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Electric- and weak magnetic-dipole-moment effects in $e^+e^- \rightarrow l^+l^-$

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We consider the weak effects in $e^+e^- \rightarrow l^+l^-$ that come, not from neutral currents, but from possible muon or heavy-lepton electric dipole moments, and from weak corrections to magnetic dipole moments. We show that in the coming experiments on this reaction, these weak effects can safely be ignored.

A major objective of the next generation of electron-positron colliding-beam machines will be to observe phenomena that are expected to arise from the exchange of heavy neutral weak bosons W^0 (Fig. 1).[1] The existence of neutrino and hadronic weak neutral currents is abundantly confirmed by neutrino scattering data, but the study of neutral weak couplings of charged leptons in e^+e^- collisions is only now beginning. Neutral weak effects [arising from the interference of Figs. 1(a) and 1(b)] are expected to be roughly of order $Gs/2\pi\alpha$ compared to one-photon exchange, and it should therefore be possible to measure them in e^+e^- collisions at $\sqrt{s} = E_{\rm c.m.} \simeq 30$ GeV.

Indeed, it is interesting to inquire whether other weak effects might not also be observable in high-energy e^+e^- collisions. Specifically, we consider in this note effects arising from weak contributions to leptonic electromagnetic form factors: magnetic and electric dipole moments. We will show that, in fact, the experimental consequences of such contributions are negligible. But this is not entirely obvious at first glance, since these weak form factors, while small, are "hard," i.e., they are approximately constant for $s/M_W^2 \lesssim 1$. In contrast to this, any corrections to the pointlike $e\gamma_\alpha$ electromagnetic coupling arising from electromagnetism are "soft"; e.g., the magnetic form factor falls as $m_{\rm lepton}^2/s$ for large s. (We show why this is so in the Appendix.)

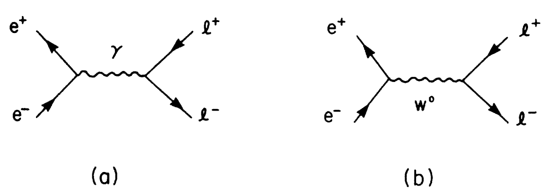

FIG. 1. (a) One-photon-exchange and (b) W^0-exchange graphs for $e^+e^- \rightarrow l^+l^-$.

The leptonic moment which is most poorly constrained experimentally is the muon electric dipole moment D_μ; the old upper limit[2] is $D_\mu \leq 2\times 10^{-17}$ cm. Suppose, just for the sake of argument, that indeed $D_\mu = 2\times 10^{-17}\ {\rm cm} \simeq 10^{-3}\ {\rm GeV}^{-1}$. Let us now make a very naive estimate of the contribution to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ of the interference between the diagram in which the virtual photon couples to the e pair by the usual charge coupling but to the μ pair by the D_μ coupling, and the analogous diagram with charge coupling at both ends. The result, taking account of the fact that the electric-dipole vertex,[3] $eD_\mu\sigma_{\alpha\beta}q^\beta\gamma_5$, involves an extra power of energy relative to the charge vertex, $e\gamma_\alpha$, is that this term makes a fractional correction to the standard QED expression of order $10^{-3}\ E$/GeV $\simeq 3\%$ for $E = 30$ GeV. This is comparable to the expected W^0-exchange contribution.

We hasten to explain that this naive estimate is much too large, for at least two reasons. First, there are good reasons to believe that the muon electric dipole moment D_μ is actually much smaller than 2×10^{-17} cm, and second, the interference discussed is greatly suppressed by helicity considerations.

I. MAGNITUDE OF LEPTONIC ELECTRIC AND WEAK MAGNETIC DIPOLE MOMENTS

The muon electric dipole moment is measured as a by-product of muon $g-2$ measurements: The presence of an electric as well as a magnetic dipole moment would cause the muon to wobble as well as precess as it circulates in a magnetic field, and counters are placed above and below the plane of the orbit to look for decay electrons sent in these directions as the muon spin wobbles.[2] The presence of a muon electric dipole moment also changes the precession rate: If the muon has an electric dipole moment $eD_\mu = fe\hbar/m_\mu c$ in addition to its magnetic-moment anomaly $a = \frac{1}{2}(g-2)$, its precession rate in a muon storage ring is increased by the factor $[1+4(f/a)^2]^{1/2}$.[4] Recent muon $g-2$ measurements agree with QED calculations to within statistical errors.[5,6] If we assume that the observed precession rate actually contains a small D_μ contribution, but not so large as to disturb the agreement between QED and experiment by more than the statistical error in the experiment, then $f \leq (a\,\Delta a/2)^{1/2}$. Using[5] $\Delta a = 6\times 10^{-7} = 2\sigma$, we deduce $f \leq 2\times 10^{-5}$, or $D_\mu \leq 4\times 10^{-18}$ cm. Using the preliminary results[6] from the current $g-2$ measurement, in which $2\sigma = 6\times 10^{-18}$, we obtain $D_\mu \leq 10^{-18}$ cm. A new direct measurement of D_μ is presently in progress as part of the current muon $g-2$ experiments, and this measurement should be sensitive to D_μ of order 10^{-18} cm.

The experimental upper limit on weak contributions to the muon magnetic dipole moment μ_μ is considerably smaller than this. Since the present experimental value[6] for a_μ agrees with QED (including hadronic corrections), it seems reasonable to conclude that $a_\mu^{\text{weak}} \leq \Delta a \approx 6\times 10^{-8}$, or $\mu_\mu^{\text{weak}} \leq 10^{-20} e$ cm.

In the context of gauge models of weak and electromagnetic interactions, weak contributions to leptonic electric or magnetic dipole moments are calculable but model-dependent. Many gauge models give weak corrections to leptonic magnetic dipole moments[7,8] $\mu_l^{\text{weak}} \approx eGm_l/8\pi^2 \approx 3\times 10^{-22}\,(m_l/m_\mu)\,e$ cm, arising from Fig. 2 (with ν_μ). (Here $l = e$, μ, or a possible sequential heavy lepton L.) Gauge models containing heavy neutral leptons (L^0) coupled to the muon via charged weak currents of both left and right chirality can produce μ_l^{weak} terms considerably larger than this[7,8]; the experimental limit on a_μ^{weak} discussed above then gives constraints on such models.

Electric dipole moments are even more model-dependent than magnetic dipole moments, since they depend on the way in which the model incorporates CP violation. Although it is possible to exhibit fairly reasonable models[9] in which maximal CP violation occurs in the muon sector and $D_\mu \approx Gm_{L0}/2\pi^2 \approx (m_{L0}/\text{GeV})10^{-20}$ cm, in most models CP violation is associated with quarks or Higgs mesons[10] and D_μ is several orders of magnitude smaller than this. There appears to be no reason why the situation should be much different for a heavy lepton.

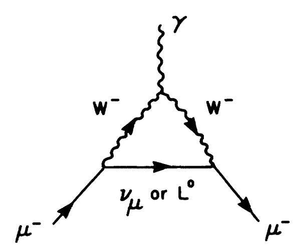

FIG. 2. Graph contributing to a_μ^{weak} (and to D_μ) in the model of Ref. 9.

Of course, these theoretical considerations do not rule out the possibility that D_μ is actually as large as the present experimental upper limit. If D_μ were that large, then the resulting CP-violating effects in $e^+e^- \to \mu^+\mu^-$ would be much larger than those expected from W^0 exchange, at least in gauge models or other weak-interaction models in which the lowest-order weak vector-boson couplings are a mixture of V and A (both of which of course are even under CP). In such models, CP violation arises from weak corrections to both graphs of Fig. 1, and one expects the CP-violating contributions from γ-exchange graphs to be larger than those from W^0-exchange graphs. Indeed, in gauge models[9] in which CP-violating couplings arise from one-loop graphs like Fig. 2, the contribution of Fig. 1(a) is $\sim G\alpha$ while that of Fig. 1(b) is $G^2 s$; at $\sqrt{s} = E_{\text{c.m.}} \simeq 30$ GeV, Fig. 1(a) dominates. It was this observation that initially motivated us to look more closely at possible electric-dipole-moment contributions.

II. SUPPRESSION OF INTERFERENCE BY HELICITY CONSIDERATIONS

There is a second reason why the naive estimate discussed above is too large; namely, that the interference between electric-charge coupling and electric- or magnetic-moment couplings is suppressed by helicity considerations. At energies where the lepton masses are negligible, dipole coupling produces only μ pairs in which the two particles have the same helicity, while charge coupling produces only pairs in which they have opposite helicities.[11] Thus, the diagrams corresponding to these two couplings cannot interfere at all in the differential cross section. The only effect to which an interference between them can possibly contribute is the transverse polarization of the outgoing muons. Only when one of the muons is transversely polarized is the μ pair in a coherent superposition of like- and unlike-helicity states.

From the vertex factors, the polarization will be of order

$$\frac{A_{\text{dipole}}A_{\text{charge}}}{(A_{\text{charge}})^2} \sim D_\mu E_{\text{c.m.}}\,,$$

where the A's are reaction amplitudes. However, detailed analysis shows that the effect vanishes unless one of the incoming beams has longitudinal polarization or one of the form factors has an imaginary part. In the first case the muon is polarized in the direction $\vec{k}_{e^+} \times \vec{k}_{\mu^+}$, where $\vec{k}_i$ is the c.m. momentum of particle i. One finds that the μ^+ polarization in this direction will be

$$P(\mu^+) = D_\mu E_{\text{c.m.}} \frac{2\sin\theta}{1+\cos^2\theta}\left(\frac{h_{e^-} - h_{e^+}}{1 - h_{e^-}h_{e^+}}\right), \qquad (1)$$

where θ is the production angle of the μ^+ relative to the e^+ beam and $h_{e^\pm}$ are the helicities of the beams. Now if $D_\mu = 2\times10^{-17}$ cm,[2] then for a completely longitudinal e^- or e^+ beam, with $E_{\text{beam}} = 20$ GeV and $\theta = 90°$, $P(\mu^+) = 8\%$, which perhaps is detectable. However, if $D_\mu < 10^{-18}$ cm as expected, then $P(\mu^+) < 1\%$, which probably makes it quite unobservable.

Another difficulty with observing $P(\mu^+)$ is that it now appears that the transverse polarization of storage-ring beams will be small at high energy.[12] Even if partial transverse polarizations are achieved, then simultaneous rotation of the polarization of both beams results in $h_{e^-} = h_{e^+}$, giving $P(\mu^+) = 0$.

One can also show that CP-violating effects can produce a correlation between the transverse polarization of $\mu^\pm$ and longitudinal polarization of $\mu^\mp$, even if the initial e^+ and e^- beams are unpolarized or transversely polarized. But such an effect would be very difficult to observe even if it were large, since a stopped μ^- is captured.

In deriving Eq. (1) we assumed the form factors to be real. Actually, for timelike s, both the charge and electric dipole form factors will have imaginary parts of order α. These produce a transverse polarization of the muon in the scattering plane even when the beams are not polarized. However, this polarization is not observable experimentally since it is expected to be even smaller (by $\sim\alpha$) than that given by Eq. (1).

Although the weak part of the magnetic dipole coupling also involves a "hard" form factor, any interference between this coupling and the charm coupling will be suppressed for the same reasons as that between the electric dipole and charge couplings.

We note, finally, that interference between electric-dipole and ordinary electromagnetic-dipole couplings $(ie/2m_\mu)\sigma_{\alpha\beta}q^\beta F_2(s)$ can affect the differential cross section itself. However, because of the "softness" of electromagnetic form factors, the effect will be very small:

$$\frac{\Delta\sigma}{\sigma} < 10^{-7} \quad \text{if} \quad D_\mu < 2\times10^{-17}\ \text{cm}.$$

In the case of heavy leptons, we do not have bounds for D_L, so the effects could conceivably be larger. On the other hand it would be difficult to see these effects. Even though the momenta of the decay products are, in general, correlated with the spin of the initial lepton L, the polarization $P(L^+)$ given by Eq. (1) has no effect on the angular or energy distribution of the lepton l resulting from the decay $L \to l\nu\bar{\nu}$. The polarization in the scattering plane does have such an effect, but it is reduced by α. The CP-violating effect producing a correlation between the transverse polarization of the $L^\pm$ and the longitudinal polarization of the $L^\mp$ conceivably could be measured by observing μ-e correlation.[13]

We conclude that e^+e^- colliding-beam experiments with ~20 GeV beams are not a good way to look for the muon electric dipole moment. On the other hand, such experiments *can* be designed to look for other subtle effects in the reaction $e^+e^- \to \mu^+\mu^-$ without any fear that these effects will be mimicked by the action of the muon moment. Note, in particular, that for $E_{\text{beam}} = 20$ GeV the expected neutral weak amplitudes for $e^+e^- \to \mu^+\mu^-$ will be at least ten times larger than the electric-dipole amplitudes if $D_\mu \le 10^{-18}$ cm. Even if $D_\mu = 10^{-17}$ cm and the amplitudes are comparable, the neutral weak phenomena for which one would like to search[1] cannot possibly be imitated by effects of the electric dipole moment, since to order $(A_{\text{dipole}})\times(A_{\text{charge}})$ there are no such effects in either the differential cross section or the polarization of one of the outgoing muons, so long as one uses the naturally occurring unpolarized or transversely polarized beams.

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APPENDIX

In this Appendix we will explain why weak contributions to form factors are expected to be "hard," i.e., approximately constant for momentum transfer $\sqrt{s}$ smaller than M_W, the intermediate-weak-vector-boson mass. The basic point is that the behavior of a Feynman integral is controlled by the largest masses and external momenta which it contains.

It is easiest to consider a particular example in order to illustrate the "hardness" of weak electric-dipole-moment (D) and magnetic-dipole-moment (F_2^W) contributions. We will use for this purpose Pais's O(4) gauge model of weak and electromagnetic interactions.[9] In this model there are two pairs of charged vector bosons $W_1^\pm$ and $W_2^\pm$; their masses are $M_1 = 37\ \text{GeV}/(\cos\theta_C)^{1/2} \approx 37$ GeV and $M_2 = 37\ \text{GeV}/(\sin\theta_C)^{1/2} \approx 75$ GeV, where θ_C is the Cabibbo angle; and $W_1^\pm$, $W_2^\pm$ couple to the muon currents

$$j_\alpha^{(1)} = -\frac{ie}{\sqrt{2}}[(\overline{\nu}_\mu - i\overline{L}^0)\gamma_\alpha P_- \mu + \sqrt{2}\beta \overline{L}^0 \gamma_\alpha P_+ \mu]\,,$$
$$\text{(A1)}$$
$$j_\alpha^{(2)} = -\frac{ie}{\sqrt{2}}[(i\overline{\nu}_\mu - \overline{L}^0)\gamma_\alpha P_- \mu - \sqrt{2}\beta \overline{L}^0 \gamma_\alpha P_+ \mu]\,,$$

where $P_\pm = (1 \pm \gamma_5)/2$ and β is a model-dependent complex number of order unity. It is of course the fact that the μ couplings are not relatively real that gives rise to CP-violating effects.

In the O(4) model, it is the Feynman diagram drawn in Fig. 2, with a heavy neutral lepton L^0 in the intermediate state and with both W_1 and W_2 graphs included, that gives the largest weak contributions to both F_2^W and D. For $s=0$ the results are[9]

$$F_2^W(0) = a_\mu^W = \frac{G}{\pi^2} m_\mu m_{L^0}(\mathrm{Im}\beta \cos\theta_C - \mathrm{Re}\beta \sin\theta_C)\,,$$
$$\text{(A2)}$$
$$D(0) = \frac{G}{2\pi^2} m_{L^0}(\mathrm{Re}\beta \cos\theta_C + \mathrm{Im}\beta \sin\theta_C)\,,$$

where we have dropped correction terms of order m_{L^0}/M_W.

Now all we wish to argue is that both $F_2^W(s) - F_2^W(0)$ and $D(s) - D(0)$ are $O(s/M_W^2)$, rather than $O(s/m_{L^0}^2)$ or $O(s/m_\mu^2)$. The necessary calculations are analogous to those in Ref. 8 (U gauge) or Ref. 7 (R_ξ gauge). For example, in the U gauge, Fig. 2 contributes terms of the form

$$\Gamma_\mu = \int d^4k \left[\overline{u}(p')\gamma_\alpha A \frac{1}{\not p - \not k - m_{L^0}} \gamma_\beta A^* v(p)\right] \times D^{\alpha\alpha'}(k+q) D^{\beta\beta'}(k) V_{\alpha'\beta'\mu}(k,q)\,, \qquad \text{(A3)}$$

where A is the appropriate coupling factor for W_1 or W_2 [from Eq. (A1)],

$$D^{\alpha\beta}(k) = \frac{k^\alpha k^\beta / M_W^2 - g^{\alpha\beta}}{k^2 - M_W^2}$$

is the vector propagator ($M_W = M_1$ or M_2), and

$$V_{\alpha\beta\mu}(k,q) = (k-q)_\alpha g_{\beta\mu} + (2q+k)_\beta g_{\alpha\mu} - (2k+q)_\mu g_{\alpha\beta}$$

is the Yang-Mills coupling. After the F_2 or D terms are identified and the momentum integral is performed using the 't Hooft–Veltman regularization, the resulting terms are of the form

$$\int dx\,dy\,dz\,\delta(1-x-y-z) N_\mu \times [(x+y)M_W^2 + z m_{L^0}^2 - z^2 m_\mu^2 - xys]^{-1}\,, \qquad \text{(A4)}$$

where N_μ involves factors of x, y, z, masses, and kinematical terms.

The point we wish to make is simply that the integral (A4) remains finite even if we set $m_\mu^2 = 0$ and $m_{L^0}^2 = 0$. Specifically, the numerator factor N_μ contains no inverse powers of m_μ or $m_{L^0}^2$, and the factors of x, y, z in N_μ cannot lead to singularities. The integral over the remaining denominator factor, with $m_\mu^2 = m_{L^0}^2 = 0$, gives

$$\frac{1}{s}\int_0^1 \frac{dv}{v(1-v)} \ln\left[1 - \frac{s}{M_W^2} v(1-v)\right] = -\frac{1}{M_W^2}\left[1 - \frac{s}{6M_W^2} + \cdots\right]\,,$$

which is clearly finite. Thus the natural expansion parameter for the weak form factors is s/M_W^2, and such form factors are consequently expected to be slowly varying for s in the range accessible at presently planned e^+e^- machines.

Unfortunately, the result of evaluating the graph of Fig. 2 is not gauge invariant except at $s=0$; it will depend, for example, upon the gauge parameter ξ. This remains true even including the contributions of the unphysical scalars $S^\pm$ in the R_ξ gauge (for $\xi = 0$).[7] Form factors are not gauge invariant; only a complete S-matrix element, which for a scattering process will include box graphs and propagator insertions as well as vertex corrections, is gauge invariant in general. Fortunately, these gauge-invariance subtleties are irrelevant for our present purposes since the D and F_2^W contributions that we have discussed are present in any gauge at $s=0$ and, as we have just argued, such weak contributions are independent of s for $s \ll M_W^2$.

It is interesting to note that the electromagnetic contributions to the Pauli form factor F_2^{em} do not share the "hardness" of the weak form factors. To see that F_2^{em} must be soft, note that when $\sqrt{s}$ is large enough so that lepton masses can be neglected, any purely electromagnetic diagram contributing to $\langle \mu^+\mu^- | J_\lambda^{em} | 0\rangle$ contains a factor of the form $\overline{u}(k_{\mu^-})\gamma_\alpha \not k_1 \gamma_\beta \not k_2 \cdots \gamma_\rho \not k_l \gamma_\tau v(k_{\mu^+})$, corresponding to the muon line in the diagram. Now if the outgoing μ^+ has a certain helicity, one may freely insert the corresponding helicity projection operator, $(1\pm\gamma_5)/2$, in front of the μ^+ wave function v. This operator can then be commuted through to operate on the μ^- wave function; one will always encounter an odd number of vertices plus propagators in the process. As a result, the helicity of the outgoing μ^- must always be opposite to that of the μ^+.[14] Now, when $\sqrt{s} \gg m_\mu$, the terms $F_1\gamma_\lambda$ and $F_2\sigma_{\lambda\nu}(iq^\nu/m_\mu)$ in the matrix element of the current produce only $\mu^+\mu^-$ pairs of opposite and like helicity, respectively. Thus we see that

$$F_2^{em}(s)\frac{\sqrt{s}}{m_\mu} \ll F_1^{em}(s)$$

in this region. That is, F_2^{em} is a "soft" form factor, as the well-known lowest-order contribution to it illustrates.

The hard and soft behaviors which we have found agree, of course, with one's naive expectations. The quantities $D(s)$ and $F_2^W(s)$ measure components of the muon's charge and current distributions which would vanish were it not for the action of weak forces with a range of order $1/M_W$. Thus, one expects these components to be confined to a small radius of this same order, which means that D and F_2^W will be hard. By contrast, $F_2^{em}(s)$ measures a distribution of currents which results from the action of electromagnetic forces of infinite range. This distribution should then be much bigger. One expects it to have a radius of order $1/m_\mu$, since this is the only scale of size available, so that F_2^{em} will be small when $s \gg m_\mu^2$.

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