2013 II Q3 SH

$$W = \frac{1}{2}\int V^2 + B^2 \, d\tau \qquad \partial W = \int \delta\vec{V}.\vec{V} + \delta\vec{B}.\vec{B} \, d\tau$$

$$H = \frac{1}{2}\int \vec{A}.\vec{B} \, d\tau \qquad \delta A = \frac{1}{2}\int \delta\vec{A}.\vec{B} + \delta\vec{B}.\vec{A} \, d\tau$$

$$= \frac{1}{2}\int \vec{A}.\delta\vec{B} + \delta\vec{B}.\vec{A} - \nabla.(\delta\vec{A}\times\vec{A}) \, d\tau$$

$$= \int \delta\vec{B}.\vec{A} \, d\tau \quad \text{as } \delta A \text{ vanishes on } S$$

$$K = \int \vec{V}.\vec{B} \, d\tau \qquad \delta K = \int \delta\vec{V}.\vec{B} + \delta\vec{B}.\vec{V} \, d\tau$$

a) $\delta(W - \mu K) = 0 \Rightarrow \int \delta\vec{V}.(\vec{V} - \mu\vec{B}) + \delta\vec{B}.(\vec{B} - \mu\vec{V}) \, d\tau$

$\Rightarrow \vec{V} = \mu\vec{B} \qquad \vec{B} = \mu\vec{V}$ $\quad \mu$ constant in τ

$\Downarrow$

$\mu = 1$

b) $\delta(W - \lambda H) = 0 \Rightarrow \int \delta\vec{V}.\vec{V} + \delta\vec{B}.(\vec{B} - \lambda\vec{A}) \, d\tau$

$\vec{V} = 0 \qquad \vec{B} = \lambda\vec{A}$

$\nabla\times\vec{B} = \lambda\vec{B}$ standard force free field.

3. $\delta(W - \mu K - \lambda H) = 0$

$$\Rightarrow \int \delta\vec{V}\cdot(\vec{V} - \mu\vec{B}) + \delta\vec{B}\cdot(\vec{B} - \mu\vec{V} - \lambda\vec{A})\, d\tau$$

$$\vec{V} = \mu\vec{B} \qquad \vec{B} - \mu^2\vec{B} - \lambda\vec{A} = 0$$

$$\vec{B} = \frac{\lambda}{1-\mu^2}\vec{A} \qquad \nabla\times\vec{B} = \frac{\lambda}{1-\mu^2}\vec{B}$$

Still force free.

4. $\lambda \to 0 \quad \vec{V} = \mu\vec{B}$ for all λ

$\mu \to 0 \quad \vec{V} \to 0 \quad \vec{B} \to \lambda\vec{A}$

$\lambda \to 0, \mu \to 0 \qquad \frac{\lambda}{1-\mu^2} \to 0 \quad \frac{\mu\lambda}{1-\mu^2} \to 0$

So $\vec{V}, \vec{B} \to 0$ if nothing is conserved, a decaying W will yield no fields.