovers earlier results for “banana-drift” (bd) fluxes.

-The in-surface currents provide an expression for the amount of shielding the plasma provides.

-The expressions for p_{\parallel} , p_{\perp} may also be used in a perturbed mhd equilibrium code, such as IPEC [4], now under development.

-Outline:

- I.Ripple transport (heuristic)
- II.Equilibrium eqns
- III.Calculation of pressure anisotropy
- IV.Perturbed flows
- V.Radial fluxes
- VI.Diagonal components & shielding
- VII.Summary

-Coordinates: Parametrize real-space position \mathbf{x} with flux coordinates

$\{q^i\} \equiv \{\rho, \theta, \zeta\}$, with $\rho(\psi) \equiv$ radial flux coordinate, which may specialize to
 $\psi \equiv$ toroidal flux/ 2π , $\chi \equiv$ poloidal flux/ 2π , or $r(\psi) \equiv (2\psi / B_0)^{1/2}$

as convenient.

-Also useful to define

$\mathbf{e}^i \equiv \nabla q^i \equiv$ covariant basis vectors,

$\mathbf{e}_i \equiv \mathcal{J} \mathbf{e}^j \times \mathbf{e}^k \equiv$ contravariant basis vectors, and

with $\mathcal{J} \equiv (\mathbf{e}^\rho \cdot \mathbf{e}^\theta \times \mathbf{e}^\zeta)^{-1} \equiv |d\mathbf{x}/d\mathbf{q}| \equiv$ Jacobian.

I. Ripple transport (heuristic): banana-drift (bd) mechanism:

-Averged nonaxisymmetric contrib to radial drift:

$$\bar{r} \simeq \hat{r} G_0$$

with $\hat{r} \equiv \delta q n \mu B_0 / (M \Omega_g r)$, "orbit-avging factor" $G_0 \simeq 1 / (q n)^{1/2}$

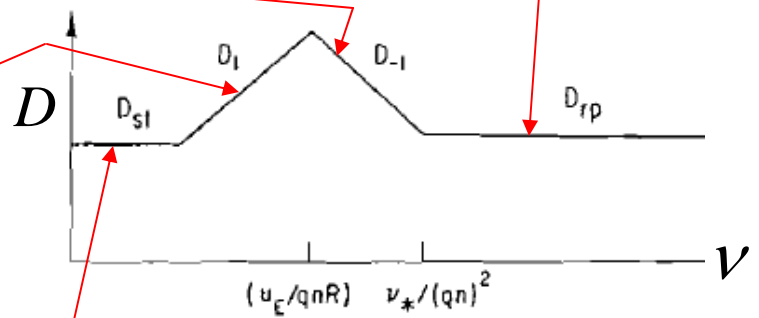
-Diffusion coef: $D \sim \epsilon^{1/2} \nu_f (\Delta r)^2$

-Ripple-plateau regime:
[Boozer, Phys. Fluids (1983).]

-Resonant case: [Linsker, Boozer, PF (1982)]

$$\Delta r \simeq \bar{r} / \nu_f, \text{ with } \nu_f \simeq \nu_t (q n)^2, \nu_t \simeq \nu / \epsilon$$

$$D_{-1} \simeq \epsilon^{1/2} \hat{r}^2 / (q n)^3 \nu_t \sim 1 / \nu$$



-Nonresonant case:[Linsker, Boozer, PF (1982)]

$$\Delta r \simeq \bar{r} / (n \Omega_d), \text{ with } \Omega_d \simeq -q \Omega_E = (q / r) (c E_r / B)$$

$$D_1 \simeq \nu_t \epsilon^{1/2} \hat{r}^2 q n / (n \Omega_d)^2 \sim \nu$$

-Stochastic regime: [Goldston, White, Boozer, PRL (1981).]

-Generalized bd transport:

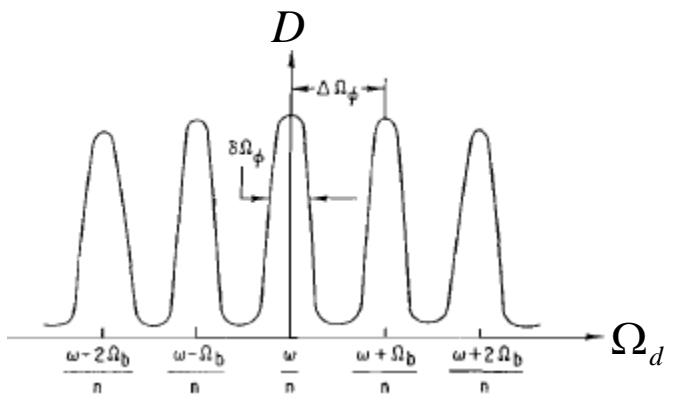
[Yushmanov, NF (83), Mynick, NF (1986)]:

Multiple bounce-harmonics l , perturbation structure including low- n MHD modes, multiple m , finite ω .

$$D = \sum_l D^l, \text{ where } D_{-1}^l \simeq \epsilon^{1/2} (\hat{r} G_l)^2 / \nu_{fl},$$

$$D_1^l \simeq \nu_{fl} \epsilon^{1/2} (\hat{r} G_l)^2 / (n \Omega_d + l \Omega_b - \omega)^2$$

$$\nu_{fl} \simeq \nu_t [(q N)^2 + (l/2)^2].$$



II. Equilibrium:

-For each species s , have force-balance equation:

$$\mathbf{j}_s \times \mathbf{B} = c\mathbf{f}_s, \quad \text{with} \quad \mathbf{f} = \nabla \cdot \mathbf{P} + en\nabla\Phi + \mathbf{f}_{xt} \quad (1)$$
$$\mathbf{P} = p\mathbf{I} + \boldsymbol{\pi} = p_{\perp}\mathbf{I} + (p_{\parallel} - p_{\perp})\hat{\mathbf{B}}\hat{\mathbf{B}}, \quad p = (2p_{\perp} + p_{\parallel})/3, \quad \text{and}$$

$$\begin{bmatrix} n \\ p_{\parallel} \\ p_{\perp} \end{bmatrix}(\mathbf{x}) = \int d\mathbf{v} \begin{bmatrix} 1 \\ Mv_{\parallel}^2 \\ Mv_{\perp}^2/2 \end{bmatrix} f(\mathbf{z})$$

-Torque on plasma:

$$\boldsymbol{\tau}(\psi) = \int_V d\mathbf{x} (\nabla \cdot \mathbf{P}) \times \mathbf{x} = \oint_S d\mathbf{S} \cdot \mathbf{P} \times \mathbf{x} \xrightarrow{\text{CGL } \mathbf{P}} \oint_S d\mathbf{S} \times \mathbf{x} p_{\perp}$$
$$\xrightarrow{\text{scalar } p} p(\psi) \oint_S d\mathbf{S} \times \mathbf{x} = 0.$$

III. Calculation of pressure anisotropy:

-III.A. 1st, if take $f(\mathbf{z})=f_0(\mathbf{I})=f_n$ of constants of motion, Hall & McNamara(75) showed[6] that

$$p_{\perp,\parallel} = p_{\perp,\parallel}(\psi, \zeta_d \equiv \zeta - q\theta, B) \quad \text{and} \quad (2)$$

$$\partial_B(p_{\parallel}/B) = -p_{\perp}/B^2, \quad \text{which implies} \quad \partial_{\ell} p_{\parallel} = (p_{\parallel} - p_{\perp})\partial_{\ell} \ln B$$

a reln which can also be shown using parallel force-balance from (1).

-However, if $p_{\perp,\parallel} = p_{\perp,\parallel}(\psi, B)$, can show[6,7] this produces 0 nonambipolar transport. Thus, nonzero torque implies pressure anisotropy, but pressure anisotropy but doesn't necessarily imply torque.

-Eg, taking $\mathbf{I} = (E, \mu, \bar{\psi})$, with $E = Mv^2/2 + e\Phi(\mathbf{x})$, then for

$$f_0(\mathbf{I}) = \bar{n}(M/2\pi T_{\parallel})^{1/2}(M/2\pi T_{\perp}) \exp[-(E - \bar{\Phi})/T_{\parallel}] \exp[-\mu\bar{B}(T_{\perp}^{-1} - T_{\parallel}^{-1})], \quad \text{find}[5]$$

$$\begin{bmatrix} n \\ p_{\parallel} \\ p_{\perp} \end{bmatrix}(\mathbf{x}) = \begin{bmatrix} 1 \\ T_{\parallel} \\ T_{\perp}/d \end{bmatrix} \bar{n} \exp(-e\Phi_1/T_{\parallel})/d, \quad \text{with} \quad (3)$$

$$\Phi_1 \equiv \Phi(\mathbf{x}) - \bar{\Phi}, \quad d \equiv \tau + (\bar{B}/B)(1 - \tau) \quad \text{and} \quad \tau \equiv T_{\perp}/T_{\parallel}$$

Eqs.(3) observe (2).

III.B. pressure anisotropy, cont'd:

-Taking $T_{\parallel} = T_{\perp}$, solve the drift-kinetic equation

$$(\partial_t + L_{H0})\delta f - C\delta f = -\bar{r}\partial_r f_0 + C f_0, \quad L_{H0} \equiv \Omega_d \partial_{\zeta_d} + \dot{\theta} \partial_{\theta} = \Omega_d \partial_{\zeta_d} + \Omega_b \partial_{\theta_b} \quad (4)$$

with banana-center radial drift velocity (for a single-harmonic pert.)

$$\bar{r} = (q/M\Omega_g r) \partial_{\zeta_d} \mu B = \hat{r} \sin \eta = \hat{r} \sum_{l=-\infty}^{\infty} G_l \sin \eta_l, \quad (5)$$

with gyrofrequency Ω_g , $\zeta_d = \zeta - q\theta$, $\hat{r} \equiv \delta q n \mu B_0 / (M\Omega_g r)$

$$\eta \equiv \eta_{nm} \equiv n\zeta - m\theta - \omega t, \quad \eta_l \equiv n\zeta_d + l\theta_b - \omega t, \quad \text{and orbit averaging factor}$$

$$G_l[qN\theta_t(y)] \equiv \oint \frac{d\theta_b}{2\pi} \exp(-il\theta_b) \exp[iqN\theta(\theta_b, y)]. \quad (qN \equiv qn - m.) \quad (6)$$

Using Krook collision operator $C_K \delta f = -\nu_f \delta f$

allows one to treat nonresonant ($D \sim \nu$) and resonant ($D \sim 1/\nu$) bd regimes together.

Use model magnetic field $B(\mathbf{x}, t) = B_s(\psi, \theta) + \delta B(\mathbf{x}, t)$, with (7)

$$B_s \equiv B_0(1 - \epsilon \cos \theta), \quad \delta B(\mathbf{x}, t) \equiv -B_0 \delta \cos \eta$$

-Can superpose results for multiple-harmonic δB 's.

-III., cont'd.

-Then one finds perturbed distribution fn

$$\delta f \simeq (\hat{r}/B_0\delta)\partial_r f_0(\tau_\Omega\delta B - \tau_\nu\partial_{n\zeta}\delta B), \quad \text{with} \quad (8)$$

$$\tau_\Omega \equiv \omega'_0/(\omega'_0{}^2 + \nu_f^2), \quad \tau_\nu \equiv \nu_f/(\omega'_0{}^2 + \nu_f^2), \quad \text{and}$$

$$\omega'_0 \equiv \omega - n\Omega_d.$$

-Putting (8) into (1), find

$$\begin{bmatrix} \delta p_{\parallel} \\ \delta p_{\perp} \end{bmatrix}(\mathbf{x}) = n_0 T \begin{bmatrix} 2I_{\nu\parallel} \\ I_{\nu\perp} \end{bmatrix} \frac{I_K}{\pi^{1/2}} \left(-\tau_\Omega \frac{\delta B}{B} + \tau_\nu \frac{\partial_{n\zeta}\delta B}{B} \right) (\hat{r}_T/\delta), \quad (9)$$

where $\hat{r}_T \equiv \hat{r}(\mu B_0 \rightarrow T) = \delta q n T / (M \Omega_g r)$, $I_{\nu\parallel} \equiv \int_{B/B_{tp}}^1 du (1-u)^{1/2} u \simeq (2/3)(1 - B/B_{tp})^{3/2}$, $I_{\nu\perp} \equiv \int_{B/B_{tp}}^1 du (1-u)^{-1/2} u^2 \simeq 2(1 - B/B_{tp})^{1/2}$, and $I_K \equiv \int_0^\infty dx x^{5/2} e^{-x} [\kappa_n + \kappa_T(x - 3/2) + \kappa_\phi] = \Gamma(7/2) \bar{\kappa}$, with $\Gamma(7/2) = (15/8)\pi^{1/2}$, $\bar{\kappa} \equiv [\kappa_n + \kappa_T(7/2 - 3/2) + \kappa_\phi]$, $\kappa_n \equiv -\partial_r \ln n$, $\kappa_T \equiv -\partial_r \ln T$, $\kappa_\phi \equiv -(\partial_r e\Phi)/T$.

IV. Perturbed flows:

From (1), have $\nabla \cdot \mathbf{P} = \nabla p + \nabla \delta p_{\perp} + (p_{\parallel} - p_{\perp})\boldsymbol{\kappa} + \hat{\mathbf{B}}\mathbf{B} \cdot \nabla((p_{\parallel} - p_{\perp})/B)$ (10)

with $\delta p_{\perp} \equiv p_{\perp} - p$ and $\boldsymbol{\kappa} \equiv \hat{\mathbf{B}} \cdot \nabla \hat{\mathbf{B}} = \text{curvature}$.

This gives perpendicular currents

$$\mathbf{j}_{\perp} = B^{-1}\hat{\mathbf{B}} \times \mathbf{f} \equiv \mathbf{j}_{\perp p} + \mathbf{j}_{\perp A} + \mathbf{j}_{\perp B}, \quad \text{where} \quad (11)$$

$$\mathbf{j}_{\perp p} \equiv (c/B)\hat{\mathbf{B}} \times (\nabla p + en\nabla\Phi),$$

$$\mathbf{j}_{\perp A} \equiv (c/B)\hat{\mathbf{B}} \times \nabla \delta p_{\perp}, \quad \mathbf{j}_{\perp B} \equiv (c/B)\hat{\mathbf{B}} \times \boldsymbol{\kappa} \delta p_{\perp\parallel},$$

$$\text{with } \delta p_{\perp\parallel} \equiv (p_{\perp} - p_{\parallel}) \simeq \delta p_{\perp}$$

As usual, compute parallel part of currents \mathbf{j}_l , (with $l \rightarrow \{p, A, B\}$), by requiring $\nabla \cdot \mathbf{j}_l = 0$. This yields magnetic differential equation

$$\mathbf{B} \cdot \nabla(g_l \equiv j_{\parallel l}/B) = -\nabla \cdot \mathbf{j}_{\perp l}. \quad (12)$$

-IV.B. Flows from scalar pressure:

-Specializing to Boozer coordinates, one has

$$\begin{aligned}\mathbf{B} &= I\nabla\theta + G\nabla\zeta + \psi'\beta_*\nabla\rho \equiv I\mathbf{e}^\theta + G\mathbf{e}^\zeta + \psi'\beta_*\mathbf{e}^\rho \\ &= \psi'\nabla\rho \times \nabla\theta + \chi'\nabla\zeta \times \nabla\rho \equiv \mathcal{J}^{-1}(\psi'\mathbf{e}_\zeta + \chi'\mathbf{e}_\theta), \quad \text{and} \quad \mathcal{J} = (G\psi' + I\chi')/B^2.\end{aligned}\tag{13}$$

Then from (11a),

$$\mathbf{j}_{\perp p} = \frac{F}{B^2}(G\nabla\zeta \times \nabla\rho - I\nabla\rho \times \nabla\theta), \quad F(\rho) \equiv c(p' + en_0\Phi').\tag{14}$$

and from (12),

$$(\chi'\partial_\theta + \psi'\partial_\zeta)g_p = -F(G\partial_\theta B^{-2} - I\partial_\zeta B^{-2}).\tag{15}$$

$$\text{Decomposing } B^{-2}, \quad B^{-2} = B_0^{-2}[1 + \Sigma'\delta_{nm} \cos \eta_{nm}],\tag{16}$$

we find the parallel portion of \mathbf{j}_p :⁹

$$g_p \equiv j_{\parallel p}/B = g_{p0} + \frac{F}{B_0^2\psi'}\Sigma'\frac{In + Gm}{n - \iota m}\delta_{nm} \cos \eta_{nm}\tag{17}$$

-IV.C. Flows from anisotropic pressure increment:

-Analogous to (14), one finds

$$\mathbf{j}_{\perp A} \simeq \frac{c\delta p'_{\perp}}{B^2}(G\nabla\zeta \times \nabla\rho - I\nabla\rho \times \nabla\theta) + \frac{c}{B^2}(I\partial_{\zeta} - G\partial_{\theta})\delta p_{\perp}\nabla\theta \times \nabla\zeta, \quad (18a)$$

$$\mathbf{j}_{\perp B} \simeq \frac{c\delta p_{\perp\parallel}}{2}(G\nabla\zeta \times \nabla\rho - I\nabla\rho \times \nabla\theta)\partial_{\rho}(B^{-2}) + [(I\partial_{\zeta} - G\partial_{\theta})(B^{-2})\nabla\theta \times \nabla\zeta, \quad (18b)$$

where the 2nd lines in (18a,b) come from the radial current j^{ρ} .

-Analogous to (15), one has, with $g_{AB} \equiv g_A + g_B$,

$$(\chi'\partial_{\theta} + \psi'\partial_{\zeta})g_{AB} = c\{B^{-2}(G'\partial_{\theta} - I'\partial_{\zeta})\delta p_{\perp} + \frac{1}{2}\delta p_{\perp}(G'\partial_{\theta} - I'\partial_{\zeta})B^{-2} - \frac{1}{2}\delta p'_{\perp}(G\partial_{\theta} - I\partial_{\zeta})B^{-2} + \frac{1}{2}(\partial_{\rho}B^{-2})(G\partial_{\theta} - I\partial_{\zeta})\delta p_{\perp}\} \quad (19)$$

V. Radial fluxes:

-Eqs.(14, 18a, 18b) give the radial component of the current:

$$j^\rho = j_p^\rho + j_A^\rho + j_B^\rho = \mathcal{J}^{-1} c [(B^{-2})(I\partial_\zeta - G\partial_\theta)\delta p_\perp + (\delta p_\perp/2)(I\partial_\zeta - G\partial_\theta)(B^{-2})] \quad (28)$$

-Flux-surface avging this, one finds the radial flux:

$$\begin{aligned} \langle j^\rho \rangle &\equiv e\langle \Gamma \rangle \equiv V'^{-1} \oint d\theta d\zeta \mathcal{J} j^\rho = -V'^{-1} c \oint d\theta d\zeta (\delta p_\perp/2) \partial_y (B^{-2}) \\ &\simeq \frac{c}{V' B_0^3} \oint d\theta d\zeta \delta p_\perp \partial_y \delta B = \frac{c p_\nu}{V' B_0^4} \oint d\theta d\zeta (\partial_{n\zeta} \delta B) (\partial_y \delta B) \\ &\simeq e \frac{q_{nm}}{q} \bar{D}_{bd} \bar{\kappa} n_0, \quad \bar{D}_{bd} \equiv \oint \frac{d\theta d\zeta}{(2\pi)^2} I_{\nu\perp} \bar{r}_T^2 \tau_\nu \Gamma(7/2) / \pi^{1/2} \sim \epsilon^{1/2} \bar{r}_T^2 \tau_\nu \end{aligned} \quad (29)$$

with $\partial_y \equiv (I\partial_\zeta - G\partial_\theta)$.

$\bar{r}_T \equiv (\hat{r}_T/\delta)(\partial_\eta \delta B/B_0)$ the thermal value of the radial drift velocity,

$\delta p_\perp = -p_\Omega(\delta B/B) + p_\nu(\partial_{n\zeta} \delta B/B)$, and $p_{\Omega,\nu} \equiv n_0 T I_{\nu\perp} (I_K/\pi^{1/2})(\hat{r}_T/\delta) \tau_{\Omega,\nu}$

VI. Diagonal components & shielding:

-Of special interest for shielding are the “diagonal” components g_{AB}^{nm} of g_{AB} , contributing at the same (n,m)-harmonic as the applied perturbation.

-An applied single-harmonic perturbation (now specializing $\rho \rightarrow r$)

$\delta \mathbf{B} \simeq \tilde{B}_r \sin \eta \nabla r$, corresponds to a current

$\delta \mathbf{j} = (c/4\pi) \nabla \times \delta \mathbf{B} \simeq (c/4\pi) \tilde{B}_r \cos \eta \nabla \eta \times \nabla r$, and gives a surface deformation

$\xi_r \simeq -\tilde{\xi}_r \cos \eta$, with

$\tilde{\xi}_r^{nres} = R\tilde{b}_r / (n - m\iota)$ (with $\tilde{b}_r \equiv \tilde{B}_r / B_0$) \equiv nonresonant amplitude

$\tilde{\xi}_r^{res} = \pm (2\tilde{b}_r R q_0^2 / m q_0')^{1/2} \equiv$ resonant amplitude, and with mod-B perturbation

$\delta B \simeq B_s(r + \xi_r, \theta) - B_s(r, \theta) \simeq (\tilde{\xi}_r / R) B_0 \cos \eta \cos \theta$.

-Diagonal components & shielding (cont'd):

-Writing (9b) as
$$\delta p_{\perp} = -p_{\Omega}(\delta B/B) + p_{\nu}(\partial_{n\zeta}\delta B/B), \quad (21)$$

the 1st & 4th terms in (19) give a nonzero diagonal contribution:

$$\partial_{\eta}g_{AB} \sim \cos\theta\partial_{\eta}(-p_{\Omega}\cos\eta\cos\theta + p_{\nu}\partial_{\eta}\cos\eta\cos\theta)$$

-One finds

$$g_{AB}^{nm} = \frac{c}{2B_0^2\psi'} \frac{[2\epsilon(G'm + I'n) + \epsilon'(Gm + In)]\tilde{\xi}_r}{n - im} \frac{1}{R}(p_{\Omega}\cos\eta_{nm} + p_{\nu}\sin\eta_{nm}). \quad (22)$$

-Comparing with (17), g_{AB}^{nm} can be comparable with the scalar-pressure contribution g_p^{nm} , depending on parameters:

$$\frac{g_{AB}^{nm}}{g_p^{nm}} \sim 2\epsilon'p_{\Omega,\nu}/p' \simeq \frac{2p\epsilon'}{p'}F_t\frac{15}{8}\bar{\kappa}(qn\rho v/r)\tau_{\Omega,\nu} \simeq \frac{30}{8}F_t\bar{\kappa}(qn v_{Bt})\tau_{\Omega,\nu}$$

with $v_{Bt} = \rho v/R$.

-Diagonal components & shielding (cont'd):

-Analogous to electrostatic shielding:

$$\delta\rho_{xt} = \delta\rho - \sum_s \delta\rho_s = (1 + \sum_s K_s)\delta\rho \quad (23)$$

with susceptibility $K_s \equiv -\delta\rho_s/\delta\rho$

one may define a current/magnetic field shielding equation

$$\delta j_{||xt} = \delta j_{||} - \sum_s \delta j_{||ABs}^{nm} = (1 + \sum_s K_{js})\delta j_{||}, \quad (24)$$

with response function K_{js} [defining complex pressure $p_c \equiv (p_\Omega - ip_\nu)$]

$$K_{js} \equiv -j_{||ABs}^{nm}/\delta j_{||} = -\frac{2\pi p_{cs}}{B_0^2} \frac{[2\epsilon(G'm + I'n) + \epsilon'(Gm + In)] \tilde{\xi}_r}{\psi'(n - im) \tilde{b}_r k_\theta R}. \quad (25)$$

→ expect appreciable shielding/amplification for $|K_j| \sim 1$

-Diagonal components & shielding (cont'd):

-Using $In/Gm \simeq (rB_p/RB_t)(n/m) \simeq \epsilon^2 u_{nm} \ll 1$, $\epsilon G'/(\epsilon'G) \ll 1$, $\tilde{\xi}_r \rightarrow \tilde{\xi}_r^{nres}$,
Eq.(25) simplifies to

$$K_{js} \simeq -\frac{2\pi p_{cs}}{B_0^2} \frac{1}{(n - \nu m)^2}. \quad (26)$$

-Heuristically extend (26) from nonresonant to resonant regime

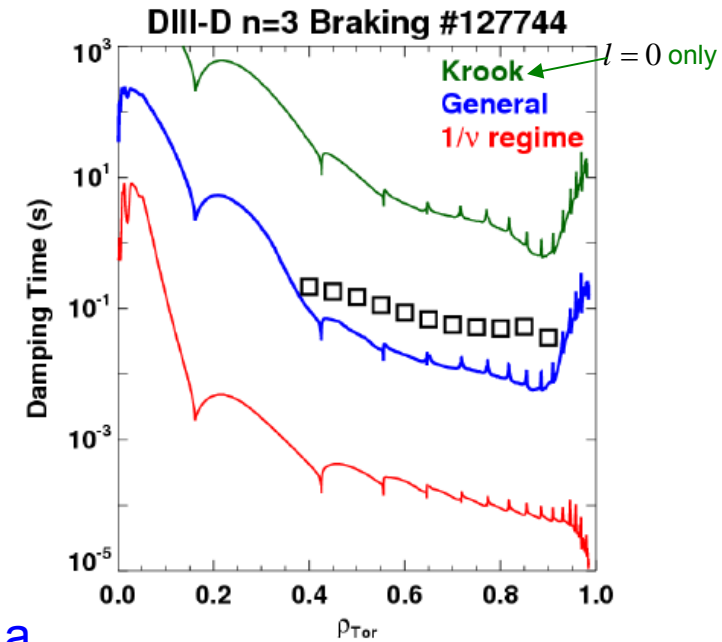
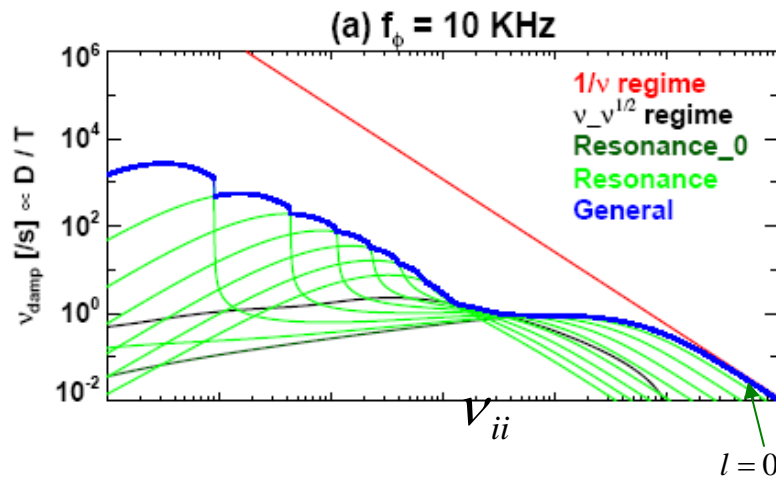
$$K_{js} \simeq -(2\pi p_{cs}/B_0^2)\mathcal{R}^2, \quad \text{with resonance factor} \quad (27)$$

$$\mathcal{R}^2(\rho, \tilde{b}_r) = (\min[|\tilde{\xi}_r^{nres}|, |\tilde{\xi}_r^{res}|]/(R\tilde{b}_r))^2 = \min[(n - \nu m)^{-2}, (2q_0^2/\tilde{b}_r Rmq'_0)].$$

K_j has peaks at resonant surface, which decrease with \tilde{b}_r as $1/\tilde{b}_r$.
Thus, for fixed pressure, as \tilde{b}_r increases the peak in $|K_j|$ will become too small,
for the plasma to fully shield the perturbation. The larger the pressure,
the larger the capacity of the plasma to produce shielding currents,
as observed experimentally.

-Other development toward a fuller theory:

-Recent work by Park and Boozer[9] incorporates the multiple bounce-harmonic contributions $D = \sum_l D^l$, into D , finding damping rates in improved agreement with experiment.



-Incorporation of pressure-anisotropy expression into a perturbed equilibrium code like IPEC would provide for a yet more complete description of RMPs in tokamaks.

-Numerical calculation of $\nabla \cdot P$, eg, via a \mathcal{F} code, would provide a yet more complete, self-consistent solution.

VII. Summary:

- We have obtained analytic expressions (9) for the pressure anisotropy induced by an applied nonaxisymmetric magnetic perturbation in a tokamak. These may be used analytically, as here, or numerically, in a perturbed mhd equilibrium code such as IPEC.
- We have found expressions (14,17,18,19) for the modifications of the equilibrium flows/currents due to this perturbation, the 1st self-consistent analytic calculation of this effect.
- Of special interest is the “diagonal” component of this, at the same (n,m) harmonic as the applied perturbation, given by Eq.(22).
- This component provides a shielding response function (25) and shielding criterion (24), for when the self-consistent response becomes comparable with the applied perturbation. The form of this function indicates that plasmas at higher pressure should be able to shield out larger applied perturbations, consistent with observations.
- Flux-surface averaging the radial component of the currents [Eq.(29)] recovers the banana-drift transport fluxes, obtained previously.[1-3]

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