PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76CH03073

PPPL-3502
UC-70

PPPL-3502

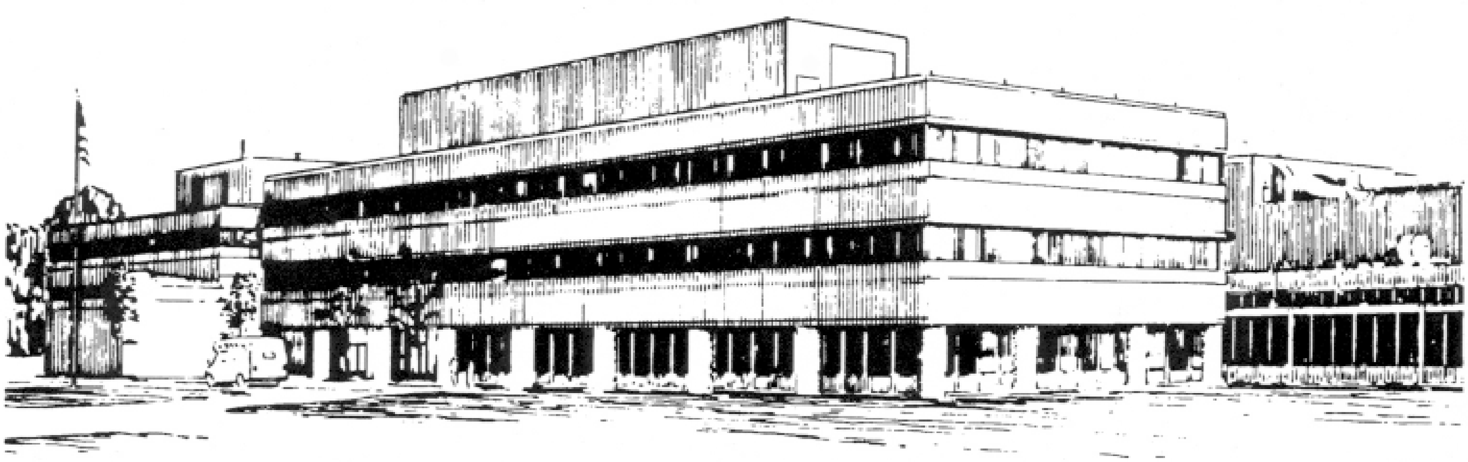
Global Stability of the Field Reversed Configuration

by

E.V. Belova, S.C. Jardin, H. Ji, R.M. Kulsrud,
W. Park, and M. Yamada

November 2000

PRINCETON PLASMA
PHYSICS LABORATORY



PRINCETON PLASMA PHYSICS LABORATORY
PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY

Global Stability of the Field Reversed Configuration

E. V. Belova, S. C. Jardin, H. Ji, R. M. Kulsrud, W. Park, M. Yamada

Princeton Plasma Physics Laboratory, Princeton, NJ 08543, USA
E-mail: ebelova@pppl.gov

Abstract. New computational results are presented which provide a theoretical basis for the stability of the Field Reversed Configuration (FRC). The FRC is a compact toroid with negligible toroidal field in which the plasma is confined by a poloidal magnetic field associated with toroidal diamagnetic current. Although many MHD modes are predicted to be unstable, FRCs have been produced successfully by several formation techniques and show surprising macroscopic resilience. In order to understand this discrepancy, we have developed a new 3D non-linear hybrid code (kinetic ions and fluid electrons), M3D-B, which is used to study the role of kinetic effects on the $n = 1$ tilt and higher n modes in the FRC. Our simulations show that there is a reduction in the tilt mode growth rate in the kinetic regime, but no absolute stabilization has been found for $\bar{s} \lesssim 1$, where $\bar{s}$ is the approximate number of ion gyroradii between the field null and the separatrix. However, at low values of $\bar{s}$, the instabilities saturate nonlinearly through a combination of a lengthening of the initial equilibrium and a modification of the ion distribution function. These saturated states persist for many Alfven times, maintaining field reversal.

1. Introduction

The FRC is a compact toroid with negligible toroidal field in which the plasma is confined by a poloidal magnetic field associated with toroidal diamagnetic current carried by the plasma. It has been known for some time that a MHD model of the FRC is unstable to many modes with large growth rates, of order of the inverse Alfven transit time. However, experimentally, FRCs have been produced successfully by several formation techniques and show surprising macroscopic resilience.
In order to understand this apparent discrepancy, we have developed a new non-linear 3D-simulation code, M3D-B [1], that is intrinsically kinetic (hybrid). The ions are treated as particles, and the electrons as a cold fluid. Quasi-neutrality is assumed. The delta-f method [2] is used to greatly reduce simulation noise and computational requirements. We have performed hybrid simulations of the $n = 1$ tilt mode to study how the kinetic effects associated with large thermal ion orbits can modify MHD predictions. The stability properties of both prolate and oblate configurations have been examined.

Important parameters for the stability of the FRC are the approximate number of ion gyroradii between the field null and the separatrix, $\bar{s}$, and the elongation of the separatrix surface, E. For large $\bar{s}$, we recover the MHD results. In the MHD limit all prolate configurations (with $E > 1$) are unstable to many modes with differing toroidal mode number n. Non-linearly, the internal tilt ($n = 1$) mode grows to large amplitude, and destroys the configuration. For oblate configurations (with $E < 1$), the $n = 1$ tilt mode becomes external, and can be stabilized in the MHD regime by a nearby conducting wall, although higher n modes remain unstable [1].

However, at low $\bar{s}$, typical of much of the experimental work to date, we find that the situation is much more complex. In prolate FRCs, linear growth rate of the tilt mode is reduced when $\bar{s} \lesssim 2$ due to finite Larmor radius (FLR) effects, and the nonlinear saturation becomes

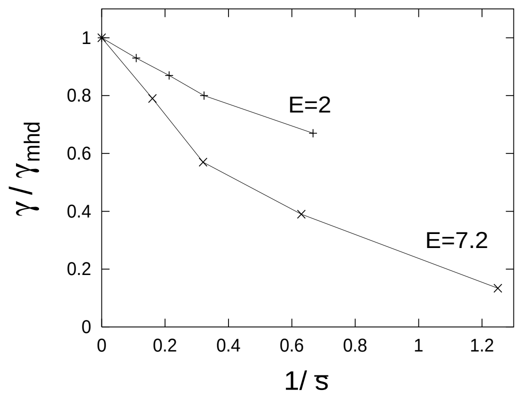


FIG.1: Variation of the normalized growth rate of $n = 1$ tilt mode with $1/\bar{s}$ parameter.

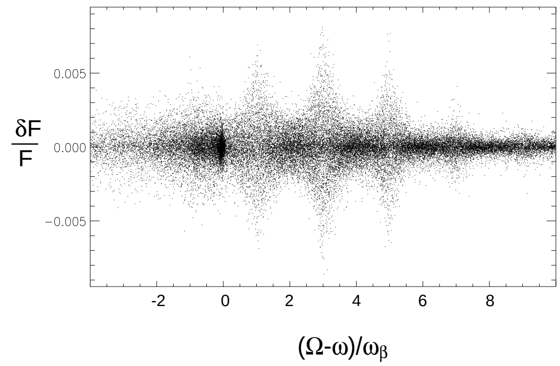


FIG.2: Scatter plot of the particle distribution from linearized simulation of the $n = 1$ tilt mode with $E = 7.2$ and $\bar{s} = 0.8$, racetrack separatrix.

possible. In contrast, the stability properties of oblate FRCs are not affected significantly by the thermal ion FLR effects even for small values of $\bar{s}$ ($\bar{s} \sim 1 - 2$).

2. Linear Stability of $n = 1$ Tilt Mode

According to analytical estimates, finite Larmor radius (FLR) stabilization of the tilt mode should occur when $\omega^* \gtrsim \gamma_0$ [3], where ω^* is the diamagnetic frequency and γ_0 is a characteristic growth rate, from which an approximate stability condition $\bar{s}/E \lesssim 0.2 - 0.5$ can be obtained. In the previous kinetic calculations based on Vlasov-fluid dispersion functional approach and using trial functions [4], a greatly reduced growth rate for $\bar{s} \approx 2$ and complete stabilization at $\bar{s} < 1.5$ ($E = 7.7$) was obtained.

Our self-consistent hybrid simulations show that there is a reduction in the tilt mode growth rate when $\bar{s}/E < 1$, but no absolute stabilization has been found for $\bar{s}/E$ values as small as 0.1. The difference between our results and the dispersion analysis results can be explained by the deviation of the true kinetic eigenfunction from an assumed MHD-like trial function [4] in the strongly kinetic regime (small $\bar{s}$). The linear stability results for two family of equilibria with $E \approx 2$ and $E = 7.2$ are summarized in Fig. 1, where the growth rate is shown for different values of $1/\bar{s}$. Notice that γ is only slightly reduced for $\bar{s} \geq 1.5$ in the simulations with small elongation $E \approx 2$. For the configurations with $E = 7.2$, there is a significant reduction in the tilt instability growth rate at small values of $\bar{s}$, however, no absolute stabilization has been found even for $\bar{s} \lesssim 1$.

One of the possible explanations of the instability existing beyond the FLR theory stability threshold is the resonant interaction of the wave with ions for which the Doppler shifted wave frequency matches the betatron frequency [5]:

$$\omega - \Omega = \pm\omega_\beta, \tag{1}$$

where ω is the tilt mode real frequency, Ω is the ion toroidal rotation frequency, and ω_β is the ion axial betatron frequency. In order to check this condition, we have calculated Ω and ω_β for each particle in the linearized simulation with $E = 7.2$ and $\bar{s} = 0.8$ (racetrack separatrix). The result is shown in Fig. 2, as a scatter plot of the particle distribution in (w, $(\Omega - \omega)/\omega_\beta$) plane (where $w = \delta F/F$ is a particle weight in the delta-f simulations). It is seen that the change in the perturbed distribution function is largest when the ratio of the ion toroidal rotation frequency in the wave frame ($\Omega - \omega$) to the betatron frequency is an odd number. Thus for a racetrack equilibria with large E, we find that multiple resonances are possible, and a generalized resonance condition is

$$(\Omega - \omega)/\omega_\beta = \pm 2l + 1, \tag{2}$$

where l is an integer. The difference between our result, Eq. (2), and the previous condition, Eq. (1), is related to the difference in the linear structure of the tilt mode in the configurations with a racetrack and an elliptical separatrix. Condition (1) was obtained assuming a rigid tilt eigenfunction, which is a good approximation for the tilt mode for an elliptic equilibrium. In this case, the axial displacement is largest near the field null, and the mode is global (perturbation is finite everywhere inside the separatrix). Thus only the ions, which satisfy condition (1), will stay in phase with the wave. In a racetrack equilibrium, the perturbation is localized near the end regions, where the curvature is large, and it is small everywhere else. Therefore, for an ion to stay in a resonance with the wave, the half-period of the ion axial motion has to be equal to an odd number of the half-periods of the toroidal rotation (note that the mode has an odd symmetry relative to the midplane). Assuming that the resonant ions are those with large weights ($|w| > 0.25 w_{max}$), we find that the fraction of the resonant ions is at least 4% of all of the confined ions, and that these resonant particles contribute about 63% to the energy balance: $dK/dt = \int (\delta \mathbf{J}_i \cdot \delta \mathbf{E})\, d^3\mathbf{x}$. Thus for $E = 7.2$ and $\bar{s} = 0.8$, the instability is predominantly a resonantly driven one.

Since multiple resonances are possible in the racetrack configuration, we conclude that such configurations generally are more unstable than the elliptic ones (in the kinetic regime). The simulation results for a different equilibrium with $E = 3.9$, $\bar{s} = 0.9$ and an elliptic separatrix are shown in Fig. 3. In this case the $(\Omega - \omega)/\omega_\beta = 1$ resonance is the most important one, and other resonances are much weaker. The fraction of the resonant ions is about 7%, and the contribution of the resonant ions to the rate of change of the total kinetic energy is about 50%.

In the previous kinetic studies of the tilt mode stability in field-reversed ion rings, the instability condition: $|\Omega - \omega| < \omega_\beta$ has been obtained [7]. Although the derivation of this condition assumes a monoenergetic ion beam, and thus it is not directly applicable to FRC with thermal ions, we were motivated to investigate the contribution of large ω_β ions to the instability drive. For the racetrack case with $E = 7.2$, we find that about 15% ions satisfy the above condition, and contribute about 12% to the energy balance. For the elliptic separatrix case with $E = 3.9$, the fraction of such ions is larger, $\sim$45%, and their contribution to the energy balance is about 47%. While in first case ($E = 7.2$) the instability is clearly a resonant one, in the elliptic separatrix case ($E = 3.9$), the resonant interaction and the large ω_β ions can both be important.

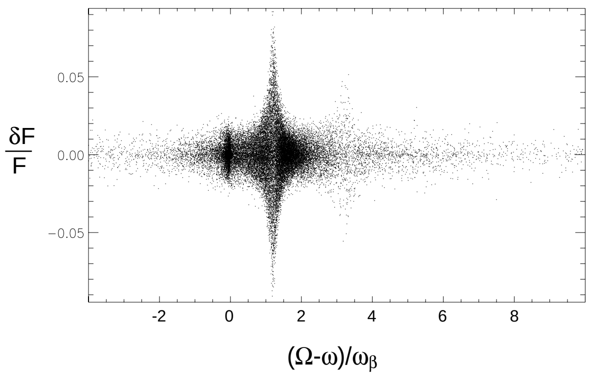


FIG.3: Scatter plot of the particle distribution from linearized simulation of the $n = 1$ tilt mode with $E = 3.9$ and $\bar{s} = 0.9$, elliptic separatrix.

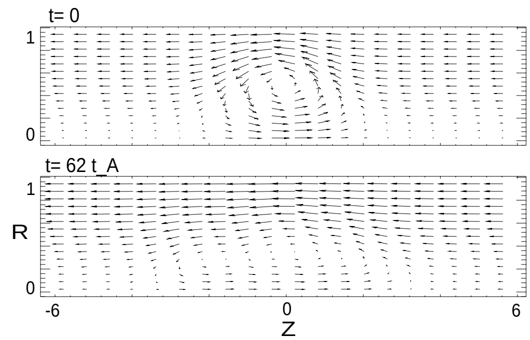


FIG.4: Vector plots of poloidal magnetic field at $t = 0$ and $t = 62t_A$ from nonlinear simulation with $E = 3.9$ and $\bar{s} = 1.9$.

3. Nonlinear Results

Our simulations indicate that the FRC stability observed in the experiments cannot be explained within linear theory. Other calculations done by Iwasawa *et al.* [6] also support this view. However, we find that the nonlinear evolution of the tilt mode at low $\bar{s}$, kinetic regime is different from that of the MHD model. The results of nonlinear hybrid simulations at low values of $\bar{s}$ indicate that the instabilities in the prolate configurations saturate through a combination of a lengthening of the initial equilibrium and a modification of the ion distribution function. These saturated states persist for many Alfven times, maintaining field reversal. Fig. 4 shows vector plots of the poloidal magnetic field at $t = 0$ and after 62 Alfven times ($t_A = R/V_A$) from the nonlinear simulations with $E = 3.9$, elliptic separatrix, and $\bar{s} = 1.9$. It is seen that FRC evolves to a new equilibrium with larger elongation, elliptic separatrix shape and larger value of the separatrix beta. The stabilization mechanism is likely to be a lengthening of the separatrix (reduction in the ion axial betatron frequencies) and the nonlinear wave-particle interaction. Note that nonlinear stabilization of a linearly unstable tilt mode may explain the observation in the low $\bar{s}$ experiments of initial $n = 1$ tilt motion that does not result in total loss of the confinement [8].

Acknowledgments

This work was supported by DOE contract DE-AC02-76CH03073. Calculations were performed at the U.S. National Energy Research Supercomputing Center.

References

[1] BELOVA E. V., et al., “Numerical Study of Tilt Stability of Prolate Field-Reversed Configurations”, to appear in Phys. Plasmas **7** N12 (2000).

[2] PARKER S. E., LEE W. W., “A Fully Nonlinear Characteristic Method for Gyrokinetic Simulation”, Phys. Fluids B **5** (1993) 77.

[3] ISHIDA A., MOMOTA H., STEINHAUER L. C., “Variational Formulation for a Multifluid Flowing Plasma with Application to the Internal Tilt Mode of a Field-Reversed Configuration”, Phys. Fluids **31** (1988) 3024.

[4] BARNES D. C., et al., “Kinetic Tilting Stability of Field-Reversed Configurations”, Phys. Fluids **29** (1986) 2616.

[5] FINN J. M., SUDAN R. N., “Betatron Resonance Instabilities in Axisymmetric Field-Reversed Systems”, Phys. Fluids **22** (1979) 1148.

[6] IWASAWA N., ISHIDA A., STEINHAUER L. C., “Tilt Mode Stability Scaling in Field-Reversed Configurations with Finite Larmor Radius Effect”, Phys. of Plasmas **7**, (2000).

[7] RUCHTI C. B., LOVELACE R. V., “Tilt Stability of Arbitrary Aspect Ratio Ion Rings”, Phys. Fluids **27** (1984) 1789.

[8] TUSZEWSKI, M., et al., “Axial Dynamics in Field-Reversed Theta Pinches. II: Stability”, Phys. Fluids B **3** (1991) 2856.

The Princeton Plasma Physics Laboratory is operated
by Princeton University under contract
with the U.S. Department of Energy.

Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2750
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: http://www.pppl.gov