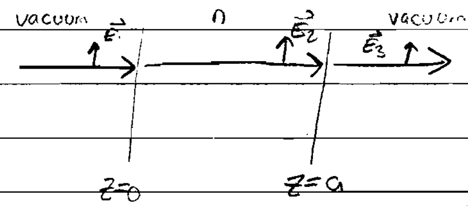
Jan 2001 #3 (EM)

incident pulse $E(z,t) = f(\frac{z}{c} - t)$



$n \approx 1$, ignore reflections

$\omega n(\omega) \approx \omega_0 + \frac{d}{d\omega}(\omega n(\omega))\Big|_{\omega_0} (\omega - \omega_0)$

$E e^{i(kz-\omega t)}$ $\quad E_2 e^{i(k'z-\omega t)}$ $\quad E_3 e^{i(kz-\omega t)}$: monochromatic picture

Continuity of $E_{\parallel}$ at the boundaries:

$E(\omega) = E_2(\omega) \qquad E_2(\omega)\, e^{ik'a} = E_3(\omega)\, e^{ika}$

$\Rightarrow E_3(\omega) = E_1(\omega)\, e^{i(k'-k)a}$

in medium 2, $\quad n = \frac{ck'}{\omega} \qquad \omega n = ck' \approx \omega_0 + \frac{d}{d\omega}(\omega n)\Big|_{\omega_0}(\omega - \omega_0)$

$= \omega_0 + \alpha(\omega - \omega_0)$

$= \omega_0(1-\alpha) + \alpha\omega$

$\Rightarrow \quad k' = \frac{\omega n}{c} = \frac{\omega_0}{c}(1-\alpha) + \frac{\alpha\omega}{c}$

$k' - k = \frac{\omega_0}{c}(1-\alpha) + \frac{\alpha\omega}{c} - \frac{\omega}{c} = \frac{\omega}{c}(\alpha - 1) - \frac{\omega_0}{c}(\alpha - 1)$

A real pulse is a wavepacket.

Compute $E(\omega)$ at $z=0$ (same for E_2, E_3)

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int dt\; E(t)\, e^{i\omega t}$$

Then add $e^{ik(\omega)z}$ to get z dependence:

$$E(z,t) = \frac{1}{\sqrt{2\pi}} \int d\omega\; E(\omega)\, e^{i(k(\omega)z - \omega t)}$$

$\Rightarrow \quad E(z,t) = E(t - \frac{z}{c}) = f(\frac{z}{c} - t)$

$\Rightarrow \quad E_2(z,t) = \frac{1}{\sqrt{2\pi}} \int d\omega\; E_2(\omega)\, e^{i(k'(\omega)z - \omega t)} = \frac{1}{\sqrt{2\pi}} \int d\omega\; E(\omega)\, e^{i(k'(\omega)z - \omega t)}$

substitute $E(\omega)$ from the integral above, switch the order of the integrals

$$\Rightarrow E_2(z,t) = \frac{1}{2\pi} \int dt'\; E(t') \int d\omega\; e^{i\omega t'}\, e^{-i\omega t}\, e^{ik'z}$$

$e^{ik'z} = e^{-i\frac{\omega_0}{c}(\alpha-1)z}\, e^{i\alpha\frac{\omega}{c}z}$

$$E_2(z,t) = \frac{e^{-i\frac{\omega_0}{c}(\alpha-1)z}}{2\pi} \int dt'\; E(t') \underbrace{\int d\omega\; e^{i\omega(t' - t + \alpha\frac{z}{c})}}_{2\pi\,\delta(t' - t + \alpha\frac{z}{c})}$$

$$\Rightarrow \boxed{E_2(z,t) = e^{-i\frac{\omega_0}{c}(\alpha-1)z}\, E\left(t - \alpha\frac{z}{c}\right) = e^{-i\frac{\omega_0}{c}(\alpha-1)z}\, f\left(\alpha\frac{z}{c} - t\right)}$$

To the extent that $\alpha = \frac{d(\omega n)}{d\omega}\Big|_{\omega_0}$ is not equal to 1, there is an extra phase to the wave in the dielectric medium. The wave also propagates at speed $\frac{c}{\alpha}$

$$E_3(z,t) = \frac{1}{\sqrt{2\pi}} \int d\omega \; E_3(\omega)\, e^{i(kz-\omega t)}$$

$$= \frac{1}{\sqrt{2\pi}} \int d\omega \; E(\omega)\, e^{i(k'(\omega)-k(\omega))a}\, e^{i(kz-\omega t)}$$

$$e^{i(k'-k)a} = e^{-i\frac{\omega_0}{c}(\alpha-1)a}\, e^{i\frac{\omega}{c}(\alpha-1)a}$$

$$E_3(z,t) = \frac{e^{-i\frac{\omega_0}{c}(\alpha-1)a}}{2\pi} \int dt'\, E(t') \underbrace{\int d\omega \; e^{i\omega t'} e^{-i\omega t} e^{i(\frac{\omega}{c}z)} e^{i\frac{\omega}{c}(\alpha-1)a}}_{2\pi\,\delta\left(t'-t+\frac{z}{c}+\frac{(\alpha-1)a}{c}\right)}$$

$$E_3(z,t) = e^{-i\frac{\omega_0}{c}(\alpha-1)a}\, E\left(t-\frac{z}{c}-\frac{(\alpha-1)a}{c}\right)$$

$$\boxed{E_3(z,t) = e^{-i\frac{\omega_0}{c}(\alpha-1)a}\, f\left(\frac{z}{c}+\frac{(\alpha-1)a}{c}-t\right)}$$

↳ Similar to as in vacuum $z<0$, with extra time delay and phase due to the dielectric

Group velocity in the dielectric?

$$ck = n\omega = \omega_0 + \alpha\omega - \alpha\omega_0$$

$$c = \alpha\frac{d\omega}{dk} \qquad v_g = \frac{c}{\alpha}$$

Negative group velocity corresponds to the pulse seemingly moving non causally