Hua Wang General Problems 4/1/2009

GPP

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a). $H = (m^2c^4 + c^2p^2)^{\frac{1}{2}} - e\phi(r)$

$$\frac{dH}{dt} = \frac{1}{2}(m^2c^4 + c^2p^2)^{-\frac{1}{2}} \cdot 2c^2 \cdot \vec{p} \cdot \frac{d\vec{p}}{dt} - e\frac{d\phi(r)}{dr} \cdot \frac{dr}{dt}$$

$$= c^2(m^2c^4 + c^2p^2)^{-\frac{1}{2}} \cdot \vec{p} \cdot \frac{d\vec{p}}{dt} + eE_r v_r$$

$$\frac{d\vec{p}}{dt} = -e\left[E_r\hat{e_r} + \frac{\vec{v}\times\vec{B}}{c}\right]$$

$$\vec{p} \cdot \frac{d\vec{p}}{dt} = -e\gamma m_e \vec{v} \cdot \left[E_r\hat{e_r} + \cancel{\frac{\vec{v}\times\vec{B}}{c}}\right]$$

$$= -eE_r\gamma m_e v_r$$

$$\frac{dH}{dt} = c^2(m^2c^4 + c^2p^2)^{-\frac{1}{2}} \cdot \vec{p} \cdot \frac{d\vec{p}}{dt} + eE_r v_r$$

$$= -eE_r v_r + eE_r v_r = 0$$

$$\frac{d\tilde{p}_\theta}{dt} = \frac{d}{dt} r\left(p_\theta - \frac{e}{c}A_\theta(r)\right)$$

$\tilde{p}_\theta$ means the canonical angular momentum

$$= r\frac{dp_\theta}{dt} + v_r p_\theta - \underbrace{\frac{e}{c} \cdot \frac{d}{dr}(rA_\theta(r))}_{-\frac{e}{c} r B_z(r)}$$

The tricky thing lies in the derivative of $\frac{dp_\theta}{dt}$, I should point out. $\left(\frac{d\vec{p}}{dt}\right)_\theta \neq \frac{dp_\theta}{dt}$, because in the cylindrical coordinates, $\hat{e_r}$, $\hat{e_\theta}$ are not constant,

$$\frac{d\hat{e_r}}{d\theta} = \hat{e_\theta}, \quad \frac{d\hat{e_\theta}}{d\theta} = -\hat{e_r}$$

$$\frac{d\hat{e}_r}{d\theta} \cdot \frac{d\theta}{dt} = v_\theta \hat{e}_\theta$$

$$\text{So } \left(\frac{d\vec{p}}{dt}\right)_\theta = \frac{dp_\theta}{dt}\hat{e}_\theta + \nu m_e v_r \frac{d\hat{e}_r}{dt} = \left(\frac{dp_\theta}{dt} + \nu m_e v_r v_\theta\right)\hat{e}_\theta$$

according to momentum equation.

$$\left(\frac{d\vec{p}}{dt}\right)_\theta = \frac{e}{c} v_r B_z(r).$$

$$\frac{dp_\theta}{dt} = \frac{e}{c} v_r B_z(r) - \nu m_e v_r v_\theta$$

$$\frac{d\hat{p}_\theta}{dt} = r\left(\frac{e}{c} v_r B_z(r) - \nu m_e v_r v_\theta\right) + r\nu m_e v_\theta v_r - \frac{e}{c} r v_r B_z(r)$$

$$= 0.$$