2005 Part II Q5 Exp.

a. Thomson Scattering cross-section

nonrelativistic equation of motion: $\dot{\vec{\beta}} = \frac{-e}{mc} \vec{E}_i$ for an electron

Radiated field: $\vec{E}_s = \frac{-e}{4\pi\epsilon_0} \left[\frac{1}{Rc} \left(\hat{s} \times (\hat{s} \times \dot{\vec{\beta}}) \right) \right]_{ret}$

$$\Rightarrow \vec{E}_s = \frac{e^2}{4\pi\epsilon_0 mc^2} \left[\frac{1}{R} \hat{s} \times (\hat{s} \times \vec{E}_i) \right]_{ret}$$

classical electron radius: $r_e = \frac{e^2}{4\pi\epsilon_0 mc^2} = 2.8 \cdot 10^{-15} m$

$$\vec{E}_s = \left[\frac{r_e}{R} \hat{s} \times (\hat{s} \times \vec{E}_i) \right]_{ret}$$

$$\frac{dP}{d\Omega} = R^2 c \epsilon_0 |E_s|^2 = r_e^2 \sin^2\theta \, c \epsilon_0 |E_i|^2$$

where θ is the angle between $\vec{E}_i$ and the observation point

Define the differential scattering cross section as the ratio of $dP/d\Omega$ to the incident power per unit area, $c\epsilon_0 |E_i|^2$

$$\Rightarrow \frac{d\sigma}{d\Omega} = r_e^2 \sin^2\theta$$

Integrated over solid angle, $\sigma = \frac{8\pi}{3} r_e^2$

b. $l = 1\,cm$ $\quad n_e = 10^{20}\, m^{-3}$ $\quad d\Omega = .01$, $\quad 90°$ scattering

total fraction of power scattered into detector $= 0.01 \times n_e \, r_e^2 \, l$

$$= 7.8 \cdot 10^{-14}$$