What is the Bogoliubov Hierarchy?

What is b_0?

How can you determine whether a given collision operator conserves entropy

The Bogoliubov Hierarchy is a hierarchy of time scales describing the relaxation of an arbitrary perturbation. A perturbed system will relax towards thermal equilibrium in 3 principal stages:

1. Pair correlations (DeBye shielding over distances λ_D) on a scale $\tau_{ac} \sim \frac{1}{\omega_p}$
2. The velocity distribution relaxes to a local Maxwellian on the collisional time scale $\tau_c = \frac{1}{\nu}$ smoothing out on scales of order λ_{mfp}
3. Hydrodynamic (diffusion) processes occur on a macroscopic space and time scale ($L \gg \lambda_{mfp}$, $t \gg \tau_c$) trying to relax the system to a global, space and time independent Maxwellian.

$b_0 = \frac{e^2}{T}$ is the distance of closest approach if you send 2 particles directly at each other. All kinetic energy has become potential energy:

$$\frac{1}{2} m v^2 = \frac{e^2}{b_0} \qquad \left(v \sim v_t = \sqrt{\frac{T}{m}}\right)$$

$$b_0 = \frac{e^2}{T} \quad \text{(ignore factor of 2)}$$

---

entropy, $S = \int f \ln f \, d\bar{v}$

so $$\left(\frac{\partial S}{\partial t}\right)_{coll} = \int \left(\frac{\partial f}{\partial t}\right)_{coll} (\ln f + 1)\, d\bar{v}$$

$$= -\int C[f]\,(\ln f + 1)\, d\bar{v}$$

Give a heuristic argument for the relationship between ν_{ie} and ν_{ei}

How can you estimate the friction force, $\vec{R}$ and what do you get?

How can you show that a collision operator, $C_s[F]$ conserves momentum?

from momentum conservation,

$$m_e n_e \left(\frac{\partial u_e}{\partial t}\right)_{coll} = -m_e n_e \left(\frac{\partial u_i}{\partial t}\right)_{coll}$$

and, by definition: $\left(\frac{\partial \bar{u}_e}{\partial t}\right)_{coll} = -\nu_{ei}(\bar{u}_e - \bar{u}_i)$

$$\left(\frac{\partial u_i}{\partial t}\right)_{coll} = -\nu_{ie}(\bar{u}_i - \bar{u}_e)$$

so: $\nu_{ie} = \left(\frac{m_e n_e}{m_i n_i}\right)\nu_{ei}$

$$\bar{R} = -\int d\bar{v}_e \, (m\bar{n})_e \, \bar{v}_e \, C_{ei}[f]$$

- with the Lorentz operator for C_{ei}, you can estimate $\bar{R}$ by taking f_e to be a shifted Maxwellian (a 0-centered Maxwellian would have zero friction force). Also assume that $u/v_{te} \ll 1$.

you get: $\bar{R} \approx -m_e n_e \nu_{ei}(\bar{u}_e - \bar{u}_i$

to show that a collision operator, $C_s[f]$ conserves momentum, you must show that:

$$\left(\frac{\partial \bar{p}}{\partial t}\right)_{coll} = -\sum_s \int d\bar{v} \, (m\bar{n})_s \, \bar{v} \, C_s[f] = 0$$

How do you show that a collision operator $C_s[f]$ conserves kinetic energy?

How do you show that a collision operator, $C_s[f]$ conserves number of particles?

Derive a heuristic scaling for ν_{ei}, the electron-ion collision frequency.

to show that a collision operator $C_s[f]$ conserves kinetic ~~energy~~ energy, you must show:

$$\left(\frac{\partial K}{\partial t}\right)_{coll} = -\sum_s \int d\vec{v}\ \frac{1}{2}(\bar{n}\,m)_s\, v^2\, C_s[f] = 0$$

~~[illegible]~~

To show that a collision operator, $C_s[f]$ conserves number of particles, you must show that:

$$\left(\frac{\partial n_s}{\partial t}\right)_{coll} = -\bar{n}_s \int d\vec{v}\ C_s[f] = 0$$

use $\nu \sim n\sigma v$,
take $n \sim n_i$, consider ions ~~[illegible]~~ nearly immobile, $v \sim v_{te}$, $\sigma \sim b_o^2$ where b_o is the distance at which potential energy of the interaction equals electron thermal energy. $b_o \sim \frac{Ze^2}{T_e}$. Also, don't forget the factor of $8 \ln \Lambda$ due to dominance of small $\measuredangle$ ~~[illegible]~~ scattering:

$$\nu_{ei} \sim n_i b_o^2 v_{te} \ln \Lambda \sim \frac{n_i Z^2 e^4}{T_e^{3/2} m_e^{1/2}} \ln \Lambda$$

Why is it kinetic energy, not total energy, that the Landau (Lorentz, Balescu-Lenard, etc) collision operator conserves?

How would this inner product be defined in the Chapman-Enskog procedure:

$$\langle \psi | \hat{L} | \chi \rangle = ?$$

What is this:

$$C[f] = -\nu(v) \frac{\partial}{\partial \lambda} (1-\lambda^2) \frac{\partial f}{\partial \lambda}$$

- What is λ?
- what is $\nu(v)$?
- where did it come from?
- What does it describe?

because we assumed straight line trajectories, ~~the~~ with constant v.

To first order, there was no change in potential energy

for an entire plasma fluid, in 3-D:

$$\langle \psi | \hat{L} | \chi \rangle \equiv \sum_s \int d^3v \, \psi(\vec{v}) \, \hat{L} \, \chi(\vec{v}) \, f_0(\vec{v})$$

$\hat{L}$ acts on (χf_0)

for a test particle, in 1-D:

$$\langle \psi | \hat{L} | \chi \rangle \equiv \int dv \, \psi \, \hat{L} \, \chi f_0$$

<u>Lorentz collision operator</u>: It came from the Landau collision operator, with the assumption that $m_e \ll m_i$.

$\lambda = \frac{v_\parallel}{v} = \cos\theta$ is the pitch angle parameter

$\nu(v) = \nu_{ei} \left(\frac{v_{te}}{v} \right)^3$ where ν_{ei} is as derived with a heuristic scaling argument.

note: $\frac{1}{v^3}$ dependence can lead to runaway electrons.

The Lorentz operator conserves electron kinetic energy - it describes the scattering of electrons off of immobile ions - occurs at the constant electron energy "on the energy shell"

How could you find <u>all</u> conservation laws of a given collision operator?

What does the Landau operator conserve?

What is the definition of conductivity

(Tensor and Scalar)

to find all quantities which are conserved by a given collision operator, write:

$$\int d^3v \; W(\vec{v}) \, C[f] = 0$$

and try to solve for W for arbitrary f.

→ Try to get in form: $\int d^3v \; G(W) \cdot f = 0$. Then $G(W) = 0$

* integrate by parts to remove all derivatives from f.

The Landau operator conserves density momentum and kinetic energy and nothing else.

conductivity tensor is defined by:

$$\vec{J}_{\vec{k},\omega} = \overleftrightarrow{\sigma}_{\vec{k},\omega} \cdot \vec{E}_{\vec{k},\omega}$$

with no asymmetry, this reduces to:

$$\vec{J}_{\vec{k},\omega} = \sigma(\vec{k},\omega) \, \vec{E}_{\vec{k},\omega} \qquad \text{where } \sigma \equiv \hat{k} \cdot \overleftrightarrow{\sigma} \cdot \hat{k}$$

and the $\vec{E}$ field is given by:

$$\vec{E}(x,t) = \vec{E}_{\vec{k},\omega} \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right)$$

How are $\langle v^2 \rangle$ and T related in thermal equilibrium?

What does $f_\alpha(\vec{r}, \vec{v}, t)\, d\vec{r}\, d\vec{v}$ represent?

In the collisionless limit, does the conductivity have a dissipative part?

$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} T$ in thermal equilibrium

$f_\alpha(\vec{r}, \vec{v}, t)\, d\vec{r}\, d\vec{v}$ represents the number of particles of species α in a 6-D volume element $d\vec{r}\, d\vec{v}$

Yes; it is due to Landau damping.

(can use Vlasov dielectric in the collisionless limit)

What are runaway particles?

Why is it possible to have different electron and ion temperatures in one plasma

How is the dielectric function defined, in terms of total and test fields/potentials?

Runaway particles are particles that accelerate indefinitely and never reach a steady state. This can arise because the Coulomb scattering cross section decreases rapidly at high energies

$$\sigma \sim \left(\frac{v_{te}}{v}\right)^3$$

It is possible to have different electron and ion temperatures in one plasma because the large mass difference ensures that each particle species will thermalize with itself much sooner than the two will thermalize together.

$$\vec{E}^{(tot)}_{\vec{k},\omega} = \frac{\vec{E}^{(test)}_{\vec{k},\omega}}{\mathcal{D}(\vec{k},\omega)}$$

or, by poisson's eqn:

$$\delta \phi^{(tot)}_{\vec{k},\omega} = \frac{\delta \phi^{(test)}_{\vec{k},\omega}}{\mathcal{D}(\vec{k},\omega)}$$

What does the test particle superposition principle state, and what does it allow you to compute?

What is the charge density of a test particle
- in configuration space?
- in fourier space?

What is the total charge density of a single quasiparticle?

The test particle superposition principle states that, to lowest order in the plasma parameter ϵ_p, one may consider the plasma to be composed of a collection of quasiparticles: statistically independent shielded test particles.

→ correlation effects accounted for by shielding.

It can be used to compute two-point quantities (e.g. fluctuation spectrum of the electric field) correctly through $O(\epsilon_p)$

$$\rho^{test}(\vec{x},t) = Q\,\delta(\vec{x}-\vec{x}_0-\vec{v}t)$$

$$\rho^{test}(\vec{k},\omega) = \int d\vec{x}\int dt\, e^{-i(\vec{k}\cdot\vec{x}-\omega t)}\, Q\,\delta(\vec{x}-\vec{x}_0-\vec{v}t)$$

$$= 2\pi Q\, e^{-i\vec{k}\cdot\vec{x}_0}\,\delta(\omega-\vec{k}\cdot\vec{v})$$

$$\rho^{QP}(\vec{k},\omega) = \frac{\rho^{test}(\vec{k},\omega)}{\mathcal{D}(\vec{k},\omega)}$$

total is sum of induced + test.

What is the electric field of a quasiparticle?

How do you use the assumption of statistical independence of quasiparticles to simplify:

$$\langle E^{tot}(\vec{k},t)\, E^{tot}(-\vec{k},t')\rangle = \sum_i \sum_j \langle E_i^{QP}(\vec{k},t)\, E_j^{QP}(-\vec{k},t')\rangle$$

If A_i and A_j are statistically independent, how can you express:

$$\sum_i \sum_j A_i A_j = ?$$

$$i\vec{k}\,\vec{E}^{QP}(\vec{k},\omega) = 4\pi\rho^{QP}(\vec{k},\omega)$$

or, defining $\vec{\epsilon}_{\vec{k}} = \frac{4\pi i \vec{k}}{k^2}$

$$\vec{E}^{QP}(\vec{k},\omega) = \vec{\epsilon}_{\vec{k}}\,\rho^{QP}(\vec{k},\omega)$$

$$\langle \vec{E}(\vec{k},t)\,\vec{E}(-\vec{k},t')\rangle = \sum_i \sum_j \langle E_i^{QP}(\vec{k},t)\,E_j^{QP}(-\vec{k},t')\rangle$$

$$= \underbrace{\sum_i \langle E_i^{QP}(\vec{k},t)\rangle \sum_{i\neq j} \langle E_j^{QP}(-\vec{k},t')\rangle}_{\text{mean field}} + \underbrace{\sum_i \langle E_i^{QP}(\vec{k},t)\,E_i^{QP}(-\vec{k},t')\rangle}_{\text{fluctuations (what you are interested in)}}$$

then use statistical independence + normalization to say

$$\sum_i \langle\,\rangle \rightarrow \sum_s N_s \int f_s(\vec{v})\,d\vec{v}$$

$$\sum_i \sum_j A_i A_j = \sum_i A_i \sum_{j\neq i} A_j + \sum_i A_i^2$$

Explain the meaning of the right hand side:

$$\frac{\partial f_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(v_\beta f_\alpha) + \frac{\partial}{\partial v_\beta}\left(\frac{F_{\alpha\beta} f_\alpha}{m_\alpha}\right) = C_\alpha[f_\alpha]$$

How are the classical Langevin equations,

$$\frac{d\tilde{x}}{dt} = \tilde{v}$$

$$\frac{d\tilde{v}}{dt} + \nu\tilde{v} = \delta a(t)$$

derived, and what physical effects do the terms represent?

In the Langevin Calculation, what is D_v?

How can this be related to the "typical step size" for ΔV and Δt?

The right-hand side of the kinetic equation is the "collisional" term - it accounts for the rapidly fluctuating microfields + forces in the plasma, which arise when particles come close to each other.
$f_a(\vec{r}, \vec{v}, t)$ is a smoothed density averaged over a volume containing a large # of particles.
The force F_a is a smoothed macroscopic force and represents an average over time + distance

They came from Newton's Laws, assuming acceleration ~~due to ① Polarization drag: the $-\nu\vec{v}$ term~~

② velocity space diffusion - random δv kicks from passing through DeBye spheres. You can assume that $\omega_p^{-1} \ll \Delta t \ll \nu^{-1}$ (coarse graining) and that DeBye spheres are <u>independent</u> → then acceleration may be considered to be Gaussian white noise, $\delta a(t)$ with $\langle \delta a(t) \rangle = 0$

$$\langle \delta a(t) \delta a(t') \rangle = 2D_v \, \delta(t-t') \, \bar{\bar{I}}$$

$D_v = \nu V_t^2$ expect $D_v \sim \frac{(\Delta v)^2}{\Delta t}$ But be careful

ν^{-1} and V_{th} are <u>NOT</u> ΔV and Δt! Recall the process of scattering off of DeBye spheres: $\Delta t \sim \omega_p^{-1} \sim \epsilon/\nu$
- setting $\frac{(\Delta V)^2}{\Delta t}$ gives $(\Delta V)^2 \sim \epsilon_p V_t^2$

→ Langevin calculation takes $\epsilon_p \to 0$ in just such a way that the velocity space diffusion coefficient remains finite while the elementary steps go to zero!

What is the Einstein relation, found from the Langevin calculation in a ($\vec{B}$-field free) plasma?

Describe the Green's function technique for solving 1st order ODE's

Describe the (unmagnetized) Langevin dynamics on short and long time scales

Einstein relation is for D_v - find by using energy equipartition → Assume that a long times, a test particle has reached equilibrium with the background bath, such that

$$\frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} T$$

Langevin calc. gave $\langle \delta v^2 \rangle = D/\nu$ in long time limit
set equal to get

$$\boxed{D_v = \nu v_{th}^2}$$

Suppose you have a 1st order DE with a driving term

$$\hat{O}\, \psi(t) = S(t).$$

You want to find a Green's function such that:

$$\hat{O} \circ G(t;t') = \delta(t-t')$$

Then the solution for ψ is given by:

$$\psi(t) = \int_{-\infty}^{\infty} d\bar{t}\; G(t;\bar{t})\, S(\bar{t}) \quad + \text{ initial conditions}$$

short times $\nu t \ll 1$		$\nu t \gg 1$ long times	
$\langle v \rangle = (1-\nu t) v_0$	collisional slowing down	$\langle v \rangle = 0$	randomization of velocity
$\langle x \rangle = x_0 + v_0 t$	free streaming	$\langle x \rangle = x_0 + \lambda_{mfp}$	
$\langle \delta v^2 \rangle = 2 D_v t$	velocity space diffusion	$\langle \delta v^2 \rangle = v_{th}^2$	thermalized
$\langle \delta x^2 \rangle = \frac{2}{3} D_v t^3$	"orbit diffusion"	$\langle \delta x^2 \rangle = 2 D_x t$	
$D_v = v_t^2 \nu$		$D_x = \frac{v_{th}^2}{\nu}$	

What equation relates heat flux to a temperature gradient?

How do you estimate the coefficient?

What equation relates particle flux to a density gradient?

How do you estimate the coefficient?

Explain the $\bar{u}$ dependent electron heat flux $\bar{q}_u$ in the unmagnetized, and magnetized cases.

$$q = -\kappa \frac{\partial T}{\partial x}$$

thermal conductivity

$$\kappa \sim \frac{n(\Delta x)^2}{\Delta t}$$

$$\Gamma = -D \frac{\partial n}{\partial x}$$

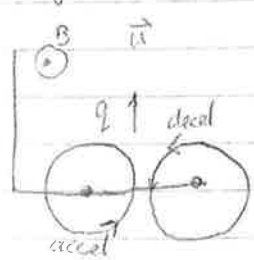
diffusion coefficient:

$$D \sim \frac{(\Delta x)^2}{2(\Delta t)}$$

- unmagnetized: faster electrons predominantly carry current (due to $\nu \sim \frac{1}{v^3}$) → more fast electrons moving along $\bar{u}$, slower ones against. Though electron fluxes cancel, energy fluxes do not. Net energy flux in $\bar{u}$ direction ($\vec{q}$ along $\bar{u}$)

Magnetized:

electrons alternately accelerated/decelerated by friction force as they move along/against $\bar{u}$. Net effect: faster electrons moving in y → heat flux

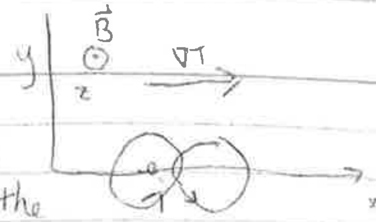


note $\vec{q} \perp$ to $\bar{B}$ and $\bar{u}$

How does the $\vec{R}_{\nabla T}$ force differ in the magnetized case from the unmagnetized case?

Derive a heuristic scaling for the temperature gradient dependent $\vec{R}_{\nabla T}$ force. (unmagnetized case)

What is the physical mechanism for the $\vec{\nabla} T$ dependent $\vec{R}$ force? (unmagnetized case) in the strongly magnetized case:



- The particles coming from regions of different temp (x) are moving in the $\hat{y}$ direction when the force imbalance occurs $\Rightarrow$ Force is now in $\hat{y}$

- typical distance of travel is now ρ_L, not λ_{mfp} $\rightarrow$ magnitude of force : $\delta R \sim \delta(m n v_{th} \nu_{ei})$

$$\sim \frac{1}{\rho_L} \frac{\partial}{\partial x}(m n v_t \nu_{ei})$$

previously, this was $\sim \frac{1}{\lambda_{mfp}}$ magnitude differs by $\frac{\rho_L}{\lambda_{mfp}}$

$$\vec{R} \sim m n v_t \nu_{ei} \quad \rightarrow \delta \vec{R} \sim \delta(m n v_{th} \nu_{ei})$$

$$\sim \lambda_{mfp} \frac{\partial}{\partial x}(n m v_t \nu_{ei})$$

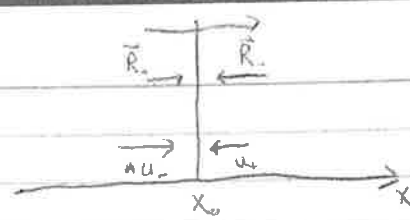
temp dependent

$$\nu_{ei} \sim \frac{n_i e^4}{T^2}\sqrt{\frac{T}{m}}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{T}\right) \sim \frac{1}{T^2}\frac{dT}{dx} \qquad \frac{\partial}{\partial x}\left(n m v_t \nu_{ei}\right) \sim \frac{\partial}{\partial x}\left(\frac{n^2 e^4}{T}\right)$$

... put all together:

$$\delta \vec{R} \sim -n \frac{\partial T}{\partial x}$$



if particles arriving at x_0 with same $\vec{u}$ (so no $\vec{R}_u$ force) - consider those arriving from right + those from left.

from right have higher energy, fewer collisions. smaller friction force.

$\Rightarrow$ Net friction force opposite $\vec{\nabla} T$.

How is the friction force related to $\Delta \vec{u}_{\parallel}$ and $\Delta \vec{u}_{\perp}$ in a strong magnetic field and why?

What equation relates momentum flux to a velocity gradient?
How do you estimate the coefficient?

Given the electric field of a single quasiparticle, $\vec{E}_i^{qp}(\vec{k},\omega)$ in fourier space, what is
- the total electric field due to all quasiparticles?
- the two-time correlation function?

$$\vec{R}_{\Delta\vec{u}} = -m_e n_e \nu_{ei} \left(0.51\, \Delta u_\parallel + \Delta u_\perp \right)$$

friction force lower in $\parallel$ direction, due to high energy electron tail (coeff ~1 for shifted maxwellian)

$\vec{B}$ field prevents tail from developing

$$\Pi_{xy} = -\eta \frac{\partial V_y}{\partial x}$$

viscosity, $\eta \sim \frac{mn(\Delta x)^2}{\Delta t}$

the total electric field due to all quasiparticles is:

$$\vec{E}(\vec{k},t) = \sum_i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{E}_i^{qp}(\vec{k},\omega)$$

and the two time correlation function is:

$\langle \vec{E}(\vec{k},t)\vec{E}(-\vec{k},t) \rangle$

$$= \sum_{ij} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left\langle \vec{E}_i^{qp}(\vec{k},\omega)\, \vec{E}_j^{qp}(-\vec{k},\omega') \right\rangle * e^{-i\omega t - i\omega' t'}$$

How do you calculate a dielectric function by inserting a test charge?

What is the answer for the Vlasov dielectric?

What is the Vlasov dielectric?

What does it reduce to in the high frequency limit?

How can the (scalar) conductivity be expressed in terms of the dielectric function?

Why shouldn't you evaluate σ using the Vlasov dielectric for the hydrodynamic regime?

insert a test charge, $\delta\rho^{test}$ → related to a test potential by poisson: $\nabla^2 \delta\varphi^{test} = -4\pi\,\delta\rho^{test}$.

Also know, $\delta\varphi^{tot} = \delta\varphi^{ind} + \delta\varphi^{test}$ (1)

use an eqn. for the distribution function (like Vlasov or gyrokinetic eqn) → E in it is $E^{tot} = -ik\,\delta\varphi^{tot}$

Linearize the eqn. to solve for f_1 → then get $\delta\rho^{ind} = q\int f_1 dv$. use this and (1) to eliminate $\delta\varphi^{ind}$ → get relation between $\delta\varphi^{tot}$ and $\delta\varphi^{test}$:

$$\delta\varphi^{tot}_{k,\omega} = \frac{\delta\varphi^{test}_{k,\omega}}{\mathcal{D}(\vec{k},\omega)}$$

for Vlasov case:

$$\mathcal{D}(\vec{k},\omega) = 1 + \sum_s \frac{\omega_{ps}^2}{k^2}\int d\vec{v}\,\frac{\vec{k}\cdot\frac{\partial f_0}{\partial \vec{v}}}{\omega - \vec{k}\cdot\vec{v}}$$

Vlasov Dielectric:

$$\mathcal{D}(\vec{k},\omega) = 1 + \sum_s \frac{\omega_{ps}^2}{k^2}\int d\vec{v}\,\frac{\vec{k}\cdot\frac{\partial f}{\partial \vec{v}}}{\omega - \vec{k}\cdot\vec{v}}$$

$$= 1 + \sum_s \frac{\omega_{ps}^2}{k^2}\left[P\int d\vec{v}\,\frac{\vec{k}\cdot\frac{\partial f}{\partial \vec{v}}}{\omega - \vec{k}\cdot\vec{v}} - i\pi\int d\vec{v}\,\delta(\omega - \vec{k}\cdot\vec{v})\,\vec{k}\cdot\frac{\partial f}{\partial \vec{v}}\right]$$

for $\omega \gg k v_{te}$

$$\approx 1 - \frac{\omega_{pe}^2}{\omega^2} - i\pi\sum_s \frac{\omega_{ps}^2}{k^2} F'\left(\frac{\omega}{k}\right)$$

$$\sigma(\vec{k},\omega) = \frac{\omega}{4\pi i}\left[\mathcal{D}(\vec{k},\omega) - 1\right]$$

Shouldn't use the Vlasov dielectric in the hydrodynamic (low frequency) regime because it was derived assuming high frequency regime - specifically, it's not ok to replace the resonant denominator $(\omega - \vec{k}\cdot\vec{v} + i\epsilon)^{-1} \rightarrow (\omega - \vec{k}\cdot\vec{v} + i\nu)^{-1}$ for constant ν, because such a ν does not respect the conservation properties of the true collision operator.

How do you use the Lorentz collision operator to find plasma conductivity?

Estimate the Dreicer limit for thermal runaway

What is Boltzmann's H-theorem?

/

write the kinetic equation:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \vec{a}\cdot\nabla_v f = -C[f]$$

- drop 1st 2 terms, taking "long λ, low ω limit"
- use chapman-enskog-like ordering, + $f = f_0 + \epsilon f_1$ to get: $C[f_0] = 0$, $\frac{q}{m}\vec{E}\cdot\frac{\partial f_0}{\partial \vec{v}} = -C[f_1]$

use the lorentz collision operator and solve for f_1. then use $\vec{u} = \int d\vec{v}\,\vec{v}\,f_1(\vec{v})$, and $\vec{J} = ne\vec{u}$ → get relationship between $\vec{J}$ and $\vec{E}$. This gives σ!

Dreicer limit is field at which thermal particles may run away → estimate by saying a particle will runaway if it doubles its thermal speed between collision times →

$$V_{te}\nu_{ei} = \frac{q}{m}E_c \Rightarrow E_c \sim \frac{m}{e}\nu_{ei}V_{te}$$

Boltzmann's H theorem states that if a distribution function changes only by virtue of collisions, that no matter what the initial conditions, the distribution function must approach a Maxwellian in the course of time.

(approach of distribution function to Maxwellian by means of collisions is called 'relaxation').

What is the general form for the Fokker-Planck equation for the PDF of a variable β?

Make a table showing:

- velocity
- range of force
- duration of scattering event
- angular scatter in one event
- coupling parameter
- cross section for 90° scatter
- τ_{inter}/τ_c

for Boltzmann gas and the weakly coupled plasma.

How does the cross section for Rutherford scattering scale with b_0 and θ? What happens at small angles?

$$\frac{\partial f}{\partial t}(\beta,t) + \frac{\partial}{\partial \beta}\left[V(\beta,t)\, f(\beta,t)\right] - \frac{\partial^2}{\partial \beta^2}\left[D(\beta,t)\, F(\beta,t)\right] = 0$$

where $V(\beta,t) \equiv \lim_{\Delta t \to 0} \dfrac{\langle \Delta\beta \rangle}{\Delta t}$

$D(\beta,t) \equiv \lim_{\Delta t \to 0} \dfrac{\langle \Delta\beta^2 \rangle}{2\Delta t}$

	V	Range of force	Duration of Scattering event	Angle scatter per event	Coupling parameter	σ^{90}	t_{inter}/τ
Boltzmann Dilute gas	V_{th}	b_o	$\frac{b_o}{V_t}$	$\Delta\theta \sim 1$	$\epsilon_n = n b_o^3$	b_o^2	ϵ_n
Weakly coupled plasma	V_{th}	λ_D	ω_p^{-1}	$\Delta\theta \ll 1$	$\epsilon_p = \frac{1}{n\lambda_D^3}$	$b_o^2 \ln\Lambda$	$\epsilon_p \ln \Lambda$

$$\sigma_R(\theta) = \frac{b_o^2}{4\sin^4\left(\frac{\theta}{2}\right)}$$

at small angles $\sigma_R \sim \dfrac{b_o^2}{\theta^4} \rightarrow$ divergent for small angles

What is the definition of a Markov process

In the unmagnetized Langevin calculation, name two ways in which you can 'coarse grain' the time axis, and what you see in each.

What is the Klimontovich Eqn?

How does it differ from the Vlasov Eqn?

A sequence x_n of discrete random variables is markov if the probability of observing x_n conditional on knowing the values of all the $n-1$ other variables depends in fact on just the value of x_{n-1}

→ independent events → if present is known, future is independent of past.

coarse-graining for diffusion in velocity space: $\tau_{ac} \ll \Delta t \ll \nu^{-1}$

coarse graining for diffusion in x-space $\nu^{-1} \ll \Delta t \ll t_{macro}$

The Klimontovich eqn. looks just like the Vlasov eqn, but in in $\tilde{N}$ instead of f →

$$\tilde{N}(z,t) = \frac{1}{n} \sum_{i=1}^{N} \delta(z - \tilde{z}_i(t))$$

$\langle \tilde{N} \rangle = \langle \delta(z - \tilde{z}(t)) \rangle$ = one particle PDF

Klimontovich includes all effects - collision, turbulence, etc, but is nonlinear in $\tilde{N}$ because E depends on $\tilde{N}$.

Vlasov eqn is from mean field theory - contains no fluctuation effects $(\epsilon_p \to 0)$

What is the Liouville eqn?

What is Liouville's Theorem?

What are the Bogoliubov time and spatial scales?

What happens on these scales?

What is this:

$$\hat{C}_{ss}\chi = -2\pi\left(\frac{ne^2}{\bar{n}m}\right)_s (ne^2)_s \ln\Lambda \frac{\partial}{\partial\vec{v}}\cdot f_M$$

$$\left\{\left[a(v)\left(\overset{1}{\mathbb{1}}-\hat{v}\hat{v}\right)+\overset{2}{b(v)}\hat{v}\hat{v}\right]\cdot\left(\frac{1}{m}\frac{\partial\chi}{\partial\vec{v}}\right) - \int d\vec{\bar{v}}\, f_M(\vec{\bar{v}})\, \overset{3}{U}(\vec{v}-\vec{\bar{v}})\cdot\left(\frac{1}{m}\frac{\partial\bar{\chi}}{\partial\vec{\bar{v}}}\right)\right\}$$

where did it come from, and what physical effects correspond to terms 1, 2 and 3 ?

Liouville Eqn: $\frac{\partial P_N}{\partial t} + \nabla_{6N} \cdot (\vec{V}_N P_N) = 0$

a continuity eqn →
system points never disappear.

Liouville's theorem states that phase space volume is conserved if $\nabla_N \cdot \vec{V}_N = 0$

(incompressible flow)

$\tau_{ac} (\sim \omega_p^{-1}) \ll \tau_c (\sim \nu)^{-1}) \ll \tau_h$

$\lambda_D \ll \lambda_\nu \ll L$

set up debye shielding	Collision time - velocity space diffusion	hydrodynamic time: Position Space diffusion

It is the collision operator for like species collision, linearized ~~around a Maxwellian distribution: $f = f_M + \chi$~~

① , which has $(\mathbb{1} - \hat{v}\hat{v})$ describes Pitch angle scattering

② which has $\hat{v}\hat{v}$ describes energy diffusion

③ which has $\int d\vec{v} \; (\;) \cdot \frac{\partial \bar{\chi}}{\partial \vec{v}}$ ensures momentum conservation

What is the Multivariate Fokker-Planck equation?

What is the Chapman Kolmogorov Equation? What does it assume?

What is the Master Equation?

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \vec{x}} \circ (\vec{v} f) - \frac{\partial^2}{\partial \vec{x} \partial \vec{x}} : (D f) = 0$$

where the drift and diffusion coefficients are:

$$\vec{V} = \lim_{\Delta t \to "0"} \frac{\langle \Delta \vec{x} \rangle}{\Delta t}, \quad D = \lim_{\Delta t \to "0"} \frac{\langle \Delta \vec{x} \Delta \vec{x} \rangle}{2 \Delta t}$$

$\vec{x}$ is vector of independent variables.

The Chapman Kolmogorov Eqn gives transition ~~probability for a Markov Sequence:~~

$$f(n|s) = \int d\vec{r} \, f(n|r) \, f(r|s)$$

→ integrate over all possible intermediate states. It is NOT exact, because it takes the probability of going from $r \Rightarrow n$ as being independent of s → assumes a Markov Process

in general:

$$f(n|s) = \int d\vec{r} \, f(n|r,s) \, f(r|s)$$

The Master Equation is the Chapman-Kolmogorov equation in the continuous time limit

What is this:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} = -C[f]$$

$$C[f] \equiv \frac{\partial}{\partial v} \cdot \left[\left(\frac{q}{m} \vec{E}^{(p)}(\vec{v}) \right) f - D(\vec{v}) \cdot \frac{\partial f}{\partial \vec{v}} \right]$$

and where did it come from?

What is the heat gained by electrons due to collisions with ions?

What is Hydrodynamics?

That is the general fokker · Planck equation for an unmagnetized, weakly coupled plasma.
It came from taking the 'jump moments' for:

$$\frac{d\tilde{x}}{dt} = \tilde{V} \quad \frac{d\tilde{V}}{dt} = \delta\tilde{a}(\hat{x},t) + \left(\frac{q}{m}\right)\vec{E}^{(p)}(\tilde{V})$$

It is still general and $E^{(p)}$ and D depend on fluctuation spectrum (which depends on f)

E field 'kick' in debye sphere — velocity space diffusion.

Polarization drag

C[f] is the Plasma fokker Planck collision operator

minus sign

$$Q_e = -Q_i - \vec{R} \cdot \vec{u}$$

energy exchang in Scattering (vanishes in Lorentz approximation)

transfer of directed momentum into heat — is finite in Lorentz approximation + contains ohmic heating.

Hydrodynamics is the study of the long wavelength, low frequency behavior of the plasma.

What are the linearized normal modes of the one component plasma?

What is the OCP?

What is the Plemelj Formula?

How do you show self-adjointness of a collision operator?

- two shear modes

- one thermal diffusion mode

- tw plasma oscillations.

Note: the one component plasma is an electron fluid with a cold, neutralizing ion background

$$\frac{1}{\omega - kv \pm i\epsilon} = P\left(\frac{1}{\omega - kv}\right) \mp \delta(\omega - kv) i\pi$$

- want to work with the linearized operator $\hat{C}$, where $f = f_0(1+\chi)$,

$C[f] = \hat{C}|\chi\rangle \rightarrow$ find w/ $C[f] = C[f_0 \chi]$

To show self-adjointness, must show that

$$\langle \psi | \hat{C} | \chi \rangle = \langle \chi | \hat{C} | \psi \rangle$$

→ use integration by parts

In conductivity, what is the Spitzer problem?

What are the solvability constraints in the Chapman-Enskog procedure?

What is the Chapman-Enskog ordering?

When is it valid?

Spitzer problem is conductivity in special case of zero frequency and wave number.

- write the first order eqn. in the form:
$|(\ )\rangle = -\hat{C}|\chi\rangle$, then the

solvability constraints amount to
$\langle e_i|(\ )\rangle = 0$
where e_i are the null eigenfunctions of the adjoint operator → for a self adjoint operator, the null eigenfunctions are all of the functions conserved by the collision operator. → there are as many solvability constraints as there are functions conserved by the collision operator

The Chapman-Enskog ordering is:

$\frac{1}{\nu}\frac{\partial}{\partial t} \sim \mathcal{O}(\epsilon) \ll 1$ note: $\epsilon \neq \epsilon_p$

$\lambda_{mfp}\vec{\nabla} \sim \mathcal{O}(\epsilon) \ll 1$

$\lambda_{mfp} \equiv \frac{V_{th}}{\nu}$

It is valid in the Hydrodynamic regime.

How do you go about finding the Chapman-Enskog Equations?

Given a kinetic equation, how do you find the corresponding Langevin equations?

What is the expression for the friction force, $\vec{R}$?

- use the Chapman-Enskog (hydrodynamic) ordering:

$$\frac{1}{\nu}\frac{\partial}{\partial t} \sim \mathcal{O}(\epsilon) \qquad \frac{v}{\nu}\frac{\partial}{\partial x} \sim \mathcal{O}(\epsilon)$$

to put ϵ's in front of those terms in your eqn. Then write $f = f_0 + \epsilon f_1 \rightarrow$ solve the eqns. order by order.

(likely to get $C[f_0] = 0$)

- put the eqn. into Fokker-Planck form:

$$\frac{\partial}{\partial \mu}(Vg) - \frac{\partial^2}{\partial \mu^2}(Dg) = 0$$

then general Langevin form is:

$\frac{\partial \mu}{\partial t} + V + \delta a$, where δa satisfies:

$$\langle \delta a(t)\, \delta a(t') \rangle = 2D\,\delta(t-t')$$

$$\vec{R} \equiv \frac{\partial}{\partial t}\left(m_e n_e \vec{u}_e\right)_{coll}$$
$$= -\int d\vec{v}_e\, (m\bar{n})_e\, \vec{v}_e\, C_{ei}[f]$$