TRANSP Current Drive Algorithms

Michael Kraus
(michael.kraus@ipp.mpg.de)

April 27, 2011

In these notes we give an overview of the derivation of the TRANSP current drive algorithms by Hirshman and Sigmar (refs. [1, 2, 3], `NMCURB=1`) and by Lin-Liu and Hinton as well as Kim, Callen and Hamnen, (refs. [5, 4], `NMCURB=3`). The latter one was derived independently by Lin-Liu et al., and earlier by Kim et al., whereby both take a somewhat different approach but arrive at the same results. The Lin-Liu version is a short derivation, so we won't repeat it here. The Kim version is scattered throughout the original paper, wherefore we summarise it. The Hirshman algorithm is scattered throughout an anthology of papers, and doesn't seem to be written down in one piece anywhere but here.
Both algorithms obey different restrictions. The one by Kim contains assumptions which are only valid in the banana regime. The one by Hirshman is valid in the banana as well as in the plateau regime, but only for inverse aspect ratios of $\epsilon \leq 0.15$. Kim's algorithm is valid for arbitrary aspect ratios, he only assumes an elliptic plasma. Hirshman doesn't comment on the plasma shape.

References

[1] S. P. Hirshman, 1978, Neoclassical current in a toroidally-confined multispecies plasma, *Physics of Fluids*, **21** 1295–1301.

[2] S. P. Hirshman, 1978, Moment equation approach to neoclassical transport theory, *Physics of Fluids*, **21** 224–229.

[3] S. P. Hirshman and D. J. Sigmar, 1981, Neoclassical transport of impurities in tokamak plasmas, *Nuclear Fusion*, **21** 1079.

[4] Y. B. Kim, J. D. Callen, and H. Hamnen, 1988, Neoclassical current and transport in auxiliary heated tokamaks, *JET Report*, **R4688**.

[5] Y. R. Lin-Liu and F. L. Hinton, 1997, Trapped electron correction to beam driven current in general tokamak equilibria, *Physics of Plasmas*, **4** 4179–4181.

1. Derivation of the algorithm by Kim et al.

The flux surface averaged total current through NBI heating is

$$\langle J_\parallel B\rangle_{total} = \langle J_\parallel B\rangle_F + \langle J_\parallel B\rangle_H \tag{1}$$

with $\langle J_\parallel B\rangle_F$ the (unshielded) current through fast ions

$$\langle J_\parallel B\rangle_F = e_f \langle n_f B V_{\parallel,f}\rangle \tag{2}$$

$\langle J_\parallel B\rangle_H$ the shielding current of the electrons

$$\langle J_\parallel B\rangle_H = -\frac{e\,\tau_{ee}}{m_e}\left(\Lambda_0^{NC}\langle B F_{e,f1}\rangle + \Lambda_1^{NC}\langle B F_{e,f2}\rangle\right) \tag{3}$$

the coefficients

$$\Lambda_0^{NC} = \frac{\mu_3^{(e)} + l_{22}^{(e)}}{D} \tag{4}$$

$$\Lambda_1^{NC} = -\frac{l_{12}^{(e)} + \mu_2^{e}}{D} \tag{5}$$

$$D = \left(\mu_1^{(e)} + l_{11}^{(e)}\right)\left(\mu_3^{(e)} + l_{22}^{(e)}\right) - \left(\mu_2^{(e)} + l_{12}^{(e)}\right)^2 \tag{6}$$

and the approximations

$$\langle B F_{e,f1}\rangle \cong \frac{m_e\, e_f^2}{\tau_{ee}\, e^2}\langle n_f B V_{\parallel,f}\rangle \tag{7}$$

$$\langle B F_{e,f2}\rangle \cong \frac{3}{2}\langle B F_{e,f1}\rangle \tag{8}$$

Therefore one finds

$$\langle J_\parallel B\rangle_H = -\frac{e\,\tau_{ee}}{m_e}\langle B F_{e,f1}\rangle\left(\Lambda_0^{NC} + \frac{3}{2}\Lambda_1^{NC}\right) \tag{9}$$

$$= -\frac{e_f^2}{e}\langle n_f B V_{\parallel,f}\rangle\left(\Lambda_0^{NC} + \frac{3}{2}\Lambda_1^{NC}\right) \tag{10}$$

and thus

$$\langle J_\parallel B\rangle_{total} = e_f\langle n_f B V_{\parallel,f}\rangle\left(1 - \frac{e_f}{e}\left(\Lambda_0^{NC} + \frac{3}{2}\Lambda_1^{NC}\right)\right) \tag{11}$$

$$= Z_f\, e\langle n_f B V_{\parallel,f}\rangle\left(1 - \frac{Z_f\, e}{Z_{\text{eff}}\, e}F\right) \tag{12}$$

$$= Z_f\, e\langle n_f B V_{\parallel,f}\rangle\left(1 - \frac{Z_f}{Z_{\text{eff}}}F\right) \tag{13}$$

$$F \equiv Z_{\text{eff}}\left(\Lambda_0^{NC} + \frac{3}{2}\Lambda_1^{NC}\right) \tag{14}$$

Inserting Λ_j^{NC}, one obtains for F

$$F = Z_{\text{eff}}\frac{\mu_3^{(e)} + l_{22}^{(e)} - \frac{3}{2}l_{12}^{(e)} - \frac{3}{2}\mu_2^{(e)}}{\left(\mu_1^{(e)} + l_{11}^{(e)}\right)\left(\mu_3^{(e)} + l_{22}^{(e)}\right) - \left(\mu_2^{(e)} + l_{12}^{(e)}\right)^2} \tag{15}$$

The viscosity matrix is defined by

$$[\mu^{(e)}] = \begin{bmatrix} \mu_1^{(e)} & \mu_2^{(e)} \\ \mu_2^{(e)} & \mu_3^{(e)} \end{bmatrix} \tag{16}$$

its coefficients are given by Kim et al. as

$$\mu_1^{(e)} = g\left(Z_{\mathrm{eff}} + \sqrt{2} - \ln(1+\sqrt{2})\right) \tag{17}$$

$$\mu_2^{(e)} = g\left(\tfrac{3}{2}\,Z_{\mathrm{eff}} + 4/\sqrt{2} - \tfrac{5}{2}\,\ln(1+\sqrt{2})\right) \tag{18}$$

$$\mu_3^{(e)} = g\left(\tfrac{13}{4}\,Z_{\mathrm{eff}} + \tfrac{39}{4}\,\sqrt{2} - \tfrac{25}{4}\ln(1+\sqrt{2})\right) \tag{19}$$

with the ratio of trapped and untrapped particles

$$g \equiv \frac{f_t}{1-f_t} \tag{20}$$

The friction matrix is defined by

$$[l^{(a)}] = \begin{bmatrix} l_{11}^{(a)} & l_{12}^{(a)} \\ l_{21}^{(a)} & l_{22}^{(a)} \end{bmatrix} \tag{21}$$

with

$$l_{11}^{(e)} = Z_{\mathrm{eff}} \tag{22}$$

$$l_{12}^{(e)} = l_{21}^{(e)} = \tfrac{3}{2}\,Z_{\mathrm{eff}} \tag{23}$$

$$l_{22}^{(e)} = \sqrt{2} + \tfrac{13}{4}\,Z_{\mathrm{eff}} \tag{24}$$

Inserting everything, one obtains for F

$$F = \frac{(Z_{\mathrm{eff}}^2 + 1.41\,Z_{\mathrm{eff}}) + g\,(Z_{\mathrm{eff}}^2 + 0.45\,Z_{\mathrm{eff}})}{(Z_{\mathrm{eff}}^2 + 1.41\,Z_{\mathrm{eff}}) + g\,(2\,Z_{\mathrm{eff}}^2 + 2.66\,Z_{\mathrm{eff}} + 0.75) + g^2\,(Z_{\mathrm{eff}}^2 + 1.24\,Z_{\mathrm{eff}} + 0.35)} \tag{25}$$

In the limit of $g \to 0 \Rightarrow F \to 1$ the shielding current of the electrons equals the Ohkawa current (where neoclassical effects are neglected):

$$\langle J_\parallel\, B\rangle_{Ohkawa} = -\frac{Z_f^2}{Z_{\mathrm{eff}}}\, e\,\langle n_f\, B\, V_{\parallel,f}\rangle \tag{26}$$

$$\langle J_\parallel\, B\rangle_{total} = \langle J_\parallel\, B\rangle_F + \langle J_\parallel\, B\rangle_{Ohkawa} \tag{27}$$

Physical quantities:

$\langle J_\parallel\, B\rangle_{total}$	total NBI driven current
$\langle J_\parallel\, B\rangle_F$	current through fast ions
$\langle J_\parallel\, B\rangle_H$	shielding current of the electrons (neoclassical)
$\langle J_\parallel\, B\rangle_{Ohkawa}$	Ohkawa shielding current (non-neoclassical)
m_e	electron mass
τ_{ee}	electron collisionality
Z_{eff}	effective nuclear charge of plasma
Z_f	nuclear charge number of fast ions
$e_f = Z_f\, e$	charge of fast ions
e	unit charge (positive)

2. Derivation of the TRANSP algorithm

The TRANSP current drive algorithm corresponds to the one in the paper of Hirshman and Sigmar [3] with the K-coefficients from the paper of Hirshman [1]. Thereby the definitions of $\mu_j^{(e)}$, $l_{ij}^{(e)}$ and $K_{ij}^{(e)}$ are different in both papers. The factors are distributed differently from $\widehat{\mu}_j^{(e)}$ and $\widehat{l}_{ij}^{(e)}$ to $\mu_j^{(e)}$, $l_{ij}^{(e)}$ and $K_{ij}^{(e)}$. In the calculation of the shielding factor the factors cancel in both papers.
Besides, both papers give different equations for $K_{ij}^{(e)}$. In [3] an equation for the banana and for the plateau regime is given, respectively. In [1] an equation valid for both regimes is given.

The current drive through fast ions is given in Hirshman and Sigmar [3] (p. 1174, eqs. (8.27)-(8.28)) as (G here corresponds to F in the paper):

$$\langle J_\parallel B\rangle = \langle \boldsymbol{J}_f \cdot \boldsymbol{B}\rangle = n_f\, e_f\, \langle V_{\parallel,f}\, B\rangle\, G \tag{28}$$

$$G = 1 - \frac{Z_f}{Z_{\text{eff}}} + \left(\frac{Z_f}{Z_{\text{eff}}} + \frac{n\,|e|\,V_{i,\theta}\,\langle B^2\rangle}{n_f\,e_f\,\langle V_{\parallel,f}\,B\rangle}\right)\left(\frac{\widehat{l}_{12}^{(e)}\,\widehat{\mu}_2^{(e)} + \widehat{l}_{22}^{(e)}\,\widehat{\mu}_1^{(e)}}{\widehat{l}_{11}^{(e)}\,\widehat{l}_{22}^{(e)} - (\widehat{l}_{12}^{(e)})^2}\right) \tag{29}$$

with

$$\widehat{\mu}_j^{(e)} = \frac{3\,\left\langle(\boldsymbol{n}\cdot\nabla B)^2\right\rangle}{\langle B^2\rangle}\,\mu_j^{(e)} \tag{30}$$

$$\widehat{l}_{ij}^{(e)} = -\frac{m_e\,n_e}{\tau_{ee}}\,l_{ij}^{(e)} \tag{31}$$

the viscosity coefficients([3], p. 1108, eqs. (4.20)-(4.22)):

$$\mu_1^{(e)} = K_{11} \tag{32}$$

$$\mu_2^{(e)} = K_{12} - \tfrac{5}{2}\,K_{11} \tag{33}$$

$$\mu_3^{(e)} = K_{22} - 5\,K_{12} + \tfrac{25}{4}\,K_{11} \tag{34}$$

and the friction coefficients ([3], p. 1174, below eq. (8.28)):

$$l_{11}^{(e)} = Z_{\text{eff}} \tag{35}$$

$$l_{12}^{(e)} = l_{21}^{(e)} = \tfrac{3}{2}\,Z_{\text{eff}} \tag{36}$$

$$l_{22}^{(e)} = \sqrt{2} + \tfrac{13}{4}\,Z_{\text{eff}} \tag{37}$$

The K-coefficients are defined for the banana regime as ([3], p. 1114, eq. (4.61))

$$K_{ij}^{(e,B)} = \frac{f_t}{1-f_t}\,\frac{n_e\,m_e}{\tau_{ee}}\left\{x_e^{2(i+j-2)}\,\nu_D^e\,\tau_{ee}\right\}\frac{\langle B^2\rangle}{3\,\langle(\boldsymbol{n}\cdot\nabla B)^2\rangle} \tag{38}$$

and in the plateau regime as ([3], p. 1117, eq. (4.72))

$$K_{ij}^{(e,P)} = f_t\,\frac{n_e\,m_e}{\tau_e}\left\{x_e^{2(i+j-2)}\,\nu_{tot}^e(v)\,\tau_e\right\}\frac{\langle B^2\rangle}{3\,\langle(\boldsymbol{n}\cdot\nabla B)^2\rangle} \tag{39}$$

Hirshman and Sigmar argue the second term in the first brackets (the one which is proportional to the poloidal component of the ion velocity $V_{i,\theta}$) is of the order $\mathcal{O}\left(\sqrt{m_e/m_i}\right)$ and thus negligible:

$$G = 1 - \frac{Z_f}{Z_{\text{eff}}} + \left(\frac{Z_f}{Z_{\text{eff}}} + \mathcal{O}\left(\sqrt{\frac{m_e}{m_i}}\right)\right)\left(\frac{\widehat{l}_{12}^{(e)}\,\widehat{\mu}_2^{(e)} + \widehat{l}_{22}^{(e)}\,\widehat{\mu}_1^{(e)}}{\widehat{l}_{11}^{(e)}\,\widehat{l}_{22}^{(e)} - (\widehat{l}_{12}^{(e)})^2}\right) \tag{40}$$

$$= 1 - \frac{Z_f}{Z_{\text{eff}}}\left(1 - \frac{\widehat{l}_{12}^{(e)}\,\widehat{\mu}_2^{(e)} + \widehat{l}_{22}^{(e)}\,\widehat{\mu}_1^{(e)}}{\widehat{l}_{11}^{(e)}\,\widehat{l}_{22}^{(e)} - (\widehat{l}_{12}^{(e)})^2}\right) = 1 - \frac{Z_f}{Z_{\text{eff}}}\,(1 - H) \tag{41}$$

In the TRANSP `JBSHLD` algorithm one finds:

$$j_{\text{NBCD}} \equiv j_f\left[1 - \frac{Z_f}{Z_{\text{eff}}}\,(1 - \widetilde{H})\right] \tag{42}$$

$$\widetilde{H} = f_t\,\frac{\frac{3}{2}\,Z_{\text{eff}}\,(\widetilde{K}_{12} - \frac{5}{2}\,\widetilde{K}_{11}) + (\sqrt{2} + \frac{13}{4}\,Z_{\text{eff}})\,\widetilde{K}_{11}}{(\sqrt{2} + \frac{13}{4}\,Z_{\text{eff}})\,Z_{\text{eff}} - (\frac{3}{2}\,Z_{\text{eff}})^2} \tag{43}$$

With the relations from [1] following below this becomes

$$\widetilde{H} = \frac{l_{21}^{(e)}\,\overline{\mu}_2^{(e)} + l_{22}^{(e)}\,\overline{\mu}_1^{(e)}}{l_{11}^{(e)}\,l_{22}^{(e)} - (l_{12}^{(e)})^2} \tag{44}$$

$$\overline{\mu}_j^{(e)} = \frac{\tau_{ee}}{m_e\,n_e}\,\frac{3\,\langle(\boldsymbol{n}\cdot\nabla B)^2\rangle}{\langle B^2\rangle}\,\widetilde{\mu}_j^{(e)} \tag{45}$$

Hirshman defines the coefficients of the viscosity matrix in the regime of large mean free path (banana/plateau regime) in analogy to [2]:

$$\widetilde{\mu}_j^{(e)} = f_t\,m_e\,n_e\,\frac{\langle B^2\rangle}{3\left\langle\left(\boldsymbol{n}\cdot\nabla B\right)^2\right\rangle}\left\{\left(x_e^2 - \tfrac{5}{2}\right)^{j-1}\,\nu_t^{(e)}\right\} \tag{46}$$

Multiplying this with τ_{ee}/τ_{ee} one finds the factors from 30, 31, 38 and 39

$$\widetilde{\mu}_j^{(e)} = f_t\,\frac{m_e\,n_e}{\tau_{ee}}\,\frac{\langle B^2\rangle}{3\left\langle\left(\boldsymbol{n}\cdot\nabla B\right)^2\right\rangle}\left\{\left(x_e^2 - \tfrac{5}{2}\right)^{j-1}\,\nu_t^{(e)}\,\tau_{ee}\right\} \tag{47}$$

The braces denote the velocity integration operator:

$$\{A(v_e)\} = \frac{8}{3\sqrt{\pi}}\int_0^\infty dx\,x^4\,\exp\left(-x^2\right)A(v_{th,e}\,x) \tag{48}$$

with the ratio x of velocity v and thermal velocity v_{th}:

$$x_e^2 = \frac{v_e^2}{v_{th,e}^2} = \frac{v_e^2}{2\,T_e/m_e} \tag{49}$$

$$v_{th,e} = \sqrt{\frac{2\,k_B\,T_e}{m_e}} \tag{50}$$

Hirshman defines the K-coefficients as

$$\widetilde{K}_{1j}^{(e)}(\nu_e^*, Z_{\text{eff}}) = \left\{x_e^{2(j-1)}\,\nu_t^{(e)}\,\tau_{ee}\right\} \tag{51}$$

and uses this to express the coefficients of the viscosity matrix:

$$\widetilde{\mu}_0^{(e)} = f_t \, \frac{m_e \, n_e}{\tau_{ee}} \, \frac{\langle B^2 \rangle}{3 \left\langle \left(\boldsymbol{n} \cdot \nabla B \right)^2 \right\rangle} = 1.38 \, p_e \, \nu_e^* \, \frac{q \, R_{\mathrm{maj}}}{v_{th,e}} \tag{52}$$

$$\widetilde{\mu}_1^{(e)} = \widetilde{\mu}_0^{(e)} \, \widetilde{K}_{11} \tag{53}$$

$$\widetilde{\mu}_2^{(e)} = \widetilde{\mu}_0^{(e)} \, (\widetilde{K}_{12} - \tfrac{5}{2} \, \widetilde{K}_{11}) \tag{54}$$

The normalised electron collisionality is given in analogy to [**?** **?**]:

$$\nu_e^* = \frac{\sqrt{2} \, q \, R_{\mathrm{maj}}}{\epsilon^{3/2} \, v_{th,e} \, \tau_{ee}} \tag{55}$$

With the thermal velocity of the electrons $v_{th,e}$ and the collisionality of the electrons $\nu_{ee} = \tau_{ee}^{-1}$

$$\nu_{ee} = \frac{16 \sqrt{\pi}}{3} \, \frac{n_e \, e^4 \, \ln \Lambda_e}{m_e^2 \, v_{th,e}^3} \tag{56}$$

this becomes

$$\nu_e^* = \frac{16 \sqrt{2\pi}}{3} \, \frac{q \, n_e \, e^4 \, \ln \Lambda_e \, R_{\mathrm{maj}}}{\epsilon^{3/2} \, m_e^2 \, v_{th,e}^4} = \frac{\sqrt{32 \, \pi} \, e^4}{3 \, k_B^2} \, \frac{q \, n_e \, \ln \Lambda_e}{T_e^2} \sqrt{\frac{R_{\mathrm{maj}}^5}{R_{\mathrm{min}}^3}} \tag{57}$$

The numerical solution of eq. 51 is then fitted with the following expression:

$$K_{ij} = \frac{K_{ij}^{(0)}}{(1 + \sqrt{A_{ij} \, \nu_e^*} + B_{ij} \, \nu_e^*) \, (1 + \sqrt{C_{ij} \, \nu_e^* \, \epsilon^{3/2}} + D_{ij} \, \nu_e^* \, \epsilon^{3/2})} \tag{58}$$

what from the following coefficients result

$$K_{11} = \frac{0.53 + Z_{\mathrm{eff}}}{(1 + \sqrt{A_{11} \, \nu_e^*} + B_{11} \, \nu_e^*) \, (1 + \sqrt{C_{11} \, \nu_e^* \, \epsilon^{3/2}} + D_{11} \, \nu_e^* \, \epsilon^{3/2})} \tag{59}$$

$$A_{11} = 3.44 \, Z_{\mathrm{eff}} + \frac{0.52 - 0.42 \, Z_{\mathrm{eff}}}{1 + 1.35 \, Z_{\mathrm{eff}}}$$

$$B_{11} = 0.56 + 0.96 \, Z_{\mathrm{eff}}$$

$$C_{11} = 0.25 \, Z_{\mathrm{eff}} + \frac{0.14 + 0.55 \, Z_{\mathrm{eff}}}{1 + 5 \, Z_{\mathrm{eff}}}$$

$$D_{11} = 0.51 \, Z_{\mathrm{eff}} + \frac{0.7 + 0.78 \, Z_{\mathrm{eff}}}{1 + Z_{\mathrm{eff}}}$$

$$K_{12} = \frac{0.71 + Z_{\mathrm{eff}}}{(1 + \sqrt{A_{12} \, \nu_e^*} + B_{12} \, \nu_e^*) \, (1 + \sqrt{C_{12} \, \nu_e^* \, \epsilon^{3/2}}) + D_{12} \, \nu_e^* \, \epsilon^{3/2}} \tag{60}$$

$$A_{12} = 0.31 \, Z_{\mathrm{eff}} + \frac{0.1 + 0.084 \, Z_{\mathrm{eff}}}{1 + 1.3 \, Z_{\mathrm{eff}}}$$

$$B_{12} = 0.26 + 0.35 \, Z_{\mathrm{eff}}$$

$$C_{12} = 0.081 \, Z_{\mathrm{eff}} + \frac{0.072 + 0.15 \, Z_{\mathrm{eff}}}{1 + 3 \, Z_{\mathrm{eff}}}$$

$$D_{12} = 0.29 \, Z_{\mathrm{eff}} + \frac{0.42 + 0.62 \, Z_{\mathrm{eff}}}{1 + 1.42 \, Z_{\mathrm{eff}}}$$

with an error of ≤ 6% for $0.01 \le \epsilon \le 0.15$.
For low collisonality $\nu_e^* = 0$ one finds for the K-coefficients

$$K_{11}^{(0)} = 0.53 + Z_{\text{eff}}$$
$$K_{12}^{(0)} = 0.71 + Z_{\text{eff}}$$

and thus for $H^{(0)}$

$$\begin{aligned} H^{(0)} &= f_t \frac{1.5\,Z_{\text{eff}}\left(0.71 + Z_{\text{eff}} - 2.5\left(0.53 + Z_{\text{eff}}\right)\right) + \left(1.41 + 3.25\,Z_{\text{eff}}\right)\left(0.53 + Z_{\text{eff}}\right)}{Z_{\text{eff}}\left(1.41 + 3.25\,Z_{\text{eff}}\right) - 2.25\,Z_{\text{eff}}^2} \\ &= f_t \frac{0.75 + 2.21\,Z_{\text{eff}} + Z_{\text{eff}}^2}{1.41\,Z_{\text{eff}} + Z_{\text{eff}}^2} \end{aligned} \tag{61}$$

which corresponds to the equation for the banana regime from Hirshman and Sigmar [3] (p. 1175, eq. (8.29)):

$$F^{(0)} = 1 - \frac{Z_f}{Z_{\text{eff}}} + f_t \left(\frac{Z_f}{Z_{\text{eff}}} + \frac{n\,|e|\,V_{i,\theta} < B^2 >}{n_f\,e_f < V_{\parallel,f}\,B >}\right)\left(\frac{0.75 + 2.21\,Z_{\text{eff}} + Z_{\text{eff}}^2}{1.41\,Z_{\text{eff}} + Z_{\text{eff}}^2}\right) \tag{62}$$

Neglecting terms propotional to $V_{i,\theta}$, this corresponds to the TRANSP formula with $H^{(0)}$:

$$F^{(0)} = 1 - \frac{Z_f}{Z_{\text{eff}}}\left(1 - f_t \frac{0.75 + 2.21\,Z_{\text{eff}} + Z_{\text{eff}}^2}{1.41\,Z_{\text{eff}} + Z_{\text{eff}}^2}\right) = 1 - \frac{Z_f}{Z_{\text{eff}}}\left(1 - H^{(0)}\right) \tag{63}$$

Physical quantities:

$\boldsymbol{n} = \boldsymbol{B}/B$	unit vector of the magnetic field
Z_{eff}	effective nuclear charge of plasma
Z_f	nuclear charge number of fast ions
f_t	fraction of trapped particles
$V_{i,\theta}$	poloidal component of the ion velocity
$\nu_D^{(e)}$	90° scatter frequency
$\nu_{tot}^{(e)}$	total neoclassical collision frequency
$\nu_t^{(e)}$	neoclassical collision frequency stress anisotropy relaxation

A. The JBSHLD algorithm from TRANSP

```
C------------------------------------------------------------------------
C  JBSHLD -- CLASSICAL/NEOCLASSICAL BEAM CURRENT SHIELDING FACTOR
C
C   TAKEN OUT OF FOKKER AND PUT HERE DMC SEPT 1990
C
      SUBROUTINE JBSHLD
C
      use nbi_com
C
!============
! idecl:  explicitize implicit INTEGER declarations:
      IMPLICIT NONE
      INTEGER, PARAMETER :: R8=SELECTED_REAL_KIND(12,100)
      INTEGER j
!============
! idecl:  explicitize implicit REAL declarations:
      REAL*8 zrzon,zrboun,zrbounp,zrmajor,zcurr,zvstae,zdelta,zd1m
      REAL*8 zft,zef,zd32,za11,zb11,zc11,zd11,zk11,za12,zb12,zc12
      REAL*8 zd12,zk12
!============
      LOGICAL ILSPIZ
C
C-------------------------------------------------------------
C  D. MC CUNE IMPLIMENT NEOCLASSICAL BEAM CURRENT OPTION
C    COLLISIONALITY, TRAPPING FRACTION FORMULAE FROM "RESIS.FOR"
C    A,B, KO COEFFICIENTS FROM HIRSHMAN, PHYS. FLUIDS VOL 21
C    NO. 8 AUG 1978.
C  CLASSICAL JB
C
C  DMC OCT 1989 -- DECIDE ON N.C. OR CLASSICAL DRIVEN CURRENTS
C
      ILSPIZ=.FALSE.
      IF(NMCURB.EQ.2) THEN
        ILSPIZ=.TRUE.
      ELSE IF(NMCURB.EQ.3) THEN
        ILSPIZ=.FALSE.
      ENDIF
C
      DO 100 J=LCENTR,LEDGE
        XJBFAC(J)=(1.0_R8-XZBEAMI/XZEFFC(J,2))
C
C  IF RESISTIVITY IS CLASSICAL, USE CLASSICAL BEAM CURRENT
C  (NLSPIZ=.TRUE. ==> CLASSICAL, SPITZER RESISTIVITY S.R. RESIS)
C  *** MOD DMC OCT 1989 ***
C   CAN FORCE CLASSICAL/N.C. BEAM DRIVEN CURRENT INDEPENDENTLY
C   SEE NAMELIST CONTROL NMCURB, AND DEFN. OF ILSPIZ ABOVE ***
```

```
C
      IF(ILSPIZ) GO TO 2995
C
         ZRZON=RMNRMP(J,2)
         ZRBOUN=RMNRMP(J,1)
         ZRBOUNP=RMNRMP(J+1,1)
         ZRMAJOR=RMJRMP(J,2)
C
C  NEOCLASSICAL JB
         ZCURR=2.5E4_R8*BZXR/ZRMAJOR**2
                 *ZRZON*(ZRBOUN/QGEO(J)+ZRBOUNP/QGEO(J+1))
C  COLLISIONALITY NU*E
         ZVSTAE=3.46E-9_R8*RHOEL(J,2)*CLOGE(J)*BZXR*
    1      SQRT(ZRMAJOR*ZRZON)/(ZCURR*TE(J,2)*TE(J,2))
         ZDELTA=ZRZON/ZRMAJOR
         ZD1M=1._R8-ZDELTA
C  TRAPPING FRACTION
         ZFT=1._R8-ZD1M*ZD1M/(SQRT(1._R8-ZDELTA*ZDELTA)*
    1      (1._R8+1.46_R8*SQRT(ZDELTA)))
C  K11 COEFFICIENT
         ZEF=XZEFFC(J,2)
         ZD32=ZDELTA*SQRT(ZDELTA)
         ZA11=3.44_R8*ZEF+(.52_R8-.42_R8*ZEF)/(1._R8+1.35_R8*ZEF)
         ZB11=.56_R8+.96_R8*ZEF
         ZC11=.25_R8*ZEF+(.14_R8+.55_R8*ZEF)/(1._R8+5.0_R8*ZEF)
         ZD11=.51_R8*ZEF+(.7_R8+.78_R8*ZEF)/(1._R8+ZEF)
         ZK11=(.53_R8+ZEF)/
           ((1._R8+SQRT(ZA11*ZVSTAE)+ZB11*ZVSTAE)*
            (1._R8+SQRT(ZC11*ZVSTAE*ZD32)+ZD11*ZVSTAE*ZD32))
C  K12 COEFFICIENT
         ZA12=.31_R8*ZEF+(.1_R8+.084_R8*ZEF)/(1._R8+1.3_R8*ZEF)
         ZB12=.26_R8+.35_R8*ZEF
         ZC12=.081_R8*ZEF+(.072_R8+.15_R8*ZEF)/(1._R8+3._R8*ZEF)
         ZD12=.29_R8*ZEF+(.42_R8+.62_R8*ZEF)/(1._R8+1.42_R8*ZEF)
         ZK12=(.71_R8+ZEF)/
           ((1._R8+SQRT(ZA12*ZVSTAE)+ZB12*ZVSTAE)*
            (1._R8+SQRT(ZC12*ZVSTAE*ZD32)+ZD12*ZVSTAE*ZD32))
C  NEOCLASSICAL BEAM CURRENT CORRECTION
         XJBFAC(J)=XJBFAC(J)+XZBEAMI/ZEF*ZFT*
     (1.5_R8*ZEF*(ZK12-2.5_R8*ZK11)+
     (1.414214_R8+3.25_R8*ZEF)*ZK11)
     /(ZEF*(1.414214_R8+3.25_R8*ZEF)-2.25_R8*ZEF*ZEF)
2995   CONTINUE
C
      XJBFAC(J)=MAX(1.E-5_R8,XJBFAC(J))
      xjbfacs(j,lsbeam)=xjbfac(j)  ! multi-species
C
100  CONTINUE
```

```
C
C----------------------------------------------------------
C
      RETURN
      END
```